On the Logical Inconsistency of the Special Theory of Relativity

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6 March, 2017

ABSTRACT

Einstein's Special Theory of Relativity employs both a system of clock-synchronised stationary observers and the Lorentz Transformation. Without both the Theory of Relativity fails. A system of clock-synchronised stationary observers is proven inconsistent with the Lorentz Transformation. The Theory of Relativity is therefore invalid due to an intrinsic logical contradiction.

1 Introduction

Physicists since Einstein have assumed that a system of clock-synchronised stationary observers is consistent with the Lorentz Transformation. Without both, the Theory of Relativity fails. Herein I assure a stationary system of observers by mathematical construction and then prove that a system of clock-synchronised stationary observers is inconsistent with the Lorentz Transformation.

2 Einstein's synchronisation of clocks

In §1 of his 1905 paper, Einstein [1] defined the 'common time' for the points A and B in a space:

> "*We have so far defined only an 'A time' and a 'B time.' We have not defined a common 'time' for A and B, for the latter cannot be defined at all unless we establish* by definition *that the 'time' required by light to travel from A to B equals the 'time' it requires to travel from B to A. Let a ray of light start at the 'A time' t_A from A towards B, let it at the 'B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A.*
>
> "*In accordance with definition the two clocks synchronize if*
>
> $$t_B - t_A = t'_A - t_B.$$"

Einstein [1, §3] then produced the Lorentz Transformation:

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \quad && \xi = \beta\left(x - vt\right), \\ &\eta = y, && \zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, \end{aligned} \tag{1}$$

where x, y, z, t, pertain to the 'stationary system' and v is the uniform rectilinear speed between the two systems of coordinates in the direction of the positive x-axis.

Einstein [1, §3] synchronised his clocks for both his 'stationary system' K and his 'moving system' k:

> "*... let the time t of the stationary system be determined for all points thereof at which there are clocks by means of light signals in the manner indicated in §1 ; similarly let the time τ of the moving system be determined for all points of the moving system at which there are clocks at rest relatively to that system by applying the method, given in §1, of light signals between the points at which the latter clocks are located.*
>
> "*To any system of values x, y, z, t, which completely defines the place and time of an event in the stationary system, there belongs a system of values ξ, η, ζ, τ, determining that event relatively to the system k*".

Hence, for any given 'event', by his sychronisation method, the 'stationary system' is K, with coordinates x, y, z, t, and the 'moving system' is k, with corresponding coordinates ξ, η, ζ, τ. All points in Einstein's 'stationary system' K have the common time t and all points in his 'moving system' k have the common time τ.

3 The Lorentz Transformation

A system of clock-synchronised stationary observers is an essential feature of Special Relativity. Einstein [1, §3] maintained that the Lorentz Transformation associates coordinates x, y, z, t of the 'clock-synchronised stationary system' K with the coordinates ξ, η, ζ, τ of the 'moving system' k. A system of clock-synchronised stationary observers and the Lorentz Transformation are also the bases for Einstein's time dilation and length contraction. It is regarded in general by physicists [2, §12.1] that clocks which are synchronised when at rest are not synchronised when they all move together with respect to the 'stationary system' K, as illustrated in figure 1.

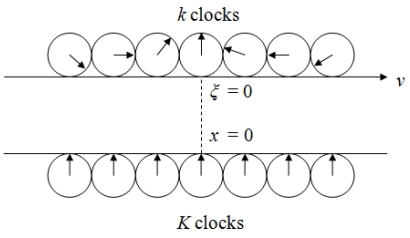

Fig. 1: All the synchronised clocks in Einstein's 'stationary system' K read the same time at all positions in the K system. All the clocks in the 'moving system' k do not read the same time according to the K system, despite being synchronised with respect to the k system. Only at $x = \xi = 0$ do the clocks read the same time in both systems, where $t = \tau = 0$.

Clocks to the left of the central clock in the 'moving system' k are 'ahead' of the central clock and those to the right of it 'lag' it, according to the 'stationary system' K where all the clocks therein always read the same time t. This is the so-called 'relativity of simultaniety' [1, §2] [2, §12.1]. After a time $t > 0$ the moving clocks advance to the right and the hands on the moving clocks advance, but they do not read the same time τ, according to the 'statonary system' K. As time t increases all the hands of the 'stationary' clocks advance by the same amount and all clocks in K still read the same time t - they are synchronised. However, for any x and t in the 'stationary system' K there is in general a place $x^* \neq x$ with a clock that reads $t^* \neq t$, thereby contradicting the assumption of a clock-synchronised system of stationary observers.

To assure by mathematical construction a stationary system of observers K set,

$$\begin{aligned} x^* &= \sigma x, \\ t^* - v\sigma x/c^2 &= t - vx/c^2, \end{aligned} \tag{2}$$

where $0 \leq \sigma$. From the second of equations (2),

$$t^* = t + \frac{(\sigma - 1)\, vx}{c^2}. \tag{3}$$

The following is a table of sample values:

σ	x^*	t^*	τ
0	0	$t - vx/c^2$	$\beta\left(t - vx/c^2\right)$
1/2	$x/2$	$t - vx/2c^2$	$\beta\left(t - vx/c^2\right)$
1	x	t	$\beta\left(t - vx/c^2\right)$
2	$2x$	$t + vx/c^2$	$\beta\left(t - vx/c^2\right)$
3	$3x$	$t + 2vx/c^2$	$\beta\left(t - vx/c^2\right)$

All x^* are independent of time - they constitute a stationary system of observers K. Note that for any time $t > 0$ of the 'stationary system' K there is, in general, a place $x^* \neq x$ therein with a clock reading $t^* \neq t$, thereby contradicting the assumption of clock synchronisation in K. Every observer x^* sees the same time $\tau^* = \tau$ at its associated ξ^* and t^*:

$$\begin{aligned} \xi^* &= \beta\,(x^* - vt^*)\,, \\ \tau^* &= \beta\left(t^* - vx^*/c^2\right) = \beta\left(t - vx/c^2\right) = \tau. \end{aligned} \tag{4}$$

According to the stationary system K all the clocks in the system k are synchronised but not stationary and all the clocks in the system K are stationary but not synchronised. Hence, by *reductio ad absurdum*, a system of clock-synchronised stationary observers is inconsistent with the Lorentz Transformation - they are mutually exclusive.

4 Conclusions

By his clock synchronisation method Einstein attempted to ensure that time at all places within a given system is the same, despite subsequently invoking the Lorentz Transformation. A system of clock-synchronised stationary observers is however inconsistent with the Lorentz Transformation. Consequently, Special Relativity is inconsistent with the Lorentz Transformation due to an insurmountable logical contradiction.

Einstein [1, §1] defined time by means of his clocks. However, time is no more defined by a clock than pressure is defined by a pressure gauge, speed by a speedometer, heat by a thermometer, or gravity by a spring. Measuring instruments are invented to measure something other than themselves. Einstein's clocks measure only themselves.

References

[1] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905

[2] Griffiths, D.J., Introduction to Electrodynamics, 4th Ed., Pearson Education Inc., 2013