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A Quadruplet of Numbers

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Three real numbers are introduced via related infinite series. With e, together they complete a quadruplet.

The fundamental mathematical constant *e* is well defined, among other ways, via the sum of the infinite series

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots \approx 2.7182818\ldots$$

Following that way one might also examine the next, rather interesting, series of primes

$$e_p = \sum_{p} \frac{1}{p!} = \frac{1}{2!} + \frac{1}{3!} + \frac{1}{5!} + \cdots \approx 0.6751984\ldots$$

This series, i.e. the number e_p, to the best author's knowledge and surprisingly enough, has not been in focus of a proper attention[i] if any at all. Anyway, the questions regarding general characteristics of e_p, such as its irrationality, transcendence or possible connections with other constants, could have been raised as well. It is especially so when we know about long term interest in primes, their distribution etc.

Along the similar line of reasoning two more convergent series as well as related numerical constants can be introduced equally. Now we use the function *primorial*[ii] over both natural and prime numbers, hence

$$\tau = \sum_{n=0}^{\infty} \frac{1}{n\#} = 1 + \frac{1}{1\#} + \frac{1}{2\#} + \frac{1}{3\#} + \cdots \approx 2.9200509\ldots$$

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and

$$\tau_p = \sum_p \frac{1}{p\#} = \frac{1}{2\#} + \frac{1}{3\#} + \frac{1}{5\#} + \cdots \approx 0.7052301 \ldots$$

We may speculate here about meaning of τ let alone τ_p. Could we say that τ, in analogy to e, rules over one to it related mathematics of primes? Next, all the questions about irrationality and transcendence of both τ and τ_p are opened too. Accordingly, we may ask ourselves about the nature and meanings of, e.g. τ^e, τ^π, τ^τ, e^τ, log_τ and so forth.

Finally, noticing kind of equivalence $e \sim e_p$ <=> $\tau \sim \tau_p$, we intuitively guess the existence of a numerical quadruplet.

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Notes

[i] The question, however, has also been raised elsewhere; see at Sum of reciprocals of primes factorial, Nov 2013.

[ii] We use this version of it, i.e.

$$n\# \equiv \prod_{i=1}^{\pi(n)} p_i \; ; \; \pi(n) \text{ is the prime-counting function.}$$

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