Two Applications of Riccati ODE in Nonlinear Physics and Their Computer Algebra Solutions

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Abstract

In this paper, we will solve 2 Riccati ODEs using Maxima computer algebra package with applications in: (a) generalized Gross-Pitaevskii equation, (b) cosmology problem. The results presented below deserve further investigations in particular for comparison with existing analytical solutions.

1. Introduction

The Riccati equation, named after the Italian mathematician Jacopo Francesco Riccati, is a basic first-order nonlinear ordinary differential equation (ODE) that arises in different fields of mathematics and physics.[4]

Riccati differential equations are known to have many applications in nonlinear physics [1]. In this paper, we will explore only 4 of possible applications of Riccati ODE in literature, i.e. (a) generalized Gross-Pitaevskii equation, (b) KdV-Burgers equation, (c) Ramanujan differential equation, and (d) cosmology problem.

Instead of using standard solution method to solve Riccati ODE, we will use Maxima computer algebra package.

We hope that our results may stimulate further serious investigation on finding numerical solutions of Riccati ODE in various domains of nonlinear physics, number theory, and cosmology.

2. Problem 1: Generalized Gross-Pitaevskii equation (GPE)

The authors in [4] presented the generalized GPE in (3+1)D for the BEC wave function u(x,y,z,t) with distributed time-dependent coefficients: [4]

$$i\partial_t u + \frac{\beta(t)}{2}\Delta u + \chi(t)|u|^2 u + \alpha(t) r^2 u = i\gamma(t)u, \tag{1}$$

Which can be transformed easily into a Riccati ODE form as follows:

$$\frac{da}{dt} + 2\beta(t)a^2 - \alpha(t) = 0 \tag{2}$$

The above Riccati ODE (2) can be rewritten as follows:[3]

$$a(t)' + 2.b(t)\cdot a(t)^2 - c(t) = 0. \tag{3}$$

Maxima expression of Riccati ODE (3) is as follows:[2]

'diff(a(t),t)+2*b(t)*a(t)^2-c(t)=0 (4)

The Maxima result for this problem is as shown below:

(%i14) 'diff(y,x)+2*b*y^2-c=0;

(%o14) $\frac{d}{dx} y + 2\, b\, y^2 - c = 0$

(%i16) ode2(%,y,x);

Is b c positive or negative?negative;

(%o16) $-\frac{\operatorname{atan}\left(\frac{\sqrt{2}\, b\, y}{\sqrt{-b\, c}}\right)}{\sqrt{2}\sqrt{-b\, c}} = x + \%c$

3. Problem 2: Cosmology problem

It can be shown that in Friedmann-Robertson-Walker spacetime the set of Einstein's equations with the cosmological constant set to zero reduce to differential equations for scale factor a(t), which is a function of comoving time t.[5] Choosing the equation of state to be barotropic and after some transformation and introducing conformal time, the equation reduces to a Riccati equation as follows:[5]

$$u' + cu^2 + kc = 0, \tag{5}$$

The above equation of cosmological Riccati equation has been obtained previously by Faraoni, see [5].

Equation (5) can be rewritten for Maxima as follows:

‘diff(a(t),t)+c*a(t)^2+k*c=0 (4)

The result is given below:

(a) Option 1: k=negative constant

(%i24) 'diff(y,x)+c*(y^2+k)=0;

$$\text{(\%o24)} \quad \frac{d}{dx} y + c(y^2 + k) = 0$$

(%i25) ode2(%,y,x);

Is k positive or negative? negative;

$$\text{(\%o25)} \quad -\frac{\log\left(-\frac{\sqrt{-k}-y}{y+\sqrt{-k}}\right)}{2\,c\sqrt{-k}} = x + \%c$$

(b) Option 2: k=positive constant

(c) (%i27) 'diff(y,x)+c*(y^2+k)=0;

$$\text{(\%o27)} \quad \frac{d}{dx} y + c(y^2 + k) = 0$$

(%i28) ode2(%,y,x);

Is k positive or negative? positive;

$$\text{(\%o28)} \quad -\frac{\operatorname{atan}\left(\frac{y}{\sqrt{k}}\right)}{c\sqrt{k}} = x + \%c$$

4. Concluding remarks

In this paper, we solve 2 Riccati ODEs using Maxima computer algebra package with applications in: (a) generalized Gross-Pitaevskii equation, (b) cosmology problem. The results as presented below deserve further investigations in particular for comparison with existing analytical solutions.

It is highly recommended to verify these results with other computer algebra packages, such as Maple or Mathematica.

References:

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