Regarding the scalar perturbations of small bodies; a link between gravitational nonlocality and quantum indeterminacy.

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Using the Friedman-Lemaitre-Robertson-Walker (FLRW) universe as a background metric, purely General relativistic (classical) scalar metric perturbations are investigated for small bodies. For the approximation of a point-like perturbing mass in the closed FLRW universe, the scalar perturbation may be written in a form obeying precisely the Dirac equation up to a factor playing the role of Planck's constant. A physical interpretation suggests the scalar perturbation in this form *is* the wavefunction of quantum mechanics. Such an interpretation indicates the nonlocality of gravitational energy/momentum in General relativity leads naturally to the indeterminacy of quantum mechanics. Some physical consequences and predictions are discussed and briefly explored.

INTRODUCTION

In General Relativity there is a very-well-known problem originating with the equivalence principle. In defining the energy/momentum density of the gravitational field at a point in space and time, one must resort to use of a gravitational pseudotensor.

The energy/momentum densities associated with the pseudotensor are decidedly unphysical as they may be chosen to take on any number of values at a given point in space. Instead for physical values, such quantities are defined *en-total*, by integrating said densities globally over an appropriate 3-volume. In this manner a total energy and momentum may be obtained. Such behavior is generally referred to as the *non-local nature* of gravitational energy and momentum [1–3].

Meanwhile in quantum mechanics, one may not, with any physical meaning, specify momentum/energy at a point in space/time. Instead for physical quantities, one integrates such momenta/energy (written as densities) globally, such that a total (average) energy/momentum is obtained. This is, at it's most general, referred to as quantum indeterminism (as often embodied by the Heisenberg uncertainty principle).

Regarding the gravitational energy/momentum, the principle of equivalence demands that a noninertial observer will witness a non-zero gravitational pseudotensor. Such a situation will come about even when there are no gravitational sources present (as first pointed out by Bauer [3]). In this manner a non-inertial (accelerating) observer necessarily calculates a non-zero energy in the space around them. By contrast, for any inertial frame, the equivalence principle requires the local vanishing of the gravitational pseudotensor.

The physical interpretation of this non-local behavior was a matter of some debate at general relativity's inception.

Meanwhile in quantum field theory, one has that a noninertial (accelerating) observer must necessarily witness a thermal energy in the surrounding space (often referred to as "Unruh radiation" or the "Unruh effect") [4, 5, 34]. It is the case however; that for any inertial observer the intensity of such radiation (as the Unruh) is observed to be zero.

The physical interpretation of this has been a matter of some debate among physicists.

There are clear parallels between these phenomena. On the General relativistic side, one has such behavior being ascribed to the non-local nature of the gravitational field; while on the other hand one refers to corresponding phenomena as being ultimately a result of the inherent indeterminacy of quantum physics.

The precise nature of the aforementioned gravitational phenomena necessarily depends upon the background metric and ultimately the global spacetime geometry.

It is natural then to consider the possibility of a spacetime geometry in which the two phenomena manifest in precisely the same way. Should such a spacetime (or class of spacetimes) exist, it then seems reasonable to entertain the notion that this could correspond to the physical spacetime of our universe.

The current paper is an (albeit imperfect) attempt to further explore this notion. The focus will be on attempting to find a formalism for writing the gravitational energy and momentum of a body, that coincides with that of a particle in quantum theory.

PROCEDURE

Historically physicists have tended to formulate the equations of General Relativity in terms of quantities which suppress or otherwise circumvent the non-local nature of the gravitational field. Some examples of this include Komar quantities, the ADM formalism, and various proposed notions of quasilocal mass/energy (a thorough introduction is given in [3].

For the current approach integral quantities of the metric (and it's perturbations) will be considered physical. In this respect the current treatment bears an initial similarity to those mentioned above.

A standard first order linearization of Einstein's field

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equation is used. This necessitates choosing a definite background metric. In the interest of large scale homogeneity and isotropy, the Friedmann-Lemaitre-Robertson-Walker (FLRW) universe is chosen.

The eigenfunction series expansions inherent in quantum theory and such procedures as the box quantization (in second quantization) which yield quantum field theory, lead the author naturally to consider spatial boundary conditions that are periodic. To correspond to this condition, the closed FLRW universe is chosen.

In considering a point-like mass in the closed FLRW universe, it is found that the scalar metric perturbation may be written in a form that obeys precisely the Dirac equation (up to the factor $\hbar$).

A physical interpretation necessitates that the scalar perturbation in this form *is* the wavefunction of quantum mechanics. This leads to a natural relationship between the nonlocality of gravitational energy/momentum and the indeterminism of quantum physics. Physical consequences of such a relationship are briefly discussed and predictions mentioned.

Throughout this paper the $(-+++)$ metric convention is adhered to. For the sake of brevity, four-volume elements of a manifold M will often be expressed as $\mathrm{d}\Omega$ and three-volume elements of foliations (∂M) of M will be denoted by $\mathrm{d}\Sigma$. The Einstein summation convention is utilized unless specified otherwise. Finally, for reasons that will become apparent, all physical constants will be kept in their standard form (i.e. *not* set equal to unity).

THE EINSTEIN-HILBERT ACTION.

The starting point for deriving the Einstein field equation (EFE) is the well-known Einstein-Hilbert action:

$$\mathcal{S} = \int\limits_M \left(\frac{c^4}{16\pi G_N} R + \mathfrak{L}_M\right)\sqrt{-g}d^4x \qquad (1)$$

Where R is the Ricci scalar curvature (the trace of the Ricci tensor R^μ_μ which itself is the contraction of the Riemann tensor $R^\alpha_{\mu\alpha\nu}$) and $\mathfrak{L}_M$ is the Lagrangian density of the matter/fields. When varied with respect to $g^{\mu\nu}$,

$$\delta\mathcal{S} = \int\limits_M \left(\frac{c^4}{16\pi G_N}\frac{\delta}{\delta g^{\mu\nu}}\left[R\sqrt{-g}\right] + \frac{\delta}{\delta g^{\mu\nu}}\left[\mathfrak{L}_M\sqrt{-g}\right]\right) d^4x\delta g^{\mu\nu} \qquad (2)$$

(1) is known to yield the famous Einstein field equation, written here as an integrand:

$$0 = \int\limits_M \left[\frac{c^4}{8\pi G_N} G_{\mu\nu} - T_{\mu\nu}\right]\sqrt{-g}d^4x\delta g^{\mu\nu} \qquad (3)$$

Where Hilbert's stress energy tensor is defined to be $T_{\mu\nu}\sqrt{-g} = \frac{\delta}{\delta g^{\mu\nu}}\left[L_M\right]\sqrt{-g} + L_M\frac{\delta}{\delta g^{\mu\nu}}\left[\sqrt{-g}\right]$, and the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$. Upon introducing a small perturbation (3) can be written as:

$$0 = \int\limits_M \left[\frac{c^4}{8\pi G_N}\left\{G_{\mu\nu} + \delta G_{\mu\nu}\right\} - \left\{T_{\mu\nu} + \delta T_{\mu\nu}\right\}\right]\sqrt{-g}d^4x\delta g^{\mu\nu}$$

By subtracting equation (3) from this we obtain our Einstein equation for a perturbation:

$$0 = \int\limits_{Ms} \left[\frac{c^4}{8\pi G_N}\delta G_{\mu\nu} - \delta T_{\mu\nu}\right]\sqrt{-g}d^4x\delta g^{\mu\nu} \qquad (4)$$

As (4) will hold for any variation $\delta g^{\mu\nu}$, one may write this as:

$$0 = \int\limits_M \left[\frac{c^4}{8\pi G_N}\delta G_{\mu\nu} - \delta T_{\mu\nu}\right]\sqrt{-g}d^4x \qquad (5)$$

Equation (5) kept within the integral, will be our starting point in considering perturbations to the metric.

Part I
The perturbed FLRW metric

Let us consider a perturbation to first order in the metric of the Friedmann-Lemaitre-Robertson-Walker (FLRW) Universe. As is standard, we consider a metric $\bar{g}_{uv}$ of the form:

$$\bar{g}_{uv} = (g_{uv} + h_{uv}) \qquad and \qquad \bar{g}^{uv} = (g^{uv} - h^{uv}) \qquad (6)$$

Where g^{uv} is the background metric (FLRW for our case), and h_{uv} is a small perturbation such that $\mid h_{uv} \mid \ll 1$. Such a system has been thoroughly studied in the literature. The general perturbation is known to be of the form [6]:

$$h_{\mu\nu} = \begin{bmatrix} 2g_{00}\phi & \omega^\mu \\ \omega_\mu & 2\gamma_{\mu\nu}\psi\delta^\mu_\nu + H_{\mu\nu}\end{bmatrix} \qquad (7)$$

Where ϕ and ψ are genuine scalar perturbations and $\gamma_{\mu\nu}$ refers to spatial components of the metric. The vector perturbation, $\omega = \omega^\perp + \omega^\parallel$ consists of rotational $\omega^\perp$ and irrotational $\omega^\parallel$ parts, and $H_{\mu\nu}$ represent the traceless tensor perturbations.

A. The Lorenz/harmonic Gauge

Choosing the Lorenz or harmonic gauge $\partial_\nu\bar{h}^{\mu\nu} = 0$ and utilizing the trace-reversed perturbation $\bar{h}_{\mu\nu} = h_{\mu\nu} -$

$\frac{1}{2}g_{\mu\nu}h$. The linearized Einstein field equation (EFE) is known [7, 8], to be of the form:

$$\int_M \{\frac{c^4}{16\pi G_N}(\Box\bar{h}^{\mu\nu} + 2R^{\mu\ \nu}_{\ \alpha\ \beta}\bar{h}^{\alpha\beta}) + T^{\mu\nu}\}\,\mathrm{d}\Omega = 0 \quad (8)$$

Where the field equation has been kept within the integral over the full spacetime manifold (M), as derived by varying the Einstein-Hilbert Action.

Equation (8), in flat space ($R^{\mu\ \nu}_{\ \alpha\ \beta} = 0$), is a typical starting point for demonstrating that General relativity reduces to Newtonian gravity in the appropriate weak-field and nonrelativistic approximation [23].

Since we are considering scalar perturbations, it is convenient to work with the trace of (8), which is of the form:

$$\int_M \{\frac{c^4}{16\pi G_N}(g_{\mu\nu}\Box\bar{h}^{\mu\nu} + 2g_{\mu\nu}R^{\mu\ \nu}_{\ \alpha\ \beta}\bar{h}^{\alpha\beta}) + T^{\mu}_{\mu}\}\,\mathrm{d}\Omega = 0 \quad (9)$$

Note that in taking the trace, the linearized Einstein equation is written entirely in terms of the trace of both the perturbed Einstein tensor δG, and the stress energy tensor of the perturbation source T. In this manner equation (9) deals only with gauge invariant quantities (as discussed by Bardeen in [33]).

Let us examine the first term of (9) separately, one has:

$$g_{\mu\nu}\Box\bar{h}^{\mu\nu} = g_{\mu\nu}\partial_\mu\partial^\mu\bar{h}^{\mu\nu} \quad (10)$$

This may be written as:

$$g_{\mu\nu}\partial_\mu\partial^\mu = \left[\partial_\mu - \frac{1}{4}g^{\mu\nu}\left(\partial_\mu g_{\mu\nu}\right)\right]g_{\mu\nu}\partial^\mu$$

Where the covariant metric $g_{\mu\nu}$ has been moved to the other side of the partial derivative. Similarly, one can also write:

$$g_{\mu\nu}\partial_\mu\partial^\mu = \left[\partial_\mu - \frac{1}{4}g^{\mu\nu}\left(\partial_\mu g_{\mu\nu}\right)\right]\left[\partial^\mu - \frac{1}{4}g^{\mu\nu}\left(\partial^\mu g_{\mu\nu}\right)\right]g_{\mu\nu}$$

Utilizing this expression in equation (10), one obtains:

$$-\partial_\mu\partial^\mu\left(\bar{h}^\mu_\mu\right) + \frac{1}{4}g^{\mu\nu}\left(\partial_\mu g_{\mu\nu}\right)\left(\partial^\mu g_{\mu\nu}\right)\bar{h}^{\mu\nu} - \partial_\mu\left[\left(\partial^\mu g_{\mu\nu}\right)\bar{h}^{\mu\nu}\right]$$

Finally taking the trace $\bar{h}^\mu_\mu = \bar{h} = -h$ and enforcing the gauge condition $\partial_\mu\bar{h}^{\mu\nu} = 0$ this becomes:

$$= -\left(\partial_\mu\partial^\mu h\right) - \left[\left(\partial_\mu\partial^\mu g_{\mu\nu}\right) - \frac{1}{4}g^{\mu\nu}\left(\partial_\mu g_{\mu\nu}\right)\left(\partial^\mu g_{\mu\nu}\right)\right]\bar{h}^{\mu\nu}$$

Which can be written as:

$$= -\left(\partial_\mu\partial^\mu h\right) - \left[\left(\partial_\mu\partial^\mu g_{\mu\nu}\right) - \frac{1}{4}\left(\partial^\nu g_{\mu\nu}\right)\left(\partial^\mu g_{\mu\nu}\right)\right]\bar{h}^{\mu\nu}$$

For the condition of harmonic coordinates, the term in square brackets simplifies drastically (see Chow and Knopf [32]):

$$= -\left(\partial_\mu\partial^\mu h\right) + 2R_{\mu\nu}\bar{h}^{\mu\nu}$$

Utililzing this, equation (9) then takes the form:

$$\int_M \left\{\frac{c^4}{16\pi G_N}\left(-\partial_\mu\partial^\mu h + 4R_{\mu\nu}\bar{h}^{\mu\nu}\right) + T\right\}\mathrm{d}\Omega = 0$$

Examining the background Ricci tensor, one expects from observation of the universe that $R_{\mu\nu} \ll 1$ (i.e. the universe is extremely large). One can justify ignoring this term (since $R_{\mu\nu}\bar{h}^{\mu\nu} \lll 1$), leaving:

$$\int_M \left(\frac{c^4}{16\pi G_N}\right)\partial_\mu\partial^\mu h\,\mathrm{d}\Omega = \int_M T\,\mathrm{d}\Omega \quad (11)$$

Application of the divergence theorem on the left-hand-side of (11) takes the integral to a bounding three-surface $M \longrightarrow \partial M$.

$$\oint_{\partial M}\left(\frac{c^4}{16\pi G_N}\right)\partial^\mu h n_\mu\,\mathrm{d}\Sigma = \int_M T\,\mathrm{d}\Omega \quad (12)$$

Where n_μ is the unit basis normal to ∂M.

In specifically considering the closed expanding FLRW universe, any closed spacelike surface ∂M at some particular time naturally encloses all of M up to that time. In the spirit of general covariance, we will refrain from choosing an explicit 3-slice.The above can be rewritten as:

$$\oint_{\partial M}\left(\frac{c^4}{16\pi G_N}\right)g^{\mu\alpha}\partial_\alpha h n_\mu\,\mathrm{d}\Sigma = \int_M T\,\mathrm{d}\Omega \quad (13)$$

Examining the left hand side of (13), the contravariant metric $g^{\mu\alpha}$ can be pulled through the partial derivative such that the above becomes:

$$\oint_{\partial M}\{\left(\frac{c^4}{16\pi G_N}\right)\left[\partial_\alpha - \frac{1}{4}g_{\mu\alpha}(\partial_\alpha g^{\mu\alpha})\right]g^{\mu\alpha}h n_\mu\,\mathrm{d}\Sigma \quad (14)$$

Because we are only dealing with the trace of the perturbation (h) , it is clear (from equation 7) only the scalar perturbation will enter into equation (14). In the rest frame h is simply 4ϕ (where ϕ is the Newtonian potential divided by c^2)[15, 18]. Accordingly we will denote: $h = 4\phi$.

$$= \oint_{\partial M} \{ \frac{c^4}{4\pi G_N} \left[\partial_\alpha - \frac{1}{4} g_{\mu\alpha} (\partial_\alpha g^{\mu\alpha}) \right] g^{\mu\alpha} \phi n_\mu \, \mathrm{d}\Sigma \tag{15}$$

Choosing a spacelike bounding surface with normal n_0 and writing the metric in the form $g^{\mu\alpha} = e^\mu \cdot e^\alpha$ one obtains:

$$= \oint_{\partial M} \{ \frac{c^4}{4\pi G_N} \left[\partial_\alpha - \frac{1}{4} g_{\mu\alpha} (\partial_\alpha g^{\mu\alpha}) \right] \phi \frac{1}{c} e^\alpha \, \mathrm{d}\Sigma$$

As with the metric earlier, e^α can be pulled through the derivative leaving a connection type term behind:

$$= \oint_{\partial M} \{ \frac{c^3}{4\pi G_N} e^\alpha \left[\partial_\alpha + \frac{1}{2} e_\alpha (\partial_\alpha e^\alpha) - \frac{1}{4} g_{\mu\alpha} (\partial_\alpha g^{\mu\alpha}) \right] \phi \, \mathrm{d}\Sigma$$

Where we have used the fact that $e^\mu e_\mu = -1+1+1+1 = 2$, thus we obtain:

$$= \oint_{\partial M} \{ \frac{c^3}{4\pi G_N} e^\alpha \left[\partial_\alpha + \frac{1}{2} e_\alpha (\partial_\alpha e^\alpha) - \frac{1}{4} g_{\mu\alpha} (\partial_\alpha g^{\mu\alpha}) \right] \phi \, \mathrm{d}\Sigma$$

For brevity, let us denote.

$$\Gamma_\alpha = \frac{1}{2} \left[e_\alpha (\partial_\alpha e^\alpha) - \frac{1}{2} g_{\mu\alpha} (\partial_\alpha g^{\mu\alpha}) \right] \tag{16}$$

The scalar metric perturbation (equation (11)) is then represented by:

$$\oint_{\partial M} \{ \frac{c^3}{4\pi G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \phi \, \mathrm{d}\Sigma = \int_M T \, \mathrm{d}\Omega \tag{17}$$

B. A brief note on the three-sphere, or closed FLRW universe

On can consider the 3-sphere of radius a as being embedded in a four-dimensional Euclidean space. One has in this view the condition for any coordinate system with origin at the center of the 3-sphere:

$$a^2 = g_{\mu\nu} x^\mu x^\nu$$

Note that the metric here is Riemannian (not pseudoriemannian). In Cartesian coordinates for example this reads:

$$a^2 = x^2 + y^2 + z^2 + w^2$$

This is simply the condition that the coordinates lie somewhere on the 3-sphere. For a 3-sphere of changing radius a, one can write this infinitesimally as:

$$da^2 = g_{\mu\nu} dx^\mu dx^\nu$$

Where it is clear that a will be a function of some external parameter (time or conformal time). Let us consider the integral over the 3- ball (M) of which the 3-sphere is the boundary (∂M). For the closed and expanding FLRW universe, this is equivalent to the integral over all space and time (due to the parameterization of a by time). For some arbitrary function $f(x^\mu)$ one has then that:

$$\int_M f(x^\mu) \, \mathrm{d}\Omega$$

This can be written as an integral over ∂M which is then integrated over the radial coordinate da:

$$\int_M f(x^\mu) \, \mathrm{d}\Omega = \int_0^a \left[\oint_{\partial M} f(x^\mu) d\Sigma \right] da$$

For the FLRW universe, after integration, a may be replaced by it's time parameterization. While this may appear entirely trivial, it will be found useful in the following section.

C. a Point-Like Particle

Considering the latter half of (17) now, let us consider our perturbing mass to be point-like. The Lagrangian density $\mathfrak{L}_{particle}$ then has the standard form:

$$\mathfrak{L}_{particle} = mc\sqrt{-g_{\mu\nu} \dot{x^\mu} \dot{x^\nu}} \delta^3 (x^\mu - X^\mu) \tag{18}$$

Which we can write as the total Lagrangian $L_{particle}$.

$$L_{particle} = \oint_{\partial M} \mathfrak{L}_{particle} \, \mathrm{d}\Sigma = mc\sqrt{-g_{\mu\nu} \dot{x^\mu} \dot{x^\nu}}$$

Being a scalar, the action $\mathcal{S}_{particle}$ must be an invariant quantity.

$$\mathcal{S}_{particle} = \int L_{particle} c dt \tag{19}$$

This is clearly seen by examining the square root portion:

$$= mc \int \sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial(ct)} \frac{\partial x^\nu}{\partial(ct)}} cdt$$

$$= mc \int \sqrt{-g_{\mu\nu} \partial x^\mu \partial x^\nu} = mc \int \mid ds \mid$$

One observes parameterization independence and the integral is just the invariant interval between the limits of integration.

Utilizing now the definition for Hilbert's stress energy tensor (varying (19) with respect to $g_{\mu\nu}$):

$$\int_M T_{\mu\nu} d\Omega \delta g^{\mu\nu} = \int_M \left(2 \frac{\delta \mathfrak{L}_M}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathfrak{L}_M \right) \sqrt{-g} d^4x \delta g^{\mu\nu}$$

one obtains:

$$= \int_M \left(-mc \frac{\dot{x}_\mu \dot{x}_\nu \delta^3(x^\mu - X^\mu)}{\sqrt{-g_{\alpha\beta} \dot{x^\alpha} \dot{x^\beta}}} \right) \sqrt{-g} d^4x \delta g^{\mu\nu}$$

Taking the trace of $T_{\mu\nu}$, the latter half of (17) becomes simply:

$$\int_M T d\Omega = \int_M \left(mc \frac{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}{\sqrt{-g_{\mu\nu} \dot{x^\mu} \dot{x^\nu}}} \delta^3(x^\mu - X^\mu) \right) \sqrt{-g} d^4x$$

$$= \int_M mc \delta^3(x^\mu - X^\mu) \sqrt{-g_{\mu\nu} \frac{\partial x^\mu}{\partial(t)} \frac{\partial x^\nu}{\partial(t)}} cdt \, \mathrm{d}\Sigma$$

The integral over the hypersurface ∂M is a constant, hence one can separate the integrals:

$$= \int \left[\oint_{\partial M} mc \delta^3(x^\mu - X^\mu) \, \mathrm{d}\Sigma \right] \sqrt{-g_{\mu\nu} \partial x^\mu \partial x^\nu}$$

Once again we have that our integral may be written in terms of the invariant interval $\mid ds \mid$:

$$= \left[\oint_{\partial M} mc \delta^3(x^\mu - X^\mu) \, \mathrm{d}\Sigma \right] \int \mid ds \mid \tag{20}$$

In general, a parameterization of ds may be given in terms of the scale factor $\mid ds \mid = dS(a)$, which in turn (for the FLRW universe) may be given by solutions to the Friedmann equations [14]. Note that from a geometric point of view, $\mathbb{S}$ is the path interval between the singularity and any point on the three sphere at some later time. In this respect, for the comoving frame, $\mathbb{S}$ is necessarily independent of position at a given cosmological time.

. Equation (20) then becomes:

$$\int_M T d\Omega = \left[\oint_{\partial M} mc \delta^3(x^\mu - X^\mu) \, \mathrm{d}\Sigma \right] \int_0^a dS(a)$$

Note that $\oint_{\partial M} \delta^3(x^\mu - X^\mu) \, \mathrm{d}\Sigma = 1$ so that the a dependence of $d\Sigma$ does not come into play here.

$$\int_M T d\Omega = \mathbb{S}(a) \oint_{\partial M} mc \delta^3(x^\mu - X^\mu) \, \mathrm{d}\Sigma \tag{21}$$

Where we have denoted the quantity:

$$\mathbb{S}(a) = \int_0^a dS(a)$$

$\mathbb{S}(a)$ has units of length and the character $\mathbb{S}$ is chosen to reflect that it is in fact measuring the absolute invariant interval (from the onset of expansion to the present).

Using equation (21) we can write the equation for our metric perturbation (17) entirely on ∂M:

$$\oint_{\partial M} \left[\frac{c^3}{4\pi G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \phi - \mathbb{S}(a) c \rho \right] \mathrm{d}\Sigma = 0$$

Where $\rho = m\delta^3(x^\mu - X^\mu)$. Because the scale parameter a represents the radius of curvature of spacetime (for the closed FLRW universe), $\mathbb{R}$ is chosen here to denote a.

$$\oint_{\partial M} [\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \phi - \rho c] \, \mathrm{d}\Sigma = 0 \tag{22}$$

Equation (22) will be our starting point for further developments. Note that one can (not quite appropriately) expand the rest mass density out in terms of four-momentum density components $\rho c = \sqrt{-p_\alpha g^{\alpha\beta} p_\beta} = \sqrt{-e^\alpha p_\alpha e^\beta p_\beta} = i e^\alpha p_\alpha$. In this manner, one can identify the scalar perturbation with components of the total four-momentum:

$$P_\alpha = \oint_{\partial M} [\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} (-i e^\alpha) \{ \partial_\alpha + \Gamma_\alpha \} \phi \, \mathrm{d}\Sigma \tag{23}$$

Where there is no summation over α. Because the energy/momentum density of the perturbation ϕ is unphysical, only the integral quanties are physically relevant. Equation (23) is notably similar in form to the pseudotensor formula for 4-momentum [1].

D. The Landau-Lifshitz approach

Equation (22) may be motivated from an entirely different perspective, that of the gravitational pseudotensor. That such an entity is nonunique is well-known. The gravitational energy-momentum pseudotensor of Landau and Lifshitz for example, is given by:

$$t^{\mu\nu} = \frac{-c^4}{8\pi G_N} G^{\mu\nu} + \frac{-c^4}{16\pi G_N(-g)} \{(-g)(g^{\mu\nu} g^{\alpha\beta} - g^{\mu\alpha} g^{\nu\beta})\}^{,\alpha\beta} \tag{24}$$

In accordance with the equivalence principle $t_{\mu\nu}$ must vanish in a local inertial frame, thus it contains only first derivatives of the metric. Together with the stress-energy tensor of the source energy/matter a conservation law is well-known [1]:

$$\frac{\partial}{\partial x^\nu} \left[-g(t^{\mu\nu} + T^{\mu\nu})\right] = 0 \tag{25}$$

Enforcing this over the entirety of a spacetime manifold M, and writing the above as an integrand one obtains:

$$\int\limits_M \frac{\partial}{\partial x^\nu} \left[-g(t^{\mu\nu} + T^{\mu\nu})\right] \mathrm{d}\Omega = 0$$

Utilizing the divergence theorem this can be expressed on a three-slice ∂M bounding M with volume element $d\Sigma$ and unit normal n_ν.

$$\oint\limits_{\partial M} -g(t^{\mu\nu} + T^{\mu\nu}) n_\nu \, \mathrm{d}\Sigma = 0 \tag{26}$$

Choosing a spacelike surface and thus timelike normal:

$$\oint\limits_{\partial M} (t^{\mu 0} + T^{\mu 0}) \, \mathrm{d}\Sigma = 0$$

$$t^{\mu 0} + cP^0 = 0$$

$$\oint\limits_{\partial M} (\frac{1}{c} t^{\mu\nu} + P^\mu) e_\mu \, \mathrm{d}\Sigma = 0$$

$$iP^\mu e_\mu = +\sqrt{-P_\mu e^\mu P_\nu e^\nu} = +\sqrt{-P_\nu P^\nu} = \rho c$$

$$\oint\limits_{\partial M} (it_{\mu 0} e^\mu + \rho c) \, \mathrm{d}\Sigma = 0 \tag{27}$$

Because $t^{\mu 0}$ contains only first derivatives of the metric, a form of $t^{\mu 0}$ may be chosen so that (27) coincides with (22) (after performing an appropriate first order linearization). Furthermore, because the energy/momentum densities associated with the gravitational pseudotensor have no proper physical interpretation [5] [3], one expects only integral quantities in (27) to be physically relevant.

E. Orthogonal Series Expansions & Fredholm Theory

A well-behaved function $\Phi(x^\mu)$ subject to periodic boundary conditions (as one finds in a spatially closed universe) may be expressed as an infinite series of orthogonal functions $\psi(x^\mu)_n$ weighted by coefficients a_n (a simple Fourier series expansion or "inverse Fourier transform"). For three dimensions, each with a periodic boundary condition, this takes the form:

$$\Phi(x^\mu) = \sum_{n,l,m} a_{nlm} \psi(x^\mu)_{nlm} \tag{28}$$

In considering the mass density ρ in (22), it is clearly subject to the physical requirement that $\rho \geq 0$ everywhere (the dominant energy condition). Without altering any physics, one may specify a class of functions Φ exist such that $\rho =\mid \Phi \mid^2$ everywhere. In the most general case Φ will be complex. One can expand Φ into an orthogonal series expansion as in equation (28) such that:

$$\rho = \Phi^\dagger \Phi = \{\sum_{nlm}^{\infty} a^\dagger_{nlm} \psi^\dagger(x)_{nlm}\}\{\{\sum_{nlm}^{\infty} a_{nlm} \psi(x)_{nlm}\} \tag{29}$$

Where $\Psi^\dagger$ is the Hermitian adjoint of Ψ. Clearly Ψ is not uniquely specified, something we will address shortly. One may always choose $\psi(x)_{nlm}$ to be an orthonormal set of basis such that:

$$\oint\limits_{\partial M} \psi^\dagger_{nlm} \psi_{nlm} \, \mathrm{d}\Sigma = 1 \tag{30}$$

(Note: For the particular case of the 3-sphere, Alertz [13] does a thorough job in discussing explicit orthonormal basis.)

Φ can also always be rescaled such that $\rho = m\Phi^\dagger\Phi$. Under such a rescaling, $\Phi^\dagger\Phi$ necessarily possesses units of 3-density and contains all information regarding the distribution of m on ∂M.

$$m = \oint\limits_{\partial M} \rho \, \mathrm{d}\Sigma = m \oint\limits_{\partial M} \Phi^\dagger \Phi \, \mathrm{d}\Sigma \tag{31}$$

For this choice of basis one has the following conditions:

$$\oint_{\partial M} \Phi^\dagger \Phi \, \mathrm{d}\Sigma = 1 \qquad \textit{and} \qquad \sum_{n,l,m}^{\infty} | \, a_{nlm} \, |^2 = 1 \tag{32}$$

While this procedure may seem entirely $ad-hoc$, for the case of the point-like particle one may demonstrate this explicitly. From both Fredholm theory and mathematical physics, it is known that the Dirac delta and delta Kronecker functions may be written as [14, 15]:

$$\delta(x - X) = \sum_{n}^{\infty} \bar{a}_n a_n \bar{\psi}_n(x) \psi_n(X) \tag{33}$$

$$\delta_n^m = \bar{a}_n a_m \int_b^a \bar{\psi}_n(x) \psi_m(x) dx$$

Where the ψs are any complete and orthogonal set of functions and the a_n are normalization coefficients. Generalized to three dimensions the first equation is written as:

$$\delta^3(x^\mu - X^\mu) = \sum_{n,l,m}^{\infty} \bar{a}_{nlm} a_{nlm} \bar{\psi}_{nlm}(x) \psi_{nlm}(X)$$

Examining this expression under the integral and taking into account the second part of (33), it is clear that:

$$1 = \int \delta^3(x^\mu - X^\mu) = \int \bar{\Phi} \Phi \, \mathrm{d}\Sigma$$

$$= \int \{ \sum_{nlm}^{\infty} a_{nlm}^\dagger \psi^\dagger(x)_{nlm} \} \{ \sum_{nlm}^{\infty} a_{nlm} \psi(x)_{nlm} \} \, \mathrm{d}\Sigma$$

Here Φ has the same definition as in (28) above. Cross terms vanish due to orthogonality and we can use the full independently summed series. Note that the conditions chosen in (32) are automatically fulfilled.

Inserting ρ back into equation (22), one obtains:

$$\oint_{\partial M} [\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \phi - mc \Phi^\dagger \Phi] \, \mathrm{d}\Sigma = 0 \tag{34}$$

Using the same method as with ρ, ϕ is then expanded out as the absolute value squared of an orthonormal set (In fact any positive (or negative) definite, function may be expanded out this way [29]).

It is stressed that this changes nothing physically; however, the freedom to choose Φ, and Ψ (i.e. their non uniqueness) proves of great utility.

$$\phi = \Psi^\dagger \Psi = \{ \sum_{nlm}^{\infty} b_{nlm}^\dagger \psi_{nlm}^\dagger \} \{ \sum_{nlm}^{\infty} b_{nlm} \psi_{nlm} \} \tag{35}$$

Where we have chosen the same orthonormal basis ψ_{nlm} as for Φ. Note that Ψ is unitless, in contrast to Φ which is a three-density. Substituting (35) into (34) now:

$$\oint_{\partial M} [\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \Psi^\dagger \Psi - mc \Phi^\dagger \Phi] \, \mathrm{d}\Sigma = 0 \tag{36}$$

(36) may be written as four equations by expanding out mc in terms of four-momentum components:

$$mc = \sqrt{-P_\mu g^{\mu\nu} P_\nu} = \sqrt{-P_\mu e^\mu P_\nu e^\nu} = i e^\mu P_\mu \tag{37}$$

Using this in (36) one obtains:

$$\oint_{\partial M} \left[\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} e^\alpha \{ \partial_\alpha + \Gamma_\alpha \} \Psi^\dagger \Psi - i e^\alpha P_\alpha \Phi^\dagger \Phi \right] \mathrm{d}\Sigma = 0 \tag{38}$$

It is evident that $e^\alpha \partial_\alpha = e_\alpha \partial^\alpha$ is self-adjoint. Together with the completeness of ψ, this indicates our operator is Hermitian (which of course would be required anyway for real physical eigenvalues). Hence we must have that:

$$\Psi^\dagger e^\alpha \partial_\alpha \Psi = (e^\alpha \partial_\alpha \Psi^\dagger) \Psi \tag{39}$$

Using the above, (44) can be rewritten as:

$$\oint_{\partial M} \left[\Psi^\dagger \left(\frac{c^3}{4\pi \mathbb{S}(\mathbb{R}) G_N} \right) e^\alpha \{ 2\partial_\alpha + \Gamma_\alpha \} \Psi - i e^\alpha P_\alpha \Phi^\dagger \Phi \right] \mathrm{d}\Sigma = 0$$

Factoring out the 2, the above becomes:

$$\oint_{\partial M} \left[\Psi^\dagger \left(\frac{c^3}{2\pi \mathbb{S}(\mathbb{R}) G_N} \right) e^\alpha \{ \partial_\alpha + \frac{1}{2} \Gamma_\alpha \} \Psi - i e^\alpha P_\alpha \Phi^\dagger \Phi \right] \mathrm{d}\Sigma = 0$$

Let us absorb the $\frac{1}{2}$ into the connection term (equation (16)) $\frac{1}{2}\Gamma_\alpha \Longrightarrow \Gamma_\alpha$, such that the equation of motion for our perturbation can be written as:

$$\oint_{\partial M} e^\alpha \left[\Psi^\dagger \left(\frac{c^3}{2\pi \mathbb{S}(\mathbb{R}) G_N} \right) \{ \partial_\alpha + \Gamma_\alpha \} \Psi - i P_\alpha \Phi^\dagger \Phi \right] \mathrm{d}\Sigma = 0 \tag{40}$$

We will now consider just one of the four α components (no summation over α). Being orthogonal and linearly independent, we can examine just a single term of the series expansion:

$$e^{\alpha}\left[\frac{c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}k_{\alpha(nlm)} \mid b_{nlm}\mid^2 - iP_\alpha \mid a_{nlm}\mid^2\right] = 0 \tag{41}$$

Here the derivative term has been replaced with it's eigenvalue $k_{\alpha(nlm)}$ on the left side (note this requires that our basis ψ_{nlm} now must be eigenfunctions to the operator $\{\partial_\alpha + \Gamma_\alpha\}$). Ψ and Φ have been written as their series expansions (equations (28), and (35) respectively).

The term in brackets must vanish, allowing one to solve for the eigenvalue of any term n, l, m in the series.

$$\frac{c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}k_{\alpha(nlm)}b^\dagger_{nlm}b_{nlm} - iP_\alpha a^\dagger_{nlm}a_{nlm} = 0$$

$$k_{\alpha(nlm)} = i\left[P_\alpha\frac{\mid a_{nlm}\mid^2}{\mid b_{nlm}\mid^2}\right]\left(\frac{c^3}{4\pi\mathbb{S}(\mathbb{R})G_N}\right)^{-1} \tag{42}$$

Let us denote the quantity in square brackets by:

$$P_\alpha\frac{\mid a_{nlm}\mid^2}{\mid b_{nlm}\mid^2} = \frac{P_{\alpha(nlm)}}{\mathbf{1}} \tag{43}$$

Where dimensional analysis requires $\mathbf{1}$ to have units of 3-volume (denoted in bold to reflect this property).

One then has for (42) that $P_{\alpha(nlm)}$ takes on the role of the eigenvalue.

$$k_{\alpha(nlm)} = iP_{\alpha(nlm)}\left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\right)^{-1}$$

It is interesting to observe that there is a natural relation between the wavenumber $k_{\alpha(nlm)}$ and corresponding four-momentum component $P_{\alpha(nlm)}$ such that:

$$\left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\right)k_{\alpha(nlm)} = iP_{\alpha(nlm)}$$

Making use of the relation $P_{\alpha(nlm)} \mid b_{nlm}\mid^2 = \mathbf{1}P_\alpha \mid a_{nlm}\mid^2$ from (43), equation (40) may then be written entirely in terms of the scalar perturbation:

$$\oint_{\partial M} e^{\alpha}\left[\Psi^\dagger\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\{\partial_\alpha + \Gamma_\alpha\}\Psi - iP_{\alpha(nlm)}\mid\Psi\mid^2\right]\,\mathrm{d}\Sigma = 0$$

It is important to note here that the total physical momentum in the α direction for our particle is then of the form:

$$P_\alpha = \sum_{nlm}^{\infty} P_{\alpha(nlm)}b^\dagger_{nlm}b_{nlm}$$

Note also, that for any given mode nlm, one can specify the condition that $\sqrt{-P_{\alpha(nlm)}P^{\alpha}_{(nlm)}} = mc$ (this follows from the nonuniqueness of Φ and Ψ). This has the immediate consequence that:

$$mc = ie^{\alpha}P_\alpha = \sum_{nlm}^{\infty} ie^{\alpha}P_{\alpha(nlm)}\mid b_{nlm}\mid^2 = mc\sum_{nlm}^{\infty}\mid b_{nlm}\mid^2$$

Which implies a normalization condition $\sum_{nlm}^{\infty} \mid b_{nlm}\mid^2 = 1$.

$$\oint_{\partial M}\Psi^\dagger\left[\left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\right)e^{\alpha}\{\partial_\alpha + \Gamma_\alpha\} - mc\right]\Psi\,\mathrm{d}\Sigma = 0 \tag{44}$$

Which is subject to the conditions:

$$\Gamma_\alpha = \frac{1}{4}[e^{\alpha}(\partial_\alpha e_\alpha) + e_\sigma\Gamma^\sigma_{\alpha\mu}e^{\mu}]$$

$$\oint_{\partial M}\Psi^\dagger\Psi\,\mathrm{d}\Sigma = 1 \tag{45}$$

The perturbation in this form should be familiar; it greatly resembles the Dirac equation. Though the form of the integrand has been changed, the integral (physical) quantities of (44), are precisely the same as for (22).

F. The Dirac Equation

Using the tetrad hypothesis, $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$, we can examine (44) in a local frame. As is standard, manifold indices are denoted by Greek letters and Lorentzian indices by Latin. We have then locally: $e^{\alpha} \implies e^{\alpha}_b\gamma^b$, where the γ^a are the standard gamma matrices satisfyings $\{\gamma^a, \gamma^b\} = -2\eta^{ab}I_4$ (via our choice of the $(-+++)$ metric). Examining Γ_α :

$$\Gamma_\alpha = \frac{1}{4}\left[e_\alpha(\partial_\alpha e^{\alpha}) - g_{\mu\alpha}\Gamma^\alpha_{\alpha\sigma}g^{\mu\sigma}\right] \tag{46}$$

Locally this becomes:

$$\Gamma_\alpha = \frac{1}{4}\left[e^a_\alpha\gamma_a(\partial_\alpha e^\alpha_b\gamma^b) - e^a_\mu e^b_\alpha\{\gamma_a\gamma_b\}\Gamma^\alpha_{\alpha\sigma}e^\mu_c e^\sigma_d\{\gamma^c\gamma^d\}\right]$$

$$= \frac{1}{4}\left[e_{\alpha a}\gamma^a(\partial_\alpha e^\alpha_b\gamma^b) - e^b_\alpha\{\gamma_a\gamma_b\}\Gamma^\alpha_{\alpha\sigma}e^\sigma_d\{\gamma^a\gamma^d\}\right]$$

With a bit of rearranging, and making use of the relations $e^a_\mu e^\mu_c = \delta^a_c$ and $\gamma_b\gamma^a = \delta^b_a$, one may write this as:

$$\Gamma_\alpha = \frac{1}{4}\left[e^{\mu a}(\partial_\alpha e^b_\mu) - e^{\nu a}\Gamma^\sigma_{\alpha\nu}e^b_\sigma\right]\gamma_a\gamma_b$$

The term in brackets can be recognized as none other than the spin-connection ω^{ab}_μ.

$$\Gamma_\alpha = \frac{1}{4}\omega^{ab}_\alpha\gamma_a\gamma_b = \frac{1}{8}\omega^{ab}_\alpha\sigma_{ab} \tag{47}$$

Where $\sigma_{ab} = [\gamma_a\gamma_b - \gamma_b\gamma_a]$. Equation (47) can be recognized as the spin vector term [16, 17] making it evident that Ψ transforms as a spinor. One is left with the set of conditions:

$$\Psi^\dagger\left[\left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\right)e^\alpha_b\gamma^b\{\partial_\alpha + \frac{1}{8}\omega^{ab}_\alpha\sigma_{ab}\} - mc\right]\Psi = 0 \tag{48}$$

$$\oint_{\partial M}\Psi^\dagger\Psi\,\mathrm{d}\Sigma = 1$$

Up to the value of the factor $\frac{\mathbf{1}c^3}{\mathbb{S}(\mathbb{R})2\pi G_N}$, (48) is precisely the Dirac equation.

I. INTERPRETATION

It is evident that the scalar metric perturbation (in the form of Ψ) has assumed characteristics typically associated with the quantum mechanical wavefunction, including momentum-position and energy-time uncertainty relations (required by the Dirac-like structure of 48). Such properties have come about as a direct result of the non-local nature of the energy/momentum of the metric perturbation. It becomes then entirely redundant to have an independent quantum hypothesis, the two phenomena appear to be one and the same.

Any physical interpretation of (48) unavoidably demands a relationship with the Dirac equation such that:

$$\hbar = \left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\mathbb{R})G_N}\right) \tag{49}$$

Where $\hbar$ is the Planck constant divided by 2π. Only when equation (49) holds can one make physical sense of the scalar perturbation in the form of equation (48). One has then in this interpretation that the quantum wavefunction must be a manifestation of the metric perturbation of the particle.

Let us examine this qualatatively. In considering a point particle, one can say that the metric perturbation is inextricably associated with it's source particle. Yet one also has that the energy/momenta of the perturbation are necessarily non-local. Writing the energy and momentum of the particle in terms of the perturbation then must lead to non local or "quantum-like" characteristics.

One has here indeterminacy arising from nonlocality, which itself is a direct result of the equivalence principle. In this respect, there is a certain (much needed) satisfaction within this explanation of the quantum wavefunction.

The current paper has approached this problem from the consideration of some equivalence between gravitational nonlocality and quantum indeterminism (though no assumption was made until the conjecture of equation (49)).

Equation (49) however, posesses the strongly dissatisfying assertion that the Planck constant for some massive particle is dependent upon it's path history $\mathbb{S}$. One can approximate a path history as being comoving, as spatial momenta of massive particles are redshifted towards the comoving frame in the expanding universe. There would remain the issue though, that two particles would have a Plancks constant that diverged from one another on a level many orders of magnitude below the value of $\hbar$ itself (which is reminiscent of Weyl's theory {ref}).

It would appear that pursuing this formalism beginning with the pseudotensor approach (as mentioned in section D) might leave the value of $\hbar$ dependent only upon $\mathbb{R}$. This comes about because the pseudotensor portion of equation (26) to first order should only be dependent upon $\mathbb{R}$. Being an invariant related to the scalar curvature this dependence would possess none of the drawbacks of the current approach.

Previous mentions in the literature.

The notion that Planck's constant and/or quantization may be related to global spacetime geometry is not unprecedented in the literature. As far back as 1939 Schrodinger writes [18]:

> "*Wave mechanics imposes an a priori reason for assuming space to be closed; for then and only then are it's proper modes discontinuous and provide an adequate description of the observed atomicity of matter and light.*"

As the father of the wavefunction, Schrodinger's view merits consideration (Incidentally, it was a similar line of thought which led the author to Schrodinger's paper and eventually the present article).

Later, mathematicians Folland and Stein [19] demonstrated that a pseudo-elliptic operator on a real hypersurface is best approximated not by the unitary group but rather the Heisenberg group.

Further building upon Folland and Stein, Buliga, indicates (in a comprehensive article [20]) that:

> "*A measurement process should correspond to trying to make an Euclidean chart of this dynamical system..... Planck constant might*

be the effect of this fact, namely it could measure the distance from the (metric profile or dilatation profile) and best Euclidean approximations. After reading this paper one can be sensible to the idea that the Planck constant could measure a distance between curvatures."

Most recently Lipovka [32], has utilized a wholly independent approach involving adiabatic invariants. Lipovka's analysis ends concluding that the Planck's constant (h) is necessarily an adiabatic invariant of the expanding universe. He derives a relationship such that the Planck constant is necessarily inversely proportional to the cube of the radius of curvature of spacetime ($h \propto \mathbb{R}^{-3}$).

II. THE FORM OF $S(\mathbb{R})$

In order to for the current interpretation to be valid $S(\mathbb{R})$ must follow certain criteria. For example we expect that the Planck constant has maintained the level of consistency in it's value as determined by observations of our universe.

In order to check that this is indeed the case (and also to proceed with an analysis) we must know the form of $S(\mathbb{R})$. This requires knowing the form of it's parameterization. For the closed FLRW universe, the form of the metric in terms of the scale parameter is given by solutions to the Friedmann equations [14]:

$$-ds^2 = \left(\frac{3c^4}{8\pi G_N}\right) \frac{-d\mathbb{R}^2}{\left(\rho_{r0}\mathbb{R}_0^4\right)/\mathbb{R}^2 + \left(\rho_{m0}\mathbb{R}_0^3\right)/\mathbb{R} - \left(\frac{3c^4}{8\pi G_N}\right)} + \mathbb{R}^2 d\Sigma^2 \tag{50}$$

Where the $\rho' s$ denote the density of radiation and matter respectively at some value of the scale parameter (denoted $\mathbb{R}_0$). Let us denote the quantities $\rho_{r0}\mathbb{R}_0^4 = E_{rl}$ and $\rho_{m0}\mathbb{R}_0^3 = E_m$, which refer to the total radiation energy and mass/energy respectively. We have also that $d\Sigma$ indicates an object moving over the three sphere as it expands. One then has that equation (50) takes the form:

$$ds^2 = \left(\frac{3c^4}{8\pi G_N}\right) \frac{(\mathbb{R})^2 d\mathbb{R}^2}{E_{rl} + \mathbb{R}E_m - \left(\frac{3c^4}{8\pi G_N}\right)\mathbb{R}^2}$$

The total (absolute) interval is then:

$$\mathbb{S} = \left(\frac{3c^4}{8\pi G_N}\right)^{1/2} \int_0^{\mathbb{R}_p} \mathbb{R}\sqrt{\frac{1}{E_{rl} + aE_m - \left(\frac{3c^4}{8\pi G_N}\right)\mathbb{R}^2}}\, d\mathbb{R}$$

Where $\mathbb{R}_p$ is the scale parameter in the present time. For simplicity, let us denote the constant $\frac{8\pi G_N}{3c^4} = C$, the solved integral is then of the form:

$$\mathbb{S}(\mathbb{R}) = \left(\frac{E_m}{2C}\right) arcsin\left(\frac{\mathbb{R} - (E_m/2C)}{\sqrt{(E_{rl}/C) + (E_m/2C)^2}}\right)_0^{\mathbb{R}_p}$$

$$-\left\{\sqrt{-\mathbb{R}^2 + \mathbb{R}(E_m/C) + (E_{rl}/C)}\right\}_0^{\mathbb{R}_p}$$

At the current time t_p, the interval $\mathbb{S}_p$ must be on the order of 10^{68} meters (from (49)). In this manner we can write:

$$\mathbb{S}(\mathbb{R}) = \mathbb{S}_p\left[\frac{\mathbb{S}(\mathbb{R})}{\mathbb{S}_p}\right] = \mathbb{S}_p\mathbb{S}'(\mathbb{R}) \tag{51}$$

Where $\mathbb{S}'_p = 1$ in the present. We are now in a position to proceed in seeing how the Planck "constant" would evolve with the scale parameter.

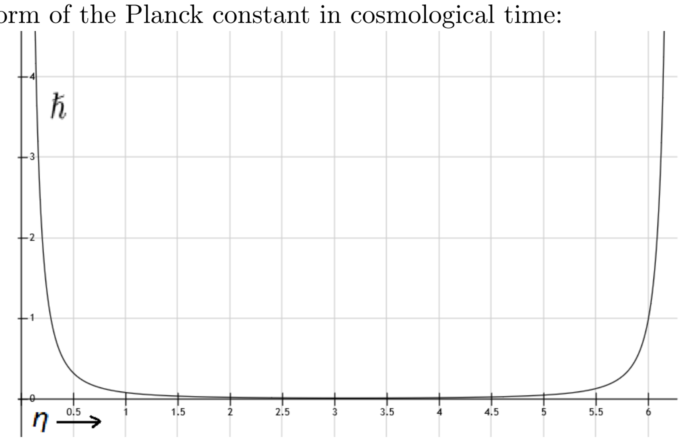
If we suppose (as observation suggests) that the mass density has been much greater than the radiation density for most of the universe's lifetime; then we may approximate the scale parameter $\mathbb{R}$ by solutions to matter dominated Friedmann's equations [14] (we could alternatively use the radiation dominated solution for the early universe).

$$\mathbb{R}(\eta) = \frac{\mathbb{R}_{max}}{2}(1 - cos(\eta)) \tag{52}$$

In this manner, the Planck "constant" may be written in terms of conformal or cosmological time.

$$\hbar = \left(\frac{\mathbf{1}c^3}{2\pi\mathbb{S}(\eta)G_N}\right)$$

Using this with (52) and (51) we may graph the general form of the Planck constant in cosmological time:



Where, for ease of viewing, the amplitude of $\hbar$ has been over-represented by a factor of $\sim 10^{32}$. Figure 1. indicates that the value of $\hbar$ remains quite consistent over most of the lifetime of the closed universe (in agreement with observation).

III. SOME CONSEQUENCES AND POTENTIAL EXPERIMENTAL VERIFICATIONS THEREOF.

The above relationship between General Relativity and quantum mechanics offers many consequences, the most immediately evident of which is the geometrical nature of Planck's constant and consequent dependence upon the scale factor $a(\eta)$. Thus an atom for example would not be exempt from energy loss in an expanding universe but should rather experience diminishment in a manner similar to that of a propagating electromagnetic wave. Let us examine this from an observational point of view. The following sections should be considered crude models in that any experiment would necessarily need to perform a more in-depth analysis to check results

A. The Dirac equation over cosmological time periods

While under the present interpretation $\hbar$ possesses a dynamic geometric meaning, in quantum physics it is considered strictly constant. This can be shown to be merely a matter of choosing how to manage the time dependence of $\hbar$. Examining equation (48):

$$\{\hbar(\eta)e^{\alpha}(\eta)(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\}\Psi=0 \tag{53}$$

Denoting $\hbar(\eta)$ in the present as $\hbar_p$, the above may be written as:

$$\{\hbar_p\left(\frac{\hbar(\eta)}{\hbar_p}\right)e^{\alpha}(\eta)(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\}\Psi=0 \tag{54}$$

This may be rewritten as:

$$\{\hbar_p e^{\alpha}(\eta)(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\left(\frac{\hbar_p}{\hbar(\eta)}\right)\}\Psi=0$$

After rearranging, the above is written as:

$$\left\{\hbar_p e^{\alpha}(\eta)(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\right\}\Psi-mc\left(\frac{\hbar_p}{\hbar(\eta)}-1\right)\Psi=0 \tag{55}$$

It becomes convenient here to write the entirety of (55) in terms of conformal time. Let us write the scale parameter as $\mathbb{R}(\eta)=\mathbb{R}_p a(\eta)$, where $a(\eta)$ is the unitless scale parameter.

Then $a(\eta)$ chosen as unity in the present, sets $\mathbb{R}_p$ as the Radius of curvature of space in the present. The scale parameter $a(\eta)$ is given by solutions to the Friedmann equations. as we are considering a closed (matter dominated) universe, such solutions are known to be of the form [14]:

$$a(\eta)=\frac{a_{max}}{2}(1-cos(\eta)) \tag{56}$$

For simplicity, let us take our frame as the comoving one. Pulling the scale factor out $e^{\alpha}(\eta)\rightarrow a(\eta)^{-1}e^{\alpha}$ (note that pulling $a(t)$out of the metric automatically resets the metric to conformal time η), We have that we can write (55) as:

$$\left\{\hbar_p e^{\alpha}(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\right\}\Psi-mca(\eta)\left(\frac{\hbar_p}{\hbar(\eta)}-1\right)\Psi=0$$

The term is curly brackets is simply the Dirac equation as encountered in quantum mechanics. The latter term, required to be zero in the present, acts as an effective potential $V_1(\eta)$ for times varying from the present.

$$V_1(\eta)=-mca(\eta)\left(\frac{\hbar_p}{\hbar(\eta)}-1\right)\Psi$$

$$\{\hbar_p e^{\alpha}(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\}\Psi-V_1\Psi=0 \tag{57}$$

Within the Spin connection component, for the comoving frame one has a term on the order of $\sim\frac{\dot{a}(\eta)}{a(\eta)}$ (the Hubble constant) [30]. the Dirac equation takes on the form:

$$\left\{\hbar_p e^{\alpha}(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\right\}\Psi+\left\{\sim\hbar_p\frac{\dot{a}(\eta)}{a(\eta)}+V_1\right\}\Psi=0$$

Using equation (52) the "connection potential" is of the form:

$$\hbar_p\frac{\dot{a}(\eta)}{a(\eta)}=\frac{sin(\eta)}{1-cos(\eta)}=V_0$$

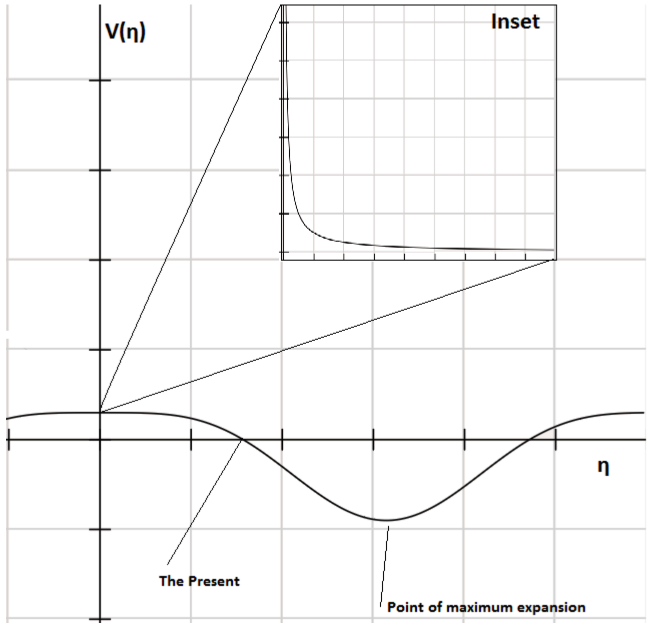
Where we've denoted this term V_0. One has then that (53) may be written as:

$$\left\{\hbar e^{\alpha}(\partial_{\alpha}+\frac{1}{8}\omega_{\alpha}^{ab}\sigma_{ab})-mc\right\}\Psi+\{V_0+V_1\}\Psi=0 \tag{58}$$

The first term is clearly the standard Dirac equation expressed in conformal time η, while the latter terms acts as an effective potential $V(\eta)=V_0+V_1$.

$$V(\eta)=\hbar\frac{Sin(\eta)}{(1-cos(\eta))}-mca(\eta)\left(\frac{\hbar_p}{\hbar(\eta)}-1\right)$$

As required,$V(\eta)$ vanishes in the present (aside from a small Hubble term). Let us examine the form of this potential as it varies with cosmological time η.



Examining $V(\eta)$ (Figure 2) first very close to the origin (expanded in the inset), we see that the potential (dominated by V_0) diverges to $+\infty$ as one approaches ever near the onset of expansion (the singularity). While our perturbation analysis surely fails very near the singularity, one might qualatatively say that the term V_0 acts as a strong potential, rapidly dropping towards zero roughly on the order of $\eta \sim 10^{-34}$. Such a potential is highly reminiscent of the so-called inflationary field, which is theorized to appear on a very similar time scale in the early universe.

Away from the origin V_1 dominates as viewed in the main body of Figure 1 (The amplitude of V_1 and V_0 are both over-represented for ease of viewing). The profile is interesting in that it starts out flat and at some period well after the onset of expansion begins to drop significantly.

Interpreted as a real potential, $V(\eta)$ would indicate that at some period into it's life the universe began to accelerate in it's expansion.

This is in agreement with recent observations indicating that our universe began accelerating in it's expansion some time in the recent past. A proper analysis using observed data could ascertain whether this potential is a proper fit to observation or not.

One has in both of these cases that requiring $\hbar$ to be fixed leads an observer to require that there exists a potential acting to drive accelerated expansion of the universe. This complements well the constant $\hbar$ interpretation of the redshift which indicates an accelerating expansion of the universe as discussed in in the previous section.

It is clear however, that $V(\eta)$ should be considered as merely a coordinate artifact resulting from one's insistence on utilizing the current radius of curvature $\mathbb{R}_p$ within Planck's constant for times differing significantly from the present. In this manner, $V(\eta)$ can be interpreted as a ficticious field in the same sense that the centrifugal or even gravitational force itself are ficticious forces resulting from choice of coordinates.

B. The cosmic rate of expansion as inferred by the redshift of distant bodies

We will begin by considering the Hubble redshift of distant galaxies. Generally this is interpreted as indicating an expansion rate of the universe. The redshift factor for a given light source is determined by finding the atomic spectra corresponding to known elements and figuring out how much said spectra have been shifted. The spectra of hydrogen, for example, are well-known to be given (sufficiently for our purposes) by the Rydberg formula.

$$\frac{1}{\lambda} = \frac{m_e e^4}{8\epsilon_0^2 h^3 c}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{59}$$

Where λ is the wavelength of radiation emitted/absorbed, n_1 and n_2 are any positive integers $n_1 \neq n_2$; $n_1 < n_2$ *and* $n_2 < n_1$ implying absorption and emission respectively.

In light of equation (55) however; one can no longer regard h as constant over cosmic time-scales. The spectra of an atom will necessarily be dictated by the Cosmological scale factor of spacetime at the moment (denoted *then*) of absorption/emmission. The Rydberg formula then becomes:

$$\frac{1}{\lambda_{then}} = \frac{m_e e^4}{8\epsilon_0^2 c}\left(\frac{G_N \mathbb{S}_{then}}{c^3}\right)^3\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{60}$$

For convenience, let us absorb all of (57) except $\mathbb{R}^3$ into a factor, denoted λ_0 such that:

$$\lambda_{then} = \lambda_0 \mathbb{S}_{then}^{-3} \tag{61}$$

One may now follow the standard prescription used to determine the Hubble factor from the redshifting of light. Prior to any calculation, it is evident that the obtained Hubble factor will be much different. In the present time (denoted *now*), we have that the emission spectra from the same atomic species will follow:

$$\lambda_{now} = \lambda_0 \mathbb{S}_{now}^{-3} \quad where: \quad \mathbb{S}_{now} > \mathbb{S}_{then} \tag{62}$$

The equation used for determining cosmic redshift is given by[24]:

$$z + 1 = \frac{\lambda_{now}}{\lambda_{then}} = \frac{a_{now}}{a_{then}} \tag{63}$$

However, λ_{then} is considered in all redshift surveys, to be the same as the spectra that an atom has *now* (λ_{now}).

Under the present treatment however, this is not the case. One can correct for this:

$$\lambda_{then} = \lambda_{now} \left[\frac{\lambda_{then}}{\lambda_{now}} \right] = \lambda_{now} \left[\frac{\mathbb{S}_{now}}{\mathbb{S}_{then}} \right]^3 \tag{64}$$

When substituted into (63) this yields:

$$\frac{\lambda_{arriving}}{\lambda_{now}} \left[\frac{\mathbb{S}_{now}}{\mathbb{S}_{then}} \right]^{-3} = \frac{a_{now}}{a_{then}} \tag{65}$$

and rearranged:

$$z + 1 = \frac{\lambda_{arriving}}{\lambda_{now}} = \left[\frac{a_{now}}{a_{then}} \right] \left[\frac{\mathbb{S}_{now}}{\mathbb{S}_{then}} \right]^3 \tag{66}$$

Because $\mathbb{S}$ is of first order or greater in $\mathbb{R}$, it becomes evident that in observing the redshift of distant bodies and assuming the constancy of atomic spectra, one would be forced to conclude the universe is expanding at an accelerated rate. It may be however that the universe slowing in it's acceleration and the former interpretation is but a mirage brought about by the changing of atomic spectra over cosmological timescales. Using the procedure very roughly outlined here, one could potentially check the fit here against known observational data. Note that this complements well the discussion in the previous section.

C. Macroscopic wavefunctions; a local experiment

The former two sections are both complimentary and require observations on a cosmological scale. It would be more satisfying to have a local test of this paper's main point.

Let us consider a localized mass density ρ_a subject to a Newtonian potential φ. The differential element of force is given as:

$$\overrightarrow{dF_a} = \rho_a dvol(-\overrightarrow{\nabla}\varphi) \tag{67}$$

Consider now the presently posited relationship between the wavefunction and the scalar metric perturbation. We will denote the source mass of φ as m_b. Near the rest frame of m_b, the Newtonian potential is of the form $\varphi = -c^2\mathbf{1} \mid \Psi_b \mid^2$ (where Ψ_b is the wavefunction of m_b). The mass density ρ_a may be similarly written in terms of it's own wavefunction $\rho_a = m_a \mid \Psi_a \mid^2$. 67 is then written as:

$$\overrightarrow{dF_a} = \mid \Psi_a \mid^2 \overrightarrow{\nabla} \mid \Psi_b \mid^2 \tag{68}$$

Integrating 68, we arrive at an expression for the total force:

$$\vec{F_a} = \mathbf{1} m_a c^2 \int \mid \Psi_a \mid^2 \overrightarrow{\nabla} \mid \Psi_b \mid^2 d\Sigma \tag{69}$$

For a wavefunction possessing a non-zero gradient, one ought observe a Newtonian-like potential acting upon another mass (m_a) immersed within it. A proper calculation of the total force necessitates an overlap integral involving the two wavefunctions.

Given a single particle perturbation source, such an effect would be infinitesimal. The most feasable experiment would necessarily involve a gradient induced within a macroscopic wavefunction such as a superfluid, superconductor, or some other manifestation of Bose-Einstein Condensate.

For this case in particular, the high absolute-value of the wavefunction coupled with the relative ease of manipulating said wavefunction over a macroscopic distance, one might hope to obtain an observable result.

In such a situation, it would appear that the superfluid wavefunction is effectively altering the geodesics of the background metric (locally). Under a proper analysis of vector and tensor perturbations more interesting effects could, in principle, result.

FURTHER THOUGHTS AND REMARKS

The scalar perturbation in reality is but a part of the metric perturbation, whose full analysis would require a similar treatment of the vector and tensor perturbation components (Of course the linear perturbation itself is but a first order approximation to the full Einstein field equation). Such an analysis could have terms contributing to the proposed physical phenomena mentioned in the former sections. A similar treatment of the vector and tensor components might shed light on the origin of other quantum phenomena. It would be of great interest to see if other components of the standard model could be thus derived.

If one proceeds in so-called second quantization using the present formalism several interesting points arise:

1. One has in the present article that the wavefunction is a perturbation to the background metric. In the context of field theory, the vacuum state would necessarily correspond to the background metric (or rather it's factored harmonic expansion). In this manner a creation operator corresponds to adding a set of weighted harmonics to the background metric. One can contrast this with typical QFT in which the vacuum is quantized at each point as a set of harmonic oscillators.

2. The vacuum state energy density would necessarily correspond to the Ricci scalar of the background metric. In terms of harmonic expansions, the series expansion for vacuum energy would have to converge on the Ricci scalar (times a factor). In

this way there is a natural suppression of higher order modes, thus avoiding ultraviolet divergences.

It would be of great interest to pursue such a formulation (the pseudotensor approach being the most promising).

AFTERWORD

The present article has presented a case that the non-local nature of gravitational energy and momentum is directly responsible for quantum indeterminacy. In particular it has been posited that the quantum mechanical wavefunction of a massive particle can be interpreted as a manifestation of the scalar spacetime metric perturbation. Further analysis appears to be warranted.

After reading this paper, one might begin to understand why attempts at quantizing gravity have proven largely intractible. Global spacetime structure appears to bequeath quantization (to paraphrase Schrodinger), but is not in and of itself quantized.

It is nostalgic to consider here Eratosthenes of Cyrene (circa ∼240BC). Using his knowledge of geometry and the observation of shadows cast by the sun, Eratosthenes was able accurately ascertain the scale of the earth's circumference (from within ancient Egypt) [31]. In an analogous manner perhaps the Planck constant is a means through which one might glimpse the true scale and geometry of the cosmos.

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