# ROTATION CURVES OF THE COSMIC OBJECTS AND ATTRACTIVE FORCE IN THE UNIVERSE

Tamara Kudykina[1], Alexander Pervak

## ABSTRACT

A hydrodynamic model of the motion of galaxies, planets and moons is proposed in this paper. The solution of the Euler equation gives the description of the rotation curves with the positive parts of the Bessel functions $J_1(\beta r)$, where $r$ is the distance from the object to the axis of rotation, and $\beta$ — is the parameter depending upon the angular velocities, the dimension of the system and the velocity of the progressive motion of the system. In the dimensionless units the case with $\beta >> 1$ corresponds to the rotation curves of the planets and moons and in this limit coincides with the Kepler-Newton law. In the case of parameters $\beta \leq 1$ we have the rotation curves of the galaxies. The hydrodynamic theory describes the rotation curves of both the galaxies and the planets systems without invoking dark matter hypothesis.

Within the limit of motion of the cosmic objects in the ideal medium, the expression of the generalized attractive force is derived. In the case of planets ($\beta >> 1$), the form of the attractive force coincides with the Newton law; in the case of the galaxies, the attractive force differs sufficiently from the Newton law.

The distribution of the energy-density for the cosmic objects is obtained. For the Solar system, the distances between the planets and the Sun are derived. For small planets, the calculation agrees well with the observed values at the parameters $\beta \approx 80 \div 90$ and for the planets-giants — at $\beta \approx 40 \div 50$.

*Key words:* hydrodynamics; galaxies: kinematics and dynamics; cosmology: dark matter; gravitation.

[1] "Ukraina" University, Kiev, 04071, Ukraine

## 1. INTRODUCTION

The planets of the Solar system have a rotation curve which is well-described by the Kepler-Newton law: the orbital velocity, $V(R)$, is equal to $V(R) = \sqrt{\frac{MG}{R}}$ where $M$ is the central mass, $G$ is the gravitation constant and $R$ is the distance from the planet to the Sun.

The attempts to use the Kepler-Newton law for the description of the rotation curves of the galaxies, have led to failure. The galaxies rotation curves are varied and differ strongly on the Keplerian form. L.Volders was one of the first scientists who noticed this distinction. In 1959, he demonstrated (Volders 1959), that the rotation curve of the spiral galaxy M33 differed strongly from the Kepler law. During the 1960s-1980s, V.Rubin and K.Ford, and O.Sofue , V.Rubin investigated the spiral galaxies(Rubin & Ford 1970), (Sofue & Rubin 2001). They showed that there are three types of the rotation curves, and all of them differ from the Keplerian form . Their results have been confirmed over subsequent decades. (Faber & Gallagher 1979); (Bosma et al. 1992); (Zasov & Hoperskov 2003); ( Stabile & Capozziello 2013); (Roscoe 1997); (Noordermeer 2007 ); (Begeman et al. _1991); (Moran et al. 2007); (Simard & Pritchet 1998); (Wechakama & Ascasibar 2011); (Lukash et al. 2011); (Doroshkevich et al. 2012);

Previously, astronomers thought that disk galaxies had mass distribution similar to the observed shining distributions of stars and gas, therefore the orbital speed would decline with the increasing distances in the same way as do the planets of the Solar System or moons of Earth, Jupiter et al. But it is not the case. Moreover, the rotation curves of the spiral galaxies are often asymmetric.

The galaxies masses calculated from the observed rotation curves and law of gravity, and the mass profiles of galaxies, calculated from the luminosity profiles and the mass-to-light ratio in the stellar disks, do not match one another. The rotation curves imply that the mass continues to increase linearly with radius. Therefore, it has been postulated that a large amount of dark matter what extends galaxy into the galaxy's halo and permits to explain the observed rotation curves.

To calculate the speeds of the stars in the galaxies and the gas clouds speeds, the Doppler Effect is used. The measurement of the frequency (more exactly- the frequency shift relative to its position in free reference frame) is carried out on different spectrographs and optical interferometers. However, these methods allow to measure the spectrum velocities not of one star (there are a lot of stars in the galaxy), but some average integrated spectrum of emission, emitted by the large amount of indistinguishable emitters-stars.

The stars velocities are determined using the optical range of spectrum. The velocities of the gas clouds are measured also with the use of radio irradiation (usually with the help of frequency of the neutral hydrogen, it is the most widespread in the Universe, ($\lambda = 21$ cm), and of CO-molecules line in mm – range).

Unfortunately, the accuracy of such a method of calculation $V(R)$ is small due to the contribution to the frequency shift both the Doppler effect (due to the approximately progressive orbital motion) and the Sagnac effect (Sagnac 1913, 1914), which is caused by the star rotation around its axis. Perhaps, namely the considerable influence of the Sagnac effect leads to the asymmetry of the rotation curves, which is often observed.

Therefore, the calculation of true rotation curve $V(R)$ for the galaxies on the basis of the observed data, averaged on a large amount of stars, is very difficult.

Several alternative hypotheses were proposed to explain the discrepancies of the observed rotation curves from the Kepler-Newton form.

1. **F. I. Cooperstock and S. Tieu (Cooperstock & Tieu 2005)**

thought that the Newton linear gravity is not applicable to the galaxies and it is necessary to account the relativity nonlinearity, although the velocities of the stars and gas clouds in galaxies are not large enough, the relativistic effects to take place. They have proposed the stationary model of galaxy as liquid which don't undergo pressure and have the axial symmetry. From the equations of the general theory of relativity for this model, they have obtained two equations (linear and nonlinear) which connect the angular velocity and density of liquid. For some galaxies (the Milky Way, NGC 3031, NGC 3198, NGC 7331) they have obtained the rotation curves similar to observed ones, which correspond well to the distribution of the mass density of the usual visible matter, which are present in the galaxies disks.

F. I. Cooperstock and S. Tieu concluded that it is unnecessary to introduce the dark matter.

But their theory cannot explain the rotation curves of a lot of galaxies.

2. **John Moffat (Brownstein & Moffat 2006)**

have proposed a nonsymmetrical gravitational theory with incorporated a symmetric field (gravity) and an antisymmetric field. He supposed that the antisymmetric component is another manifestation of gravity, and it may be massive. These two fields modify the strength of gravity at large distances. This theory describes the rotation curves and the mass profiles of X-ray galaxy clusters without invoking dark matter.

**3. Mordehai Milgrom (Milgrom 1983)** proposed the “modified Newton dynamics”, “modified gravity”, MOND, to explain the fact that the observed velocities in the galaxies are greater than the Newton mechanics prescribes. He proposed that the gravity force is not proportional to the centripetal acceleration but to its square. Then it is possible to match the calculated rotation curves to the observed ones in some cases. For example, for spiral galaxies.

There are several other prepositions to avoid invoking dark matter.

We propose the new natural approach to this problem, without artificial introduction of doubtful fields and accelerations: the hydrodynamic description of the motion of all cosmic objects: galaxies, stars, planets, moons. This has led to fruitful results.

## 2. THE MOTION OF THE COSMIC OBJECTS

The galaxies consist of a huge community of stars, quasars, gas clouds, consisting of (according to modern knowledge) high temperature plasmas, the gaseous clouds of hydrogen, helium and some other light elements, molecules CO et al. (heavy elements and solid state materials, perhaps, are present only on small and cold cosmic objects- planets, asteroids et al.). Therefore, it is natural to use hydrodynamics to describe the motion of cosmic objects.

The hydrodynamic model is productive for the description of motion of the cosmic objects. Most of them rotates (with the angular velocity $\vec{\Omega}$) and simultaneously move progressively with some velocity $V_0$. This is a motion along the screw line. Consider the rotation around $z$ axis. The left and right screw lines are possible. It’s convenient to use the cylindrical coordinates. The parametric equations for the motion along the screw lines:

$$\rho = a; \varphi = \pm\Omega t; z = V_0 t$$

We’ll consider one of them with sign (+). Introduce the vector of velocity for an element of the cosmic matter: $\vec{U}(U_r, U_\varphi, U_z)$. At first, we consider the propagation of the cosmic object through the ideal media, without collisions. It can be considered as incompressible liquid. In the case of rotation, the Euler equation has the form:

$$\frac{\partial \vec{U}}{\partial t} + (\vec{U}\cdot\nabla)\cdot\vec{U} + 2\cdot[\vec{\Omega}\times\vec{U}] = -\frac{1}{\rho_f} grad(p) \quad (1)$$

Suppose, that small vibrations of media take place when the cosmological object moves, and neglect the second term on the left side (Landau & Lifshitz 1986). The Euler equation has the form:

$$\frac{\partial \vec{U}}{\partial t} + 2\cdot[\vec{\Omega}\times\vec{U}] = -\frac{1}{\rho_f}\nabla p' \qquad (2)$$

Here the second term on the left side is the Coriolis force (with the sign (-)), $\nabla p'$ — is the gradient of the variable part of the pressure in a medium, included the centrifugal force $\frac{1}{2}\nabla[\vec{\Omega}\times\vec{r}]^2$ and other possible forces in the Universe (Landau & Lifshitz 1986).

The solution of the equation (2) is given in Appendix 1. In the dimensionless values the components of the velocity are equal to:

$$U_r(r,t) = C\cdot\exp(i\omega t)\cdot J_1(r\cdot\beta)$$

$$U_\varphi(r,t) = iC\exp(i\omega t)\cdot\frac{2\Omega}{\omega}\cdot J_1(r\beta) \qquad (3)$$

$$U_z(r,t) = iC\cdot\exp(i\omega t)\cdot\sqrt{\frac{4\Omega^2}{\omega^2}-1}\cdot J_0(r\beta)$$

Here $r = \frac{r_1}{R_0}; \beta = \frac{\omega R_0}{V_0}\sqrt{\frac{4\Omega^2}{\omega^2}-1};$

$r_1$ is a dimension coordinate, $R_0$ – is some characteristic dimension of the cosmic object. As the velocity is positive value, then only positive parts of the Bessel functions have the physical sense. The possible meanings of the frequency $\omega$ are restricted with the condition $\omega < 2\Omega$, because only in this case the equation for $U_r$ has the finite solutions. As a result, we have the quantized function on $r$ for all three components of velocity.

## 3. THE ROTATION CURVES OF THE COSMIC OBJECTS

The obtained results for the velocity components allow conducting an analysis of the possible rotation curves of the cosmic objects. The orbital velocity is proportional to $U_\varphi(r,t)$ component: $U_\varphi \sim \frac{2\Omega}{\omega}\cdot J_1(r\beta)$. The Bessel function *J1(x),* has asymptotic form for *x>>1* :

$$J1(x) \to \sqrt{\frac{2}{\pi}}\cdot\frac{1}{\sqrt{x}}\cdot\cos(x-\frac{3\pi}{4}) \qquad (4)$$

The positive part of it coincides with the Kepler-Newton law. In Fig.1, the rotation curves for the Solar system are given: symbols — the orbital velocities of nine planets of the Solar system; dotted line — the Kepler law and solid line — the Bessel function *J1(x)* with $r = \frac{r_1}{R_0} = 0 \div 1; \beta = 200; R_0 = 40 a.u.$. In the case of the planets $\beta >> 1$.

The form of the rotation curves depends on the parameter $\beta$ ( fig 2).

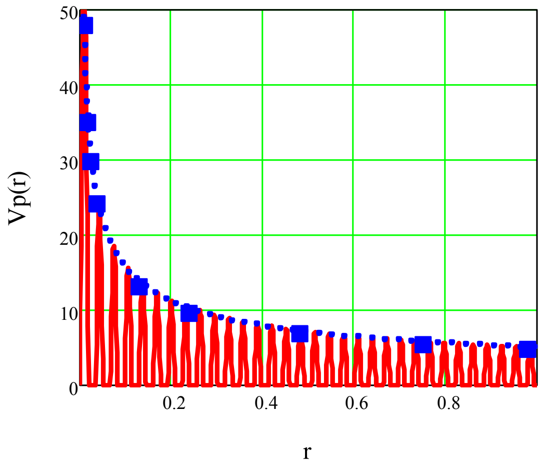



**Fig. 1 The rotation curves for the Solar system are given: symbols – the orbital velocities of nine planets of the Solar system; dotted line – the Kepler-Newton law and solid line – the Bessel function with** $\beta = 200$.

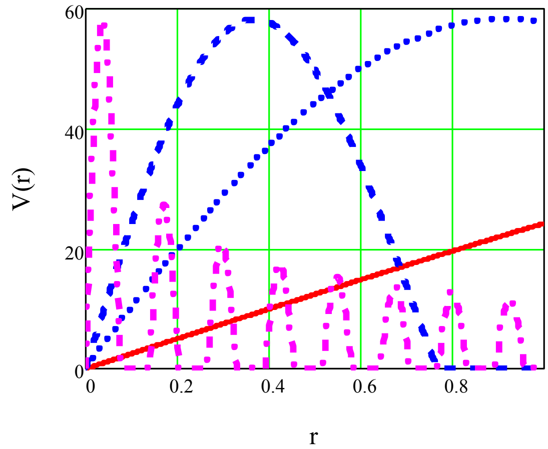



**Fig. 2 The rotation curves for the parameter = 0.5 (solid line); = 2 (dotted line); = 5 (dashed line); = 50 (dot-and-dash line).**

In Fig. 2, it is seen that the form of the rotation curves depends on the parameter $\beta$. The case $\beta >> 1$ corresponds to the rotation curves of the planets and moons and coincides with the Kepler law. Hence, the Kepler approach is good for the planets. The other limited case, $\beta \le 1$, corresponds to the rotation curves of the stars and gas clouds in the galaxies.

In Fig.3 the rotation curves are given in the case of a group of stars with the different parameters $\beta$. It is seen, that they are close to the measured ones. It agrees with the fact that the measured curves corresponds to the group of different stars with different rotation parameters $\beta$ .

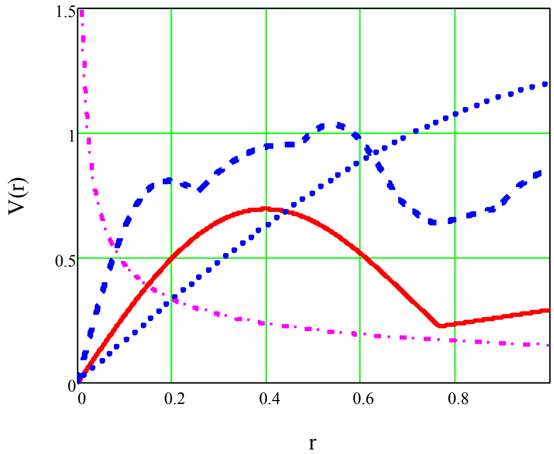



**Fig. 3 The rotation curves at the sum of the stars parameters: β=0.1; 0.5;5 (solid line); β=0.1; 0.5; 0.7; 2 (dotted line); β=0.1 ; 0.5; 2; 5; 15 (dashed line); β=50 — dot-and-dash line (the Kepler-Newton law).**

If the motion of the galaxy takes place in medium with some "viscosity", we have to use the Navier-Stokes equation. At the same conditions, the linear approach and the incompressible medium ($div\vec{U} = 0$) the equation has the form:

$$\frac{\partial \vec{U}}{\partial t} + 2 \cdot [\vec{\Omega} \times \vec{U}] - \vec{D} \cdot \Delta \vec{U} = -\frac{1}{\rho} grad(p) \quad (5)$$

Where the components of the coefficient of "viscosity" $D_i$ , generally speaking, can be different for different coordinates. In the ideal medium all coefficients of "viscosity" are equal to zero and we return to the Euler equation. If we take account the coefficients of "viscosity" for three coordinates, we'll have the equation of high order which can be solved only numerically. Therefore, to estimate the influence of "viscosity", we consider the more simple case — of the "viscosity" only along radial direction (as the most relevant).

The solution of the equation (5) is given in *Appendix 2*. In the dimensionless values the components of the velocity are equal:

$$U_r(r,t) = C \cdot \exp(i\omega t) \cdot J_1(r \cdot b)$$

$$U_\varphi = -C_2 \frac{2\Omega}{i\omega} \cdot \exp(i\omega t) \cdot J_1(r \cdot b) \quad (6)$$

$$U_z(r,t) = iC_3 \cdot \frac{V_0}{i\omega} \exp(i\omega t) \cdot b \cdot J_0(rb)$$

Here

$$b = R_0 \cdot \sqrt{\frac{4\Omega^2 - \omega^2 + i\omega D_r \dfrac{\omega^2}{V_0^2}}{V_0^2 - i\omega D_r}} \to \beta$$

$$(when(D_r \to 0))$$

Now the parameter $b$ — is complex, therefore, in this case, the solutions of the equation are the Bessel functions of the complex variable.

The approximate solutions of another two cases — the "viscosity" in the directions $\varphi, z$ , are the Bessel functions of the complex variable as well, with something different parameters $b$ .

In Fig.4 the calculated rotation curves at the small "viscosity" in the radial direction are given for different parameters $b$. (Only positive parts of $b$ are present).

Probably, in the inner parts of the galaxies the "viscosity" is absent and the case of zero velocity at $r \to 0$ and the ideal medium is realized (as in Fig. 5, at $r \to 0$ ). For many galaxies the increase of the velocity near the center of the axis of rotation is observed, then — the flat regions, which can correspond to the parameters $b \approx 1 \div 5$ .

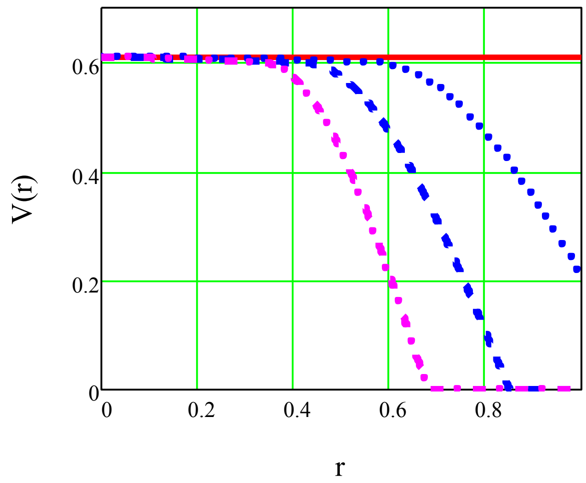



**Fig.4. The rotation curves of galaxies with accounting of the small "viscosity" at the parameters: $b$ =0.1 (solid line); $b$ =3 (dotted line); $b$ =4 (dashed line); $b$ =5 - (dot-and-dash line).**

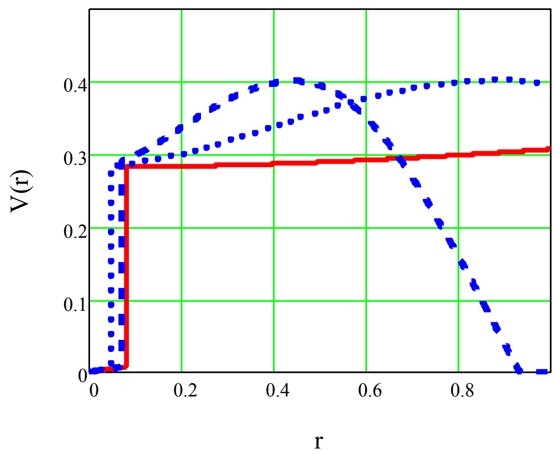



**Fig.5. The rotation curves of galaxies at accounting of the small "viscosity", except the region near the center of the galaxy, at the parameters: *b*=0.5 (solid line); *b*=2 (dotted line); *b*=4 (dashed line).**

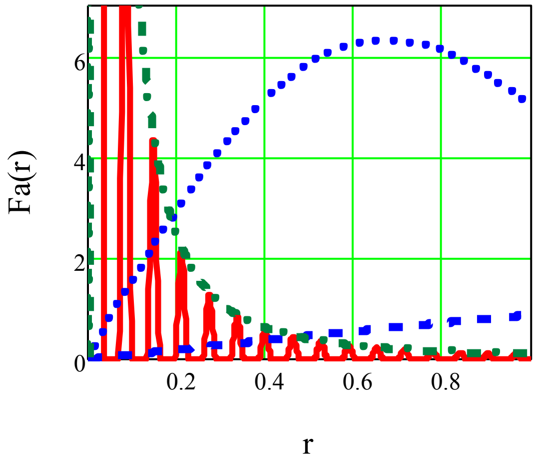
## 4. THE ATTRACTIVE FORCE

Let us estimate the attractive force, which is peculiar both to the stars in the galaxies and the planets in the star systems. Consider the motion of the cosmic object on the circular orbit around the galaxy (or star) center and assume that the attractive force is balanced against the centripetal force of this object. If the medium is ideal, the velocity of an object is determined by the Euler equation and the orbital velocity is proportional to the positive part of the Bessel function: $U_\varphi(r,t) = iC\exp(i\omega t)\cdot\frac{2\Omega}{\omega}\cdot J_1(r\beta)$ . The attractive force is equal to: $F_a = \rho V\frac{U_\varphi^2}{r}$ , where $V$ is a volume of the object.

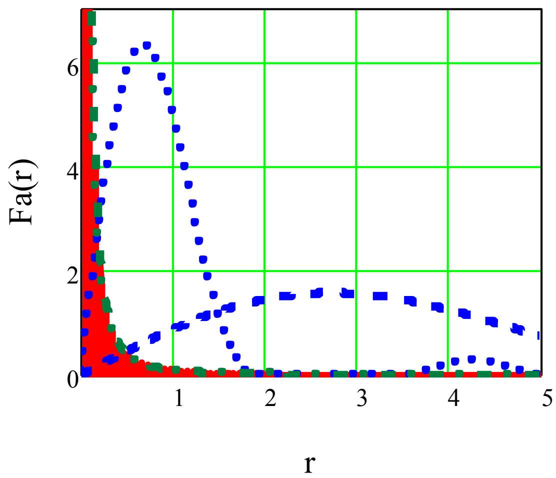
Note the positive part of the Bessel function $J_1(r\beta)$ as $G_1(r\beta)$ . The attractive force is equal:

$$F_a(r) = \rho V\frac{4C^2\Omega^2}{\omega^2}\cdot\frac{1}{r}\cdot[G_1(r\beta)]^2 \qquad (7)$$

In the limit of big parameter $\beta$ ($\beta >> 1$ for the planet), we have the Newton law: $F_a(r) = \frac{GMm}{r^2}$ . Here the parameter $GM$ corresponds to $\frac{4C^2\Omega^2}{\omega^2}$ .




(a)




(b)

**Fig.6. The dependence of the attractive force $F_a(r)$ on the distance from the rotation axis. The parameters: β=100 (planets, solid line); and for galaxies: β=2 (dotted line); β=0.5 (dashed line). The Newton law — dot-and-dash line.**

**a) Corresponds to the maximum distance $R_0$; b) the maximum distance 5 $R_0$**

In Fig. 6 the dependences of the attractive force $F_a(r)$ on the distance from the center of the rotation axis are given for the cosmic objects, both the planets and stars. It is seen that for the planet the attractive force coincides with the Newton law, at the edge of a system $F_a(r) \to 0$, whereas for the galaxies it is not the case: the attractive force is extended far from the edge of a system, it is not monotone. After zero point there are regions where forces are still strong enough. The form of $F(r)$ depends strongly on the parameter $\beta$.

The attractive force for the group of neighboring stars with different parameters $\beta$, can have more complicated form, for example, is present in Fig. 7. The stars attractive forces are extended far into space, much further than the Newton force and it can capture from the space other cosmic objects. Perhaps, owing to this fact, the large rarefied regions of gas clouds, asteroids and small planets exist far from the lighting regions of the galaxies.

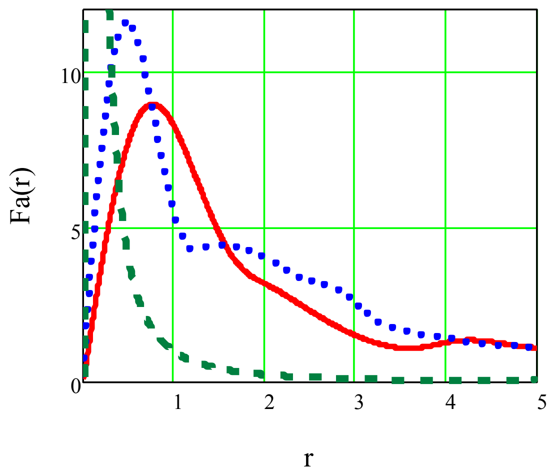



**Fig.7. The dependence of the attractive force $F_a(r)$ on $r$ in the case of the sum of different parameters β: β=(0.1 +0.3 + 1 + 2)(solid line); β=(0.1 + 0.5 + 1 + +3) (dotted line). Dashed line – the Newton law for the planets.**

## 5. THE PLANETS OF THE SOLAR SYSTEM

The obtained solutions mean that during the motion of the cosmic matter through space the regions with high density of matter and empty regions take place.

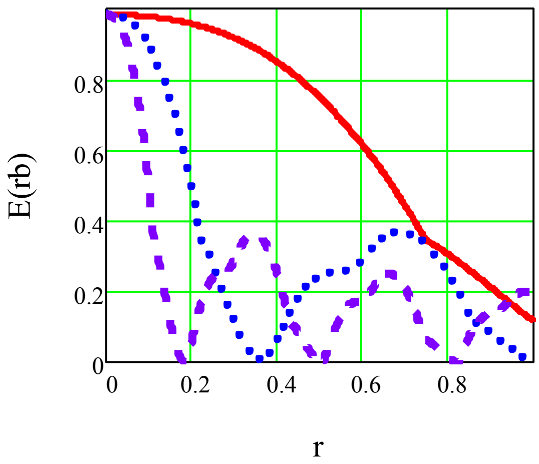
In Fig. 8 the dependence of the energy-density of the matter *W(r)* on radial coordinate *r* is present (in dimensionless coordinates):

$$W(r) = U_r^2(r) + U_\varphi^2(r) + U_z^2(r)$$

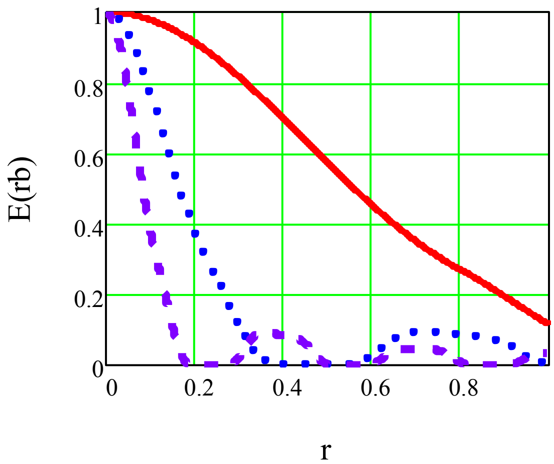
$$W(r) = -A^2[\frac{4\Omega^2}{\omega^2} - 1]\{J_1^2(r\beta) + J_0^2(r\beta)\} =$$

$$= -A^2[\frac{4\Omega^2}{\omega^2} - 1] \cdot E(r\beta) \quad (8)$$

It is seen that in the absence of "viscosity", the spherical part exists in the center, then the empty ring, then the ring of a matter and so on. This description is valid both for the galaxies and planets. Taking account of "viscosity" (scatter) leads to "washing out" of the outer rings.




(a)




(b)

**Fig.8. The dependence of the energy-density of matter *E(rβ)* on radial coordinate *r*. β=3 (solid line); β=10 (dotted); β=20 (dashed line). (a) — the ideal medium; (b) — the medium with the small "viscosity".**

The distances between the planets of the Solar system and Sun, evaluated according the maxima of the function dependence *E(rβ)*, with *β*=80 agree well for small planets and differ considerably for Jupiter and other gas giants (Table 1) .

**Table 1. The distances between the planets and the Sun.**

| *Planet* | Radius of the orbit, a.u., calculated | Radius of the orbit, a.u., measured |
|---|---|---|
| Mercury | 0.37 | 0.39 |
| Venus | 0.69 | 0.72 |
| Earth | 1 | 1 |
| Mars | 1.37 | 1.52 |
| Asteroid belt | 1.69 | 2.2-3.6 |
| Ceres | 2.02 | 2.096 |
| Jupiter | 2.37 | 5.2 |
| Saturn | 2.73 | 9.54 |
| Uranus | 3.05 | 19.22 |
| Neptune | 3.42 | 30.06 |
| Pluto | 3.73 | 39.5 |

The distances for the group of the planets-giants can be described with the maxima of the same dependence *E(rβ)*, but with lower values of the parameter *β*: these planets are more similar to stars in their size and content. If for small planets the parameter *β* is equal to $\beta \approx 80 \div 90$, then in the case of large planets, it is equal to $\beta \approx 40 \div 50$. Both solutions are "matched" in the region of the asteroid belt between Jupiter and Mars.

Probably, in the region of the "matching", neither the stationary orbit for the small, nor for the large planet can exist.

It is possible that the strange motion of the space crafts "Pioneer" is due to the additional forces near the planets-giants.

## 6. CONCLUSION

The hydrodynamic model of the motion of the cosmic objects (the Euler equation for the motion in the ideal medium and the Navier-Stokes equation in the"viscosity" medium) explains the rotation curves of both the galaxies and planet systems. The positive part of the Bessel functions describes the law of the motion. The Kepler-Newton law, which describes well the rotation curves of the planets and moons, is one of the limited cases ( $\beta >> 1$) of the general expression for the rotation curves. In the other limit case $\beta \le 1$, the obtained expression describes the rotation curves of the stars in the galaxies.

The approximate accounting of the "viscosity" of the medium leads to description of the motion with the Bessel functions with the complex variable and some differing parameters. (In the absence of the "viscosity" we return to the former solutions).

The research has shown that in the ideal medium, the distribution of the cosmic matter during its motion presents the systems of rings of matter and empty rings, around the density center. In the central part the density decreases at the edge of this region.

Our analysis has shown that in the galaxy center, the medium can be considered as ideal and the rotation curve starts with zero meaning. In regions farther from the galaxies centers, the account of the "viscosity" often leads to better results. The measured rotation curves correspond to a large group of stars with different parameters of rotation.

The proposed hydrodynamic model shows that the observed rotation curves can be explained without invoking dark matter.

The expression for the attractive force which is applicable both for the stars in the galaxies and the planets in the stars systems, was derived. It was obtained for the case of ideal medium and the motion of the cosmic object on the circular orbit around the center of rotation. In the case of $\beta >> 1$, which takes place for planets, this expression coincides with the Newton law. When $\beta \le 1$, the attractive force differs considerably from the Newton law and spreads much longer than to the edge of the galaxy.

The evaluation of the distances between the planets of the Solar system and Sun, shows that for the small planets there is good agreement with the observed ones with $\beta \approx 80 \div 90$, but not for the planets-giants. The agreement can be achieved if for the small planets we take the parameter $\beta \approx 80 \div 90$ and for the large planet $\beta \approx 40 \div 50$. The both solution can be "matched" in the region of the asteroid belt between the Mars and Jupiter. Probably, in the region of the "matching", the stationary orbits are absent.

The strange motion of the space crafts "Pioneer" can be explained with the influence of the force of the planets-giants.

Our investigation has shown that dark matter, dark energy and black holes don't exist in the Universe; the attractive force is caused by the motion without any special gravity field.

## APPENDIX 1

Let us solve the equation (2) in the parametric form. Write three components of this equation and the equation of the discontinuity, in variables $r,t$ (for the screw line $z = V_0 t$):

1. $$\frac{\partial U_r}{\partial t} - 2 \cdot \Omega U_\varphi = -\frac{1}{\rho} \cdot \frac{\partial p'}{\partial r}$$

2. $$\frac{\partial U_\varphi}{\partial t} + 2 \cdot \Omega U_r = 0$$

3. $$\frac{\partial U_z}{\partial t} = -\frac{1}{\rho \cdot V_0} \cdot \frac{\partial p'}{\partial t}$$

4. $\frac{\partial U_r}{\partial r}+\frac{1}{r}U_r+\frac{1}{V_0}\frac{\partial U_z}{\partial t}=0$

After transformations, we have the equations for all three components.

For the radial component $U_r$ :

$$\frac{1}{V_0^2}\cdot\frac{\partial^2 U_r}{\partial t^2}+[\frac{\partial^2 U_r}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial U_r}{\partial r}+$$
$$+(\frac{4\Omega^2}{V_0^2}-\frac{1}{r^2})\cdot U_r]=0 \quad \text{(A1)}$$

For the component $U_z$ we have the equation, which can be solved after the solving the equation (A1) :

$$\frac{\partial U_z}{\partial t}=-V_0\cdot\frac{1}{r}\frac{\partial(rU_r)}{\partial r} \quad \text{(A2)}$$

The third component is determined by the equation:

$$U_\varphi=\frac{1}{2\cdot\Omega}[\frac{\partial U_r}{\partial t}-V_0\cdot\frac{\partial U_z}{\partial r}] \quad \text{(A3)}$$

It can be solved, if the components $U_r$ and $U_z$ are known.

The equation (A1) can be solved with the division of the variables:

$$U_r(t,r_1)=T(t)\cdot R(r_1)$$

Taking account the finiteness of the functions, we have the solution in the Bessel functions:

$$U_r(r_1,t)=C\cdot\exp(i\omega t)\cdot J_1(r_1\cdot\beta_1)$$

where $\beta_1=\frac{\omega}{V_0}\sqrt{\frac{4\Omega^2}{\omega^2}-1}$

It is convenient to use the dimensionless values. Therefore, we introduce

$$r=\frac{r_1}{R_0};\beta=\beta_1\cdot R_0$$

where $R_0$ — is the approximate dimension of the cosmic object. Then we have:

$$U_r(r,t)=C\cdot\exp(i\omega t)\cdot J_1(r\cdot\beta)$$

$$U_\varphi(r,t)=iC\exp(i\omega t)\cdot\frac{2\Omega}{\omega}\cdot J_1(r\beta) \quad \text{(A4)}$$

$$U_z(r,t)=iC\cdot\exp(i\omega t)\cdot\sqrt{\frac{4\Omega^2}{\omega^2}-1}\cdot J_0(r\beta)$$

The constant of the integration $C$ can be found from the boundary conditions.

## APPENDIX 2

Let us solve the equation (5) in the parametric form. Due to the axial symmetry, all velocity components don't depend on the variable $\varphi$ . Write three components of this equation and the equation of discontinuity, in variables $r,t$ :

1. $\frac{\partial U_r}{\partial t}-2\cdot\Omega U_\varphi=-\frac{1}{\rho}\cdot\frac{\partial p'}{\partial r}+D_r\cdot[\Delta U_r-\frac{U_r}{r^2}]$

2. $v_0\frac{\partial U_\varphi}{\partial z}+2\cdot\Omega U_r=D_\varphi\cdot[\Delta U_\varphi-\frac{U_\varphi}{r^2}]$

(A5)

3. $\frac{\partial U_z}{\partial t}=-\frac{1}{\rho}\cdot\frac{\partial p'}{\partial z}+D_z\cdot\Delta U_z$

4. $\frac{1}{r}\cdot\frac{\partial(r\cdot U_r)}{\partial r}+\frac{\partial U_z}{\partial z}=0$

Suppose that the "viscosity" is only for the radial direction. Then, we have the system of equations:

1. $\frac{\partial U_r}{\partial t}-2\cdot\Omega U_\varphi=-\frac{1}{\rho}\cdot\frac{\partial p'}{\partial r}+D_r\cdot[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial U_r}{\partial r})+$
$+\frac{\partial^2 U_r}{\partial z^2}-\frac{U_r}{r^2}]$

2. $\frac{\partial U_\varphi}{\partial t} + 2 \cdot \Omega U_r = 0$

3. $\frac{\partial U_z}{\partial t} = -\frac{1}{\rho} \cdot \frac{\partial p'}{\partial z}$

4. $\frac{1}{r} \cdot \frac{\partial (r \cdot U_r)}{\partial r} + \frac{\partial U_z}{\partial z} = 0$

Taking into account the fact that in the case of ideal medium all components of the velocity are proportional to $\sim \exp(i\omega t)$, suppose the same time dependence in this case as well. Then we have the system of equations:

1.

$$i\omega U_r - 2 \cdot \Omega U_\varphi = -\frac{1}{\rho} \cdot \frac{\partial p'}{\partial r} + D_r \cdot [\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial U_r}{\partial r}) + \frac{\partial^2 U_r}{\partial z^2} - \frac{U_r}{r^2}] \quad \text{(A6)}$$

2. $i\omega U_\varphi + 2 \cdot \Omega U_r = 0$ (A7)

3. $i\omega U_z = -\frac{1}{\rho} \cdot \frac{\partial p'}{\partial z}$ (A8)

4. $\frac{1}{r} \cdot \frac{\partial (r \cdot U_r)}{\partial r} + \frac{\partial U_z}{\partial z} = 0$ (A9)

From the equation (A9) we have:

$$U_z = -\frac{V_0}{i\omega}\frac{1}{r} \cdot \frac{\partial (r \cdot U_r)}{\partial r} \quad \text{(A10)}$$

From the equation (A7) we have:

$$U_\varphi = -\frac{2\Omega}{i\omega} U_r \quad \text{(A11)}$$

The substitution of (A10) and (A11) into the equation (A6) gives the equation for the radial component $U_r$:

$$i\omega U_r + \frac{4\Omega^2}{i\omega} \cdot U_r = -\frac{V_0^2}{i\omega} \cdot \frac{\partial}{\partial r}[\frac{1}{r} \cdot \frac{\partial (r \cdot U_r)}{r}] + D_r \cdot [\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial U_r}{\partial r}) - \frac{\omega^2}{V_0^2} U_r - \frac{U_r}{r^2}]$$

After transformation:

$$\frac{\partial^2 U_r}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial U_r}{\partial r} + [b^2 - \frac{1}{r^2}] \cdot U_r = 0;$$

where $b = R_0 \cdot \sqrt{\frac{4\Omega^2 - \omega^2 + i\omega D_r \frac{\omega^2}{V_0^2}}{V_0^2 - i\omega D_r}}$ .

The solution of this equation is the Bessel function of the first order on the complex variable. The parameter $b$ is complex now.

$$U_r(r,t) = C \cdot \exp(i\omega t) \cdot J_1(r \cdot b)$$

$$U_\varphi = -C\frac{2\Omega}{i\omega} \cdot \exp(i\omega t) \cdot J_1(r \cdot b) \quad \text{(A12)}$$

$$U_z(r,t) = iC \cdot \frac{V_0}{i\omega} \exp(i\omega t) \cdot b \cdot J_0(rb)$$

All components are quantized here as well.

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