On the Logical Inconsistency of Einstein's Time Dilation

Stephen J. Crothers

Tasmania, Australia
thenarmis@yahoo.com

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ABSTRACT

Time dilation is a principal feature of the Special Theory of Relativity. It is purported to be independent of position, being a function only of uniform relative velocity, via the Lorentz Transformation. However, it is not possible for a 'clock-synchronised stationary system' of observers K to assign a definite time to any 'event' relative to a 'moving system' k using the Lorentz Transformation. Consequently, the Theory of Relativity is false due to an insurmountable intrinsic logical contradiction.

1 Introduction

In a previous paper [1] I proved that a system of clock-synchronised stationary observers is inconsistent with the Lorentz Transformation. Assuming both leads to a contradiction. Herein I synchronise clocks in Einstein's 'stationary system' K by mathematical construction and prove that his 'stationary system' K cannot then assign any definite time τ anywhere in his 'moving system' k for any given position x and time t in his 'stationary system' K. From this it follows immediately that Einstein's 'time dilation' is false because there is no common determinable time dilation for all observers in Einstein's 'stationary system' K.

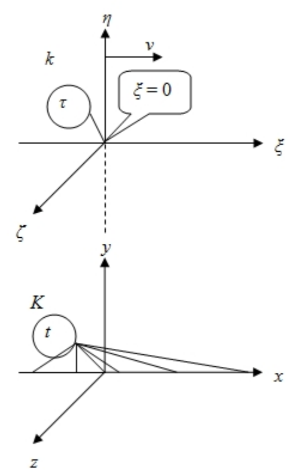

Fig. 1: All the synchronised clocks in Einstein's 'stationary system' K read the same time t at all positions x in the K system. The clock at the origin of the 'moving system' k, where $\xi = 0$, reads $\tau = 0$ when the y and η axes coincide, so $t = 0$ and $x = 0$ too.

2 Stationary and moving clocks

In §4 of his 1905 paper, Einstein [2] compared one clock 'at rest' relative to the 'moving system' k, with all the synchronised identical clocks 'at rest' relative to his 'stationary system' K:

> "*... we imagine one of the clocks which are qualified to mark the time t when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of the co-ordinates of k, and so adjusted that it marks the time τ. What is the rate of this clock, when viewed from the stationary system?*
>
> "*Between the quantities x, t, and τ, which refer to the position of the clock, we have, evidently, $x = vt$ and*
>
> $$\tau = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t - vx/c^2\right).$$
>
> "*Therefore,*
>
> $$\tau = t\sqrt{1 - v^2/c^2} = t - \left(1 - \sqrt{1 - v^2/c^2}\right) t$$
>
> "*whence it follows that the time marked by the clock (viewed in the stationary system) is slow by $1 - \sqrt{1 - v^2/c^2}$ seconds per second, ...*" [2, §4]

In Einstein's notation the coordinates of his assumed system of clock-synchronised stationary observers K are x, y, z, t, those corresponding to the 'moving system' k are ξ, η, ζ, τ, illustrated in figure 1, for his initial conditions.

The Lorentz Transformation is,

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \quad &&\xi = \beta\left(x - vt\right), \\ &\eta = y, &&\zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}. \end{aligned} \qquad (1)$$

Note that according to the Lorentz Transformation the time τ depends upon both t and x. Einstein specifically set $x = 0 = \xi$ for $\tau = t = 0$, shown in figure 1.

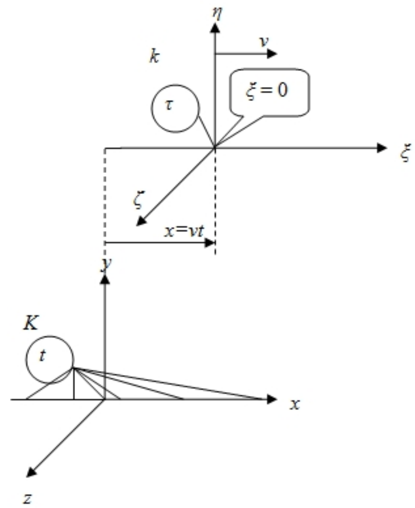

Fig. 2: After a time $t > 0$ all the synchronised clocks in Einstein's 'stationary system' K read the same time t at all positions x in the K system. The clock at the origin of the 'moving system' k, where $\xi = 0$, reads $\tau > 0$. The origin $\xi = 0$ has advanced a distance $x = vt$.

After a time $t > 0$ the origin of Einstein's 'moving system' k has advanced a distance $x = vt$, illustrated in figure 2. At this time t all the observers in Einstein's 'stationary system' K read the same time t on their clocks no matter where they are located, because their clocks are synchronised. The clock at Einstein's $\xi = 0$ of the 'moving system' k reads time $\tau > 0$. According to Einstein's time dilation, $\tau = t/\beta$.

All of Einstein's 'stationary observers' in K are entitled to look at the same moving clock. An observer located at any $x^* \neq x$ in Einstein's 'stationary system' K can observe the clock in the 'moving system' k at any synchronised time t of Einstein's 'stationary system' K, to see what the clock reads. However, Einstein's assumption that a system of clock-synchronised stationary observers is consistent with the Lorentz Transformation is demonstrably false. An observer x^* does not find the same τ or the same ξ as observer x does.

3 Systems of stationary observers

The Lorentz Transformation between systems of observers stationary with respect to their own systems is [1, §2],

$$\begin{aligned} &\tau = \beta\left(t_\kappa - vx_\kappa/c^2\right), \; x_\kappa = \kappa x_1, \; \eta = y, \; \zeta = z, \\ &\xi_\kappa = \beta\left(x_\kappa - vt_\kappa\right) = \beta\left[\left(\kappa/\beta^2 + v^2/c^2\right)x_1 - vt_1\right], \\ &t_\kappa = t_1 + (\kappa - 1)\,vx_1/c^2, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, \; \kappa \in \Re. \end{aligned} \tag{2}$$

The Inverse Stationary Lorentz Transformation is [1, §2] is,

$$\begin{aligned} &t = \beta\left(\tau_\kappa + v\xi_\kappa/c^2\right), \; \xi_\kappa = \kappa\xi_1, \; y = \eta, \; z = \zeta, \\ &x_\kappa = \beta\left(\xi_\kappa + v\tau_\kappa\right) = \beta\left[\left(\kappa/\beta^2 + v^2/c^2\right)\xi_1 + v\tau_1\right], \\ &\tau_\kappa = \tau_1 - (\kappa - 1)\,v\xi_1/c^2, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, \; \kappa \in \Re. \end{aligned} \tag{3}$$

By means of the Inverse Stationary Lorentz Transformation (3),

$$\Delta\tau_\kappa = \Delta\tau_1 = \frac{\Delta t}{\beta} = \Delta t\sqrt{1 - \frac{v^2}{c^2}}.$$

This is Einstein's time-dilation equation. However, if l_0 is the length of a 'rigid' rod in the moving system k, according to the system K its length is [1, §4],

$$\Delta x = \beta l_0 = \frac{l_0}{\sqrt{1 - \frac{v^2}{c^2}}},$$

which is length expansion, not length contraction. Thus, although no observer in the stationary system K is clock-synchronised, every observer x_κ of the stationary system K observes the same time-interval Δt in K and the same time-dilated interval $\Delta\tau$ in k, but at the expense of length contraction and clock-synchronisation [1, §4]. This is irreconcilable with Einstein's theory.

4 Clock-synchronised observers

The Clock-Synchronised Lorentz Transformation is [1, §5],

$$\begin{aligned} &\tau_\kappa = \beta\left(t - vx_\kappa/c^2\right) = \kappa\tau_1, && \xi_\kappa = \beta\left(x_\kappa - vt\right), \\ &x_\kappa = (1 - \kappa)\,c^2t/v + \kappa x_1, && \eta = y, \; \zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, && 1 - v/c < \kappa < 1 + v/c. \end{aligned} \tag{4}$$

Although all observers in K are clock-synchronised to a common time t, only x_1 is not a function of the time t. Thus, only x_1 is stationary. All other observers in K cannot be stationary.

The Inverse Clock-Synchronised Lorentz Transformation is [1, §5],

$$\begin{aligned} &t_\kappa = \beta\left(\tau + v\xi_\kappa/c^2\right) = \kappa t_1, && x_\kappa = \beta\left(\xi_\kappa + v\tau\right), \\ &\xi_\kappa = (\kappa - 1)\,c^2\tau_1/v + \kappa\xi_1, && y = \eta, \; z = \zeta, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, && 1 - v/c < \kappa < 1 + v/c. \end{aligned} \tag{5}$$

Although all observers in k are clock-synchronised to a common time τ, only ξ_1 is not a function of the time τ. Thus, only ξ_1 is stationary. All other observers in k cannot be stationary.

From this it follows that, from their vantage points, no two observers in the 'stationary system' K read either the same time or same time-interval on the moving clocks in system k;

examples tabulated:

κ	x_κ	τ_κ	τ_1
0	c^2t/v	0	$\beta\left(t - vx/c^2\right)$
$1/\beta^2$	$\left(\beta^2 - 1\right)c^2t/v\beta^2 + x_1/\beta^2$	τ_1/β^2	$\beta\left(t - vx/c^2\right)$
$1/\beta$	$(\beta - 1)\,c^2t/v\beta + x_1/\beta$	τ_1/β	$\beta\left(t - vx/c^2\right)$
1	x_1	τ_1	$\beta\left(t - vx/c^2\right)$

Since all observers are clock-synchronised with respect to their own systems, all observers in the K system observe the common clock time-interval Δt. Observer x_κ of system K watches the clock of ξ_κ in the 'moving system' k and observes the clock time-interval $\Delta\tau_\kappa$ of ξ_κ in system k. Then from (4),

$$\Delta\tau_\kappa = \kappa\beta\Delta t.$$

Thus, after a time-interval Δt in K any observer x_κ in the clock-synchronised system K reads the time-interval $\Delta\tau_\kappa$ at ξ_κ in the k system. Each observer x_κ observes a different time and a different time-interval on the corresponding clock held by observer ξ_κ in system k. For example, the observer $\kappa = 1$ located at x_1 in system K observes not time dilation at ξ_1 but time expansion at ξ_1: $\Delta\tau_1 = \beta\Delta t$. The observer $\kappa = 1/\beta$ located at $x_{1/\beta}$ in system K observes no change in the time-interval of the clock at $\xi_{1/\beta}$ in system k: $\Delta\tau_{1/\beta} = \Delta t$. The observer $\kappa = 1/\beta^2$ located at x_{1/β^2} observes the time-interval $\Delta\tau_{1/\beta^2}$ at ξ_{1/β^2} in system k:

$$\Delta\tau_{1/\beta^2} = \frac{\Delta t}{\beta} = \Delta t\sqrt{1 - \frac{v^2}{c^2}},$$

which is Einstein's time dilation equation. In all cases, $\Delta\tau_\kappa = \kappa\tau_1$, in accordance with (4), since $\Delta\tau_1 = \beta\Delta t$. Conversely, all observers ξ_κ in the k system read a common time τ and common time-interval $\Delta\tau$, finding that the clock at x_κ in K reads, from (5), the time-interval,

$$\Delta t_\kappa = \kappa\beta\Delta\tau.$$

Neither system K nor system k can assign any particular time-interval to one another because no observer in the one observes the same time-interval at all locations in the other.

5 Conclusions

Einstein's 'clock-synchronised stationary system' K cannot assign any common time dilation to the observers in his 'moving system' k. A system of clock-synchronised stationary observers is not consistent with the Lorentz Transformation. Einstein's time dilation is inconsistent with the Lorentz Transformation. It is therefore false. Hence, the Theory of Relativity is false.

References

[1] Crothers, S.J., On the Logical Inconsistency of the Special Theory of Relativity, 6th March 2017, http://vixra.org/abs/1703.0047

[2] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905