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Thoughts on the Causality of the Maxwell Equations

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Abstract

The Maxwell equations are not causal, says Oleg Jefimenko. In this paper, the causality of the Maxwell equations is explored and explained, and new causal equations formulated, according to Jefimenko's important work in this area, strongly inspired by Oliver Heaviside's genius work. The causal equations take into account what is commonly called 'relativistic' velocities, and in fact replace the need for any relativity theory.

Keywords: Electromagnetism, Maxwell equations, causality, Oleg Jefimenko, Coulomb law, Oliver Heaviside.

1. What is causality?

The Maxwell equations are sometimes incorrectly interpreted. For time-dependent applications, they cannot directly be used. We will see why in this paper.
The Maxwell equations are:

$$\nabla \cdot \mathbf{E} = \rho / \varepsilon_0 \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \tag{4}$$

wherein $\mathbf{J}$ is the flow density of electrons through the considered surface.

Generally, we can say that the right hand induces or is responsible for the left hand of the equation. However, Jefimenko is more precise: the right hand of the equations is only responsible for the left hand "here and now". But not "there and then". This means that the right and the left hand can only occur simultaneously and locally [1].

And, consequently, the equations aren't causal equations. If they were, that would mean that the left hand could be found at a certain distance from the source, some time after the emission.

But what is the source in reality? Can the right hand of eq.(3) be the source that generates the equation's left hand?

Strictly speaking, it can't. Both the magnetic and the electric field must be generated by a third entity, a distribution of electrons and a flow of electrons. In other words, a pure magnetic field cannot generate an electric field nor vice versa. There must be a presence of electrons (or protons) that causes both fields simultaneously.

Jefimenko succeeds to purely deduce the source equations that are responsible for the fields: they are functions that only depend on the static electric densities and on the electric flows at the place p_0 and the moment t_0 , with the corresponding constants of the aether medium. The fields are found at a place p_1 at a distance r from p_1 to the source p_0 , and at a moment t_1 , which is of course regulated by the speed of propagation c.

$$\mathbf{E}\Big|_{p_1,t_1} = f_a\left(\rho, \mathbf{J}, r\right)\Big|_{p_0,t_0} \tag{5}$$

$$\mathbf{B}\Big|_{p_1,t_1} = f_b\left(\mathbf{J}, r\right)\Big|_{p_0,t_0} \tag{6}$$

The equations, fully written down in [1], are the causal equations related to the Maxwell equations. Eq.(5) and (6) are causal equations, and in principle the only valid equations when we speak of non-steady systems, where the observer is at a distance from the source, or for cases where the source or the observer is moving with varying distance between-in.

The equations (5) and (6), in the most general form, written in full are:

$$\mathbf{E}\Big|_{p_1,t_1} = \left[\frac{1}{4\pi\varepsilon_0}\int\left(\frac{\rho}{r^2}+\frac{\partial\rho}{rc\,\partial t}\right)\frac{\mathbf{r}}{r}\,\mathrm{d}\,v - \frac{1}{4\pi\varepsilon_0 c^2}\int\frac{1}{r}\left(\frac{\partial \mathbf{J}}{\partial t}\,\mathrm{d}\,v\right)\right]\Bigg|_{p_0,t_0}$$

$$\mathbf{B}\Big|_{p_1,t_1} = \left[\frac{\mu_0}{4\pi}\int\left(\frac{\mathbf{J}}{r^2}+\frac{\partial \mathbf{J}}{rc\,\partial t}\right)\times\frac{\mathbf{r}}{r}\,\mathrm{d}\,v\right]\Bigg|_{p_0,t_0} \qquad (7)\,(8)$$

The eq. (1) to (4) can only be used for steady state systems, while still keeping in mind that the origin of the fields can only be the electrons themselves.

Remark that the first integral of (7) takes into account static charge densities but variable charge densities too.

The second integral shows a minus sign. The signification of that second integral with a minus sign is the induced field caused by the first integral, which is the inducing field. As the reader can see, the denominator of the second integral shows c^2, which is typical for induced fields, such as Lenz's law. As a matter of fact, it *is* Lenz's law.

For a single charge q moving at a velocity v, the eq.(7) reduces to the well-known Heaviside equation, which expresses the electric field in terms of the present position of the charge [1] [4] [5]:

$$\mathbf{E}\Big|_{p_1,t_1} = \frac{q\left(1-v^2/c^2\right)\mathbf{r}_0}{4\pi\varepsilon_0 r_0^3\left[1-\left(v^2/c^2\right)\sin^2\theta\right]^{3/2}}\Bigg|_{p_1,t_1} \qquad (9)$$

wherein the right hand is the value of the present position and r_0 the distance between the present position of the source and the observer, and θ is the present angle between the velocity vector and the observer (Fig.1).

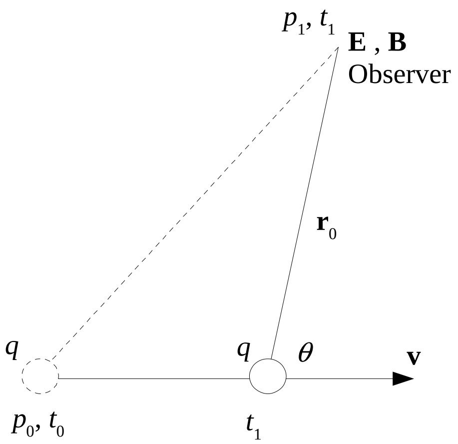

Fig.1. Explanation of the symbols in eq.(9).

This equation fully takes into account the retardation of the fields by the speed of light.

Jefimenko also finds the well-known equation out of eq.(8):

$$\mathbf{B}\Big|_{p_1,t_1} = \frac{\mathbf{v}\times\mathbf{E}}{c^2}\Bigg|_{p_1,t_1} \qquad (10)$$

wherein $\mathbf{E}$ is the value in eq.(9).

Only the equations (9) and (10) are valid at high velocities as well as for low velocities, and dismiss the need for any kind of relativity theory.

For a static situation, eq.(9) reduces further to the Coulomb law and eq.(10) to zero.

2. Conclusion

The Maxwell equations are only valid locally and for steady systems. When a charge is in linear motion, the retarded electric field and the corresponding magnetic field can easily be expressed in terms of present position parameters, according to the eqs.(9) and (10).

References

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