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Key point of the proof (Global in Time Solvability of Incompressive NSIVP in the Whole Space)

Global in Time Solvability is confirmed, based on nonincreasing of $\boldsymbol{L}^2$-norm of the solution $\|\boldsymbol{u}\|_2 \le \|\boldsymbol{a}\|_2$ gotton as a priori estimation, by means of upper limit estimation of solution based on integral equation with regard to the solution.

However, estimation by means of simple combination of following integral equation (1.3) and Hölder inequality and Young's inequarity has limitation and can't give result to be proved.

$$(1.3) \quad \boldsymbol{u}_t = K_t * \boldsymbol{a} - \int_0^t d\tau\, \mathcal{P}(\boldsymbol{\partial} K_{t-\tau} * \cdot \boldsymbol{u}_\tau \boldsymbol{u}_\tau)$$

For examble, simply using $\|\boldsymbol{u}\|_2 \le \|\boldsymbol{a}\|_2$ and equation above, following pro forma inequality is confirmed.

$$\|\boldsymbol{u}_t\|_q \le \|\boldsymbol{a}\|_q + C\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \|\boldsymbol{\partial} K_{t-\tau}\|_q$$

However, considering $\|\boldsymbol{\partial} K_{t-\tau}\|_q = c(t-\tau)^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$, for $n \ge 3, q \in (n, \infty]$, this factor diverges in case of integrating with regard to τ.

In the paper, avoiding divergence like the abobe by means of time transfered integral equation (1.4), usuful estimation is confirmed and global in time solvability is proived.

$$(1.4) \quad \boldsymbol{u}_t^{\Phi} = K_t^{\Phi} * \boldsymbol{a} - \int_0^t d\tau\, \mathcal{P}(\boldsymbol{\partial} K_{t-\tau}^{\Phi} * \varphi_\tau \cdot \boldsymbol{u}_\tau^{\Phi} \boldsymbol{u}_\tau^{\Phi})$$

By means of this relation and $\|\boldsymbol{u}\|_2 \le \|\boldsymbol{a}\|_2$, following relation is confirmed.

$$\|\boldsymbol{u}_t^{\Phi}\|_q \le \|\boldsymbol{a}\|_q + C\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \varphi_\tau \|\boldsymbol{\partial} K_{t-\tau}^{\Phi}\|_q$$

Based on $\|\boldsymbol{\partial} K_{t-\tau}^{\Phi}\|_q = c(\Phi(t-\tau))^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$ and using appropriate time transformation funstion $\Phi = \Phi(t)$, even though for $n \ge 3, q \in (n, \infty]$, factor $\|\boldsymbol{\partial} K_{t-\tau}^{\Phi}\|_q$ doesn't diverge for integrating with regard to τ and with integral factor φ integral keeps finete, useful estimation can be confirmed.

$$\|\boldsymbol{u}_t^{\Phi}\|_q \le \|\boldsymbol{a}\|_q + Cc\|\boldsymbol{a}\|_2^2 \int_0^t d\tau\, \varphi_\tau {\Phi_{t-\tau}}^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$$

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