Solving Coupled Riccati ODEs as Solution of Incompressible Non-stationary 3D Navier-Stokes equations

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Abstract

In a recent paper, Ershkov derived a system of two coupled Riccati ODEs as solution of nonstationary 3D Navier-Stokes equations. Now in this paper, we will solve these coupled Riccati ODEs using Maxima computer algebra package. The result seems to deserve further investigation in particular for finding nonstationary Navier-Stokes equations for real fluid.

Introduction

In a recent paper, Ershkov derived a system of two coupled Riccati ODEs as solution of nonstationary 3D Navier-Stokes equations. The coupled Riccati ODEs read as follows:[1]

$$a' = \frac{w_y}{2} \cdot a^2 - (w_x \cdot b) \cdot a - \frac{w_y}{2}(b^2 - 1) + w_x \cdot b, \tag{1}$$

$$b' = -\frac{w_x}{2} \cdot b^2 - (w_y \cdot a) \cdot b - \frac{w_x}{2}(a^2 - 1) + w_z \cdot a \tag{2}$$

We are going to rewrite the above coupled equations in Maxima language.

Computer algebra solution

The above coupled Riccati ODEs (1) and (2) can be rewritten as follows:[3]

$$a(t)' = \frac{v}{2} \cdot a(t)^2 - (u \cdot b(t)) \cdot a(t) - \frac{v}{2}(b(t)^2 - 1) + w \cdot b(t), \tag{3}$$

$$b(t)' = -\frac{u}{2} \cdot b(t)^2 - (v \cdot a(t)) \cdot b(t) - \frac{u}{2}(a(t)^2 - 1) + w \cdot a(t) \tag{4}$$

Maxima expression of coupled Riccati ODEs (1) and (2) are as follows:[3]

‘diff(a(t),t)=v/2*a(t)^2-(u*b(t))*a(t)-v/2*(b(t)^2-1)+w*b(t), (5)

‘diff(b(t),t)=-u/2*b(t)^2-(v*a(t))*b(t)-u/2*(a(t)^2-1)+w*a(t). (6)

The Maxima results are as shown below:

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(%i3) 'diff(a(t),t)=v/2*a(t)^2-(u*b(t))*a(t)-v/2*(b(t)^2-1)+w*b(t);
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$$(\%o3)\quad \frac{d}{dt}a(t) = b(t)\,w - \frac{(b(t)^2-1)\,v}{2} + \frac{a(t)^2\,v}{2} - a(t)\,b(t)\,u$$

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(%i4) 'diff(b(t),t)=-u/2*b(t)^2-(v*a(t))*b(t)-u/2*(a(t)^2-1)+w*a(t);
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$$(\%o4)\quad \frac{d}{dt}b(t) = a(t)\,w - a(t)\,b(t)\,v - \frac{b(t)^2\,u}{2} - \frac{(a(t)^2-1)\,u}{2}$$

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(%i5) desolve([%o3,%o4],[a(t),b(t)]);

(%o5) [a(t)=ilt(-(
((laplace(b(t)^2,t,g34120)-laplace(a(t)^2,t,g34120))*v+2*laplace(a(t)*b(t),t,g34120)*u-2*a(0))*
g34120^2+(
(2*laplace(a(t)*b(t),t,g34120)*v+(laplace(b(t)^2,t,g34120)+laplace(a(t)^2,t,g34120))*u-2*b(0))*
w-v)*g34120-u*w)/(2*g34120^3-2*w^2*g34120),g34120,t),b(t)=ilt(-(
(2*laplace(a(t)*b(t),t,g34120)*v+(laplace(b(t)^2,t,g34120)+laplace(a(t)^2,t,g34120))*u-2*b(0))*
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g34120^2+(

((laplace(b(t)^2,t,g34120)-laplace(a(t)^2,t,g34120))*v+2*laplace(a(t)*b(t),t,g34120)*u-2*a(0))*

w-u)*g34120-v*w)/(2*g34120^3-2*w^2*g34120),g34120,t)]

Concluding remarks

Using Maxima package we solve the two coupled Riccati ODEs as solution of nonstationary Navier-Stokes equations.

It is advisable to verify this result with other computer algebra packages, such as Maple or Mathematica.

References:

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