An Elementary Proof for infinitely many twin primes

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Abstract

Prime numbers are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved. A kaleidoscope can produce an endless variety of colorful patterns and it looks like magic, but when you open one and examine it, it contains only very simple, loose, colored objects such as beads or pebbles and bits of glass. Humans are very easily cheated by 2 words, infinite and anything, because we never see infinite and anything, and so we always make a simple thing complex. The pattern of prime numbers similar to a "kaleidoscope" of numbers, if we divide primes into 4 groups, twin primes conjecture becomes much simpler. Based on the fundamental theorem of arithmetic and Euclid's proof of endless prime numbers, we have proved there are infinitely many twin primes.

Introduction

Prime numbers[1] are the basic numbers and are crucial important. There are many conjectures concerning primes are challenging mathematicians for hundreds of years and many "advanced mathematics tools" are used to solve them, but they are still unsolved.

I believe that prime numbers are "basic building blocks" of the natural numbers and they must follow some very simple basic rules and do not need "advanced mathematics tools" to solve them. One of the basic rules is the "fundamental theorem of arithmetic" and the "simplest tool" is Euclid's proof of endless prime numbers.

Fundamental theorem of arithmetic:

The crucial importance of prime numbers to number theory and mathematics in general stems from the fundamental theorem of arithmetic[2], which states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.[1] Primes can thus be considered the "basic building blocks" of the natural numbers.

Euclid's proof[3] that the set of prime numbers is endless

The proof works by showing that if we assume that there is a biggest prime number, then there is a contradiction.

We can number all the primes in ascending order, so that $P_1 = 2$, $P_2 = 3$, $P_3 = 5$ and so on. If we assume that there are just **n** primes, then the biggest prime will be labeled $\mathbf{P_n}$. Now we can form the number Q by multiplying together all these primes and adding 1, so

$$\mathbf{Q = (P_1 \times P_2 \times P_3 \times P_4 ... \times P_n) + 1}$$

Now we can see that if we divide Q by any of our n primes there is always a remainder of 1, so Q is not divisible by any of the primes, but we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or Q must be divisible by primes that are larger than P_n.

Our assumption that P_n is the biggest prime has led us to a contradiction, so this assumption must be false, so there is no biggest prime and the set of prime numbers is endless.

Discussions

Twin Prime Conjecture: There are infinitely many twin primes.

A twin prime is a prime number that is either 2 less or 2 more than another prime number — for example, the twin prime pairs (11 and 13; 17 and 19; 41 and 43). In other words, a twin prime is a prime that has a prime gap of two.

Twin primes become increasingly rare as one examines larger ranges, in keeping with the general tendency of gaps between adjacent primes to become larger as the numbers themselves get larger. However, it is a longstanding conjecture that there are infinitely many twin primes. Work of Yitang Zhang[4] in 2013, as well as work by James Maynard, Terence Tao and others, has made substantial progress towards proving this conjecture, but at present it remains unsolved.

If a large number N is not divisible by 3 or any prime which is smaller or equal to N/3, it must be a prime. 1/3 of all numbers that are divisible by 7 can be divisible by 3, 1/3 of all numbers that are divisible by 11 can be divisible by 3 and 1/7 of all numbers that are divisible by 11 can be divisible by 7, 1/3 of all numbers that are divisible by 13 can be divisible 3, 1/7 of all numbers that are divisible by 13 can be divisible by 7, and 1/11 of all numbers that are divisible by 13 can be divisible by 11, so on, so we have terms: 1/3, 1/7x2/3, 1/11x2/3x6/7, 1/13x2/3x6/7x10/11…,

Let N_o represent any odd number, the chance of N_o to be a non-prime is: [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +
(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +
(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +
(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +
(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +
(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) +
(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +
(1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) +
(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +
(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …] ----------Formula 1

Any odd number cannot be divisible by 2 and any odd number with 5 as its last digit is not a prime except 5, and so these primes are omitted.

Let ∑ represent the sum of the infinite terms and Δ=1-∑, according to Euclid's proof[2] that the set of prime numbers is endless. Δ is the chance of any odd number to be a prime. ∑ may be very close to 1 when N is growing to ∞, but is always less than 1. Let Δ=1-∑, when N is approaches ∞, Δ may be very close to 0, but always more than 0 according to Euclid's proof that the set of prime numbers is endless. If Δ is 0, then there is no prime, and we know that is not true.

The sum of first 20 terms = [(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) +
(1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+
(1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) +
(1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) +
(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +
(1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) +
(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +
(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +
(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +

(1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) =[0.333333 + 0.095238 + 0.051948 + 0.039960 +0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.010328 + 0.009069 + 0.008436 + 0.007538 + 0.006543 + 0.005766 + 0.005483 + 0.004910 + 0.004564 + 0.004377 + 0.003989] =0.688872

For the first 20 term: ∑ = 0.688872, Δ=1-∑ =0.311128

The chance of N_o to be a prime is: Δ=1-[(1/3) + (1/7x2/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …] ------------Formula 2

Let $1 represent a prime with 1 as its last digit, such as 11, 31, 41, 61, 71, 101, 131, 151, 181, 191,…; let $3 represent a prime with 3 as its last digit, such as 3, 13, 23, 43, 53, 73, 83, 103, 113, 163, 193….; let $7 represent a prime with 7 as its last digit, such as, 7, 17, 37, 47, 67, 97, 107, 127, 137, 157, 167, 197…; and let $9 represent a prime with 9 as its last digit, such as 19, 29, 59, 79, 89, 109, 139, 149, 179, 199,….

Let O1 represent an odd number with 1 as its last digit, such as 11, 21, 31, 41, 51, 61, 71,…; let O3 represent an odd number with 3 as its last digit, such as 3, 13, 23, 33, 43, 53, 63, 73,….; let O7 represent an odd number with 7 as its last digit, such as, 7, 17, 27, 37, 47, 57, 67, 77…; and let O9 represent an odd number with 9 as its last digit, such as 9, 19, 29, 39, 49, 59, 69, 79,….

The fundamental theorem of arithmetic states that every integer larger than 1 can be written as a product of one or more primes in a way that is unique except for the order of the prime factors.

Every odd number (O1) with 1 as its last digit is a product of unlimited terms, such as $1x$1, $3 x $7, $9 x $9, $1 x $1 x $1, …, $1 x $3 x $7,…, $3 x $3 x $3 x $3,…., $7x$7x$7x$7…, but we can only consider $1, $7, and $9 because they decide how large other $1s, $3s, $7s, and $9s can be. Let the number 600 be the example. For $1x$1, the smallest $1 is 11 which means that another $1 cannot be larger than 41 (11 x41=451<600, but 11x61=671>600 and 11 x 11 x 11=1331>600); for $3x$7, the smallest $7 is 7 which means that $3 cannot be more than 83 (7x83=581<600, 7x3x31=651>600),…; for $9x$9, the smallest $9 is 9 (3x3) which means that another $9 cannot be more than 59 (3x3x59=531<600), so the smallest $1, $7, and $9 decide the largest possible $1, $3, $7, and $9 for any O1 and the largest possible $1, $3, and $9 determine the chance of O1 being a prime

The chance of any odd number O1 to be a prime is: $\Delta_1=1-\sum_1=1-$[(1/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) ++(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/61x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59) + (1/71x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …] ------------Formula 3.

For N=600, the number of primes with 1 as its last digit=600/10 –600/10 [(1/3) + (1/11x2/3x6/7) + (1/13x2/3x6/7x10/11) + (1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/31x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29) + (1/41x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/83x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73x78/79)]=60-60[0.333333 + 0.051948 + 0.039960 + 0.023753 + 0.018590 + 0.014102 + 0.012738 + 0.009069 + 0.008436 + 0.006543 + 0.005766 + 0.004377 + 0.003597] =60-60x0.532212=28. There are 25 primes with 1 as its last digit, if we count 1. Thus, the difference between real number and the calculated number is only 2 (please see the next 5 primes: 601, 607, 613, 617, and 619, 601 just after 600). The distribution of primes is not uniform and 600 is not a big number, so the difference is reasonable. When the number N becomes larger, the difference will be ≤ 1.

Every odd number with 3 as its last digit is a product of unlimited terms, such as, $3x$1, $7x$9, $3 x $1 x $1, …, $7 x $3 x $3,…, $1 x $7 x $9,…. but we can only consider $1 and $9. Let the number 600 be the example. For $1x$3, the smallest $1 is 11 which means that $3 cannot be more than 53 (11 x53=483<600, but 11x11x13=1573>600 and 3x3x3x3x13=1053>600); for $7x$9, the smallest $9 is 9(3x3) which means $7 cannot be more than 47 (3x3x47=423<600, but 3x3x67=603>600, 19x37=703>600),…; thus, the smallest $1 and $9 decide the largest possible $1, $3, $7, and $9 for any O3 and the largest possible $3, and $7 determine the chance of O3 being a prime

The chance of any odd number O3 being a prime is: Δ_3=1-$\sum_3$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …] -------Formula 4

For N=600, the number of primes with 3 as their last digit=600/10 -600/10 [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) =60-60[0.333333 + 0.095238 + 0.039960 +0.028207 + 0.018590 + 0.010328 + 0.008436 + 0.007538 + 0.006543 + 0.004910 + 0.004377] =60-60x0.55746=26.6. There are 26 primes with 3 as their last digit. Hence the difference between the actual number and the calculated number is 0.6, which is ≤1.

Every odd number with 7 as its last digit is a product of unlimited terms, such as $7x$1, $3x$9, $3x$9x$1, $7x$1x$1,…, $3x$3x$3,…, $1x$3 x $9,…. but we can only consider $1 and $9. Let the number 600 be the example. For $1x$7, the smallest $1 is 11 which means that $7 cannot be more than 47 (11x47=517<600, but 11x11x7=847>600, 3x3x3x31=837>600, and 3x19x11=627>600); for $3x$9, the smallest $9 is 9(3x3) which means $3 cannot be more than 53 (3x3x53=477<600, but 7x3x3x19=1197>600),…; thus, the smallest $1 and $9 decide the largest possible $1, $3, $7, and $9 for any O7 and the largest possible $3, and $7 determine the chance of O7 being a prime

The chance of any odd number O7 to be a prime is: Δ_7=1-$\sum_7$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …] -------Formula 4

For N=600, the number of primes with 7 as its last digit=600/10 -600/10 [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +

(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) =60-60[0.333333 + 0.095238 + 0.039960 +0.028207 + 0.018590 + 0.010328 + 0.008436 + 0.007538 + 0.006543] =60-60x0.548173=27.1. There are 28 primes with 7 as their last digit. Thus, the difference between the actual number and the calculated number is 0.9, which is ≤1.

Every odd number (O9) with 9 as their last digit is a product of unlimited terms, such as $1x$9, $7 x $7, $3 x $3, $1 x $1 x $9, …, $1 x $7 x $7,…, $3 x $3 x $1,…., $3x$3x$3x$7…, but we can only consider $1, $7, and $3 because they decide how large other $1s, $3s, $7s, and $9s can be. Let the number 600 be the example. For $1x$9, the smallest $1 is 11 which means that another $1 cannot be more than 29 (11x29=319<600, but 11x59=649>600 and 11x11x 3x3=1089>600); for $7x$7, the smallest $7 is 7 which means another $7 cannot be more than 67 (7x67=469<600, but 7x97=679>600),…; for $3x$3, the smallest $3 is 3 which means that $9 cannot be more than 193 (3x193=579<600). And so, the smallest $9, $7, and $3 decide the largest possible $1, $3, $7, and $9 for any O9 and so the largest possible $3, $7, and $9 determine the chance of O9 being a prime

The chance of any odd number O9 being a prime is: Δ_9=1-$\sum_9$=1[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …]------------Formula 5.

For N=600, the number of primes with 9 as their last digit=600/10 –600/10[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/5

3x58/59x60/61x66/67x70/71) + (1/83x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73) + (1/103x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101) + (1/113x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107) + (1/163x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107x112/113x126/127x130/131x136/137x150/151x156/157) + (1/193x2/3x6/7x10/11x12/13x16/17x22/23x30/31x36/37x40/41x42/43x46/47x52/53x60/61x66/67x70/71x72/73x82/83x96/97x100/101x106/107x112/113x126/127x130/131x136/137x150/151x156/157x162/163x166/167x172/173x180/181x190/191)] =60-60[0.333333 + 0.095238 + 0.039960 +0.028207 + 0.023753 + 0.018590 + 0.014102 + 0.010328 + 0.008436 + 0.007538 + 0.006543 + 0.004910 + 0.004377 + 0.004222 + 0.003294 + 0.002974 + 0.001972 + 0.001618] =60-60x0.609395=23.4. There are 25 primes with 9 as their last digit, and so the difference is 1.6 (please see the next 5 primes: 601, 607, 613, 617, and 619, 619 is farther behind 600 than other primes with 1, 3, or 7 as its last digit). When N becomes larger, the difference will be ≤1.

The results above show there are almost an equal number of \$1s, \$3s, \$7s, and \$9s. ~1/4 of all primes are \$1, ~1/4 of all primes are \$3, ~1/4 of all primes are \$7, and ~1/4 of all primes are \$9. As Euclid proved primes are infinite, and an infinite number x1/4 is still infinite, \$1, \$3, \$7, and \$9 are consequently infinite. We can also prove \$1, \$3, \$7, and \$9 are infinite with Euclid's proof: We can number all the primes in ascending order (excluding 2 and 5), so that $P_{11} = 11$, $P_{12} = 31$, $P_{13} = 41$, $P_{31} = 3$, $P_{32} = 13$, $P_{33} = 23$, $P_{71} = 7$, $P_{72} = 27$, $P_{73} = 47$, $P_{91} = 19$, $P_{92} = 29$, $P_{93} = 59$, and so on. If we assume that there are just **n** primes with 1 as their last digit, **n** primes with 3 as their last digit, **n** primes with 7 as their last digit, and **n** primes with 9 as their last digit, then the largest primes with 1, 3, 7 or 9 as their last digit will be labeled $\mathbf{P_{1n}}$, $\mathbf{P_{3n}}$, $\mathbf{P_{7n}}$, and $\mathbf{P_{9n}}$. Now we can form the number Q by multiplying all of these primes together and adding 10, the difference between primes with 1, 3, 7, or 9 as their last digit is at least 10, if $\mathbf{(P_{11} \times P_{31} \times P_{71} \times P_{91}... \times P_{1n} \times P_{3n} \times P_{7n} \times P_{9n})}$ is a number with 1 as its last digit, Q +10 is also with 1 as its last digit, if $\mathbf{(P_{11} \times P_{31} \times P_{71} \times P_{91}... \times P_{1n} \times P_{3n} \times P_{7n} \times P_{9n})}$ is a number with 3 as its last digit, Q + 10 is also with 3 as its last digit, if $\mathbf{(P_{11} \times P_{31} \times P_{71} \times P_{91}... \times P_{1n} \times P_{3n} \times P_{7n} \times P_{9n})}$ is a number with 7 as its last digit, Q + 10 is also with 7 as its last digit, or if $\mathbf{(P_{11} \times P_{31} \times P_{71} \times P_{91}... \times P_{1n} \times P_{3n} \times P_{7n} \times P_{9n})}$ is a number with 9 as its last digit, Q + 10 is also with 9 as its last digit, for

$$\mathbf{Q = (P_{11} \times P_{31} \times P_{71} \times P_{91}... \times P_{1n} \times P_{3n} \times P_{7n} \times P_{9n}) + 10}$$

Now we can see that if we divide Q by any of our 4n primes there is always a remainder of 10, and so Q is not divisible by any of the primes. However, we know that all positive integers are either primes or can be decomposed into a product of primes. This means that either Q must be a prime or if Q is a number with 1 as its last digit, Q must be divisible by a prime that is larger than $\mathbf{P_{1n}}$, if Q is a number with 3 as its last digit, Q must be divisible by a prime that is larger than $\mathbf{P_{3n}}$, if Q is a number with 7 as its last digit, Q must be divisible by a prime that is larger than $\mathbf{P_{7n}}$, or if Q is a number with 9 as its last digit, Q must be divisible by a prime that is larger than $\mathbf{P_{9n}}$, thus our assumption that $\mathbf{P_{1n}}$, $\mathbf{P_{3n}}$, $\mathbf{P_{7n}}$, or $\mathbf{P_{9n}}$ are the largest prime numbers with 1, 3, 7, or 9 as their last digit has led us to a contradiction. Therefore, this assumption must be false, and so there is no largest prime number with 1, 3, 7, or 9 as its last digit and the set of prime numbers with 1, 3, 7, or 9 as their last digit is endless.

The chance for O3 following $1, to be a non-prime is: $\sum_3$=[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]

The chance for O3 following $1, to be a prime is: Δ_3=1-$\sum_3$=1-[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …] -------Formula 4

Let assume the pair, \$$1_x$,\$$3_x$ is the largest twin pair, however we know both $1 and $3 are endless, there are \$$1_{x+1}$, \$$1_{x+2}$, \$$1_{x+3}$,… \$$1_{x+n}$, and \$$3_{x+1}$, \$$3_{x+2}$, \$$3_{x+3}$,… \$$3_{x+n}$. We need only find if there is at least 1 of \$$3_{x+n}$ follow 1 of \$$1_{x+n}$. Any $1 can be divisible by only 1 and itself, so \$$1_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 14, 16, or 18) cannot be divisible by \$$1_{x+n}$ because the smallest $1 is 11 and the next $1 is 31.

The chance for O3 following a prime, \$$1_x$, to be a prime is: Δ_{x3}=1-$\sum_{x3}$=1-{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +

(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]- 1/$\$1_{x+1}$}

The O3 next to $\$1_{x+1}$ is only 2 (also is true for 4, 6, 8, 10, 12, 14, 16, or 18 difference) different from $\$1_{x+1}$, so the term is a single term 1/$\$1_{x+1}$. For limited n primes, $\$1_{x+n}$, we will have:

Δ_{x3}=1-$\sum_{x3}$=n-n{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]- [1/$\$1_{x+1}$+ 1/$\$1_{x+2}$+ 1/$\$1_{x+3}$,…+ 1/$\$1_{x+n}$]}

If the number n ≥ $\$1_{x+1}$ of primes $\$1_{x+1}$, $\$1_{x+2}$, $\$1_{x+3}$,… $\$1_{x+n}$, then: n(1/$1_{x+1}$+ 1/$\$1_{x+2}$+ 1/$\$1_{x+3}$,…+ 1/$\$1_{x+n}$)= ($\$1_{x+1}$/$\$1_{x+1}$ + $\$1_{x+1}$/$\$1_{x+2}$ + $\$1_{x+1}$/$\$1_{x+3}$,…. $\$1_{x+1}$/$\$1_{x+n}$)= (1+ $\$1_{x+1}$/$\$1_{x+2}$ + $\$1_{x+1}$/$\$1_{x+3}$,…. $\$1_{x+1}$/$\$1_{x+n}$)>1, so there is at least 1 more $\$3_{x+n}$ following one of $\$1_{x+n}$ for every n primes of $\$1_{x+n}$. If $\$1_{x+1}$ is 101, 101 primes $\$1_{x+1, \dots,}$$\$1_{x+n}$ after 101 will match at least 1 prime which is $\$1_{x+n}$ plus 2 (or plus 4, ,6 8, 10,…), our assumption that $\$1_x$,$\$3_x$ is the biggest twin prime pair has led us to a contradiction, so this assumption must be false, there is no biggest prime $\$1_x$ +2 and the twin prime numbers ($1, $1+2) is endless, in fact, ($1, $1+2) will occur much more often. According to the prime number theorem, we will have n/ln(n) twin pairs ($1, $1+2) for every n $1s, such as from 100-1000, there are 31 primes ($1) with 1 as their last digit, thus, there are 31/ln31=31/3.434=9.1 $\$3_{x+n}$, the number is same to the real number 9: (101,103), (191,193), (281,283), (311, 313), (431, 433), (461, 462), (521, 523), (821, 823), and (881, 883).

The chance for O9 following a non-prime, $7, to be a non-prime is: $\sum_9$= [(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +
(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +
(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +
(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …]

The chance for O9 following a prime, \$7, to be a prime is: $\Delta_9=1-\sum_9=1-$[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) +
(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +
(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +
(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +
(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +
(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +
(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) +
(1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …]

Let assume the pair, $\$7_x$,$\$9_x$ is the largest twin pair, however we know both \$7 and \$9 are endless, there are $\$7_{x+1}$, $\$7_{x+2}$, $\$7_{x+3}$,… $\$7_{x+n}$, and $\$9_{x+1}$, $\$9_{x+2}$, $\$9_{x+3}$,… $\$9_{x+n}$. We need only find if there is at least 1 of $\$9_{x+n}$ follow 1 of $\$7_{x+n}$. Any \$7 can be divisible by only 1 and itself, so $\$7_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 16, or 18) cannot be divisible by $\$7_{x+n}$ because the smallest \$7 is 7 and the next \$7 is 17.

The chance for O9 following a prime, $\$7_x$, to be a prime is: $\Delta_{x9}=1-\sum_{x9}=1-${[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) +
(1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) +
(1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) +
(1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) +
(1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +
(1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) +
(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …]- 1/$\$1_{x+1}$}

The O9 next to $\$7_{x+1}$ is only 2 (also is true for 4, 6, 8, 10, 12, 16, or 18 difference) different from $\$7_{x+1}$, so the term is a single term 1/$\$7_{x+1}$. For limited n primes, $\$7_{x+n}$, we will have:

Δ_{x9}=1-$\sum_{x9}$=n-n{[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) +(1/19x2/3x6/7x10/11x12/13x16/17)+ (1/23x2/3x6/7x10/11x12/13x16/17x18/19) +(1/29x2/3x6/7x10/11x12/13x16/17x18/19x22/23) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/59x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + (1/79x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71x72/73) + …] -[1/$\$7_{x+1}$+ 1/$\$7_{x+2}$+ 1/$\$7_{x+3}$,…+ 1/$\$7_{x+n}$]}

If the number n ≥ $\$7_{x+1}$ of primes $\$7_{x+1}$, $\$7_{x+2}$, $\$7_{x+3}$,… $\$7_{x+n}$, then: n(1/$7_{x+1}$+ 1/$\$7_{x+2}$+ 1/$\$7_{x+3}$,…+ 1/$\$7_{x+n}$)= ($\$7_{x+1}$/$\$7_{x+1}$ + $\$7_{x+1}$/$\$7_{x+2}$ + $\$7_{x+1}$/$\$7_{x+3}$,…. $\$7_{x+1}$/$\$7_{x+n}$)= (1+ $\$7_{x+1}$/$\$7_{x+2}$ + $\$7_{x+1}$/$\$7_{x+3}$,…. $\$7_{x+1}$/$\$7_{x+n}$)>1, so there is at least 1 more $\$9_{x+n}$ following one of $\$7_{x+n}$ for every n primes of $\$7_{x+n}$. If $\$7_{x+1}$ is 107, 107 primes $\$7_{x+1,\ldots,}$$\$7_{x+n}$ after 107 will match at least 1 prime which is $\$7_{x+n}$ plus 2 (or plus 4, ,6 8, 10,…), our assumption that $\$7_x$,$\$9_x$ is the biggest twin prime pair has led us to a contradiction and this assumption must be false, so there is no biggest prime $\$7_x$ +2 and the twin prime numbers ($7, $7+2) is endless.

The chance for O1 following $9, to be a non-prime is: $\sum_1$=[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) +

(1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]

The chance for O1 following $9, to be a prime is: $\Delta_1=1-\sum_1=1-$[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]

Let assume the pair, $\$9_{x}$,$\$1_{x}$ is the largest twin pair, however we know both $9 and $1 are endless, there are $\$9_{x+1}$, $\$9_{x+2}$, $\$9_{x+3}$,… $\$9_{x+n}$, and $\$1_{x+1}$, $\$1_{x+2}$, $\$1_{x+3}$,… $\$1_{x+n}$. We need only find if there is at least 1 of $\$1_{x+n}$ follow 1 of $\$9_{x+n}$. Any $9 can be divisible by only 1 and itself, so $\$9_{x+n}$ plus 2 (also plus 4, 6, 8, 10, 12, 14, 16, or 18) cannot be divisible by $\$9_{x+n}$ because the smallest $9 is 19 and the next $9 is 29.

The chance for O1 following a prime, $\$9_{x}$, to be a prime is: $\Delta_{x1}=1-\sum_{x1}=1-${[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) + (1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …]- 1/$\$1_{x+1}$}

The O1 next to $\$9_{x+1}$ is only 2 (also is true for 4, 6, 8, 10, 12, 14, 16, or 18 difference) different from $\$9_{x+1}$, so the term is a single term 1/$\$9_{x+1}$. For limited n primes, $\$9_{x+n}$, we will have:

$\Delta_{x1}=1-\sum_{x1}=n-n${[(1/3) + (1/7x2/3) + (1/13x2/3x6/7x10/11) + (1/17x2/3x6/7x10/11x12/13) + (1/23x2/3x6/7x10/11x12/13x16/17x18/19) + (1/37x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31) + (1/43x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41) + (1/47x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43) + (1/53x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47) +

(1/67x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61) + (1/73x2/3x6/7x10/11x12/13x16/17x18/19x22/23x28/29x30/31x36/37x40/41x42/43x46/47x52/53x58/59x60/61x66/67x70/71) + …] - [1/$\$9_{x+1}$+ 1/$\$9_{x+2}$+ 1/$\$9_{x+3}$,…+ 1/$\$9_{x+n}$]}

If the number n ≥ $\$9_{x+1}$ of primes $\$9_{x+1}$, $\$9_{x+2}$, $\$9_{x+3}$,… $\$9_{x+n}$, then: n(1/$9_{x+1}$+ 1/$\$9_{x+2}$+ 1/$\$9_{x+3}$,…+ 1/$\$9_{x+n}$)= ($\$9_{x+1}$/$\$9_{x+1}$ + $\$9_{x+1}$/$\$9_{x+2}$ + $\$9_{x+1}$/$\$9_{x+3}$,…. $\$9_{x+1}$/$\$9_{x+n}$)= (1+ $\$9_{x+1}$/$\$9_{x+2}$ + $\$9_{x+1}$/$\$9_{x+3}$,…. $\$9_{x+1}$/$\$9_{x+n}$)>1, so there is at least 1 more $\$1_{x+n}$ following one of $\$9_{x+n}$ for every n primes of $\$9_{x+n}$. If $\$9_{x+1}$ is 109, 109 primes $\$9_{x+1, \ldots,}$$\$9_{x+n}$ after 109 will match at least 1 prime which is $\$9_{x+n}$ plus 2 (or plus 4, ,6 8, 10,…), our assumption that $\$9_x$,$\$1_x$ is the largest twin pair has led us to a contradiction, so this assumption must be false, so there is no biggest prime $\$1_x$ +2 and the twin prime numbers ($9, $9+2) is endless.

It is easy to prove ($3, $7, p+4 prime), ($7, $1, p+4 prime), ($1, $7, sexy prime), ($7, $3), ($3, $9), (p, p+8), (p, p+10), (p, p+12), (p, p+14), (p, p+16), and (p, p+18) are endless.

References:

1. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Theorem 2 (https://en.wikipedia.org/wiki/Prime_number).
2. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., Section 2, Lemma 5 (https://en.wikipedia.org/wiki/Prime_number).
3. Dudley, Underwood (1978), Elementary number theory (2nd ed.), W. H. Freeman and Co., p. 10, section 2. (https://en.wikipedia.org/wiki/Prime_number).
4. Zhang, Yitang (2014). "Bounded gaps between primes".Annals of Mathematics. 179 (3): 1121–1174