Quantum Waves in Nature, a New Interpretation of Quantum Mechanics

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Abstract

We propose a new interpretation of quantum mechanics to address challenges of the Copenhagen Interpretation and to explain observations from certain double-slit experiments. We explore some characteristics of quantum mechanics and analyze a quantum mechanical model which leads us to the assumptions for our new interpretation. We show how quantized waves, like those in Schrödinger's wave equation, might exist in nature and explain the fundamentals of quantum-scale processes including: the above-mentioned double-slit experiments, wave function collapse, quantum entanglement and quantum tunneling. We classify our interpretation based on commonly used criteria. Finally, we consider some future theoretical points and list some experimental questions. Our new interpretation has the potential to facilitate new theory and experiments leading to a better understanding of fundamental processes in nature and possibly new applications for quantum theory.

Contents

1) Motivation

This paper originated from an attempt to explain interference patterns which accumulate over time in double-slit experiments when particles are sent through such an experiment one-at-a-time. This has been observed for photons[1], electrons[2] and other particles[3]. The implication is that each particle interferes with itself to produce an interference pattern. This implication challenges one to find an explanation for how this might occur in nature.

There are at least a dozen interpretations of quantum mechanics. Cramer[4] describes how the Copenhagen Interpretation was formed in the 1920's and how it became[5] a "'don't ask; don't tell' approach to the quantum formalism that fulfilled the needs of those who wanted to calculate and make predictions, but frustrated those who wanted to understand what went on behind the scenes." Regarding the Copenhagen Interpretation, Griffiths[6] states, "Among physicists it has always been the most widely accepted position." That interpretation proposes that the quantum wave function exists "in the observer's mind". It is not credible that a particle in nature would consult a quantum wave

function in some observer's mind to know where to go when creating an interference pattern. Other interpretations present similar difficulties. This prompted us to seek a new interpretation which would be accessible to nature and would be able to explain the experimental results from the double-slit experiments mentioned above.

2) Characteristics of Quantum Mechanics

Quantum mechanics is based on Schrödinger's equation[7]:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2\,m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi \qquad \text{Griffith's equation [1.1]}$$

Where i is the square root of -1, ħ is Plank's constant divided by 2π, t is time, Ψ is the complex wave function, m is mass, and V is an external energy potential possibly varying with space and time. Time, space and ħ are real numbers. Consider the following characteristics of Quantum Mechanics (QM):

Characteristic	Illustrative Examples
A. QM is accurate.	Predictions match experiments to better than 1 part per million[8].
B. QM is not symmetric with respect to space and time.	Each term in Schrödinger's equation represents energy. On the left side, $\frac{\partial\Psi}{\partial t}$ shows how energy is distributed through time. On the right side, $\frac{\partial^2\Psi}{\partial x^2}$ shows how energy is distributed through space. The wave function Ψ can also describe how an external potential energy V, is distributed in space and time.
C. QM can model mass as well as energy.	Although Schrödinger's equation models energy, mass and energy are equivalent. Feynman states[9], "An electron and a positron come together at rest, each with a rest mass m_0. When they come together they disintegrate and two gamma rays emerge, each with the measured energy of m_0c^2. This experiment furnishes a direct determination of the energy associated with the existence of the rest mass of a particle."
D. QM can model position and momentum, and therefore all classical dynamical variables	Griffiths[10], provides quantum mechanical operators for position and momentum: $\langle x\rangle = \int \Psi^*(x)\Psi dx$ Griffith's equation (1.34) $\langle p\rangle = \int \Psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\, dx$ Griffith's equation (1.35} He then states. "The fact is, all classical dynamical variables can be expressed in terms of position and momentum."
E. QM accounts for interference of amplitudes representing both Bose and Fermi particles.	Per Feynman[11], regarding a collision of identical particles "Bose particles are the photon, the mesons and the graviton. The Fermi particles are the electron, the muon, the neutrinos, the nucleons, and the baryons. We have then, that the amplitude for the scattering of identical particles is: Bose particles: $(Amplitude\ direct) + (Amplitude\ exchanged)$ Fermi particles: $(Amplitude\ direct) - (Amplitude\ exchanged)$"

F. QM can model particles[12] or waves[13] in space and time and can handle models with sums over a possibly infinite number of discrete variables …	Example of Infinite Sums of discrete variables. Griffith considers the case[14] in which (lower case) ψ is a function of x alone and (lower case) φ is a function of t alone. This means that by separation of variables, Schrödinger's equation can be split into two separate parts and rewritten as: $$i\,\hbar\,\frac{1}{\varphi}\,\frac{d\varphi}{dt} = -\frac{\hbar^2}{2m}\,\frac{1}{\psi}\,\frac{d^2\psi}{dx^2} + V \quad \text{Griffith's equation [2.3]}$$ The right side represents how energy is distributed through space. The left side represents how the energy is distributed through time. Griffiths shows how the right side can be solved exactly, and then how the contribution from the left side can be included to form a complete exact solution to his equation [2.3]: $$\psi(x,t) = \sum_{n=1}^{\infty} c_n \psi_n(x)\, e^{-iE_n t/\hbar}$$ $$= \sum_{n=1}^{\infty} c_n \Psi(x,t) \quad \text{Griffith's equation [2.17]}$$ Where $\psi_n(x,t)$ are the wave functions of time independent solutions for the various energy levels, x is space, t is time, ħ is Plank's constant divided by 2π, i is the square root of -1 and Ψ(x, t) is the complete wave function over all space and time. The functions c_n represent a set of orthogonal base states for the system being modeled.
F. (Continued) or an integral over an infinite set of continuous functions.	Example of Infinite Integrals of continuous functions. Considering the Free Particle[15] Griffiths starts with the time-independent Schrödinger equation: $$-\frac{\hbar^2}{2m}\,\frac{d^2y}{dx^2} = E\,\psi \quad \text{Griffith's equation [2.90]}$$ $$or \quad \frac{d^2y}{dx^2} = -k^2\psi \;\; where\; k \equiv \frac{\sqrt{2\,m\,E}}{\hbar}$$ His solution is an integral over an infinite number of energies: $$\psi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Phi(k) e^{\left(ikx - \frac{\hbar k^2}{2m}\right)t}\, dk \quad \text{Griffith's equation [2.100]}$$ where $\psi(x,t)$ is the wave function of space x and time t, ħ is Plank's constant divided by 2π, m is mass, E is energy and i is the square root of -1. Just below equation [2.100] Griffith states, "(The factor $1/2\pi$ is factored out for convenience; what plays the role of the coefficient c_n in Equation [2.17] is the combination $\left(1/\sqrt{2\pi}\right)\Phi(k)\,dk$. Now *this* wave function *can* be normalized (for appropriate ϕ(k)). But it necessarily carries a range of k's and hence a range of energies and speeds. We call it a wave packet.
G. QM is dependent upon the existence of an infinite dimensional Hilbert space.	As clearly described by Griffiths[16], "The collections of all functions of x constitutes a vector space, but for our purposes it is much too large. To represent a physical state, the wave function ψ must be *normalized*: $$\int |\psi|^2 \; dx = 1$$

	The set of **square-integrable functions**, on a specified interval, $$f(x)\ \ such\ that\ \int_a^b |f(x)|^2\,dx\ <\ \infty$$ constitutes a (much smaller) vector space. Mathematicians call it L_2(a, b); physicists call it Hilbert space. In quantum mechanics, then, Wave functions live in Hilbert space.**"**
H. QM is dependent upon the use of complex variables.	Per Feynman[17], a complex number c is of the form c = a + i b where a and b are real numbers and i is defined by: $i \equiv \sqrt{-1}$. Unlike the real number system, the complex number system is complete in the sense that for any combination of arithmetical operations (addition, subtraction, multiplication, division, raising to powers and taking roots) applied to a complex number of the form a + i b, results in another complex number of the same form.
I. All observables predicted by QM are real numbers.	As clearly described by Griffiths[18], "The expectation value of an observable Q(x, p) can be expressed very neatly in inner-product notation: $$\langle Q\rangle = \int \psi^* \hat{Q}\psi\,dx$$ $$= \langle\psi|\hat{Q}\psi\rangle \text{ Griffith's eqation [3.13]}$$ Now the outcome of a measurement has got to be *real*, and so *a fortiori*, is the *average* of many measurements: $$|Q| = |Q|^* \qquad \text{Griffith's eqation [3.14]}$$ But the complex conjugate of an inner product reverses the order, so $$\langle\psi|\hat{Q}\,\psi\rangle = \langle\hat{Q}\psi|\psi\rangle \qquad \text{Griffith's eqation [3.15]}$$ Thus, operators representing observables have the very special property that $$\langle f|\hat{Q}f\rangle = \langle\hat{Q}f|f\rangle\ \ for\ all\ f(x) \quad \text{Griffith's eqation [3.16]}$$ We call such operators hermitian."

3) A Quantum Mechanical Example

Let's examine a simple two-state quantum example to illustrate how energy changes through time in quantum systems. Consider the quantum example from Feynman's Volume III, Section 8-6 *The Ammonia Molecule*.[19] He describes it as follows:

> The ammonia molecule has one nitrogen atom and three hydrogen atoms located in a plane below the nitrogen so that the first molecule has the form of a pyramid, as drawn in Fig. 8-1(a). Now this molecule, like any other, has an infinite number of states. It can spin around any possible axis; it can be moving in any direction; it can be vibrating inside, and so on, and so on. It is therefore, not a two-state system at all. But we want to make an approximation that all other states remain fixed, because they don't enter into what we are concerned with at the moment. We will consider only that the molecule is spinning around in its axis of symmetry (as shown in the figure), that it has zero translational momentum, and that it is vibrating as little as possible. That specifies all conditions except one: *there are still the two possible positions for the nitrogen atom* – the nitrogen may be on one side of the plane of hydrogen atoms or on the other, as shown in Fig. 8-1(a) and (b). We will discuss the molecule as though it were a two-state system.

Feynman then develops a pair of quantum wave equations to describe the time-evolution of the two possible energy states. In the process of his derivation, he discovers that there must be two energy levels involved which he calls, E_0 and A. The equations for $C_1(t)$ and $C_2(t)$ are the wave equations for states C_1 for state |1> when the nitrogen atom is "up" and C_2 for state |2> when the nitrogen atom is "down". They show for each state how the energy changes with time:

$$C_1(t) = \frac{a}{2}\, e^{-(i/\hbar)(E_0 - A)t} + \frac{b}{2}\, e^{-(i/\hbar)(E_0 + A)t} \quad \text{Feynman's equation (8.50)}$$

$$C_2(t) = \frac{a}{2}\, e^{-(i/\hbar)(E_0 - A)t} - \frac{b}{2}\, e^{-(i/\hbar)(E_0 + A)t} \quad \text{Feynman's equation (8.51)}$$

He then says, "Suppose that at t=0, we know that a molecule is in the state |1> … What is the probability that the molecule will be found in the state |2> at the time t?" He then derives these two equations:

$$C_1(t) = e^{-(i/\hbar)E_0 t}\, \cos(At/\hbar) \quad \text{Feynman's Equation (8.52)}$$

$$C_2(t) = ie^{-(i/\hbar)E_0 t}\, \sin(At/\hbar) \quad \text{Feynman's Equation (8.53)}$$

Where $\hbar$ is Plank's constant h divided by 2 π, E_0 and A are two energy levels, and t is time.

Feynman then calculates the probability that the molecule can be found in state 2 by taking the absolute square of C2 as follows:

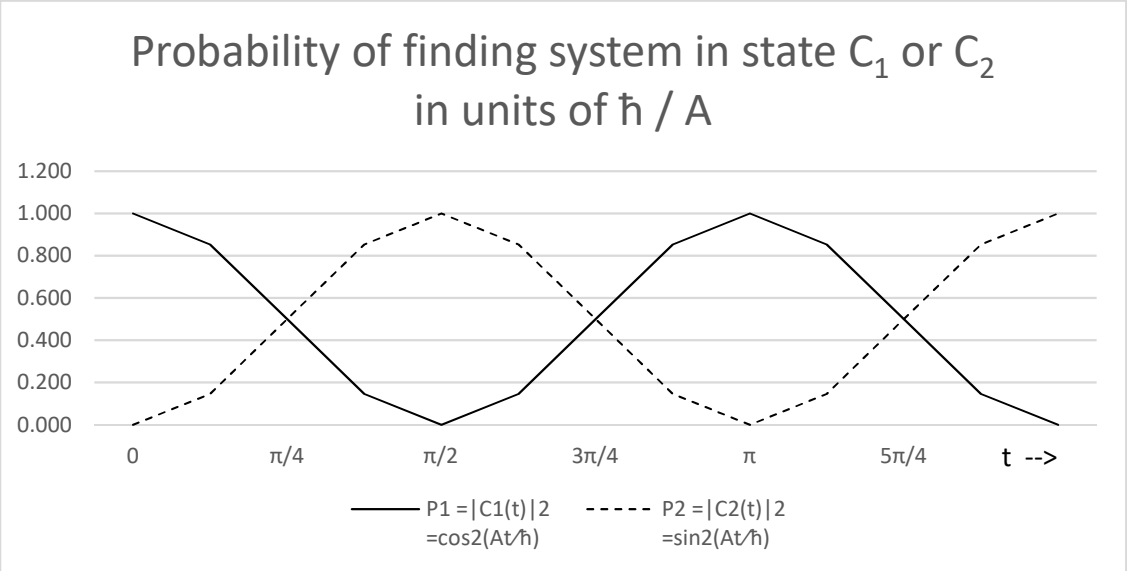
$$Probability\ to\ be\ in\ state\ 2 = |C_2(t)|^2 = sin^2(At/\hbar) \quad \text{Feynman's Equation(8.54)}$$

Similarly, we calculate:

$$Probability\ to\ be\ in\ state\ 1 = |C_1(t)|^2 = cos^2(At/\hbar) \quad \text{This papar's equation (P)}$$

Feynman then graphs the probabilities in units of $\hbar/A$. See Appendix A – Probability Data at the end of this paper to see the data underlying this graph.

Here is a rough reproduction of Feynman's Figure 8.2:



4) Analysis of the Example

Please see ***Appendix B – Derivations*** to see an expansion from Feynman's equations (8.52) and (8.53) to its component parts, the real and imaginary parts of functions $C_1(t)$ and $C_2(t)$ as shown in equations (A), (B), (C) and (D) below:

$$Re\big(C_1(t)\big) = \cos(E_0 t/\hbar)\, cos(A\ t/\hbar) \qquad \text{This papar's equation (A)}$$

$$Im(C_1(t)) = -\sin\big((E_0 t/\hbar)\big)\cos(A\ t/\hbar) \qquad \text{This papar's equation (B)}$$

$$Re(C_2(t)) = \sin(E_0 t/\hbar)\ \sin(A\ t/\hbar) \qquad \text{This papar's equation (C)}$$

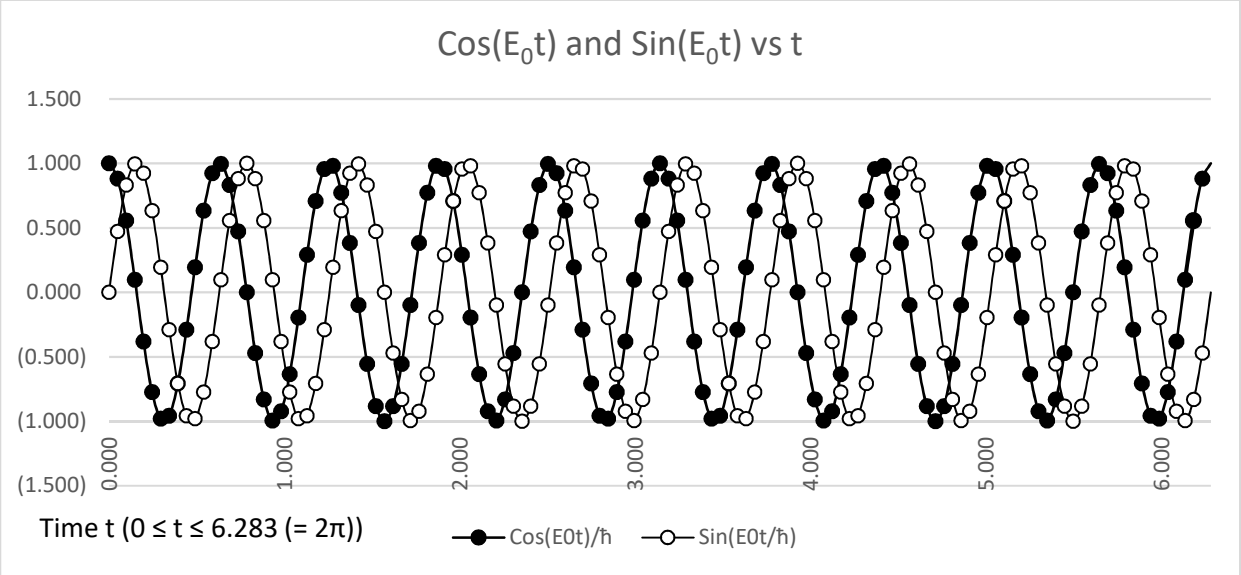
$$Im(C_2(t)) = \cos(E_0 t/\hbar)\ \sin(A\ t/\hbar) \qquad \text{This papar's equation (D)}$$

Note that these are all functions of sin and cos of (energy x time / ħ). Therefore, we can examine these functions in graphs using the same units that Feynman used to graph the probabilities $P_1(t)$ and $P_2(t)$ above. ***All the graphs in this paper show time t in units of (ħ / energy)***.

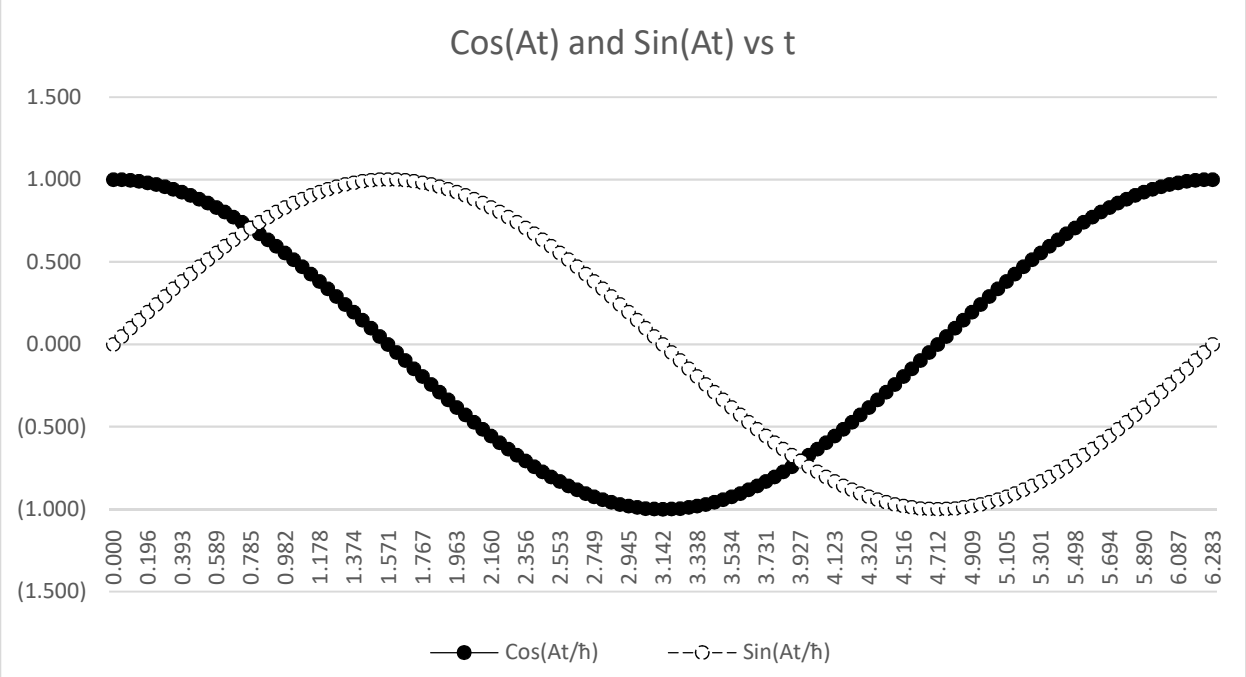
Next, we need to consider the relation between the energies E_0 and A. Feynman has said that A is much smaller than E_0. Our goal here is not to accurately model the ammonia molecule, but rather is to understand the relation of the various components of the functions $C_1(t)$ and $C_2(t)$ over time.

Since E_0 has more energy than A, the wave function of E_0 will cycle faster than that of A. We want to consider a complete cycle of both E_0 and A. It turns out that we can do this if we let $A = E_0/10$. Doing so results in 10 cycles of E_0 and one cycle of A. In our graphs below, we divide the time between 0 and 2π into 128 intervals in increments of π/64, therefore the increments of time t = .049, (rounded to 3 decimal places) in units of ħ/energy. We include one extra increment at the end to complete the cycles and return us to our starting point, so the data table contains 129 values. The formulas and the first few rows of the data table underlying the following graphs are shown in ***Appendix C – Data Table***.

Here is Graph 1, the graphs of $Cos(E_0)$ and $Sin(E_0)$ versus t with 10 complete cycles in units of $t= \hbar/E_0$:

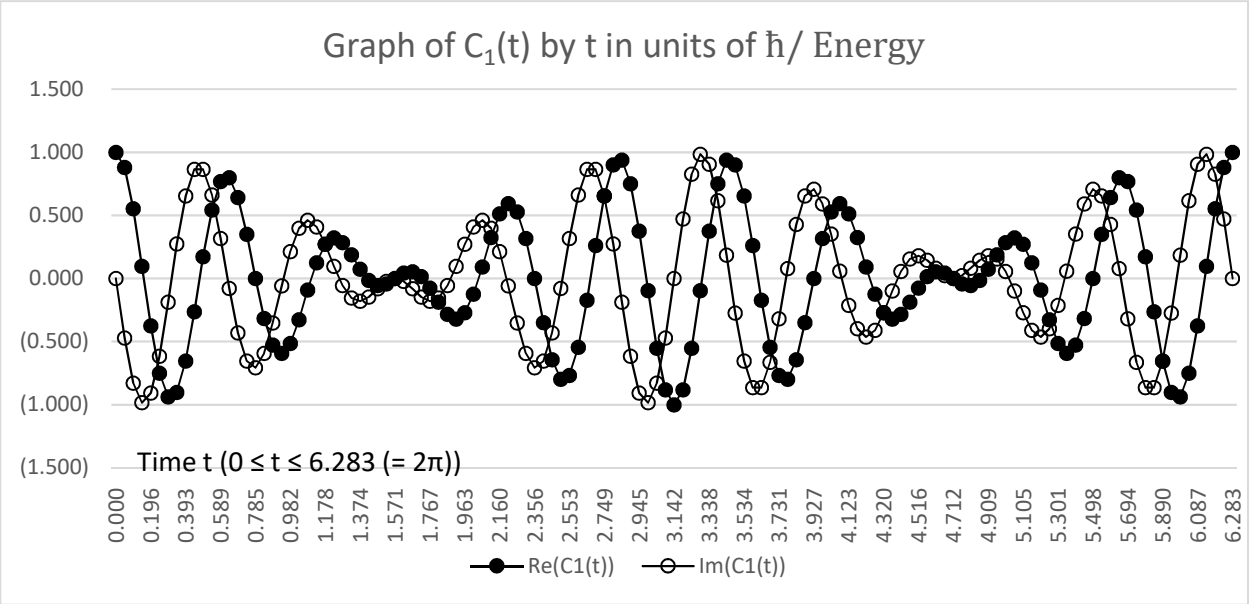


Here is Graph 2 the graphs of Cos(A) and Sin(A) versus t with one complete cycle of in units of $t= \hbar/A$:



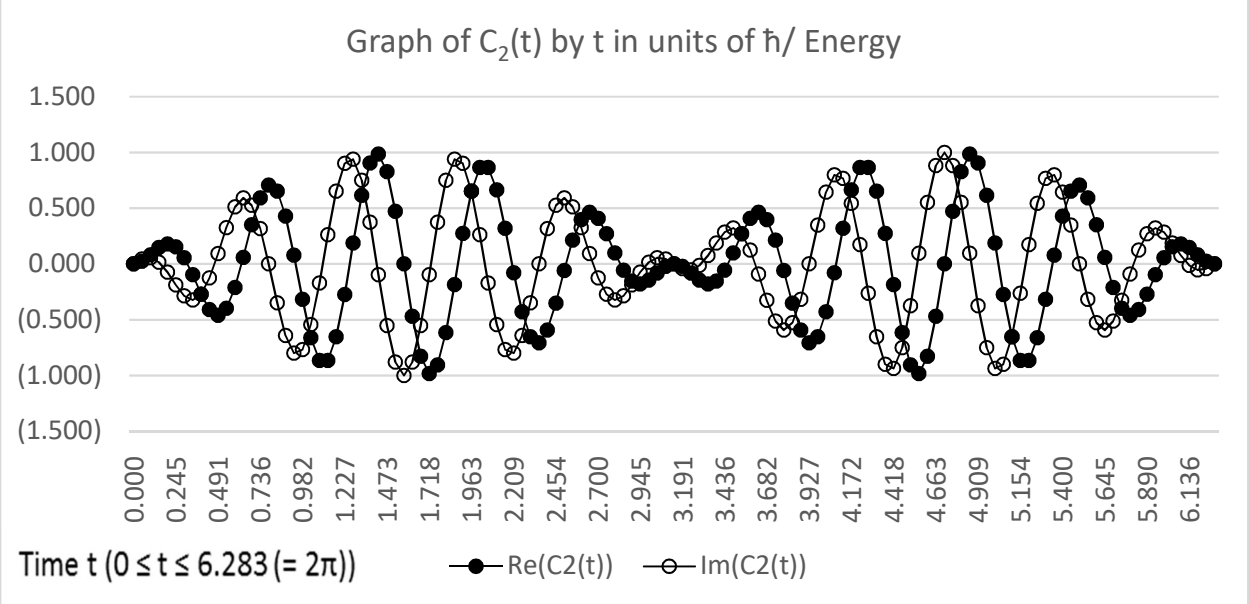
Time t (0 ≤ t ≤ 6.283 (= 2π))

Here is Graph 3, the graph of $C_1(t)$ showing its real and imaginary components:



Note that like Feynman's graph of the probabilities, this graph shows the energy in $C_1(t)$ starting at a high level for time t=0 and dropping to a low value at time t= π/2 (1.571) and then returning to a hi level again at t = π (3.142).

Here is Graph 4, the graph of $C_2(t)$ showing its real and imaginary components:

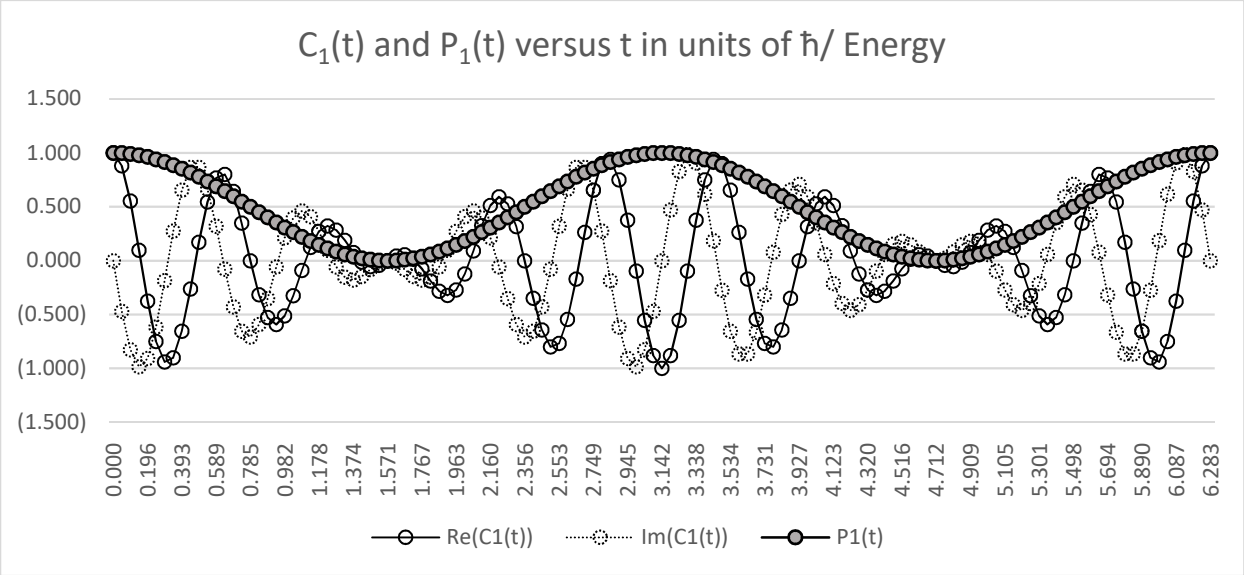


Note that for $C_2(t)$ the energy starts out at zero and increases to a large value at t= π/2 (1.571)

In the above graphs we see that the energy cycles between the two states |1> and |2> with most of the energy in state |1> at the beginning when t=0 which makes sense because Feynman assumed the

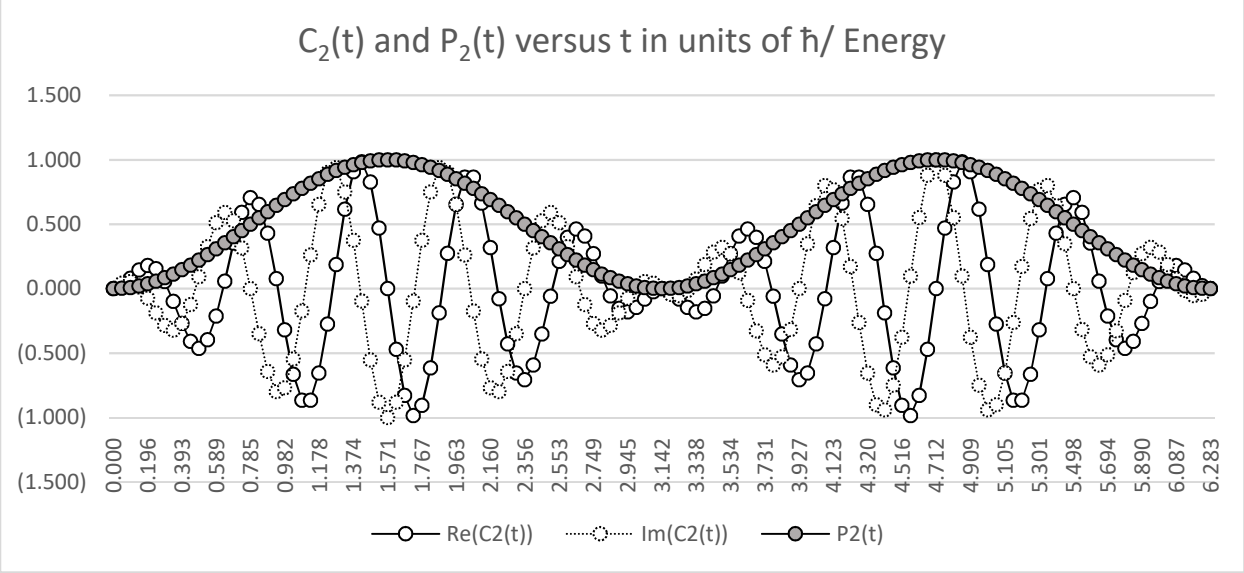
system was in state |1> at time t=0. We also see that the energy goes into state 2 near times π/2 and 3π/2, returning to state |1> at times π and 2π.

Here is Graph 5, the graph of both the components of $C_1(t)$ together with $P_1(t)$. This graph clearly shows that the probability of finding the molecule in state |1> is greatest when the wave function $C_1(t)$ contains the most energy.



Time t (0 ≤ t ≤ 6.283 (= 2π))

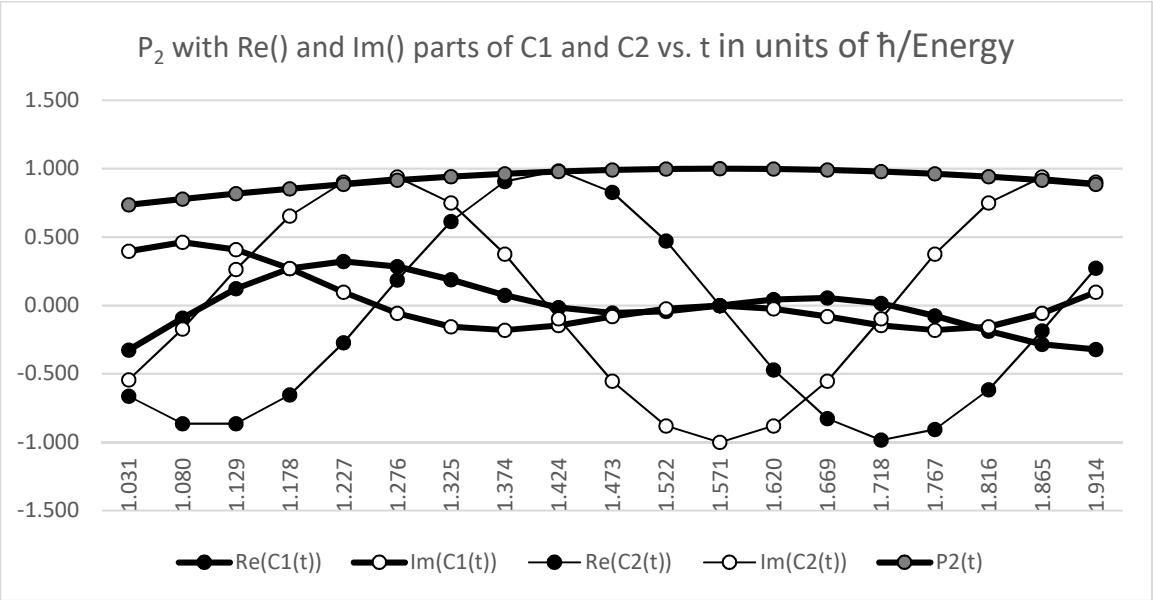
Here is Graph 6, the corresponding graph for the components of $C_2(t)$ and the probability $P_2(t)$. We see that the probability of finding the system in state |2> is greatest when the wave function $C_2(t)$ contains the most energy.



Time t (0 ≤ t ≤ 6.283 (= 2π))

At this point we take a closer look at the components of the wave functions C1(t) and C2(t). In certain regions near the highest probability of finding the system in state |2> we see that there are times when most of the energy is in the imaginary components of the functions.

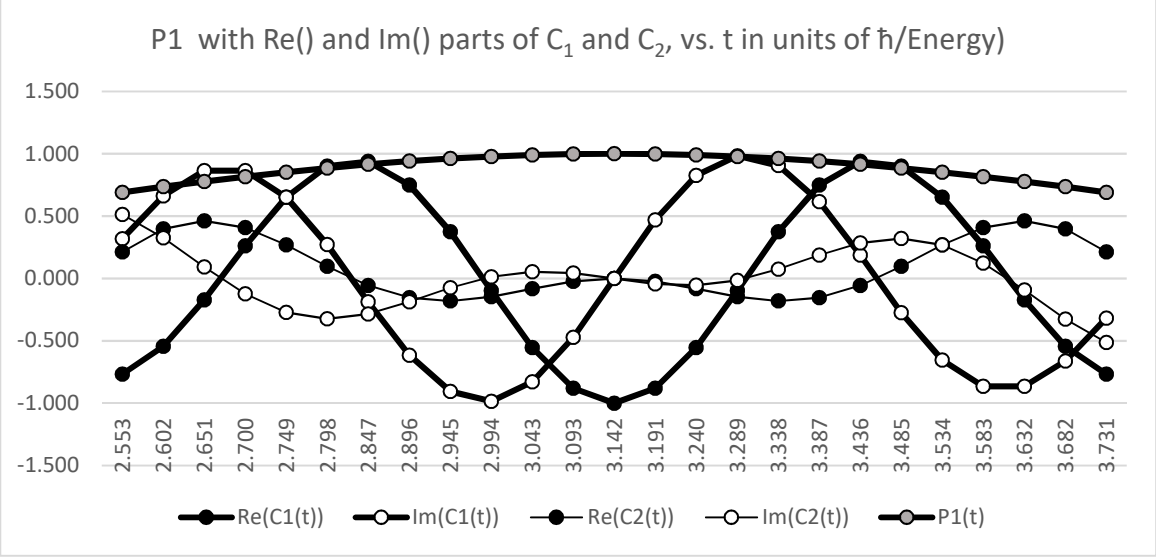
Graph 7 shows details in a region near the highest probability of finding the system in state |2>



Time t ($1.031 \le t \le 1.914$)

We see that at times t=1.227 and 1.276 most of the energy is in the imaginary component of C_2. The same is true at times t=1.522 – 1.620, when most of the energy is in the imaginary component of C_2.

The same is true for state |1> shown in Graph 8 below:



Time t ($2.553 \le t \le 3.731$)

Here we see energy in the imaginary components of C_1 at t = 2.945-3.043 and at t = 3.240-3.338.

In our example, we see Schrödinger's equation model the energy of a system based on the Hamiltonian matrix. This leads to a quantum mechanical solution involving the complex functions $C_1(t)$ and $C_2(t)$ representing the evolution in time of the energy associated with the states of the system. These functions cycle the energy back and forth through the real and imaginary components of $C_1(t)$ and $C_2(t)$ and lead to the predicted probabilities $P_1(t)$ and $P_2(t)$ for finding the system in either state.

This suggests that when using Schrodinger's equation, ***we*** must use complex numbers with their imaginary components to calculate probabilities of finding a system in a given state. It also suggests that if something like Schrödinger's equation exists in nature, then ***nature*** must use complex numbers with their imaginary components to do the same.

We would like to extend the above example just a bit further, based on comments Feynman made in the next chapter[20]. He refers to his equations 8.50 and 8.51 shown earlier and rewrites them:

$$C_1 = \frac{a}{2}\, e^{-(i/\hbar)(E_0 - A)t} + \frac{b}{2}\, e^{-(i/\hbar)(E_0 + A)t} \quad \text{Feynman's equation (9.2)}$$

$$C_2 = \frac{a}{2}\, e^{-(i/\hbar)(E_0 - A)t} - \frac{b}{2}\, e^{-(i/\hbar)(E_0 + A)t} \quad \text{Feynman's equation (9.3)}$$

and states:

> "Suppose that the molecule was initially put in a state $|\psi_{II}>$ for which the coefficient b was equal to zero. Then at t = 0 the amplitudes to be in the states |1> and |2> are identical. *and they stay that way for all time*. Their phases both vary with time in the same way – with the frequency $(E_0 - A) / \hbar$. Similarly, if we were to put the molecule into a state $|\psi_{I}>$ for which a = 0, The amplitude C_2 is the negative of C_1, and this relationship would stay that way forever. Both amplitudes would now vary with time with the frequency $(E_0 + A) / \hbar$. These are the only two possibilities of states for which the relation between C_1 and C_2 is independent of time.
>
> We have found two special solutions in which the two amplitudes *do not vary in magnitude* and, furthermore, have phases which vary at the same frequencies. These are *stationary states* as we defined them in Section 7-1, which means that they are *states of definite energy*."

One final point: These stationary states of definite energy for which the amplitudes do not vary in magnitude, and for which the total probability = 1 for all time ***still cycle energy back and forth between the real and imaginary components of the wave function!*** Consider the case in which b = 0. Then the wave function for C_2 has the exponential time-dependent form:

$$C_2 = \frac{a}{2}\, e^{-(i/\hbar)(E_0 - A)t}$$

Disregarding the factor a/2, this function starts at point (1, 0) on the (x, i) plane and traces out a circle in the clockwise direction as time t increases. This function ***is*** a function of t, and the energy spends fully half of the time mostly in the imaginary components. Specifically, in the interval $(\pi/4 < t < 3\pi/4)$ most of the energy of the wave amplitude is in the negative imaginary numbers, and in the interval $(5\pi/4 < t < 7\pi/4)$ most of the energy of the wave amplitude is in the positive imaginary numbers. The final and important point here is that unless specifically blocked somehow, the energy ***always*** cycles back and forth through the real and imaginary components, whether a system ***appears*** to be in motion or not.

5) Can Schrödinger's Wave Equation Exist in Nature?

Let's reconsider the above-mentioned characteristics of Quantum Mechanics and see if they support or challenge to the notion that some form of Schrödinger's wave equation might exist in nature:

Characteristic	Illustrative Examples
A. QM is accurate.	This supports the notion (that quantum mechanical formulas and methods might exist in nature). If quantum mechanics and nature were using different approaches, it would be difficult to get such accurate predictions of actual experimental results.
B. QM is not symmetric with respect to space and time.	This must be a fundamental characteristic of nature, because of the accuracy of QM predictions and so should be acknowledged in any interpretation of quantum mechanics.
C. QM can model mass as well as energy.	This supports the notion since mass and energy are equivalent.
D. QM can model position and momentum, and therefore all classical dynamical variables	This supports the notion because both quantum mechanics and nature generalize to the familiar classical mechanical models.
E. QM accounts for interference of amplitudes representing both Bose and Fermi particles.	This supports the notion because both of these effects are found in nature.
F. QM can model particles or waves in space and time and can handle models with sums over a possibly infinite number of discrete variables or an integral over an infinite set of continuous functions.	This requires further elaboration. See the section below: ***5. c) Particles and Waves***.
G. QM is dependent upon the existence of an infinite dimensional Hilbert space.	This requires further elaboration. See the section below: ***5. a) Hilbert Space***.
H. QM is dependent upon the use of complex variables.	This requires further elaboration. See the section below: ***5. b) Complex Dimensions and Observables***.
I. All observables predicted by QM are real numbers.	This requires further elaboration. See the section below: ***5. b) Complex Dimensions and Observables***.

5. a) Hilbert Space

The use of Hilbert space to normalize a wave function implements conservation of energy. The total probability of finding ***all of "something"*** should not change as ***the thing*** moves through space and time or is looked at from a different point of view. Hilbert space seems to be a result of the practices of changing base states and of allowing the Schrödinger wave functions to be calculated, for example, over "all space" or "all time", and later ensuring that energy is conserved. The total probability of finding something ***somewhere*** must add to 1. We assert that in nature, ***some*** amount of mass and energy exists. As it spreads out in space and time, barring interactions that might change its mass or energy, it remains "normalized" at each tiny increment and from any point of view because there is only the initial amount of mass or energy to begin with, and mass and energy are conserved.

5. b) Complex Dimensions and Observables

Historically we have thought of the world as composed of four real valued dimensions, three for space and one for time. There has been a bias against believing that complex dimensions might exist in nature. This bias may be based on traditional usage of the real number line to describe space and time in classical mechanics, thermodynamics and even electromagnetism. This bias combined with our inability to observe complex variables may have resulted in a disinclination to find a way in which complex variables, and the complete number system that comes with them, might exist in nature.

In quantum mechanics, we are unable to directly measure anything about the components of the wave function. Per "Characteristic I" from section 2 above, an observable in quantum mechanics must arise from a Hermitian operator in the form of a set of real-valued physical observations.

Therefore, if complex wave functions exist in nature, their direct observation and measurement must be hidden from us. Let's assume for the moment that nature uses a complete number system, namely a set of space dimensions described by a complex variable of the form $c = a + i\,b$, so that each real dimension "a" would be paired with an imaginary dimension "b" hidden from our view. There are at least two ways such hidden imaginary dimensions might exist in nature. A hidden dimension might be:

1. Curled-up, like a garden hose viewed from a tall building[21]
2. In the Bulk[22], if we exist on a 4-dimensional brane in a higher dimensional universe

If either of the above conditions were to exist, then the hidden imaginary dimensions would not be observable by us and we would not be able to directly measure anything about them. They would appear to us to be just like the imaginary dimensions of quantum mechanics! This supports the notion that such hidden imaginary dimensions might exist in nature. We could look to indirect experimental evidence to support or challenge this notion and for clues about the characteristics of these dimensions.

We know about metrics in the real dimensions in nature because we can measure real variables. We know that a single quantum of energy might appear as a wave, spread out over a very broad range of real-valued locations relative to its wavelength. For example, after the wave of an electron has passed through a double-slit experiment, it may end up in any of a wide range of physical locations.

We don't know anything about the shape or metrics of the hidden imaginary dimensions. For example, such dimensions might be large or compact. As energy circulates around between the real and imaginary dimensions, the wave might be widely dispersed in the real dimensions. It could be widely dispersed or quite compact in the imaginary dimensions.

If space is complex, is time also? Per special relativity, time and space are similar. For example, if an observer is at rest with respect to an emission of light, the light cone spreads symmetrically into the future and three events, A, B and C separated in space might appear to be simultaneous. However, to an observer moving in one direction, the events might appear to occur in sequence A, then B, then C. While to an observer moving in the opposite direction, the events might appear to occur in sequence C, then B, then A. If space and time are even a little bit interchangeable as suggested by special relativity, this means that if space is complex, time must also be complex. There is an obvious conflict between the asymmetry of time and space evident in Schrodinger's wave equation, and the symmetry of time and space per special relativity. This conflict is exacerbated by the notions of space and time suggested by general relativity. We note these issues but do not explore them further in this paper.

5. c) Particles and waves

First, we consider whether the infinite sums and integrals used in quantum mechanics pose a problem for the notion that Schrödinger's wave function might exist in nature.

In Griffith's example in Section 2, Characteristic F.1 above the Infinite sum on the right ranges over all space and includes the possibly uncountable number of energy levels of a system. The infinite sum on the left ranges over all time, from the infinite past to the infinite future. If one is solving equations over all space and all time, it is perfectly reasonable to assume that infinite sums might be involved and might describe the real world within the constraints of the system being modeled. We do not see a conflict in modeling such infinite sums when attempting to describe systems of possibly uncountable particles having possibly uncountable energy levels over all space and time.

In Griffith's example in Section 2, Characteristic F.2 above, the Infinite integral represents a possibly infinitesimal wave packet. Since a wave packet may have a very short length, it has an indefinite wave length and momentum. The infinite integral is necessary to include all the possible energy levels of the components of the packet. Again, we see no conflict in imagining a wave packet moving through space and time in nature.

Next, we consider some consequences of assuming Schrödinger's wave equation exists in nature and is fundamental to the behavior of physical systems on quantum scales. We've seen examples of Schrodinger's equation modeling both waves and particles. Fundamentally, though, the equation itself is a "wave equation". Therefore, in finding a way for that equation to manifest itself in nature, we must look to the wave nature of the equation, recognizing that particles can also be modeled using the wave functions in that equation. We should note at this point that although Schrodinger's equation is a wave equation, the energy represented by the wave function is quantized by Plank's constant h (or $\hbar = h/2\pi$). If Schrödinger's equation is to be found in nature, then quantized waves must exist in nature as well. We don't see an issue in assuming this, it is merely a description of what nature must be like at very small scales.

If Schrodinger's equation does exist in nature, then quantized waves form the building blocks in nature of all matter and energy. What we think of as *energy* is usually represented by wave functions "adding" as those of Bose particles do. What we think of as *matter* is usually represented by wave functions "subtracting" as those of Fermi particles do per Section 2, Characteristic E above.

Finally, let's consider the double slit experiments mentioned at the beginning of this paper. Here is a possible explanation of how a "particle" (a photon, electron or atom) might interfere with itself when sent one at a time through a double-slit experiment. Assume the particle's energy is modeled by Schrödinger's equation and includes both the space- and time-dependent components of the model. The particle is then represented by a quantized wave packet as follows:

1. Different parts of the quantized wave pass through each slit of a double-slit experiment
2. The energy of the quantized wave cycles between the real and imaginary dimensions
3. By remaining whole in the imaginary dimensions, the wave retains its quantized nature
4. The two parts of the wave interact producing interference patterns
5. The wave and its interference patterns approach the detector

This explains how, in this interpretation, a “particle” can go through both slits of a double-slit experiment and interfere with itself while remaining quantized.

Now, consider what happens as the wave approaches the detector:

1. At some point in time, the particle’s quantized wave begins to interact with the detector
2. The wave might explore a wide range of possible locations on the detector in the real dimensions relative to its wavelength
3. In each cycle, all the energy of the wave cycles from the real dimensions, through the imaginary dimensions and then back into the real dimensions
4. At some location in the real dimensions the wave finds a suitable landing spot on the detector and that spot begins to drain energy from the wave
5. As the rest of the wave returns to the real dimensions its energy is drained into that spot
6. This appears to be instantaneous, but instead it requires only one cycle of the wave to complete

This explains how, in this interpretation, a “particle”, having had its quantized wave split into two interfering parts in the real dimensions, and retaining its quantized nature via the imaginary dimensions, might find a suitable landing spot on the detector, and in one cycle transfer all its energy to that one spot in the real dimensions. This second process is commonly called “wave function collapse” but a better term would be “wave function re-localization” because nothing collapses. The wave’s energy is maintained and the wave merely moves to a different location.

Since both the above effects can be explained by our new interpretation and both are observed in double-slit experiments, we assume that our new interpretation has achieved its goal of providing a reasonable mechanism in nature to describe what happens in such experiments.

6) A New Interpretation of Quantum Mechanics

This new interpretation is based on the following assumptions:

- Complex dimensions exist in nature
- Schrödinger’s equation could exist within a set of complex dimensions in nature
- Schrödinger’s equation cycles energy between the real and imaginary dimensions
- Waves of quantized energy are fundamental to quantum scale processes in nature
- Quantized waves may be compact or may range widely in the real dimensions
- Quantized waves may be compact or may range widely in the imaginary dimensions

7) Consequences of our new interpretation

This interpretation deals with the question of wave-particle duality by asserting that fundamentally and at small scales, all matter and all energy are always quantized waves:

- Fundamentally, the photon is never a particle, it is always a quantized wave
- Fundamentally, the electron is never a particle, it is always a quantized wave
- Fundamentally, the molecule is never a particle, it is always a quantized wave

Having said this, it is sometimes convenient to think of matter and energy as particles just as Feynman did in our example from Section 3) above. At large scales for which systems can be described

sufficiently by classical mechanics this is almost always true. In many cases the use of the idea of a "particle" instead of a wave may be both convenient and appropriate.

A Mechanism for Quantum Entanglement. Let's assume that a quantized wave representing an "entangled pair of particles" might be located within a small region relative to its wavelength in the imaginary dimensions, even though the "entangled pair" might be separated by a large distance in the real dimensions. This would allow our new interpretation to explain quantum entanglement as a local phenomenon as follows:

1. The entangled quantized wave in the imaginary dimensions is local to ***both*** of its widely-separated components in the real dimensions
2. As the wave cycles energy between the real and imaginary dimensions, the entire energy of the wave can easily move to any location in the real dimensions no matter how widely separated the two entangled waves (and their "particles") might be in the real dimensions.
3. Under these circumstances, the real portions of the two widely separated but entangled quantized waves are essentially local to one another since the energy of the entangled wave continuously cycles back and forth through its local connection via the imaginary dimensions.
4. When one part of the entangled wave is "measured" (or re-localized) it necessarily absorbs that portion of the entangled wave associated with the "measurement".
5. Whatever entangled property is realized in the "measurement" is therefore cut off from the entangled wave, leaving the complimentary "un-measured" property with the remainder of the previously entangled wave.
6. As always, the re-localization should occur in one cycle of the wave.

This might provide a mechanism to explain quantum entanglement.

A Mechanism for Quantum Tunneling. Tunneling might also be explained by our new interpretation. As we saw in "Section 4)" above, a particle in a "stationary state" of "definite energy" spends half of its time with most of its energy in the imaginary dimensions. Consider two such "particles":

1. The quantized wave for one might have its energy completely in the real dimensions at time t
2. The quantized wave for the other might have its energy completely in the imaginary dimensions at the same time t
3. These two quantized waves could be cycling back and forth in the same place in both dimensions, but while the energy of one wave is mostly in the real dimensions, the energy of the other could be in the imaginary dimensions, and vice versa.

This might provide a mechanism to explain how, as Feynman[23] put it, "It is possible in quantum mechanics to sneak quickly across a region which is illegal energetically."

8) Characteristics of our new interpretation

Our interpretation of quantum mechanics can be classified as follows:

Criteria		Comment
Deterministic?	No	Quantized waves travel through available paths in the real dimensions and select suitable targets at random.
Wave function exists in nature?	Yes	Yes. Nature does not contain particles, but only quantized waves representing matter and energy. Such quantized waves exist in nature and cycle energy back and forth through the real and imaginary dimensions.
Unique history?	Yes	There are no alternate histories.
Hidden variables	Yes	Matter and energy are stored in nature as quantized complex waves. The imaginary components of these complex waves exist in nature within hidden imaginary dimensions. Time might also have a complex nature and therefore might have a hidden imaginary component.
Collapsing wave function?	No	Quantized waves in nature never "collapse", they merely "re-locate" by moving matter and energy through the hidden imaginary dimensions from one place in the real dimensions to another, once a quantized wave finds a suitable destination. This raises the question of what makes a "suitable" destination for a quantized wave.
Observer role?	No	Observers play no role in this interpretation.
Local?	Yes	This interpretation is explicitly local and assumes that locality is provided by the existence of hidden imaginary dimensions in nature.
Counterfactual definiteness?	Yes	It is possible to speak meaningfully about measurements that have not been performed.
Universal wave function exists?	Not yet	See comments on this below:

Does a universal wave function exist in this interpretation? I think the answer is, "Not yet". By this I mean that there might be a universal set rules governing how quantized waves interact. Such rules might describe all the known interactions of all known "particles". If there such rules exist, we don't know them yet. We also don't know what might be going on inside the many black holes believed to be widely scattered through our universe. So, a truly universal wave function could only be known to us locally, within our range of possible experimental observation, and even then, only knowable to the extent that one can know anything about the complimentary properties that waves possess. As Cramer[24] pointed out, a "Gaussian pulse in the electronics lab" is not able to have both a narrow time width and a narrow frequency width at the same time. This places limits on what we might be able to know about any given wave. In this interpretation, quantized waves exist in our universe. They cycle between the real and imaginary dimensions. They move around in space and time. Sometimes they interact with one another and sometimes they don't. We do not yet understand the full details of these processes even for known "particles" and their known interactions.

9) Future Theory and Experiments

It seems reasonable to assume that we will never be able to directly observe the features of the imaginary dimensions or the exact configuration of the complex quantized waves proposed in our new interpretation of quantum mechanics. The best we can do is to probe the possible features with experiments looking for patterns which build up over time due to interference, wave function re-location, entanglement, tunneling and whatever other measurable features we might manage to observe regarding the quantized waves. We see from the wave functions that energy does spend time in the imaginary dimensions. We know that matter and energy are equivalent. Therefore, it should be possible to transmit matter and energy through the imaginary dimensions if we know how the matter and energy in question are encoded into the quantized waves.

One way to test our new interpretation would be to attempt to fully model at least one type of quantum interaction and then experimentally test that model. For example, a gamma ray (or a pair of gamma rays) might produce an electron-positron pair. This is a reversible reaction because an electron and a positron can annihilate producing gamma rays. We have the following challenges: [1] How does the quantized wave of the gamma ray fully encode all its properties including energy, polarization, spin and so on. [2] How do the quantized waves of the electron and positron fully encode all their properties including energy, charge, spin and so on. Is an extra dimension needed to encode the charge? If there must be a charge dimension, is it a complex dimension? [3] How does the first type of quantized wave change into the second and vice-versa? [4] Experiments could be done to test the predictions of such a model: Do relative angles play a role? What about the phases of the interacting waves? Does the presence of an electric field affect what happens? Does it matter if there is an atomic nucleus nearby? If photons can create an electron-positron pairs and vice-versa, and if electrons can absorb and emit photons, then should photons and electrons be expected to behave similarly at low (classical) and high (quantum) energies? Do they behave similarly, or not? If not, why not?

Similar questions would apply to the encoding for all the properties of all the known Bose and Fermi “particles” and all their known interactions. One could pick one’s favorite interaction and look for a reasonable model and statistical evidence of the encoding of quantized wave characteristics as well as evidence for extra dimensions.

How does a quantized wave maintain its quantum nature when it is spread over large distances in the real dimensions? Is it something about the imaginary dimensions? It would be interesting to model how and why this might happen. Or is it something about the real dimensions, maybe the structure of space-time itself that might explain this? The concept of strings is interesting, but that raises the question, “What are the strings made of?”

If energy and mass are represented by complex quantized waves, what does that say about imaginary components of mass? Would that have any theoretical implications?

It would be interesting to model exactly what characteristics make a destination “suitable” for re-localization of a quantized wave whether considering detection of a photon, electron or molecule and then test that model experimentally.

Is there any way to measure the shape or extent of the imaginary dimensions with interference or entanglement experiments?

Are there any experiments that could be done to determine if imaginary or hidden dimensions are of different sizes or shapes for the various forces: Electric force and photon, Weak force and W and Z bosons, Strong force and gluons? Can a W or Z particle be made to interfere with itself in a double slit experiment? Can a quark be made to interfere with itself in a double slit experiment?

Is there an imaginary component of the time dimension? To test this, one would need speeds associated with the effects of special relativity. How could one indirectly observe an effect associated with such an imaginary dimension?

Regarding Experiments:

1. We know from Feynman[25] that photons on a large scale such as radio waves can be generated by using a set of dipole antennas and that by interference of the signals from those antennas, electromagnetic waves can be sent in practically any direction. One might experimentally test the hypothesis that a photon behaves the same way on a large scale with dipole antennas as it does with photons of a quantum scale such as light or gamma rays. This may have already been done, but if not, it should be. Is there a transition in behavior when the wavelength of a photon approaches quantum scales? Does the speed of re-localization depend on a wave's energy as it should if it occurs in a single cycle of the wave, or does it depend on other factors such as a distance in the real dimensions? Does re-localization occur for radio waves with long wavelengths? Can the re-localization time be measured for very long radio waves with long cycle times? The challenge of course with photons is that they are so small and one photon has so little energy that it is hard to generate or detect just one at a time.
2. A crystal can be used to cause photon interference. Is there something similar that could be used to create interference patterns with electrons or molecules of various energies?
3. How widely can two parts of a quantized wave be separated in a double-slit experiment and yet have the interfering wave retain its wave nature? Can an arrangement of slits or gratings allow a single quantized wave (photon or electron) to have a 50% probability of going one direction and a 50% probability of going another direction? If so, how far apart can they be separated? Does the re-localization time depend upon the separation distance, or only the energy? Is there a maximum separation? If there is, what happens when the maximum separation is exceeded? What happens for photons? Electrons? Molecules? The same questions could be asked of entanglement experiments.
4. Can matter or energy be transmitted from one place to another by wave function re-localization in the context of multiple-slit experiments? One would need to produce an interfering quantized wave that has a high probability of ending up in either of two very different locations. An electron? A molecule? Could one put a chain of such experiments in series to transmit matter or energy with a set of interference mechanisms, via the imaginary dimensions and spanning an arbitrarily large distance in the real dimensions?
5. Could matter or energy be transmitted from one place to another using entanglement instead of interference experiments? Can a small thing like an electron be entangled with a large thing like a molecule? Can a small molecule be entangled with a larger one? If so, then by measuring the smaller object in one place, could the larger part of the quantized wave be forced to appear somewhere else? Could one put a chain of such experiments in series to transmit matter or energy with a set of entanglement mechanisms, via the imaginary dimensions and spanning an arbitrarily large distance in the real dimensions?

6. Entangled waves and interfering waves can both span large distances in the real dimensions relative to their wavelengths. Are the distances spanned comparable for both processes? What measurable factors are experimentally associated with the distances spanned? Wavelength? Energy? Other?
7. Does the rate of quantum tunneling depend on the percent of time that energy spends in the real dimensions? What features of the quantized waves representing the molecules or their electrons in a "wall" encourage or discourage tunneling through the wall? Similarly, what features of the quantized waves representing the tunneling "particle" will increase or decrease its chances of tunneling through a "wall"?

Appendix A – Probability Data

Calculations underlying my reproduction of Feynman's Figure 8.2, the graph of P1 and P2 versus the angle θ representing time t found in section "3) A Quantum Mechanical Example."

t	P1 = Cos^2(A0 t/ ħ)	P2 =Sin^2(A0 t/ ħ)
0.00	1.000	0.000
0.39	0.854	0.146
0.79	0.500	0.500
1.18	0.146	0.854
1.57	0.000	1.000
1.96	0.146	0.854
2.36	0.500	0.500
2.75	0.854	0.146
3.14	1.000	0.000
3.53	0.854	0.146
3.93	0.500	0.500
4.32	0.146	0.854
4.71	0.000	1.000

Appendix B – Derivations

Derivation of the Real and Imaginary parts of the wave equation $C_1(t)$:

$$C_1(t) = e^{-(i/\hbar)E_0 t} \cos(A\, t/\hbar) \quad \text{Feynman's Equation (8.52)}$$

Replacing "$e^{-i\,x}$" with "cos x - i sin x" gives:

$$C_1(t) = \left[\cos\left((E_0 t/\hbar)\right) - i\, \sin\left((E_0 t/\hbar)\right)\right] \cos(A\, t/\hbar)$$

Rearranging terms:

$$C_1(t) = \cos(E_0 t/\hbar)\cos(A\, t/\hbar) - i\, \sin(E_0 t/\hbar)\, \cos(A\, t/\hbar)$$

Separating the Real from the Imaginary parts:

$$Re\left(C_1(t)\right) = \cos(E_0 t/\hbar)\, cos(A\, t/\hbar) \qquad \text{This papar's equation (A)}$$

$$Im(C_1(t)) = -\sin\left((E_0 t/\hbar)\right)\cos(A\, t/\hbar) \qquad \text{This papar's equation (B)}$$

Derivation of the Real and Imaginary parts of the wave equation $C_2(t)$:

$$C_2(t) = ie^{-(i/\hbar)E_0 t} \sin(A\, t/\hbar) \quad \text{Feynman's Equation (8.53)}$$

Replacing "$e^{-i\,x}$" with "cos x - i sin x" gives:

$$C_2(t) = i\,[\cos(E_0 t/\hbar) - i\, \sin(E_0 t/\hbar)]\, \sin(A\, t/\hbar)$$

Rearranging terms:

$$C_2(t) = i\cos(E_0 t/\hbar)\sin(A\, t/\hbar) - i^2\, \sin(E_0 t/\hbar)\, \sin(A\, t/\hbar)$$

Rearranging terms and simplifying $i^2 = -1$:

$$C_2(t) = +\, \sin(E_0 t/\hbar)\, \sin(A\, t/\hbar) + i\cos(E_0 t/\hbar)\sin(A\, t/\hbar)$$

Separating the Real from the Imaginary parts:

$$Re(C_2(t)) = \sin(E_0 t/\hbar)\, \sin(A\, t/\hbar) \qquad \text{This papar's equation (C)}$$

$$Im(C_2(t)) = \cos(E_0 t/\hbar)\, \sin(A\, t/\hbar) \qquad \text{This papar's equation (D)}$$

Appendix C – Data Table

Here are the formulas in the spreadsheet used to create the data table:

Column Formula

A. Row number, starting with 0 in increments of 1: [0, 1, 2, 3 …]
B. Pi = 3.141592654…
C. Time t = (A x B) / 64
D. $\cos(E_0 t)/\hbar = \cos(C \times 10)$ [Time is multiplied by 10 because E_0 cycles 10 times faster than A]
E. $\sin(E_0 t)/\hbar = \sin(C \times 10)$
F. $\cos(At)/\hbar = \cos(C)$
G. $\sin(At)/\hbar = \sin(C)$
H. $\mathrm{Re}(C_1(t)) = D \times F$ [Real part of $C_1(t)$]
I. $\mathrm{Im}(C_1(t)) = E \times F$ [Imaginary part of $C_1(t)$
J. $\mathrm{Re}(C_2(t)) = E \times G$
K. $\mathrm{Im}(C_2(t)) = D \times G$
L. $P_1(t) = (H \times H) + (I \times I)$ [Probability that the system is in state $|\ 1 >$]
M. $P_2(t) = (J \times J) + (K \times K)$

Here are the first few rows of the data table:

A	B	C	D	E	F	G	H	I	J	K	L	M
#	Pi	t	Cos E_0	SinE_0	Cos A	Sin A	Re(C_1)	Im(C_1)	Re(C_2)	Im(C_2)	P_1	P_2
0	3.14	.000	1.000	0.000	1.000	0.000	1.000	0.000	0.000	0.000	1.000	0.000
1	3.14	.049	0.882	0.471	0.999	0.049	0.881	(0.471)	0.023	0.043	0.998	0.002
2	3.14	.098	0.556	0.831	0.995	0.098	0.553	(0.827)	0.081	0.054	0.990	0.010
3	3.14	.147	0.098	0.995	0.989	0.147	0.097	(0.984)	0.146	0.014	0.978	0.022
4	3.14	.196	(0.383)	0.924	0.981	0.195	(0.375)	(0.906)	0.180	(0.075)	0.962	0.038
5	3.14	.245	(0.773)	0.634	0.970	0.243	(0.750)	(0.615)	0.154	(0.188)	0.941	0.059
6	3.14	.295	(0.981)	0.195	0.957	0.290	(0.939)	(0.187)	0.057	(0.285)	0.916	0.084
7	3.14	.344	(0.957)	(0.290)	0.942	0.337	(0.901)	0.273	(0.098)	(0.322)	0.887	0.113
8	3.14	.393	(0.707)	(0.707)	0.924	0.383	(0.653)	0.653	(0.271)	(0.271)	0.854	0.146
9	3.14	.442	(0.290)	(0.957)	0.904	0.428	(0.262)	0.865	(0.409)	(0.124)	0.817	0.183
10	3.14	.491	0.195	(0.981)	0.882	0.471	0.172	0.865	(0.462)	0.092	0.778	0.222
11	3.14	.540	0.634	(0.773)	0.858	0.514	0.544	0.663	(0.397)	0.326	0.736	0.264
12	3.14	.589	0.924	(0.383)	0.831	0.556	0.768	0.318	(0.213)	0.513	0.691	0.309
13	3.14	.638	0.995	0.098	0.803	0.596	0.799	(0.079)	0.058	0.593	0.645	0.355
14	3.14	.687	0.831	0.556	0.773	0.634	0.643	(0.429)	0.352	0.527	0.598	0.402
15	3.14	.736	0.471	0.882	0.741	0.672	0.349	(0.653)	0.592	0.317	0.549	0.451

References

[1] Pflegor, R.L; Mandel, L (1967) Phys. Rev **159**, p1084. Also available online at "http://journals.aps.org/pr/abstract/10.1103/physrev.159.1084"

[2] Randall, L (2006) "Warped Passages – Unraveling the Mysteries of the Universe's Hidden Dimensions" Harper Collins Publishers Ney York, NY, p134-135.

[3] Nariz, O; Arndt, M; Zellinger,A (2003) "Quantum Interference Experiments with Large Molecules" American Journal of Physics 71: 319-325. PDF available online at "https://https://vcq.quantum.at/fileadmin/Publications/2003-17.pdf"

[4] Cramer, J.G. (2016) "The Quantum Handshake – Entanglement, Nonlocality and Transactions" Section 2.6 "The Copenhagen Interpretation (1927)" p24-28 Springer International Publishing, Switzerland.

[5] Ibid. p25.

[6] Griffiths,C. (2005) "Introduction to Quantum Mechanics" Pearson Prentice Hall, Upper Saddle River NJ, 2nd Ed. P4.

[7] Ibid. Section 1.1 "The Schrödinger Equation" pp1-2

[8] Hoecker, A (CERN) Marciano, W.J (BNL) (2013) "The Muon Anomalous Magnetic Moment" available on the web at "http://pdg.lbl.gov/2015/reviews/rpp2015-rev-g-2-muon-anom-mag-moment.pdf".

[9] Feynman, R.P; Leighton,,R.B, Sands, M. (1963) "The Feynman Lectures on Physics – Mainly Mechanics, Radiation, and heat Volume I" Second Printing – July 1064, Section 15-9, "Equivalence of mass and energy" p15-10f.

[10] Griffiths,C. (2005) "Introduction to Quantum Mechanics" Pearson Prentice Hall, Upper Saddle River NJ, 2nd Ed.. Section 1.5, "Momentum" pp 15-18.

[11] Feynman,R.P; Leighton,R.B; Sands, M (1965) "The Feynman Lectures on Physics – Quantum Mechanics - Volume III" Addison-Wesley Publishing Company, Reading, Massachusetts. Section 4-1 "Bose particles and Fermi particles" p4-1ff.

[12] Griffiths,C. (2005) "Introduction to Quantum Mechanics" Pearson Prentice Hall, Upper Saddle River NJ, 2nd Ed. Section 2.2 The Infinite Square Well pp30-40

[13] Ibid. Section 2.4 "The Free Particle", pp59-67.

[14] Ibid. Section 2.1, "Stationary States", pp24-30.

[15] Ibid, Section 2.4 "The Free Particle", pp59-67.

[16] Ibid, Section 3.1, "Hilbert Space", pp93-96.

[17] Feynman, R.P; Leighton,,R.B, Sands, M. (1963) "The Feynman Lectures on Physics – Mainly Mechanics, Radiation, and heat Volume I" Second Printing – July 1064, Chapter 22, "Algebra", pp22-1 thru 22-10.

[18] Griffiths,C. (2005) "Introduction to Quantum Mechanics" Pearson Prentice Hall, Upper Saddle River NJ, 2nd Ed., Section 3.2 "Observables, pp 96-100.

[19] Feynman,R.P; Leighton,R.B; Sands, M (1965) "The Feynman Lectures on Physics – Quantum Mechanics - Volume III" Addison-Wesley Publishing Company, Reading, Massachusetts. "Section 8-6 The ammonia molecule" pp8-11 through 8-14.

[20] Ibid, Section 9.1, "The states of an ammonia molecule", pp9-1 thru 9-5.

[21] Randall, L (2006) "Warped Passages – Unraveling the Mysteries of the Universe's Hidden Dimensions" Harper Collins Publishers Ney York, NY, Section I.2 "Restricted Passages: Rolled-up Extra Dimensions" pp31-49.

[22] Ibid, Figure 86 "Infinite warped spacetime with a single Brane." P420.

[23] Feynman,R.P; Leighton,R.B; Sands, M (1965) "The Feynman Lectures on Physics – Quantum Mechanics - Volume III" Addison-Wesley Publishing Company, Reading, Massachusetts. p8-12.

[24] Cramer, J.G. (2016) "The Quantum Handshake – Entanglement, Nonlocality and Transactions" Section 2.5 Heisenberg's Microscope (1927) p22f.

[25] Feynman, R.P; Leighton,,R.B, Sands, M. (1963) "The Feynman Lectures on Physics – Mainly Mechanics, Radiation, and heat Volume I" Second Printing – July 1064, Section 28-3 "The dipole radiator" and Chapter 29, "Interference" p28-5 thru 29-7.