Physical Interpretation of the 30 8-simplexes in the E8 240-Polytope:

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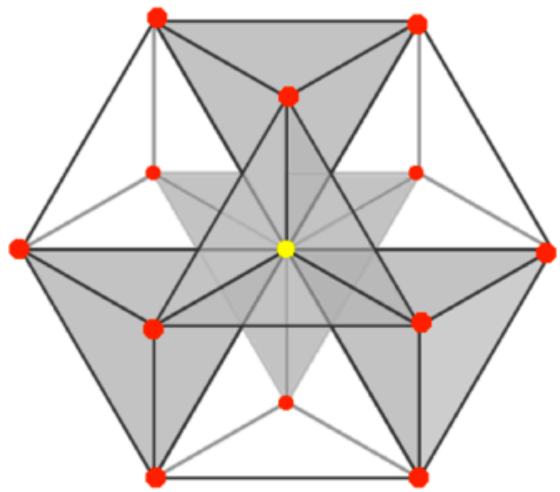
248-dim Lie Group E8 has 240 Root Vectors arranged on a 7-sphere S7 in 8-dim space.

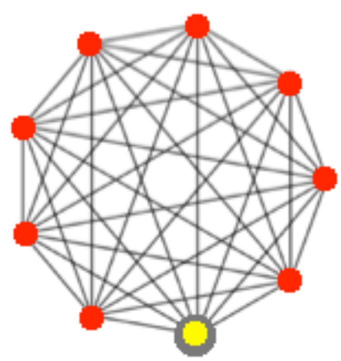
The 12 vertices of a cuboctahedron live on a 2-sphere S2 in 3-dim space.



They are also the 4x3 = 12 outer vertices of 4 tetrahedra (3-simplexes) that share one inner vertex at the center of the cuboctahedron.

This paper explores how the 240 vertices of the E8 Polytope in 8-dim space are related to the 30x8 = 240 outer vertices (red in figure below) of 30 8-simplexes whose 9th vertex is a shared inner vertex (yellow in figure below) at the center of the E8 Polytope.

The 8-simplex has 9 vertices, 36 edges, 84 triangles, 126 tetrahedron cells,
126 4-simplex faces, 84 5-simplex faces, 36 6-simplex faces, 9 7-simplex faces,
and 1 8-dim volume



The real 4_21 Witting polytope of the E8 lattice in R8 has

240 vertices;
6,720 edges;
60,480 triangular faces;
241,920 tetrahedra;
483,840 4-simplexes;
483,840 5-simplexes 4_00;
138,240 + 69,120 6-simplexes 4_10 and 4_01; and
17,280 = 2,160x8 7-simplexes 4_20 and 2,160 7-cross-polytopes 4_11.

The cuboctahedron corresponds by Jitterbug Transformation to the icosahedron.
The 20 2-dim faces of an icosahedon in 3-dim space

(image from spacesymmetrystructure.wordpress.com)

Twenty tetrahedra *almost* fitting together to make an icosahedron

are also the 20 outer faces of 20 not-exactly-regular-in-3-dim tetrahedra (3-simplexes) that share one inner vertex at the center of the icosahedron,
but that correspondence does not extend to the case of 8-simplexes in an E8 polytope, whose faces are both 7-simplexes and 7-cross-polytopes, similar to the cubocahedron, but not its Jitterbug-transform icosahedron with only triangle = 2-simplex faces.

However, since the E8 lattice in R8 has a counterpart in C4, the self-reciprocal honeycomb of Witting polytopes, a lattice of all points whose 4 coordinates are Eisenstein integers with the equivalent congruences

u1 + u2 + u3 = u2 - u3 + u4 = 0 (mod i sqrt(3)) and
u3 - u2 = u1 - u3 = u2 - u1 = u4 (mod i sqrt(3))

all of whose cells are similar, the icosahedron-type correspondence may exist for the self-reciprocal Witting polytope in C4 which has

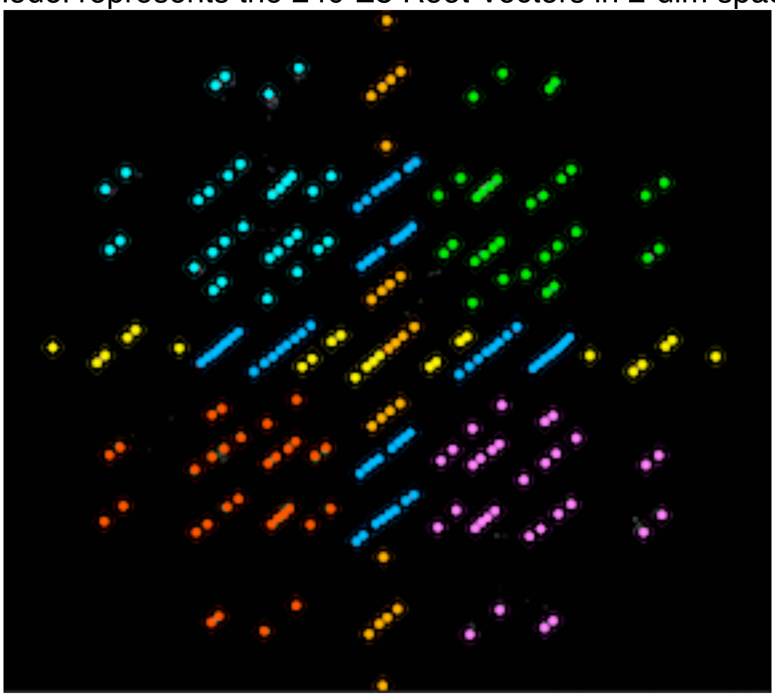
240 vertices, 2,160 edges, 2,160 faces, and 240 cells.

It has 27 edges at each vertex.
Its symmetry group has order 155,520 = 3 x 51,840.
It is 6-symmetric, so its central quotient group has order 25,920.
It has 40 diameters orthogonal to which are 40 hyperplanes of symmetry, each of which contains 72 vertices.
It has a van Oss polygon in C2, its section by a plane joining an edge to the center, that is the 3{4}3 in C2, with 24 vertices and 24 edges.

In 8-dim space it seems to me that the 8x30 outer vertices of 30 8-simplexes sharing a common vertex at the center of an E8 Polytope correspond to 240 vertices of the E8 Polytope, as is the case of 4 tetrahedra and the cuboctahedron.

However,
my E8 Physics model (viXra 1602.0319) is based on a projection to a 2-dim plane, as is the widely used 8 circles of 30 vertices each projection,
so, for the purpose of visualization in practical applications,
it seems useful to try to describe the relations of 30 8-simplexes to the E8 Polytope in terms of those projections to 2-dim.

My E8 Physics model represents the 240 E8 Root Vectors in 2-dim space as



To understand the Geometry related to the 240 E8 Root Vectors, consider that
248-dim E8 = 120-dim Spin(16) D8 + 128-dim half-spinor of Spin(16) D8
and
240 E8 Root Vectors = 112 D8 Root Vectors + 128 D8 half-spinors.

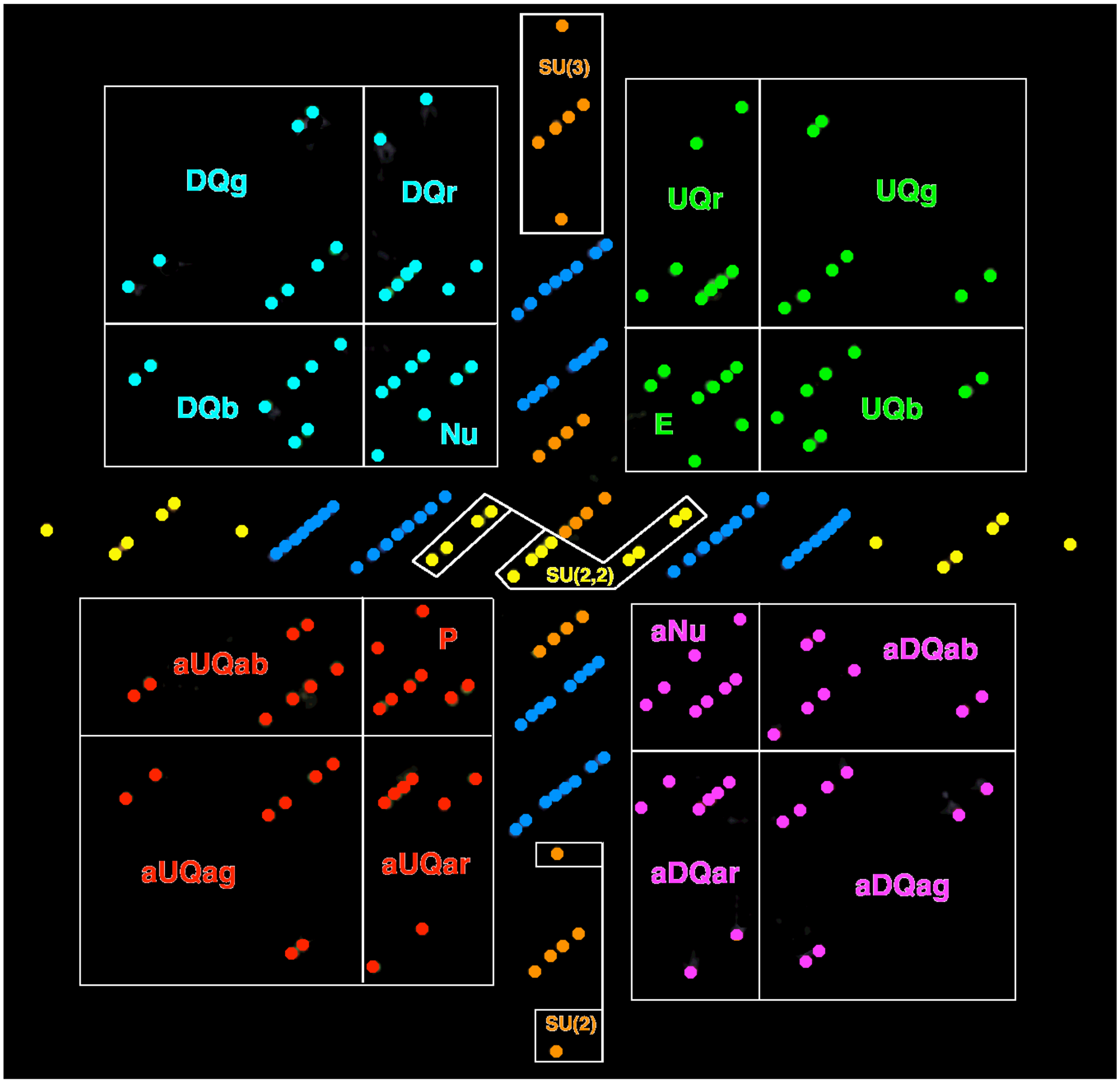
There are two ways to see a maximal symmetric subspace of E8 and E8 Root Vectors:
the symmetric space corresponding to the 128 D8 half-spinors
E8 / D8 = 128-dim Octonion-Octonionic Projective Plane (OxO)P2

and

the symmetric space corresponding to the 112 D8 Root Vectors
E8 / E7 x SU(2) = 112-dim set of (QxO)P2 in OxO)P2
where (QxO)P2 = Quaternion-Octonion Projective Planes

Also, D8 / D4 x D4 = 64-dim Grassmannian Gr(8,16)

Geometric Structure leads to physical interpretation of the E8 Root Vectors as:



E = electron,
UQr = red up quark, UQg = green up quark, UQb = blue up quark
Nu = neutrino,
DQr = red down quark, DQg = green down quark, DQb = blue down quark
P = positron,
aUQar = anti-red up antiquark, aUQag = anti-green up antiquark,
aUQab = anti-blue up antiquark
aNu = antineutrino,
aDQar = anti-red down antiquark, aDQag = anti-green down antiquark,
aDQab = anti-blue down antiquark

Each Lepton and Quark has 8 components with respect to M4 x CP2 Kaluza-Klein
where M4 = 4-dim Minkowski Physical Spacetime
and CP2 = SU(3) / SU(2)xU)(1) = 4-dim Internal Symmetry Space

The 24 orange vertices are Root Vectors of a D4 of D8 / D4xD4 that represents
Standard Model gauge bosons and Ghosts of Gravity. Denote it by D4sm.
6 orange SU(3) and 2 orange SU(2) represent Standard Model root vectors
24-6-2 = 16 orange represent U(2,2) Conformal Gravity Ghosts

The 24 yellow vertices are Root Vectors of a D4 of D8 / D4xD4 that represents
Gravity gauge bosons and Ghosts of the Standard Model. Denote it by D4g.
12 yellow SU(2,2) represent Conformal Gravity SU(2,2) root vectors
24-12 = 12 yellow represent Standard Model Ghosts

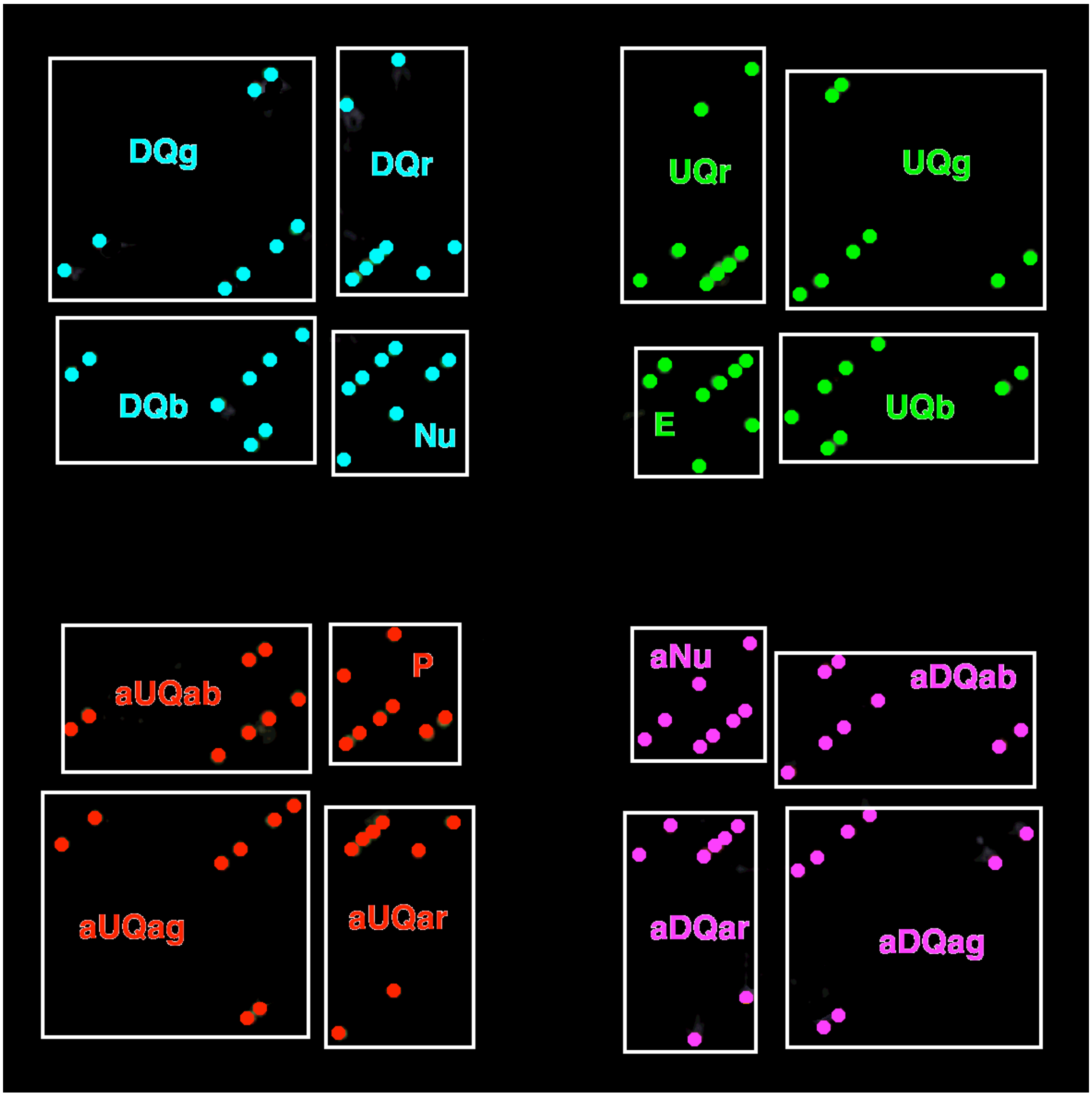
32+32 = 64 blue of D8 / D4 x D4 = 64-dim Grassmannian Gr(8,16)
represent 4+4 dim Kaluza-Klein spacetime position and momentum.

The 240-polytope has 240 vertices and 8-simplex has 9 vertices,
8 of which are outer if the 8-simplexes all share a central vertex.

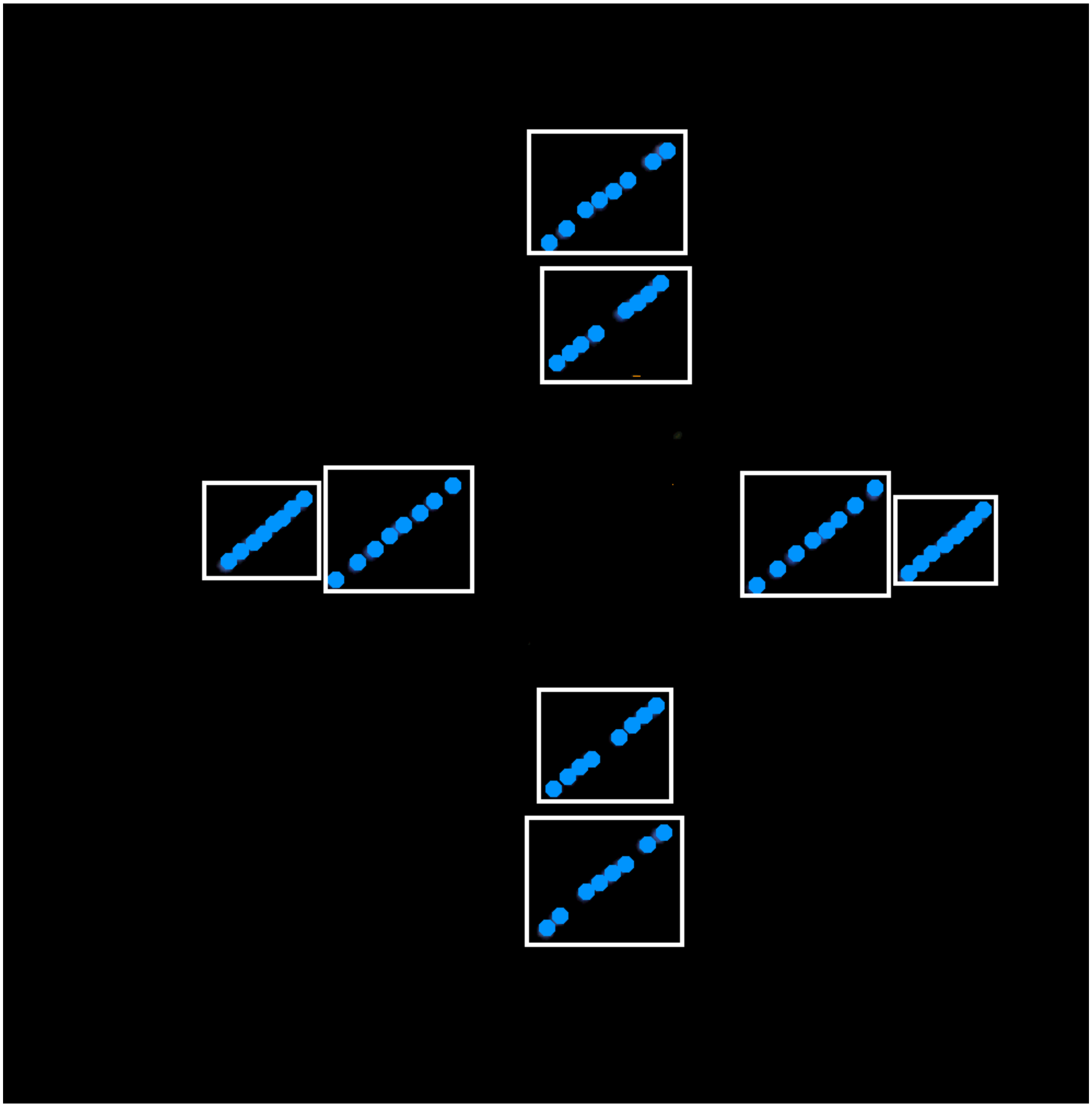
Therefore it takes 240 / 8 = 30 8-simplexes sharing a central vertex
to make up the 240 vertices of the 240-polytope.

30 sets = 16 of Fermions
+ 8 of M4 x CP2 Kaluza-Klein Spacetime
+ 3 of Standard Model and Ghosts of Gravity
+ 3 of Gravity and Ghosts of Standard Model

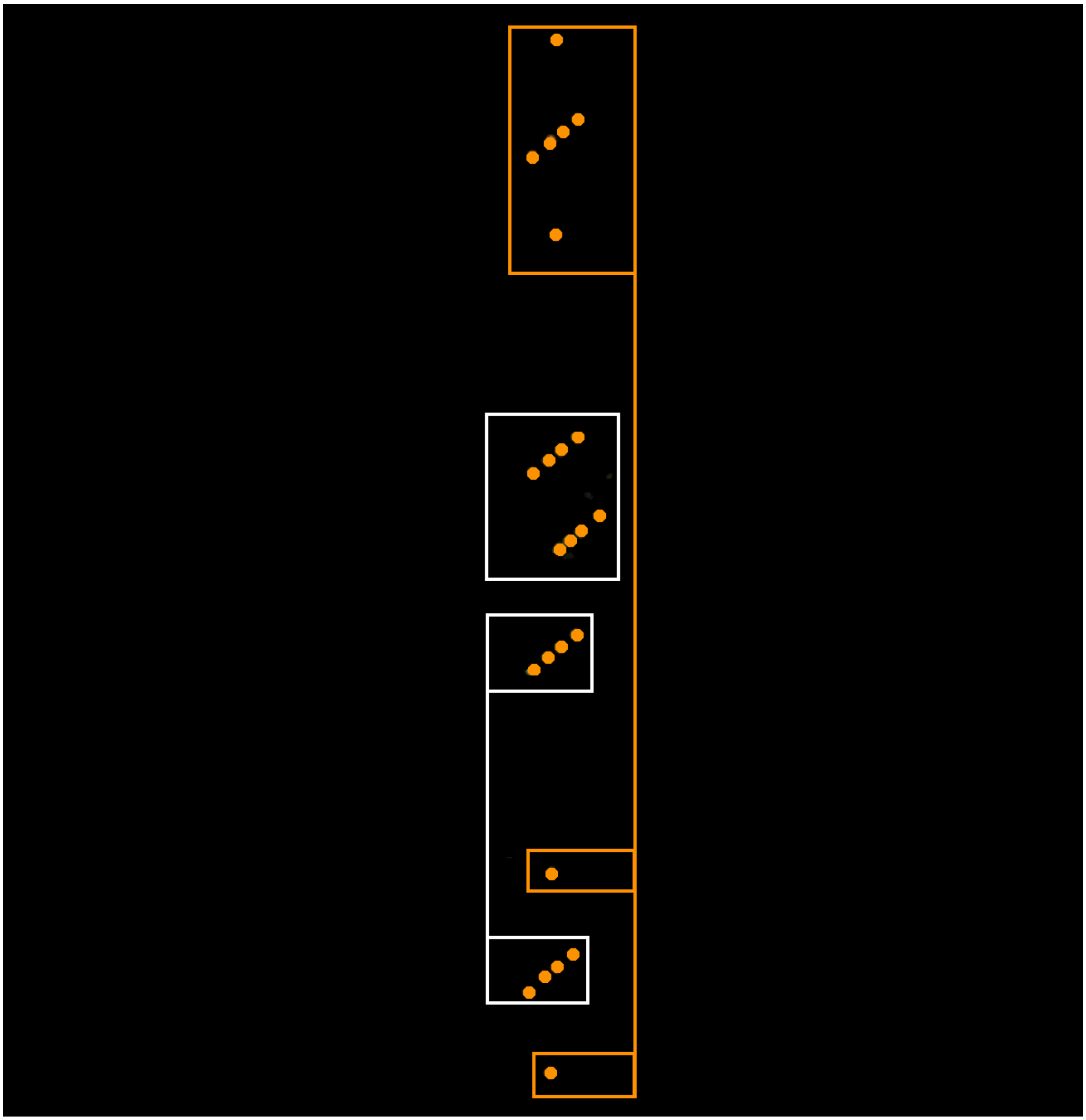
These 16 sets of 8 vertices correspond to the 8 first-generation Fermion Particles (green and cyan) and the 8 first-generation Fermion AntiParticles (red and magenta)



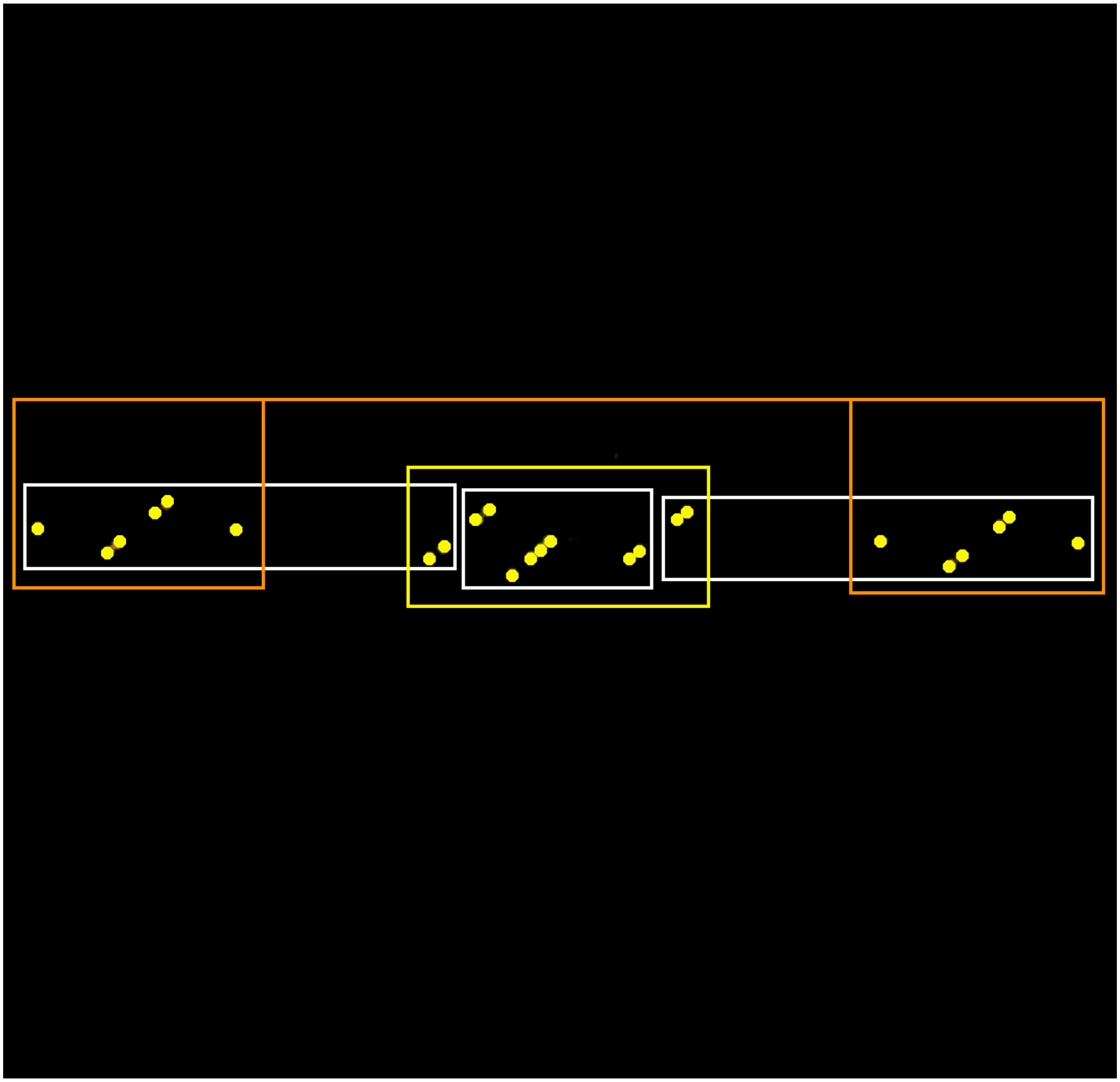
**These 8 sets of 8 vertices correspond to
the 4 dimensions of M4 Minkoski Physical Spacetime
(the 4 horizontal sets)
and
the 4 dimensions of CP2 Internal Symmetry Space
(the 4 vertical sets)**



These 3 sets of 8 vertices correspond to
8 Root Vectors of Standard Model SU(3)xSU(2)xU(1) (orange boxes)
and
Ghosts of the 16-dim Conformal Group U(2,2) of Gravity (white boxes)



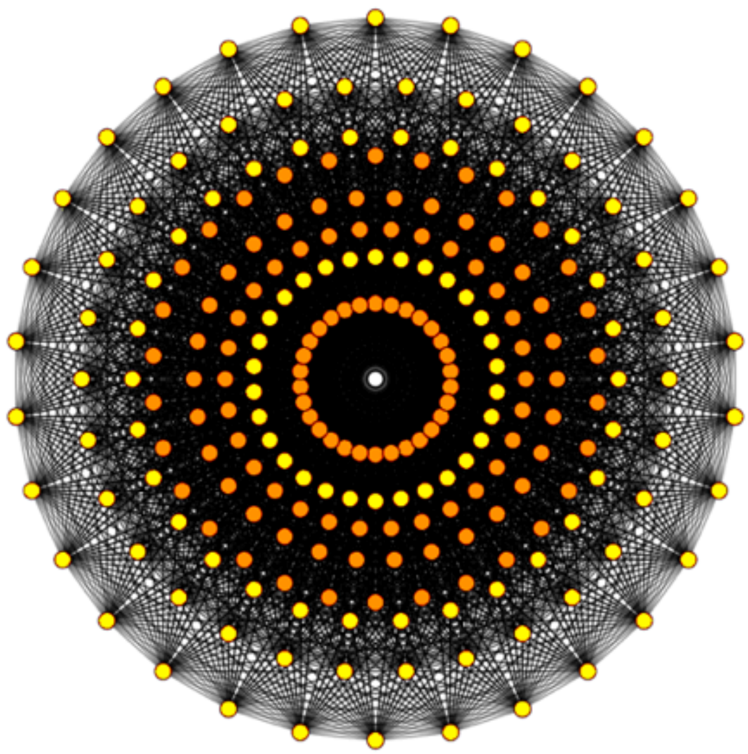
These 3 sets of 8 vertices correspond to
12 Root Vectors of Conformal Group U(2,2) of Gravity (yellow box)
and
Ghosts of the 12-dim Standard Model SU(3)xSU(2)xU(1) (orange boxes)



The 12+12 Physical Interpretation of 24 Root Vectors
does not exactly correspond to their 8+8 +8 decomposition
in terms of 8-simplexes (white boxes)

30 8-simplexes in the E8 240-Polytope can also be seen in terms of 8 Circles of 30 Root Vectors projected into 2-dim:

Consider the 240 Root Vectors of E8, based on 8-dim Octonionic spacetime being seen as 4+4 -dim Quaternionic M4 x CP2 Kaluza-Klein Spacetime:

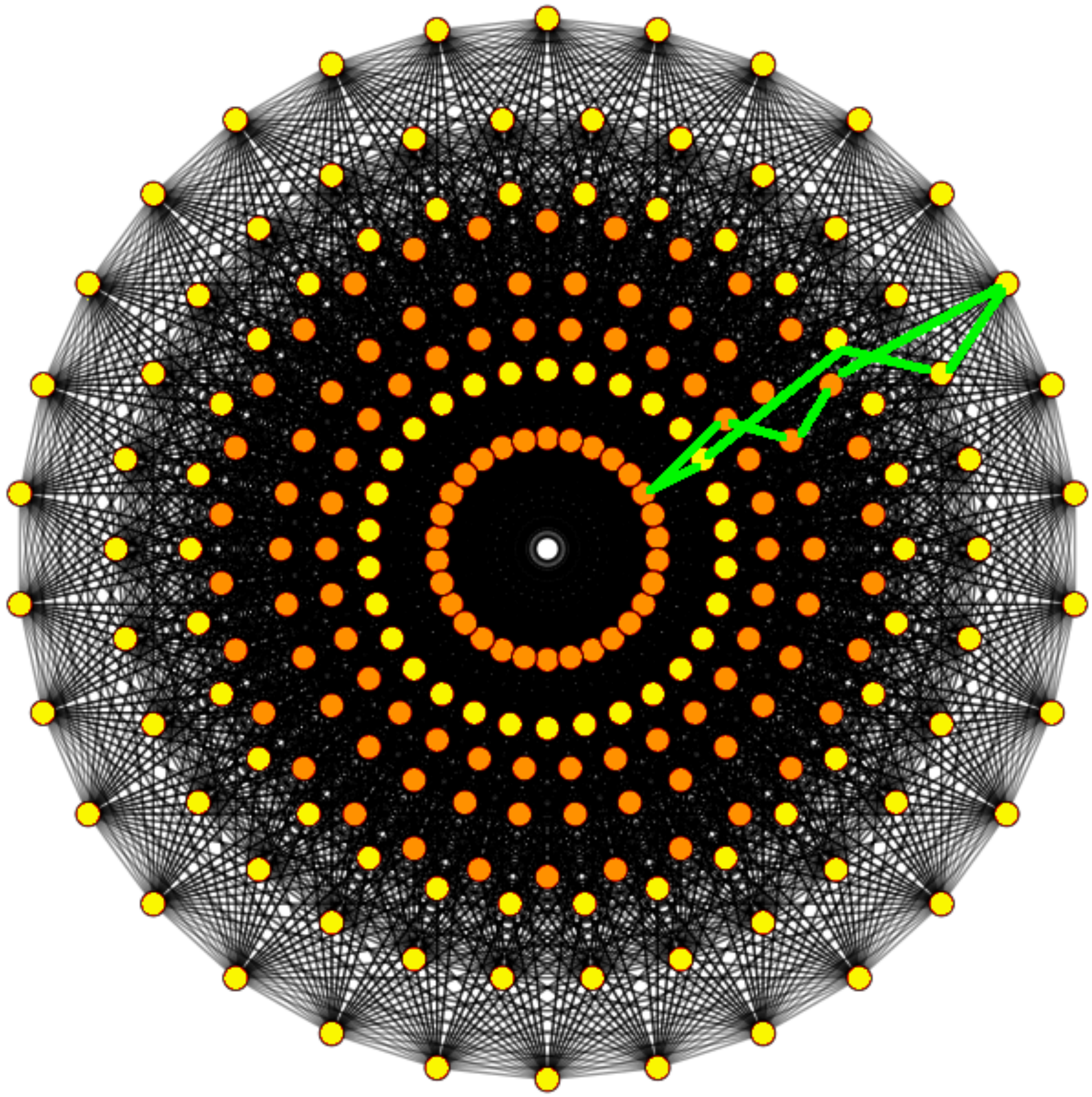


120 of the 240 (yellow dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to M4 Physical Spacetime.

120 of the 240 (orange dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space.

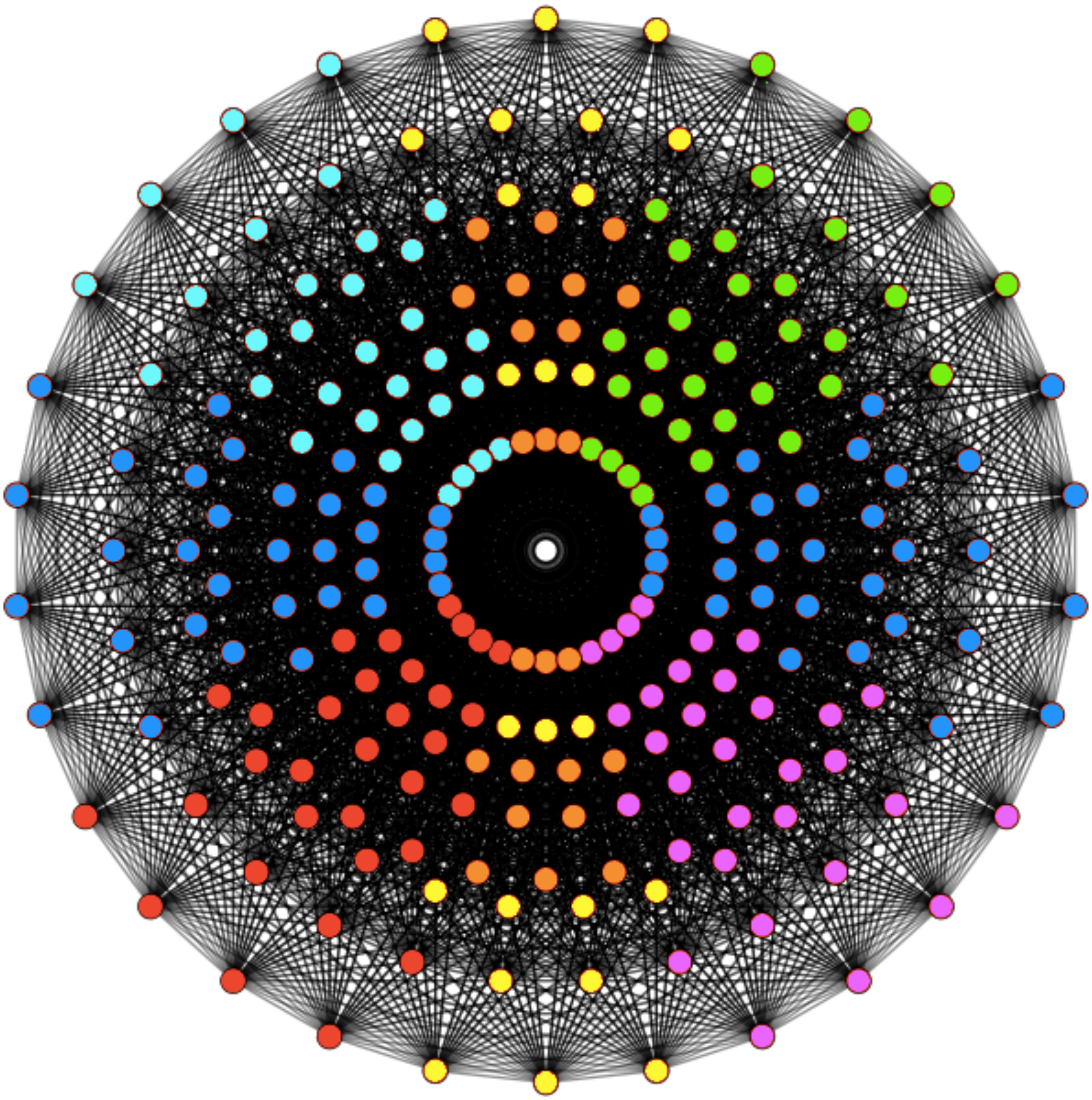
In the above 2-dim projection the M4 120 have larger radii from the center than the CP2 120 by a factor of the Golden Ratio.

If you look at the 240-polytope in the 2-dim projection of 8 circles of 30 vertices each, you can see that each of the 30 8-simplexes has outer vertices like the 8 vertices connected by green lines in the image below.

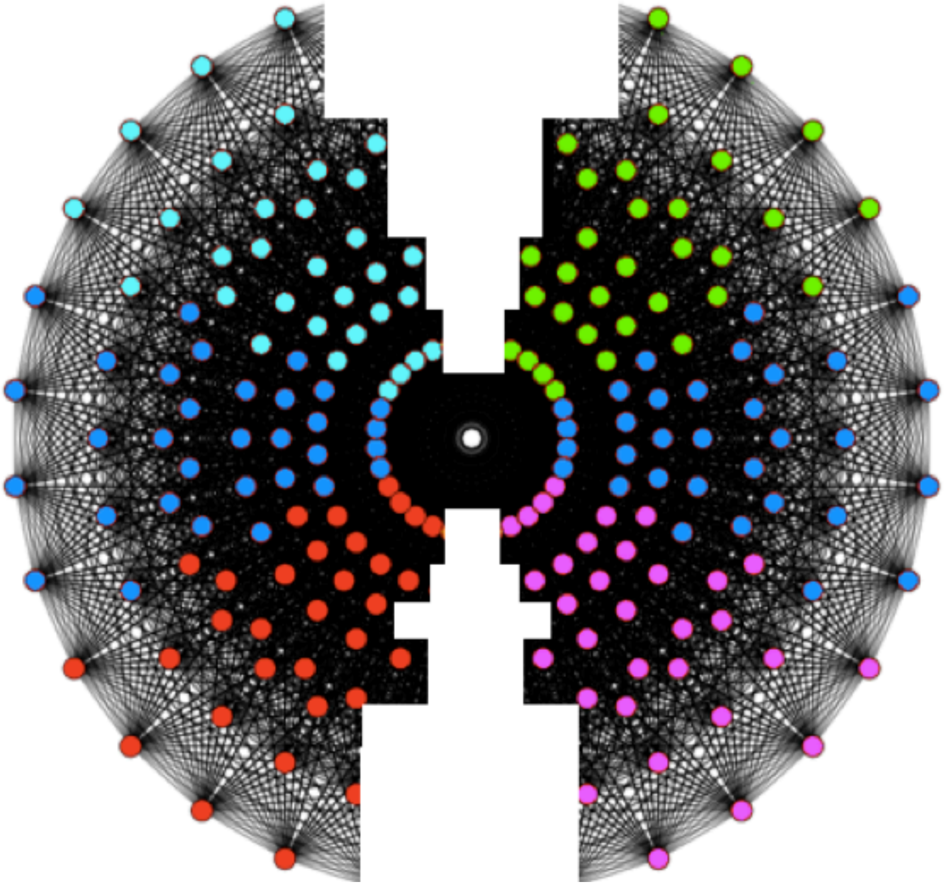


How consistent with E8 Physics is this representation of 240 as 30 x 8 outer vertices of 8-simplexes ?

My E8 Physics model Physical Interpretation of the E8 Root Vectors is:
64 blue = Spacetime
64 green and cyan = Fermion Particles
64 red and magenta = Fermion AntiParticles
24 yellow = D4g Root Vectors = 12 Root Vectors of SU(2,2) Conformal Gravity
+ 12 Ghosts of Standard Model SU(3)xSU(2)xU(1)
24 orange = D4sm Root Vectors = 8 Root Vectors of Standard Model SU(3)xSU(2)xU(1)
+ 16 Ghosts of U(2,2) of Conformal Gravity

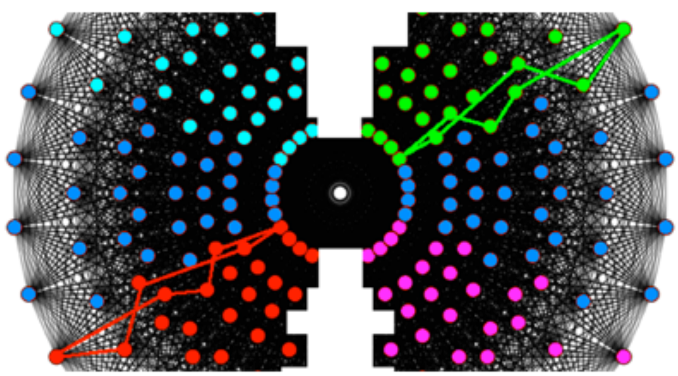


The Spacetime (blue) and Fermion (green, cyan, red, magenta) vertices

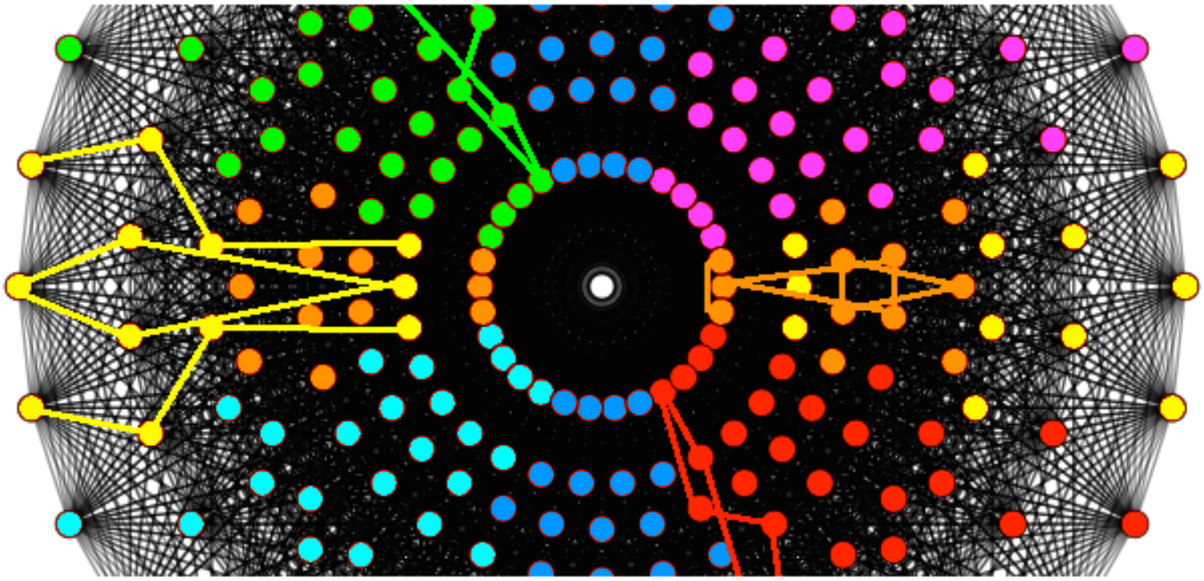


are represented by Outer Vertices of 24 of 30 8-simplexes that share a central vertex
(24 x 8 = 192 = 64 + 32 + 32 + 32 +32)

Each of those 24 sets of 8 Outer Vertices is of the form shown by red or green lines:



As to the other (30 - 24) = 6 = 3+3 sets of 8 E8 Root Vectors,
they fall into two sets of 3x8 = 24 vertices (orange and yellow).

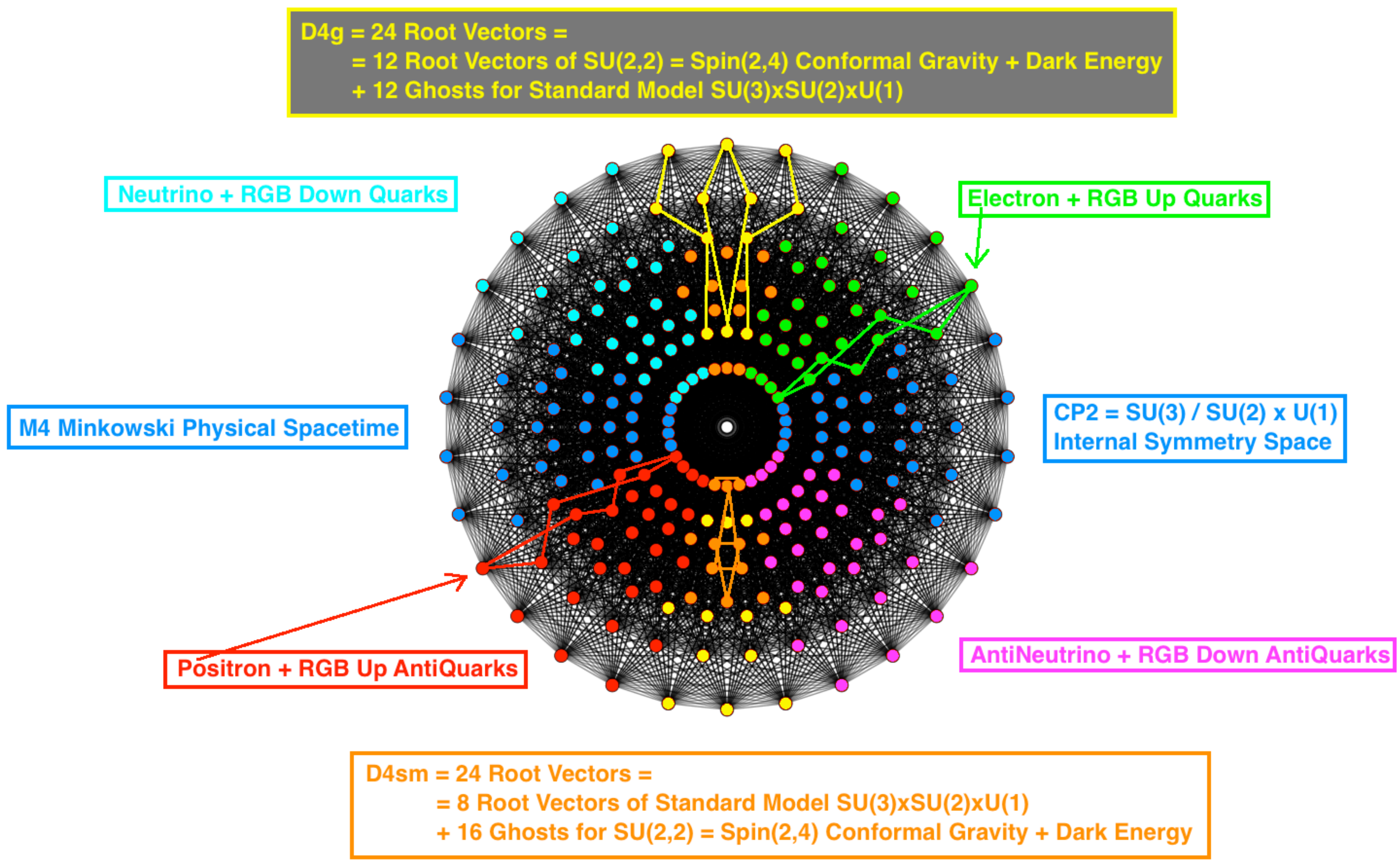


The orange 24 = D4sm Root Vectors are in the CP2 part of the E8 Polytope:
8 Root Vectors of Standard Model SU(3)xSU(2)xU(1)
+ 16 Ghosts of U(2,2) of Conformal Gravity
The 8 Root Vectors of Standard Model SU(3)xSU(2)xU(1)
fall into two sets of Root Vectors indicated by orange lines:
3+3 = 6 Root Vectors of Standard Model SU(3) and
2 Root Vectors of Standard Model SU(2)xU(1)
The 16 Ghosts of U(2,2) of Conformal Gravity fall into 2 sets:
12 for Root Vectors of SU(2,2) = Spin(2,4) of Conformal Gravity and
4 for Cartan Subalgebra elements of U(2,2)

The yellow 24 = D4g Root Vectors are in the M4 part of the E8 Polytope:
12 Root Vectors of Conformal Gravity SU(2,2) = Spin(2,4)
+ 12 Ghosts of Standard Model SU(3)xSU(2)xU(1)
The 12 Root Vectors of Conformal Gravity SU(2,2) = Spin(2,4)
The central 4 are Root Vectors of Lorentz Spin(1,3).
The two sets of 4 are Translations and Special Conformal Transformations.
The 12 Ghosts of Standard Model SU(3)xSU(2)xU(1)
are the other half of the 24

On the following page is a summary of the Physical Interpretation
of the 240 E8 Root Vectors in terms of
the 8 Circles of 30 Root Vectors projected into 2-dim:

Note that deviations from the direct correspondence
between the 240 E8 Polytope vertices and the 30x8 outer vertices of 8-simplexes
have useful Physical Interpretations.


D4g = 24 Root Vectors =
= 12 Root Vectors of SU(2,2) = Spin(2,4) Conformal Gravity + Dark Energy
+ 12 Ghosts for Standard Model SU(3)xSU(2)xU(1)
Neutrino + RGB Down Quarks
Electron + RGB Up Quarks
M4 Minkowski Physical Spacetime
CP2 = SU(3) / SU(2) x U(1)
Internal Symmetry Space
Positron + RGB Up AntiQuarks
AntiNeutrino + RGB Down AntiQuarks
D4sm = 24 Root Vectors =
= 8 Root Vectors of Standard Model SU(3)xSU(2)xU(1)
+ 16 Ghosts for SU(2,2) = Spin(2,4) Conformal Gravity + Dark Energy