Conjecture on a subset of Mersenne numbers divisible by Poulet numbers

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Abstract. The Poulet numbers (or the Fermat pseudoprimes to base 2) are defined by the fact that are the only composites n for which 2^(n – 1) – 1 is divisible by n (so, of course, all Mersenne numbers 2^(n - 1) – 1 are divisible by Poulet numbers if n is a Poulet number; but these are not the numbers I consider in this paper). In a previous paper I conjectured that any composite Mersenne number of the form 2^m – 1 with odd exponent m is divisible by a 2-Poulet number but seems that the conjecture was infirmed for m = 49. In this paper I conjecture that any Mersenne number (with even exponent) 2^(p – 1) – 1 is divisible by at least a Poulet number for any p prime, p ≥ 11, p ≠ 13.

Conjecture:

Any Mersenne number M = 2^(p – 1) – 1 is divisible by at least a Poulet number for any p prime, p ≥ 11, p ≠ 13.

Verifying the conjecture:
(for the first twenty such primes)

: for p = 11 we have M = 3*11*31 which is divisible by 341 (= 11*31), a Poulet number;

: for p = 17 we have M = 3*5*17*257 which is divisible by 4369 (= 17*257), a Poulet number;

: for p = 19 we have M = 3^3*7*19*73 which is divisible by 1387 (= 19*73), a Poulet number;

: for p = 23 we have M = 3*23*89*683 which is divisible by 2047 (= 23*89), 15709 (= 23*683) and 60787 (= 89*683), all three Poulet numbers;

: for p = 29 we have M = 3*5*29*43*113*127 which is divisible by 3277 (= 29*113), 18705 (= 3*5*29*43), 617093 (= 43*113*127), 17895697 (= 29*43*113*127), all four Poulet numbers;

: for p = 31 we have M = 3^2*7*11*31*151*331 which is divisible by 341 (= 11*31), 4681 (= 31*151), 49981 (= 151*331), all three Poulet numbers;

: for p = 37 we have M = 3^3*5*7*13*19*37*73*109 which is divisible at least by 2701 (= 37*73) and 1729 (= 7*13*19), both Poulet numbers;

: for p = 41 we have M = 3*5^2*11*17*31*41*61681 which is divisible at least by 341 (= 11*31) and 2528921 (= 41*61681), both Poulet numbers;

: for p = 43 we have M = 3^2*7^2*43*127*337*5419 which is divisible at least by 5461 (= 43*127) and 42799 (= 127*337), both Poulet numbers;

: for p = 47 we have M = 3*47*178481*2796203 which is divisible at least by 499069107643 (= 178481*2796203) and 8388607 (= 47*178481), both Poulet numbers;

: for p = 53 we have M = 3*5*53*157*1613*2731 which is divisible at least by 8321 (= 53*157) and 253241 (= 157*1613), both Poulet numbers;

: for p = 59 we have M = 3*59*233*1103*2089*3033169 which is divisible at least by 13747 (= 59*233) and 256999 (= 233*1103), both Poulet numbers;

: for p = 61 we have M = 3^2*5^2*7*11*13*31*41*61*151*331*1321 which is divisible at least by 3241 (= 11*31) and 80581 (= 61*1321), both Poulet numbers;

: for p = 67 we have M = 3^2*7*23*67*89*683*20857*599479 which is divisible at least by 2047 (= 23*89) and 137149 (= 23*67*89), both Poulet numbers;

: for p = 71 we have M = 3*11*31*43*71*127*281*86171*122921 which is divisible at least by 341 (= 11*31) and 19951 (= 71*281), both Poulet numbers;

: for p = 73 we have M = 3^3*5*7*13*17*19*37*73*109*241*433*38737 which is divisible at least by 2701 (= 37*73) and 1729 (= 7*13*19), both Poulet numbers;

: for p = 79 we have M = 3^2*7*79*2371*8191*121369*22366891 which is divisible at least by 647089 (= 79*8191) and 183207204181 (= 8191*22366891), both Poulet numbers;

: for p = 83 we have M = 3*83*13367*164511353*8831418697 which is divisible at least by 1109461 (= 83*13367), Poulet number;

: for p = 89 we have M = 3*5*17*23*89*353*397*683*2113*2931542417 which is divisible at least by 2047 (= 23*89) and 745889 (= 353*2113), both Poulet numbers;

: for p = 97 we have M = 3^2*5*7*13*17*97*193*241*257*673*65537*22253377 which is divisible at least by 18721 (= 97*193) and 129889 (= 193*673), both Poulet numbers.

Note:

A stronger enunciation for the conjecture above could be: Any Mersenne number M = 2^(p – 1) – 1 is divisible by at least two Poulet numbers q1 and q2 for any p prime, p ≥ 23, from which q1 is divisible by p and q2 is not divisible by p (which is verified true in all the cases considered up to p = 97 but one; in the case p = 83 the products of prime factors are greater than 10^12, where my table with Poulet numbers ends).