

**Key point of the proof (Proof of Global in Time Solvability of Incompressible NSIVP in Periodic Space Using Time Transformation Analysis)**

Global in Time Solvability is confirmed, based on nonincreasing of  $L^2$ -norm of the solution  $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$  gotten as a priori estimation, by means of upper limit estimation of solution based on integral equation with regard to the solution.

However, estimation by means of simple combination of following integral equation (1.3) and Hölder inequality and expanded Young's inequality has limitation and can't give result to be proved.

$$(1.3) \quad \mathbf{u}_t = K_t * \mathbf{a} - \int_0^t d\tau \mathcal{P}(\partial K_{t-\tau} * \mathbf{u}_\tau \mathbf{u}_\tau)$$

For example, simply using  $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$  and equation above, following pro forma inequality is confirmed.

$$\|\mathbf{u}_t\|_q \leq \|\mathbf{a}\|_q + C \|\mathbf{a}\|_2^2 \int_0^t d\tau \|\partial K_{t-\tau}\|_q$$

However, considering  $\|\partial K_{t-\tau}\|_q = c(t-\tau)^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$ , for  $n \geq 3, q \in (n, \infty]$ , this factor diverges in case of integrating with regard to  $\tau$ .

In the paper, avoiding divergence like the above by means of time transferred integral equation (1.4), useful estimation is confirmed and global in time solvability is proved.

$$(1.4) \quad \mathbf{u}_t^\Phi = K_t^\Phi * \mathbf{a} - \int_0^t d\tau \mathcal{P}(\partial K_{t-\tau}^\Phi * \varphi_\tau \cdot \mathbf{u}_\tau^\Phi \mathbf{u}_\tau^\Phi)$$

By means of this relation and  $\|\mathbf{u}\|_2 \leq \|\mathbf{a}\|_2$ , following relation is confirmed.

$$\|\mathbf{u}_t^\Phi\|_q \leq \|\mathbf{a}\|_q + C \|\mathbf{a}\|_2^2 \int_0^t d\tau \varphi_\tau \|\partial K_{t-\tau}^\Phi\|_q$$

Based on  $\|\partial K_{t-\tau}^\Phi\|_q = c(\Phi(t-\tau))^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$  and using appropriate time transformation function  $\Phi = \Phi(t)$ , even though for  $n \geq 3, q \in (n, \infty]$ , this factor doesn't diverge for integrating with regard to  $\tau$  and with integral factor  $\varphi$  integral keeps finite, useful estimation can be confirmed.

$$\|\mathbf{u}_t^\Phi\|_q \leq \|\mathbf{a}\|_q + Cc \|\mathbf{a}\|_2^2 \int_0^t d\tau \varphi_\tau \Phi_{t-\tau}^{-\frac{n}{2}(1-\frac{1}{q})-\frac{1}{2}}$$

**note**

Global in time solvability of nonlinear problems were proved depending typically on local in time solvability and a priori estimations.

As for NSIVP in periodic space, local in time solvability for initial value which belongs to  $L^q, q \in (n, \infty]$  and a priori estimation with regard to  $L^2$ -norm of solution are proved simply by means of well known relations like integral equation and expanded Young's inequality, under additional condition, for example, mean value of solution is  $\mathbf{0}$ .

Therefore, if appropriate a priori estimation with regard to  $L^q$ -norm of solution is confirmed, global in time solvability is proved.

However, analysis based on well known relations like integral equation and expanded Young's inequality has never provided appropriate a priori estimation with regard to  $L^q$ -norm of solution.

On the other hand, analysis based on time transformed integral equation and expanded Young's inequality provides appropriate a priori estimation with regard to  $L^q$ -norm of solution.

