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Mechanism for the Generation of a Fundamental Unit of Charge

J. P. Lestone
Computational Physics Division, Los Alamos National Laboratory
Los Alamos, NM 87545, USA
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Virtual photons, with a reduced wavelength of ƛ, are assumed to interact with isolated charged leptons with a cross section of $\pi\text{ƛ}^2$. This interaction is assumed to generate stimulated virtual photon emissions that are capable of being exchanged with other particles. This exchange of virtual photons is assumed to define the strength of electromagnetism. With the inclusion of near-field effects, the model choices presented give a calculated fundamental unit of charge of 1.60218×10^{-19} C. If these choices are corroborated by detailed calculations then an understanding of the numerical value of the fine structure constant may emerge.

I. Introduction

The electrostatic force between two charged leptons (hereafter referred to as particles) separated by a distance d can be expressed as

$$F = \frac{e^2}{4\pi\varepsilon_o d^2} = \frac{\alpha\hbar c}{d^2}. \quad (1)$$

The fine structure constant, α, is defined relative to the elementary charge e via

$$e^2 = \alpha\hbar c\, 4\pi\varepsilon_o. \quad (2)$$

Despite the central importance of the value of α to QED, and the many attempts to understand it from a theoretical perspective [1], there is still no accepted theory to explain its value and the corresponding elementary charge $e = 1.602177\times10^{-19}$ C.

In the present paper, we use semi-classical arguments without full quantum theory or detailed special relativity. By their nature, semi-classical approximations of complex phenomena can be difficult to justify. Often, new semi-classical assumptions can only be fully justified by their usefulness as an intermediate picture on our journey to a more complete understanding of the phenomenon in question. One of the most famous examples of this is the Bohr Model of the hydrogen atom, which can reproduce the corresponding atomic spectroscopy, and served as an intermediate step to the more complete picture obtainable via the Schrödinger Equation.

By invoking a far-field virtual-photon particle-interaction cross section of $\pi\text{ƛ}^2$, and including near-field effects, a universal charge of 1.602180×10^{-19} C emerges from the presented model. The size of this charge depends sensitively on the assumed photon-particle near-field effects. If these assumed properties are confirmed by detailed calculations, then an understanding of the numerical value of α may emerge.

II. Electrostatics

We assume that virtual-vacuum photons interact with charged leptons with a cross section of $\pi\text{ƛ}^2$. This cross section is assumed to be associated with the stimulated emission of an additional virtual photon. The number density of virtual vacuum photons is [2]

$$\rho_N(\omega)d\omega = \frac{\omega^2 d\omega}{\pi^2 c^3}. \quad (3)$$

The rate of stimulated virtual-photon production from an isolated particle can be determined by multiplying the number of vacuum photons in a shell of radius r and thickness dr surrounding an isolated particle by the probability that a photon originating from the shell will generate a stimulated emission. The corresponding number of generated (stimulated) virtual photons is given by

$$N_S(\omega) = \frac{\omega^2 d\omega}{\pi^2 c^3} 4\pi r^2 dr \frac{\pi\text{ƛ}^2}{4\pi r^2} = \frac{d\omega\, dr}{\pi c}. \quad (4)$$

The time it takes the photons heading towards the particle, to clear the shell is $dt=dr/c$. Substituting this into Eq. (4) gives the rate of virtual photon generation

$$R_S(\omega) = \frac{N_S(\omega)}{dt} = \frac{d\omega}{\pi}. \quad (5)$$

This is a very simple and beautiful equation that is central to the results that follow.

The power of the stimulated emission from an isolated particle is obtained by integrating over Eq. (5) multiplied by $\varepsilon = \hbar\omega$, and is given by

$$P_S = \int_0^\infty \frac{\hbar\omega d(\hbar\omega)}{\pi\hbar} = \int_0^\infty \frac{\varepsilon d\varepsilon}{\pi\hbar}. \quad (6)$$

Of course, this stimulated emission violates conservation of energy, and is not allowed from an isolated fundamental particle. However, a violation of conservation of energy by an amount ε can be allowed for a time scale given by the time-energy uncertainty principle, $\tau = \hbar/(2\varepsilon)$. If, in this time scale, the stimulated emission could find a partner particle,

then conservation of energy can be re-established and the exchange of the stimulated emission between a pair allowed. We speculate that this exchange is the origin of the repulsive force between two identical charged leptons.

The time required for a virtual photon to be exchanged between a pair of particles separated by a distance d is $t = d/c$. Assuming the probability per unit time that the virtual photon "disappears" is $1/\tau = 2\varepsilon/\hbar$, the probability that conservation of energy is allowed to be violated for the photon exchange time is $\exp(-t/\tau)$. Ignoring near-field effects, the power of the virtual exchange from particle A to particle B can be expressed as

$$P_{A\to B} = \frac{1}{\pi\hbar}\int_0^\infty \varepsilon \exp(-\frac{d}{c}\frac{2\varepsilon}{\hbar})\ \frac{\pi\lambda\!\!\!^{-2}}{4\pi d^2}\, d\varepsilon. \qquad (7)$$

The $\pi\lambda\!\!\!^{-2}/(4\pi d^2)$ term is the assumed probability of finding the partner, assuming the jump across the distance d has been made. Given Eq. (7), the force generated by the two-way exchange of stimulated virtual photons is

$$F = \frac{\hbar c}{2\pi d^2}\int_0^\infty \frac{\exp(-\varepsilon/T_{ex})}{\varepsilon}\, d\varepsilon, \qquad (8)$$

where $T_{ex} = \hbar c/(2d)$ can be thought of as an effective exchange temperature. In semi-classical models of photon emission, one often obtains the exponential term $\exp(-\varepsilon/T)$ [3] which needs to be replaced by the Planckian factor $[\exp(\varepsilon/T)-1]^{-1}$ to obtain the correct result. Without proof, we here make the same substitution. To be clear, the main reason for making this substitution is because it leads to a favorable result, after the addition of other semi-classical model choices discussed later. If the corresponding force is assumed to be the origin of electromagnetism, then the fine structure constant can be expressed as

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{1}{\varepsilon[\exp(\varepsilon/T_{ex})-1]}\, d\varepsilon. \qquad (9)$$

This integral diverges and gives an infinite strength for the repulsive force between two particles. However, the origin of the divergence is the lowest-energy photons where $\lambda\!\!\!^{-} > d$, where the cross section for the photon-particle interaction needs to be modified to lower values to correct for near-field effects.

III. Near-field Effects

It is possible that the near-field effects needed here can be calculated using electromagnetic theory. We do not proceed down this path here, but instead start with the question: how is the photon-particle interaction cross section altered when the incoming photon does not start from a source particle an infinite distance from the absorbing particle?

Some estimates of near-field corrections can be obtained via simple semi-classical arguments. For example, based on the assumed interaction cross section of $\pi\lambda\!\!\!^{-2}$, each lepton can be thought of as a sphere of radius $\lambda\!\!\!^{-}$ when interacting with a photon with a reduced wavelength of $\lambda\!\!\!^{-}$. Within this simple picture near-field effects should be strong for photons exchanged between particles separated by $d < \lambda\!\!\!^{-}$, with the virtual photons increasingly losing their ability to interact with the particles as the separation distance decreases. This behavior is analogous to the interaction properties of closely spaced classical antennas [4]. We write the near-field corrected interaction cross section as $\sigma_{nf} = f_{nf}(\lambda\!\!\!^{-},d)\cdot\pi\lambda\!\!\!^{-2}$, where $f_{nf}(\lambda\!\!\!^{-},d)$ is the near-field correction factor.

To enable an estimate of $f_{nf}(\lambda\!\!\!^{-},d)$, we imagine a distribution of photon-interaction sites around each particle. Guided by the harmonic oscillator wave function, and the need for the distribution of interaction sites to have a length scale of $\lambda\!\!\!^{-}$, we here set the distribution of interaction sites to

$$\psi(r,\lambda\!\!\!^{-}) \propto \exp(\frac{-r^2}{2\lambda\!\!\!^{-2}}). \qquad (10)$$

For a semi-classical virtual photon falling inwards towards a particle, we assume the amplitude per unit length for absorption is given by Eq. (10). This assumption leads to a near-field correction factor

$$f_{nf}(\lambda\!\!\!^{-},d) = \left|\frac{2}{\lambda\!\!\!^{-}\sqrt{2\pi}}\int_0^d \exp(\frac{-r^2}{2\lambda\!\!\!^{-2}})dr\right|^2 = \mathrm{erf}^2(\frac{d}{\lambda\!\!\!^{-}\sqrt{2}}). \qquad (11)$$

The corresponding modification to the absorption cross section is displayed by the solid curve in Fig. 1. The near-field correction provides an effective smooth low-energy cutoff.

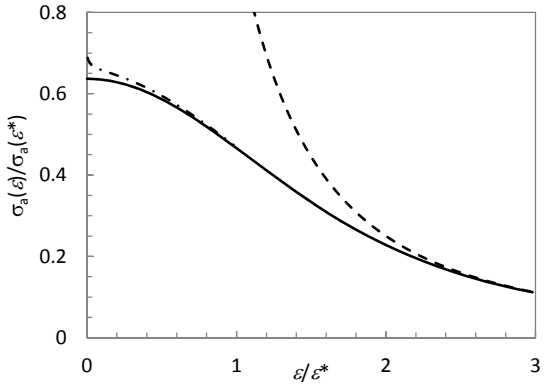

Fig. 1. Effective interaction cross sections obtained using the near-field correction factor represented by Eq. (11) (solid curve). The energy axis is in units of $\varepsilon^*=\hbar c/d$. At $\varepsilon/\varepsilon^*=1$ and 2, the separations between emitter and absorber are $\lambda\!\!\!^{-}$ and $2\lambda\!\!\!^{-}$, respectively. The dashed curve displays the far-field result. The dashed-dotted curve displays the higher-order correction discussed in section IV.

To modify the calculation of the fine structure constant given by Eq. (9) to include near-field effects, the cross section for the absorption of the stimulated virtual photons must be multiplied by f_{nf}. Perhaps less obvious, is that the cross section used to calculate the emission rate of initiating the exchange must also be multiplied by f_{nf}. This is due to time-symmetry arguments that apply equally to emission and absorption processes. Including the near-field effects at both the initiation and completion ends of the photon exchanges gives the result

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{erf}^4(\varepsilon/2^{3/2})}{\varepsilon[\exp(\varepsilon)-1]}\,d\varepsilon. \quad (12)$$

For convenience, Eq. (12) is written with energy in units of T_{ex}. The corresponding calculated inverse fine structure constant and the fundamental unit of charge are 138.9099 and 1.591333×10^{-19} C, respectively.

The force associated with the semi-classical exchange of virtual photons between two identical particles represented by Eq. (12) can only generate repulsion. However, an attractive force between oppositely charged objects can be obtained by assuming the opposite charge is associated with a hole in a Fermi-sea of negative-energy particles [2].

IV. Higher-order Corrections

The near-field correction factor as expressed by Eq. (11) does not adequately reproduce the fine structure constant obtained via experiment [5]. Therefore, if Eq. (11) is on the right track, it needs to be modified. We suggest a possible source for a modification is higher-order QED corrections to the near-field correction factor, especially for $d < \lambda\!\!\!^{-}$. For particle separation distances $d >> \lambda\!\!\!^{-}$, where the low-order correction from Eq. (11) is small, we expect the higher-order QED corrections to be very small. However, with decreasing d, we expect higher-order effects associated with the QED structure of the particles to grow logarithmically. We suggest the functional form for the higher-order near-field correction factor

$$f_{nf}(\alpha) = f_{nf}(1-\alpha\ln(f_{nf})), \quad (13)$$

where f_{nf} is the non-QED correction factor as given by Eq. (11). The corresponding modification to the absorption cross section is displayed by the dashed-dotted curve in Fig. 1. If Eq. (13) is on the right track it is likely that even higher-order corrections exist. Substituting Eq. (13) into Eq. (12) gives

$$\alpha = \frac{1}{2\pi}\int_0^\infty \frac{\mathrm{erf}^4(\varepsilon/2^{3/2})[1-\alpha\ln(\mathrm{erf}^2(\varepsilon/2^{3/2}))]^2}{\varepsilon[\exp(\varepsilon)-1]}\,d\varepsilon. \quad (14)$$

This equation can be solved numerically by iteration. The calculated inverse fine structure constant and corresponding fundamental unit of charge are 137.0355 and 1.602180×10^{-19} C, respectively. The calculated fundamental unit of charge differs from the corresponding value inferred from experiment [5] by 1 part in 5 million.

V. Summary

Using semi-classical approaches, an expression for the fine structure constant can be obtained using intuitive steps that could be easily explained to an undergraduate physics audience. Our first semi-classical estimate of the repulsive force generated by the exchange of virtual photons between a pair of particles, obtained using only the far-field interaction cross section of $\pi\lambda\!\!\!^{-2}$, is infinite. Including an estimate of near-field effects (to low order) obtained via simple arguments, leads to a force that defines a fundamental unit of charge of $\sim1.59\times10^{-19}$ C. An educated guess for higher-order corrections to near-field effects gives an inverse fine structure constant of $1/\alpha=137.0355$ and a corresponding calculated charge of 1.602180×10^{-19} C. The semi-classical choices needed to obtain this result are: a far-field cross section of $\pi\lambda\!\!\!^{-2}$ for the generation of stimulated virtual-photon emission; a probability per unit time that stimulated virtual photons "disappear", set by the time-energy uncertainty principle; the substitution of an exponential with the corresponding Planckian factor; and near-field effects controlled by a harmonic oscillator wave function with higher-order corrections as given by Eq. (13). Obviously, more work on photon-particle near-field corrections is needed. If the semi-classical choices made here can be justified by detailed calculations then an understanding of the numerical value of the fine structure constant may emerge.

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