Integrals

by

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Abstract

This note presents a collection of integrals involving pi

Resumen. En esta nota mostramos una colección de integrales definidas , relacionadas con la integral:

$$\frac{\pi}{\sqrt{2}} = 2\int_0^\infty \frac{x^2}{1+x^4}\,dx = 2\int_0^\infty \frac{1}{1+x^4}\,dx = 2\int_0^1 \frac{1+x^2}{1+x^4}\,dx = 2\int_1^\infty \frac{1+x^2}{1+x^4}\,dx$$

Las integrales se han obtenido mediante métodos elementales de integración.

Integrals:

$$\frac{\pi}{\sqrt{2}}=\int_0^1\left(\sqrt{\frac{1+\sqrt{1-x^2}}{x}}-\sqrt{\frac{1-\sqrt{1-x^2}}{x}}\right)dx \tag{1}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^{\pi/2}\left(\sqrt{\sec x+\tan x}-\sqrt{\sec x-\tan x}\right)\sin x dx \tag{2}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^{\pi/2}\left(\sqrt{\csc x+\cot x}-\sqrt{\csc x-\cot x}\right)\cos x dx \tag{3}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^{\pi/2}\left(\sqrt[4]{\frac{1+\sin x}{1-\sin x}}-\sqrt[4]{\frac{1-\sin x}{1+\sin x}}\right)\sin x dx \tag{4}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^{\pi/2}\left(\sqrt[4]{\frac{1+\cos x}{1-\cos x}}-\sqrt[4]{\frac{1-\cos x}{1+\cos x}}\right)\cos x dx \tag{5}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^1\left(\sqrt[4]{\frac{1}{x}}-\sqrt[4]{x}\right)\left(\sqrt{\frac{1}{x}}-\sqrt{x}\right)\frac{1}{(1+x)^2}dx \tag{6}$$

$$\frac{\pi}{\sqrt{2}}=\int_1^{\infty}\left(\sqrt[4]{x}-\sqrt[4]{\frac{1}{x}}\right)\left(\sqrt{x}-\sqrt{\frac{1}{x}}\right)\frac{1}{(1+x)^2}dx \tag{7}$$

$$\frac{\pi}{\sqrt{2}}=4\int_0^1\frac{\left(1-x^2\right)\left(1-x^4\right)}{\left(1+x^4\right)^2}dx \tag{8}$$

$$\frac{\pi}{\sqrt{2}}=\int_0^{\pi/2}\left(\sqrt{\cot(x/2)}-\sqrt{\tan(x/2)}\right)\cos x dx \tag{9}$$

$$\frac{\pi}{\sqrt{2}}=\left(\sqrt{2}+1\right)\int_0^1\sin^{-1}\left(\sqrt{\frac{2x}{1+x}}\right)dx \tag{10}$$

$$\frac{\pi}{\sqrt{2}}=\left(\sqrt{2}+1\right)\int_0^1\cos^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)dx \tag{11}$$

$$\frac{\pi}{\sqrt{2}}=\left(\sqrt{2}+1\right)\int_0^1\tan^{-1}\left(\sqrt{\frac{2x}{1-x}}\right)dx \tag{12}$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^1 \sin^{-1}\left(\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1-x}{1+x}}}\right)dx \qquad (13)$$

$$\frac{\pi}{\sqrt{2}} = 2\left(\sqrt{2}+1\right)\int_0^1 \cos^{-1}\left(\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{\frac{1-x}{1+x}}}\right)dx \qquad (14)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^1 \frac{1}{\left(1+x^2\right)\sqrt{1-x^2}}\,dx \qquad (15)$$

$$\frac{\pi}{\sqrt{2}} = 2+\int_0^{\sqrt{2}-1}\left(\sqrt{\frac{1+\sqrt{1-2x-x^2}}{2+x}}-\sqrt{\frac{1-\sqrt{1-2x-x^2}}{2+x}}\right)dx \qquad (16)$$

$$\frac{\pi}{\sqrt{2}} = -2+2\int_0^1 \sqrt{\frac{1+\sqrt{1+4x-4x^2}}{2x}}\,dx \qquad (17)$$

$$\frac{\pi}{\sqrt{2}} = -2+2\int_1^\infty \sqrt{\frac{1}{2x}+\frac{1}{2x}\sqrt{1+\frac{4}{x}-\frac{4}{x^2}}}\,\frac{1}{x}\,dx \qquad (18)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^{\pi/4} \frac{1}{\left(\sin x\right)^4+\left(\cos x\right)^4}\,dx \qquad (19)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^{\pi/4} \frac{\left(\sec x\right)^4}{1+\left(\tan x\right)^4}\,dx \qquad (20)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^{\pi/4} \frac{\left(\csc x\right)^4}{1+\left(\cot x\right)^4}\,dx \qquad (21)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^{\pi/2} \frac{1}{2-\left(\sin x\right)^2}\,dx \qquad (22)$$

$$\frac{\pi}{\sqrt{2}} = 2\int_0^{\pi/2} \frac{1}{1+\left(\cos x\right)^2}\,dx \qquad (23)$$

$$\frac{\pi}{2\sqrt{2}}=$$

$$=\int_0^1 \frac{3+14x+33x^2+52x^3+68x^4+88x^5+107x^6+100x^7+65x^8+26x^9+5x^{10}}{2+12x+34x^2+60x^3+75x^4+72x^5+68x^6+72x^7+75x^8+60x^9+34x^{10}+12x^{11}+2x^{12}}dx \qquad (24)$$

$$\frac{\pi}{n\sin\left(\frac{(n+1)\pi}{2n}\right)}=\int_0^1\left(\sqrt[n]{\frac{1+\sqrt{1-x^2}}{x}}-\sqrt[n]{\frac{1-\sqrt{1-x^2}}{x}}\right)dx \ \ ,n\in\mathbb{N}-\{1\} \qquad (25)$$

References

1. *Gradshteyn, I.S. and I.M. Ryzhik, Table of Integrals,Series, and Products. Seventh edition,Edited by Alan Jeffrey and Daniel Zwillinger.2007.*