On Einstein's Time Dilation and Length Contraction

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ABSTRACT

Einstein's method of synchronising clocks in his Special Theory of Relativity is inconsistent with the Lorentz Transformation, despite the latter being a fundamental component of his theory. This inconsistency subverts the very foundations of Special Relativity because it follows that Einstein's time dilation and length contraction are also quite generally inconsistent with the Lorentz Transformation. Moreover, clock synchronisation is inconsistent with the Lorentz Transformation. Clock synchronisation and the Lorentz Transformation are mutually exclusive.

1 Introduction

It has recently been proven by Engelhardt [1] that Einstein's method of synchronising clocks in his Special Theory of Relativity is inconsistent with the Lorentz Transformation. I recently extended this, proving that for any time $t > 0$ in Einstein's 'stationary system' K there is always a place ξ^* in Einstein's 'moving system' k where the time τ therein is zero, despite t and τ being synchronised according to Einstein's method [2]. This is in fact a special case, because to any time $t \geq 0$ in Einstein's 'stationary system' K there is always a place ξ^* in Einstein's 'moving system' k where the time $\tau = \kappa t$, where $0 \leq \kappa$. This has been regarded as relativity of simultaneity. However, it is in fact due to an inherent contradiction in Special Relativity, because the latter employs both clock synchronisation and the Lorentz Transformation which are nevertheless mutually exclusive; proven in §3 herein. Furthermore, this contradiction manifests in 'length contraction' because there is always a place in Einstein's 'stationary system' K where the length l'_0 of a rigid rod in his 'moving system' k equals the length l_0 of the same rigid rod in the 'stationary system' K. These facts completely subvert the foundations of Special Relativity.

2 Einstein's synchronisation of clocks

In §1 of his 1905 paper, Einstein [3] defined the 'common time' for the points A and B in a space:

> "*We have so far defined only an 'A time' and a 'B time.' We have not defined a common 'time' for A and B, for the latter cannot be defined at all unless we establish* by definition *that the 'time' required by light to travel from A to B equals the 'time' it requires to travel from B to A. Let a ray of light start at the 'A time' t_A from A towards B, let it at the 'B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A.*
>
> "*In accordance with definition the two clocks synchronize if*
>
> $$t_B - t_A = t'_A - t_B."$$

Einstein [3, §3] then produced the Lorentz Transformation:

$$\begin{aligned} &\tau = \beta\left(t - vx/c^2\right), \qquad && \xi = \beta\,(x - vt)\,, \\ &\eta = y, && \zeta = z, \\ &\beta = 1/\sqrt{1 - v^2/c^2}, \end{aligned} \tag{1}$$

where x, y, z, t, pertain to the 'stationary system' and v is the uniform rectilinear speed between the two systems of coordinates in the direction of the positive x-axis.

Einstein [3, §3] synchronised his clocks for both his 'stationary system' K and his 'moving system' k:

> "*. . . let the time t of the stationary system be determined for all points thereof at which there are clocks by means of light signals in the manner indicated in §1 ; similarly let the time τ of the moving system be determined for all points of the moving system at which there are clocks at rest relatively to that system by applying the method, given in §1, of light signals between the points at which the latter clocks are located.*
>
> "*To any system of values x, y, z, t, which completely defines the place and time of an event in the stationary system, there belongs a system of values ξ, η, ζ, τ, determining that event relatively to the system k*".

Hence, for any given 'event', by his sychronisation method, the 'stationary system' is K, with coordinates x, y, z, t, and the 'moving system' is k, with corresponding coordinates ξ, η, ζ, τ. All points in Einstein's 'stationary system' K have the common time t and all points in his 'moving system' k have the common time τ.

3 The Lorentz Transformation

Sychronisation of clocks is an essential feature of Special Relativity. Einstein [3, §3] holds that the Lorentz Transformation associates coordinates x, y, z, t of the 'stationary system' K with the coordinates ξ, η, ζ, τ of the 'moving system' k. Synchronisation and the Lorentz Transformation are the basis for Einstein's time dilation and length contraction. It is regarded in general by physicists [4, §12.1] that clocks which are synchronised when at rest are not synchronised when they all move together with respect to the 'stationary system' K, as illustrated in figure 1.

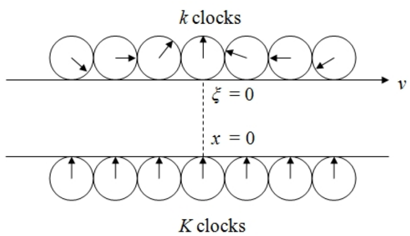

Fig. 1: All the synchronised clocks in the 'stationary system' K read the same time at all positions in the K system. All the clocks in the 'moving system' k do not read the same time according to the K system, despite being synchronised with respect to the k system. Only at $x = \xi = 0$ do the clocks read the same time in both systems, where $t = \tau = 0$.

Clocks to the left of the central clock in the 'moving system' k are 'ahead' of the central clock and those to the of it right 'lag' it, according to the 'stationary system' K where all the clocks therein always read the same time t. After a time $t > 0$ the moving clocks advance to the right and the hands on the moving clocks advance, but they do not read the same time τ. As time t increases all the hands of the 'stationary' clocks advance by the same amount and all clocks in K still read the same time t - they are synchronised. However, for any x and t in the stationary system K there is in general a place $x^* \neq x$ with a clock that reads $t^* \neq t$, yet does not disturb the values of τ and ξ of the 'moving system' k, thereby contradicting the assumption of synchronisation of clocks. Recall the Lorentz Transformations equations for the time τ in the 'moving system' k according to the 'stationary system' K:

$$\tau = \beta\left(t - \frac{vx}{c^2}\right), \tag{2a}$$

$$\xi = \beta\,(x - vt)\,. \tag{2b}$$

Assume all clocks in the 'stationary system' K to be synchronised as in figure 1. Then for any time t of K all the stationary clocks read the same time at every x in K. Similarly, assume all clocks in the 'moving system' k to be synchronised with respect to the 'moving system' k, just as Einstein prescribed. When the k-system of clocks is in motion its clocks are not synchronised with respect to the 'stationary system' K, as shown in figure 1. Now set,

$$\begin{aligned} x^* &= \sigma x \\ t^* - \frac{v\sigma x}{c^2} &= t - \frac{vx}{c^2} \end{aligned} \tag{3}$$

where $0 \leq \sigma$. From the second of equations (3),

$$t^* = t + \frac{(\sigma - 1)\,vx}{c^2}. \tag{4}$$

The following is a table of sample values:

σ	x^*	t^*	τ
0	0	$t - vx/c^2$	$\beta\left(t - vx/c^2\right)$
1/2	$x/2$	$t - vx/2c^2$	$\beta\left(t - vx/c^2\right)$
1	x	t	$\beta\left(t - vx/c^2\right)$
2	$2x$	$t + vx/c^2$	$\beta\left(t - vx/c^2\right)$
3	$3x$	$t + 2vx/c^2$	$\beta\left(t - vx/c^2\right)$

Note that for any time $t > 0$ of the 'stationary system' K there is, in general, a place $x^* \neq x$ with a clock reading $t^* \neq t$, which does not alter the values of either τ or ξ, thereby contradicting the assumption of clock synchronisation in K. Hence, by *reductio ad absurdum*, synchronisation of clocks is inconsistent with the Lorentz Transformation. Conversely, the Lorentz Transformation is inconsistent with synchronisation of clocks - they are mutually exclusive.

4 Einstein's time dilation

"*. . . we imagine one of the clocks which are qualified to mark the time t when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of the co-ordinates of k, and so adjusted that it marks the time τ. What is the rate of this clock, when viewed from the stationary system?*

"*Between the quantities x, t, and τ, which refer to the position of the clock, we have, evidently, x = vt and*

$$\tau = \frac{1}{\sqrt{1 - v^2/c^2}}\left(t - vx/c^2\right).$$

"*Therefore,*

$$\tau = t\sqrt{1 - v^2/c^2} = t - \left(1 - \sqrt{1 - v^2/c^2}\right)t$$

"*whence it follows that the time marked by the clock (viewed in the stationary system) is slow by* $1 - \sqrt{1 - v^2/c^2}$ *seconds per second, . . .*" [3, §4]

Thus, clocks A and B are first synchronised when they are both at rest with respect to the ‘stationary system’ K. At this stage the system k is not ‘moving’ and the systems K and k coincide, as shown in figure 2. Hence clocks A and B then read the same ‘time’, for the ‘stationary system’ K. The system k is then imagined to be moving with speed v along the common X-axis of x and ξ, and when their origins coincide ($x = \xi = 0$) the time is $t = \tau = 0$, again as in figure 2, except that clock B is now fixed to the origin of coordinates for the ‘moving system’ k.

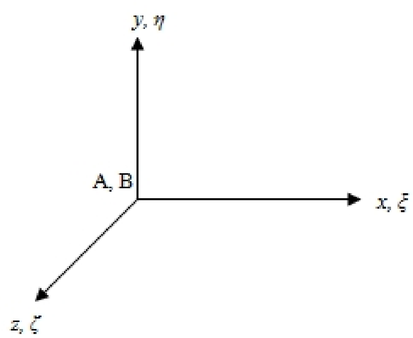

Fig. 2: Initial conditions: the origins of the ‘stationary system’ K and the ‘moving system’ k coincide, and $t = \tau = 0$. The initially synchronised clocks A and B are located at the common origin ($x = \xi = 0$).

After a time $t > 0$ the system k has advanced a distance $x = vt$ so that clock A is at the origin of the ‘stationary system’ K, although it can be located anywhere in K, and clock B at the origin of the ‘moving system’ k, as shown in figure 3.

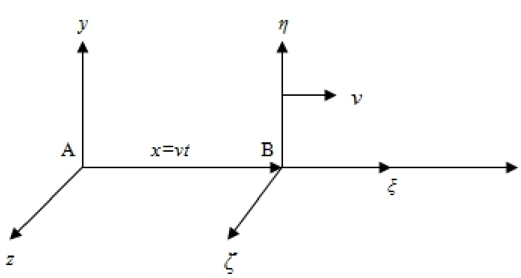

Fig. 3: Subsequent conditions: clock A reads time t for the ‘stationary system’ K and clock B reads time τ for the ‘moving system’ k. Clocks A and B are separated by the distance $x = vt$ according to the ‘stationary system’ K. Clock B is always located at $\xi = 0$. Clock A can however be located anywhere in K.

Note that in both figures, clock B is located at $\xi = 0$. Therefore, by the Lorentz Transformation (1), $x = vt$.

The ‘moving’ clock B indicates the time τ at all points in the ‘moving system’ k, according to Einstein’s method of clock synchronisation, and clock A the time at all points in the ‘stationary system’ K.

According to the Lorentz Transformation, the time τ is a function of both x and t when $v \neq 0$. Elimination of x for the ‘stationary system’ K in the Lorentz Transformation (1) for the time τ yields,

$$\tau = \frac{t}{\beta} - \frac{\xi v}{c^2}. \tag{5}$$

If $\xi = 0$, then,

$$\tau = t/\beta = t\sqrt{1 - v^2/c^2} \tag{6}$$

and so when $t = 0$, $\tau = 0$ too. Equation (6) is Einstein’s ‘time dilation’. Note that it applies *only* at $\xi = 0$ of the ‘moving system’ k, by an *ad hoc* mathematical restriction.

Setting $\tau = t$ in (5) yields,

$$\xi = \xi^* = \frac{(1-\beta)\,tc^2}{\beta v}. \tag{7}$$

Thus for all $t > 0$ there always exists a place $\xi^* \neq 0$ where $\tau = t$ in Einstein’s ‘moving system’ k, contrary to Einstein’s clock synchronisation method. The case of $t > 0$ and $\tau = 0$ at a place $\xi^* \neq 0$ has already been proven in [2]. However, the latter is really a particular case of the foregoing, since for any time $t \geq 0$ there always exists a place ξ^* in the ‘moving system’ k where $\tau = \kappa t$, κ being any real number in the range $0 \leq \kappa$. Set $\tau = \kappa t$ in (5). Then, in general, for *any* time t,

$$\xi^* = \frac{(1-\kappa\beta)\,tc^2}{\beta v} \tag{8}$$

is a place in the moving system where $\tau = \kappa t$. Setting $\kappa = 2$, for example, yields $\tau = 2t$ at $\xi^* = (1-2\beta)\,tc^2/\beta v$ in the ‘moving system’ k.

κ	τ	ξ^*
0	0	$tc^2/\beta v$
1/2	$t/2$	$(2-\beta)\,tc^2/2\beta v$
1	t	$(1-\beta)\,tc^2/\beta v$
2	$2t$	$(1-2\beta)\,tc^2/\beta v$
$1/\beta$	t/β	0

Hence the ‘stationary system’ K finds that for any time $t > 0$, τ has different values at different places in the ‘moving system’ k. Still, Einstein’s clock synchronisation is inconsistent with the Lorentz Transformation.

5 Einstein’s length contraction

According to Special Relativity a moving ‘rigid body’[*] undergoes a length contraction in the direction of its motion. If motion is in the X-direction then the length of the moving body in that direction is shortened to $l_0\sqrt{1 - v^2/c^2}$, where l_0

[*]Although Einstein utilised rigid bodies, these bodies *change their lengths* when they are in motion.

is the length of the body in the X-direction when the body is at rest. In other words, if the length of a body in the x-direction in the 'stationary system' K is l_0, then according to the 'stationary system' K the length of the very same body in the ξ-direction of the moving system k is $l'_0 = l_0\sqrt{1 - v^2/c^2}$. However, at any time $t > 0$ of the 'stationary system' K there is always a place x^* in K from which the length of the moving body is the same as in the 'stationary system' K.

Einstein [3, §4] considered a rigid sphere of radius R:

> "*We envisage a rigid sphere*[1] *of radius R, at rest relatively to the moving system k, and with its centre at the origin of co-ordinates of k. The equation of the surface of this sphere moving relatively to the system K with velocity v is*
>
> $$\xi^2 + \eta^2 + \zeta^2 = R^2.$$
>
> *The equation of this surface expressed in x, y, z at the time t = 0 is*
>
> $$\frac{x^2}{\left(\sqrt{1 - v^2/c^2}\right)^2} + y^2 + z^2 = R^2.$$
>
> *A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion - viewed from the stationary system - the form of an ellipsoid of revolution with the axes*
>
> $$R\sqrt{1 - v^2/c^2}, R, R.$$
>
> "*Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened in the ratio* $1 : \sqrt{1 - v^2/c^2}$*, i.e. the greater the value of v, the greater the shortening.*
>
> "[1] *That is, a body possessing spherical form when examined at rest.*"

Einstein's rigid sphere at rest "*relative to the moving system k*" is illustrated in figure 4. Note that the radius of a sphere at rest is R in all directions. Since Einstein's rigid sphere moves only in the X-direction, the radius R in that direction is purported to shorten to $R\sqrt{1 - v^2/c^2}$, according to the 'stationary system' K. This is easily seen by setting $y = z = 0$ in Einstein's equation for the "*ellipsoid of revolution*", from which it immediately follows that $x = R\sqrt{1 - v^2/c^2}$.

It is evident from Einstein's equation for "*an ellipsoid of revolution*" that his ellipsoid is centred at the origin of coordinates $x = y = z = 0$ for the 'stationary system' K. Hence Einstein [3, §4] superposed the two coordinate systems for K and k respectively, so that their origins coincide at the 'stationary system' K-time $t = 0$, illustrated in figure 5. In this case it is imagined that the sphere is moving at a constant speed v in the common X-direction according to the 'stationary system' K.

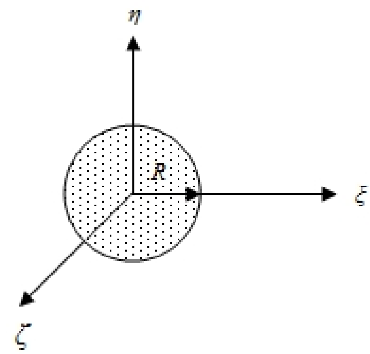

Fig. 4: Initial conditions: a rigid sphere of radius R centred at the origin of coordinates for the 'moving system' k. The sphere is at rest with respect to k. In the k system the sphere has the equation $\xi^2 + \eta^2 + \zeta^2 = R^2$. When $t = 0$ in the 'stationary system' K, the time $\tau = 0$ at the origin $\xi = 0$ but at $\xi = R$ the time is $\tau = -Rv/c^2$, by the Lorentz Transformation.

Note that Einstein set $t = 0$ at the common origin of coordinates, so that, by the Lorentz Transformation (1), $\xi = \beta x$. Consequently, at the common origin, $x = 0$ and therefore $\xi = 0$. Referring to figure 6, when $t = 0$ at all time-synchronised points in the 'stationary system' K, at $\xi = 0$ the k-time is $\tau = 0$, but at $\xi = R$ the k-time is $\tau = -Rv/c^2$, by the Lorentz Transformation. Einstein did not mention this. There is in fact no single k-time associated with the K-time $t = 0$. If $t > 0$, then $\xi = \beta(x - vt)$ and the equation of the "*ellipsoid of revolution*" according to the 'stationary system' K is,

$$\frac{(x - vt)^2}{\left(\sqrt{1 - v^2/c^2}\right)^2} + y^2 + z^2 = R^2. \tag{9}$$

This ellipsoid is centred at $x = vt, y = 0, z = 0$ of the 'stationary system' K. The first term of equation (9) is not constant, but varies with the 'time' t. To avoid this awkward problem, Einstein set $t = 0$. However, it follows from the Lorentz Transformation that for any time $t > 0$ there is always a place x^* in the 'stationary system' K, from which the moving sphere of radius R in k, is a sphere of radius R in K. In other words, there is always a place in K from which there is no 'length contraction' of the moving sphere.

Since length contraction supposedly occurs only in the direction of motion, consider a 'rigid rod' of length l_0 in the 'stationary system' K, as shown in figure 6.

Take an identical rigid rod and place it with the very same orientation in the as yet stationary system k. Now imagine the system k to have a constant speed v in the positive direction of the x-axis of K, as shown in figure 7.

Let the time t of the 'stationary system' K be reckoned from $t = 0$ when the y and η axes coincide. After a time $t = t_0$

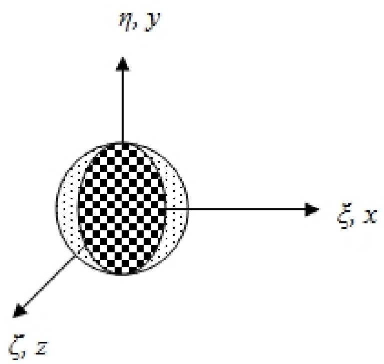

Fig. 5: Subsequent conditions: a rigid sphere of radius R centred at the origin of both coordinate systems. The sphere is at rest with respect to k but moving at a constant speed v with respect to K, in the common X-direction. The ellipsoid is the 'shortened sphere' observed from the stationary system K. In the k system the sphere has the equation $\xi^2+\eta^2+\zeta^2=R^2$. In the K system it is not a sphere, but an ellipsoid, with equation $\frac{x^2}{(1-v^2/c^2)}+y^2+z^2=R^2$. Here the time $t=0$ at all time-synchronised points in the 'stationary system' K, but for the 'moving system' k the k-time is $\tau=0$ at $\xi=0$ but $\tau=-Rv/c^2$ at $\xi=R$.

the k system advances to a distance $l_0=vt_0$ from the origin of the K system, i.e. the very length of the 'stationary' rod, shown in figure 8.

Now, according to Special Relativity, the length of the 'moving' rod l'_0 is the same at any time t and place x of the 'stationary system' K, because length contraction is independent of the value of t and position of the rod in either system, depending only on the constant relative speed v. However, according to the Lorentz Transformation, $\xi=\beta(x-vt)$. Thus, when $t=0$, $\xi=\beta x$, and so $l'_0=\beta l_0$. But when $t=t_0>0$,

$$\xi=\beta(x-vt_0)=\beta(x-l_0), \tag{10}$$

having set $l_0=vt_0$. Setting $\xi=l_0$ yields,

$$l_0=\beta(x-l_0). \tag{11}$$

Solving this for x gives,

$$x=x^*=\frac{l_0(1+\beta)}{\beta}=\frac{vt_0(1+\beta)}{\beta}. \tag{12}$$

Thus, at any $t=t_0>0$ such that $l_0=vt_0$, there is a place $x^*=l_0(1+\beta)/\beta$ from which the length of the 'moving' rod is exactly the same as the length of the 'stationary' rod. In other words, when the 'moving system' k has traversed a distance equal to the length of the 'stationary' rod, there is a place x^* in the 'stationary system' K from which the length of the 'moving' rod is the same as the length of the 'stationary' rod. In fact, there is always a place x^* in the 'stationary system' K from which the 'moving' rod has any corresponding finite length. In general, if the length of the rigid rod in

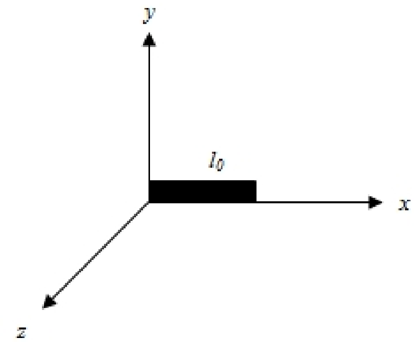

Fig. 6: A rigid rod of length l_0 in the stationary system K, and in the as yet stationary system k.

the 'stationary system' is l_0, then there is always a place x^* in the 'stationary system' K from which the very same rod, when 'moving', has the length $l'_0=\sigma l_0$, where $0\le\sigma$. Setting $\xi=\sigma l_0$ in (10), the place x^* in the stationary system is given by,

$$x^*=\frac{l_0(\sigma+\beta)}{\beta}=\frac{vt_0(\sigma+\beta)}{\beta}. \tag{13}$$

For example, set $\sigma=2$. Then $x^*=l_0(2+\beta)/\beta$. Hence, by (10), $l'_0=\beta[l_0(2+\beta)/\beta-l_0]=2l_0$. In this case the 'moving' rod becomes extended, not contracted. Similarly, set $\sigma=1$. Then $x^*=l_0(1+\beta)/\beta$ and so $l'_0=\beta[l_0(1+\beta)/\beta-l_0]=l_0$.

σ	l'_0	x^*
0	0	l_0
1/2	$l_0/2$	$l_0(1+2\beta)/2\beta$
1	l_0	$l_0(1+\beta)/\beta$
2	$2l_0$	$l_0(2+\beta)/\beta$
β	βl_0	$2l_0$

Note that only at $x^*=2l_0$ does Einstein's 'length contraction' equation hold. Therefore, only at $x^*=2R$ does Einstein's 'length contraction' hold for his moving rigid sphere, not at $x=0$ or at $x=R/\beta$, or anywhere in between. Einstein's length contraction depends upon the position of the 'stationary' observer.

6 Einstein's twins paradox

With his 'time dilation' equation in the lag form,

$$\tau=t-\left(1-\sqrt{1-v^2/c^2}\right)t, \tag{14}$$

Einstein used the following approximation, when $v^2/c^2<<1$,

$$\left(1-\frac{v^2}{c^2}\right)^{1/2}\approx\left(1-\frac{1}{2}\frac{v^2}{c^2}\right). \tag{15}$$

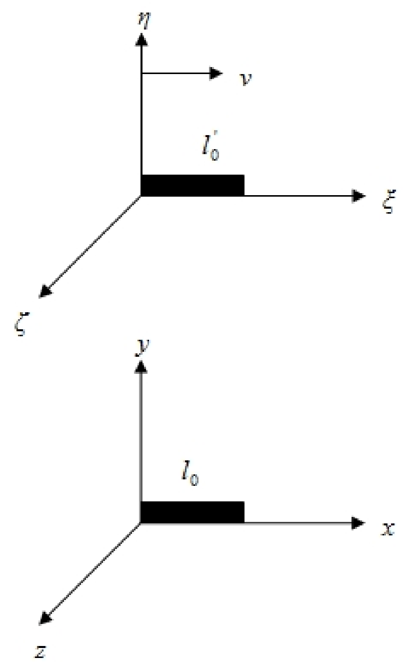

Fig. 7: A rigid rod of length l_0 in the 'stationary system' K and of length l_0' in the 'moving system' k as determined from the stationary system K. The systems are offset here because rigid rods cannot pass through one another.

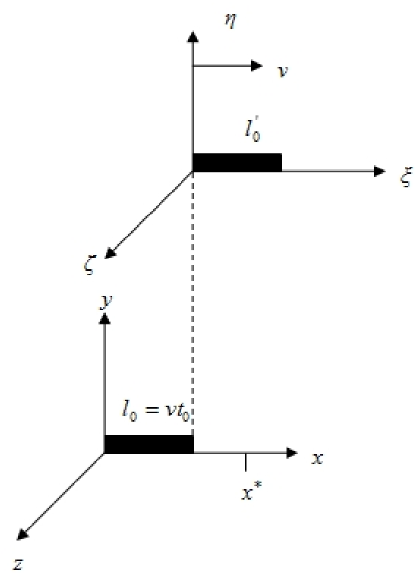

Fig. 8: After time t_0 the k system advances a distance $l_0 = vt_0$.

Putting (15) into (14) gives,

$$\tau = \frac{1}{2} t v^2 / c^2. \tag{16}$$

Then, on the basis of his 'time dilation', Einstein [3, §4] first considered twin clocks, one located at a point A, the other at a different point B, both in the 'stationary system' K. These two clocks are synchronised according to Einstein's method. Consequently they initially read the same 'time' in K. The initial situation is illustrated in figure 9, where A_C and B_C are the clocks at the points A and B respectively.

> "*If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous ; and if the clock at A is moved with the velocity v along the line AB to B, then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by* $\frac{1}{2}tv^2/c^2$ *(up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B.*" [3, §4]

The subsequently moving clock A_C constitutes the 'moving system' k. To amplify, attach the coordinate system for k to the moving clock A_C, as in figure 10.

The moving clock A_C stops at point B in the stationary system K and compares its 'time' to the clock B_C, illustrated in figure 11. According to Einstein they no longer indicate the same time: clock A_C lags clock B_C by the time $\frac{1}{2}tv^2/c^2$. However, as proven in §3 herein, for any time $t > 0$ in the 'stationary system' K there is always a place ξ^* in the 'moving system' k where a clock located there indicates the very same time as the clock B_C in the 'stationary system' K. Hence, Einstein's twins paradox is inconsistent with his clock synchronisation method.

Einstein's time dilation is reciprocal because Special Relativity is symmetric by its very definition. Indeed, concerning his length contraction, Einstein [3, §4] asserted,

> "*It is clear that the same results hold good of bodies at rest in the 'stationary' system, viewed from a system in uniform motion.*"

Consequently, the situations illustrated in figures 9, 10 and 11 can be reversed, so that clock B_C moves towards clock A_C affixed at point A in system K, so that clock B_C finally lags clock A_C. This reversal is in fact merely a copy of the first configuration, as an interchange of the A's and B's reveals. However, since only relative motion is allowed in Special Relativity, neither system can say which is really 'moving' and which is really 'stationary'. All each system can conclude is that they approach one another or recede from one another with a constant rectilinear speed, or that they neither approach nor recede. In the latter case the two systems are 'at rest'. In Einstein's twin clocks scenario each system must conclude that they approach one another with a constant rectilinear speed v. Hence, each system must finally conclude that its clock is the one that lags. That A_C lags B_C and B_C lags A_C is impossible. Einstein introduced an asymmetry into his 'time dilation'. He also asserts,

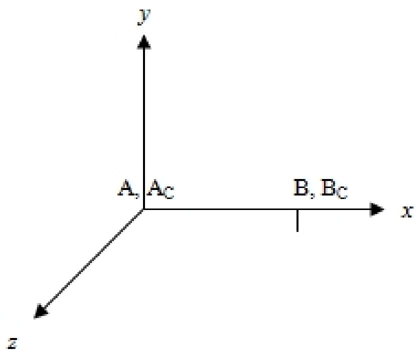

Fig. 9: Stationary clocks A_C and B_C at points A and B respectively in the 'stationary system' K, are synchronised by Einstein's method. They therefore read the same time, just as any and all other synchronised clocks in the 'stationary system' K, such clocks being able to be introduced at will.

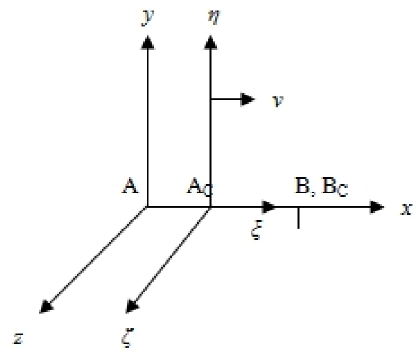

Fig. 10: Although Einstein does not say 'constant speed', owing to the very setting of Special Relativity clock A_C moves at constant rectilinear speed v towards point B to which clock B_C is affixed. Ultimately clock A_C, at the origin of the coordinate system for the 'moving system' k, stops at point B and compares its 'time' reading with the clock B_C. Points A and B, and clock B_C, constitute the 'stationary system' K.

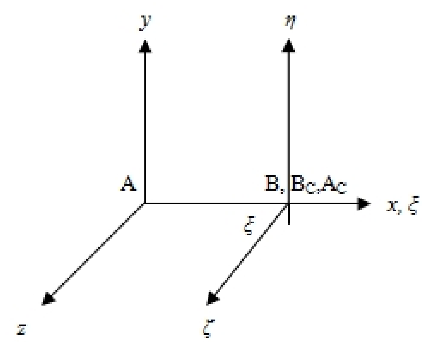

Fig. 11: Clock A_C stops at point B and its reading is compared to that of clock B_C. According to Special Relativity the clocks are no longer synchronised and so they read different times; clock A_C lags clock B_C. This conclusion is however inconsistent with Einstein's method of clock synchronisation.

> "*It is at once apparent that this result still holds good if the clock moves from A to B in any polygonal line, and also when the points A and B coincide.*" [3, §4]

The "*polygonal line*" violates a foundation of Special Relativity; that it applies only to uniform rectilinear relative motion. To move "*in any polygonal line*" dispenses with the foregoing character of Special Relativity, and the presence of 'rigid' bodies. Moreover, it is only *apparently* possible because Einstein's 'time dilation' does not depend upon position, only the time t and the relative constant speed v. Figure 12 illustrates Einstein's polygonal line motion from point A to point B when A and B do not coincide.

Note that as clock A_C moves along the dashed polygonal line on its way to point B, there is, according to Einstein, time dilation on each leg travelled, determined by his equation $\tau = t\sqrt{1 - v^2/c^2}$. As the latter is independent of position, when clock A_C reaches point B, it is no longer synchronised with the clock B_C. However, in the 'moving system' k in which clock A_C resides, there is always a place ξ^* where $\tau = \kappa t$ for any chosen value of κ, because, by the Lorentz Transformation, τ is a function of both time and position. Einstein's motion of clocks along a polygonal line is generally inconsistent with his clock synchronisation method because τ has no particular value for the 'moving system' k. Furthermore, owing to the symmetrical nature of Special Relativity, neither system can say which is really 'stationary' and which is really 'moving': there is only relative motion. Consequently, each clock experiences precisely the same effects as the other. Hence, each clock must conclude that it lags, which is again, impossible.

The case of polygonal motion between points A and B, "*when the points A and B coincide*" is depicted in figure 13.

In figure 13 clocks A_C and B_C are initially at rest at the origin of the 'stationary system' K, where points A and B coincide. Clock A_C then undertakes a polygonal journey at constant speed v on each leg. Einstein concludes from his time dilation equation that when clock A_C returns to the origin of the stationary system K, it is no longer synchronised with clock B_C, and that it lags clock B_C. But once again, by the Lorentz Transformation, the time τ of the 'moving system' is both time and position dependent in general. Consequently there is always a place ξ^* in the 'moving system' k where $\tau = \kappa t$, for any arbitrary κ such that $0 \leq \kappa$. And since Special Relativity is supposed to be symmetric, each system must experience precisely the same effects of relative motion, so that each clock finds that it lags the other, which is still impossible.

Finally, Einstein [3, §4] invokes a "*continuously curved*

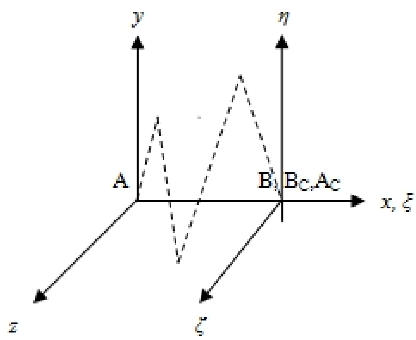

Fig. 12: Clock A_C moves to point B along a 'polygonal line' indicated by the broken lines. On this course from point A to point B the clock A_C acquires motion in the Y and Z directions as well as the X-direction. This however does not affect Einstein's time dilation owing to his *ad hoc* elimination of position.

line" in place of the polygonal line, when points A and B coincide; depicted in figure 14.

> "*If we assume that the result proved for a polygonal line is also valid for a continuously curved line, we arrive at this result : If one of two synchronous clocks at A is moved in a closed curve with constant velocity until it returns to A, the journey lasting t seconds, then by the clock which has remained at rest the travelled clock on its arrival at A will be* $\frac{1}{2}tv^2/c^2$ *second slow.*" [3, §4]

The continuously curved line traversed by clock A_C in figure 14 produces the very same time dilation effect as Einstein asserts for the polygonal line in figure 13. The continuously curved line suffers from the same inconsistencies as his polygonal line. In all cases Einstein's time clock synchronisation is inconsistent with the Lorentz Transformation.

7 Conclusions

By his clock synchronisation method Einstein attempted to ensure that time at all places within a given system is the same, despite subsequently invoking the Lorentz Transformation. By his time dilation method he attempted to assign different times to different systems by virtue of uniform relative motion, generalised in such a way as to violate the uniform rectilinear relative motion defined on Special Relativity, owing to his *ad hoc* mathematical restriction on position in the Lorentz Transformation. His method of clock synchronisation is inconsistent with the Lorentz Transformation. His time dilation and length contraction are generally inconsistent with the Lorentz Transformation, and also with his 'Principle of Relativity',

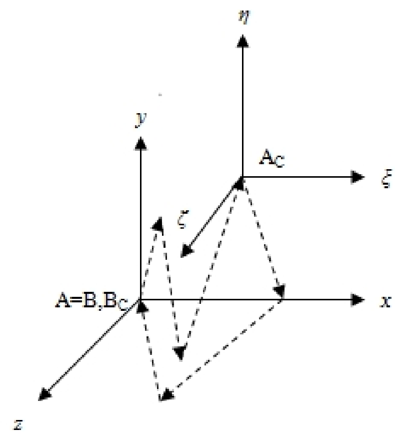

Fig. 13: Points A and B coincide at the origin of the coordinate system for the 'stationary system' K. The synchronised clocks A_C and B_C are identical twins, initially located at the origin of K, where points A and B coincide. Clock A_C then travels out along the polygonal line from the origin and returns, at a constant speed v throughout.

> "*. . . the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.*" [3]

Clock synchronisation is inconsistent with the Lorentz Transformation. Conversely the Lorentz Transformation is inconsistent with clock synchronisation. Special Relativity is inconsistent with the Lorentz Transformation in general, and therefore contains an insurmountable logical contradiction.

Einstein [3, §1] defined time by means of his clocks. However, time is no more defined by a clock than pressure is defined by a pressure gauge, speed by a speedometer, heat by a thermometer, or gravity by a spring. Measuring instruments are invented to measure something other than themselves. Einstein's clocks measure only themselves.

Nonetheless, all textbooks on the subject reiterate Einstein's arguments in equivalent if not exact, or implicit, form: for example [4–18]. They all suffer, necessarily, from the same logical inconsistencies as Einstein's 1905 paper.

References

[1] Engelhardt, W., Einstein's third postulate, *Physics Essays*, **29**, 4 (2016)

[2] Crothers, S.J., Einstein's Anomalous Clock Synchronisation, http://vixra.org/pdf/1702.0214v1.pdf

[3] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905

[4] Griffiths, D.J., Introduction to Electrodynamics, 4th Ed., Pearson Education Inc., 2013

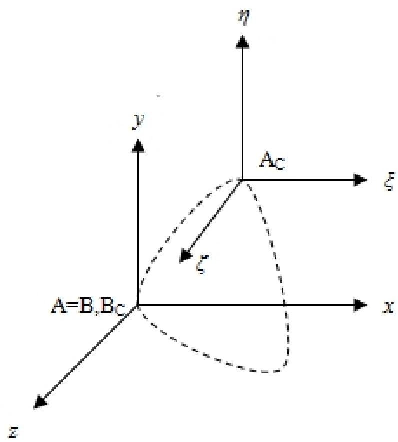

Fig. 14: Points A and B coincide at the origin of the coordinate system for the 'stationary system' K. The synchronised clocks A_C and B_C are identical twins, initially located at the origin of K, where points A and B coincide. Clock A_C then travels out along the continuously curved line from the origin and returns, at a constant speed v throughout.

[5] Einstein, A., The Meaning of Relativity, Princeton University Press, 1988

[6] Einstein, A., Relativity the Special and the General Theory, Methuen, London, 1954

[7] Taylor, E.F., Wheeler, J.A., Spacetime Physics, 2^{nd} Ed., W.H. Freeman and Company, New York, 1992

[8] d'Inverno, R., Introducing Einstein's Relativity, Oxford University Press, 1992

[9] Carroll, B.W., Ostlie, D.A., Modern Astrophysics, Addison Wesley Longman, 1996

[10] Mould, R.A., Basic Relativity, Springer, 1994

[11] Tolman, R.C., Relativity Thermodynamics and Cosmology, Dover Publications Inc., New York, 1987

[12] Radin, S.H., Folk, R.T., Physics for Scientists and Engineers, Prentice Hall Inc., 1982

[13] McMahon, D., Relativity Demystified, McGraw-Hill, 2006

[14] Schutz, B., A First Course in General Relativity, Cambridge University Press, 2009

[15] Shadowitz, A., Special Relativity, Dover Publications Inc., 1988

[16] Rindler, W., Introduction to Special Relativity, Clarendon Press, 1982

[17] Pauli, W., The Theory of Relativity, Dover Publications Inc., New York, 1981

[18] Turner, R.E., Relativity Physics, Routledge & Kegan Paul, 1984