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GENERALIZED HEISENBERG UNCERTAINTY PRINCIPLE in QUANTUM GEOMETRODYNAMICS and GENERAL RELATIVITY

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In this paper we focus on energy flows in Universe as a simple quantum system and are concentrating on the nonlinear Hamilton-Jacobi equation, which appears in the standard quantum formalism based on Schrödingier equation. The cases of the domination of radiation, barotropic fluid, and the quantum matter-energy are considered too. As a result, there is formulated the generalized Heisenberg uncertainty principle (GHUP) for a metric tensor. We also use the Kuzmichevs Geometrodynamics as a way to quantify an inter relationship between the GHUP for a metric tensor and conditions postulated as to a barotropic fluid, i.e. dust for the early Universe conditions.

Keywords: Generalized Heisenberg uncertainty principle, general relativity, universe, cosmology, quantum geometrodynamics

1. Introduction

The answer to the question brought up in the title of this paper can be provided after comparative descriptions of the Universe by classical and quantum theories. As is well known, the Universe is subject to classical theories on large space-time scales whereas on small space-time scales, comparable with Planck scales and length, it should be described from a Quantum-theoretical perspective.

The first goal of our research will be to introduce a framework about the speed of gravitons in "heavy gravity", and this is important eventually, as illustrated by C. Will [1, 2], as it could possibly be observed. Secondly, it also will involve an upper bound to the rest mass of a graviton. The third aspect of the inquiry of our manuscript will be to come up with a variant of the HUP, involving a metric tensor, as well as the Stress energy tensor, which will in time allow us to establish a lower bound to the mass of a graviton, preferably at the start of cosmological evolution. The article concludes as answering a statement by Mukhanov, in Marcel Grossman 14 as to his interpretation as to the importance of Causal barriers, in place in terms of prior to present universe transitions in cosmology. In the Mukhanov view, Causal barriers create an averaging effect of contributions from prior universe conditions to the present universe initial conditions. In fact, this means, that effectively, in the case of a multiverse, that the existence of prior universe contributions from a multiverse, would be effectively a single universe repeating itself. I.e. our view is instead very similar to an Ergodic mixing protocol. Even in the case of multiverse contributions to a present universe. This is the basis of much of our analysis. Where Mukhanov implied stating that instead of an Ergodic mixing of prior contributions from a multiverse, that causal structure would ALWAYS restrict our analysis of information from a prior ensemble to be the same as a repeating single universe model for cyclic universes. We regard the Mukhanov interpretation as indefensible. And state why in the last chapter of this article.

We reference what was done by Will in his living reviews of relativity article as to the 'Confrontation between GR and experiment". Specifically we make use of his experimentally based formula of [1,2], with $v_{graviton}$the speed of a graviton, and $m_{graviton}$the rest mass of a graviton, and $E_{graviton}$in the inertial rest frame given as:

$$\left(\frac{v_{graviton}}{c}\right)^2 = 1 - \frac{m^2_{graviton}c^4}{E^2_{graviton}} \tag{1}$$

Our take away from Formulae 1 is that if a graviton is massive, that the speed of travel of graviton drops below the value of c, the speed of light, with massless gravitons traveling at the speed of light. This in addition puts restrictions upon the energy of a graviton and argues against simple approximations like. Hence we follow [2] in terms of the following ideas as given in Formula 2, next:

$$\begin{aligned} &\tfrac{v_{graviton}}{c} = 1 - 5\times 10^{-17}\cdot\left(\tfrac{200Mpc}{D}\right)\cdot\left(\tfrac{\Delta t}{1\,\mathrm{sec}}\right)\\ &\doteq 1 - 5\times 10^{-17}\cdot\left(\tfrac{200Mpc}{D}\right)\cdot\left(\tfrac{\Delta t = \Delta t_a - (1+z)\cdot\Delta t_b}{1\,\mathrm{sec}}\right)\\ &\Leftrightarrow \tfrac{2m_{graviton}c^2}{E_{graviton}} \approx 5\times 10^{-17}\cdot\left(\tfrac{200Mpc}{D}\right)\times\\ &\times\left(\tfrac{\Delta t_a - (1+z)\cdot\Delta t_b}{1\,\mathrm{sec}}\right). \end{aligned} \tag{2}$$

Here, Δt_a is the difference in arrival time, and Δt_e is the difference in emission time in the case of the early Universe, i.e. near the big bang, then if in the beginning of time, one has, if we assume that there is an average $E_{graviton} \approx \hbar\cdot\omega_{graviton}$, and

$$\Delta t_a \sim 4.3\times 10^{17}\,\mathrm{sec}, \quad \Delta t_e \sim 10^{-33}\,\mathrm{sec}, \quad z\sim 10^{50}. \tag{3}$$

Then, $\left(\frac{\Delta t_a - (1+z)\cdot\Delta t_b}{1\,\mathrm{sec}}\right) \sim 1$, and if $D \sim 4.6\times 10^{26} meters = radii(universe)$, so one can set

$$\left(\frac{200Mpc}{D}\right) \sim 10^{-2}. \tag{4}$$

And if one sets the mass of a graviton [3] into Eq. (1), then we have in the present era, that if we look at primordial time generated gravitons, that if one uses the

$$\Delta t_a \sim 4.3\times 10^{17}\,\mathrm{sec}, \quad \Delta t_e \sim 10^{-33}\,\mathrm{sec}, \quad z\sim 10^{55}. \tag{5}$$

Note that the above given frequency for the graviton is for the present era, but it starts assuming an initial genesis from an (initial) inflationary starring point which is not a space-time singularity.

Note this comes from a scale factor, if $z\sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.

We will next discuss the implications of this point in the next section, of a non zero smallest scale factor. Secondly the fact we are working with a massive graviton, as given will be given some credence as to when we obtain a lower bound, as will come up in our derivation of modification of the values [3]

$$\begin{aligned} &\left\langle (\delta g_{uv})^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \tfrac{\hbar^2}{V^2_{Volume}}\\ &\xrightarrow[uv\to tt]{} \left\langle (\delta g_{tt})^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \tfrac{\hbar^2}{V^2_{Volume}}\\ &\&\ \ \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+. \end{aligned} \tag{6}$$

The reasons for saying this set of values for the variation of the non g_{tt}metric will be in the 3^{rd} section and it is due to the smallness of the square of the scale factor in the vicinity of Planck time interval.

Leading to nonzero initial entropy as stated in **Appendix A**, we also examine a Ricci scalar value at the boundary between Pre Planckian to Planckian regime of space-time, setting the magnitude of Ricci Scalar k as approaching flat space conditions right after the Planck regime. Furthermore, we have an approximation as to initial entropy production $S_{initial(graviton)} \sim 10^{37}$. Then we get an initial version of the cosmological "constant" as it is shown in the **Appendix D** which is linked to initial value of a graviton mass. **Appendix E** is written for the Riemannian- Penrose inequality, which is either a nonzero NLED scale factor or quantum bounce as of LQG. Finally, **Appendix F** gives conditions so that a pre Planckian kinetic energy (inflaton) value greater than Potential energy occurs, which is foundational to the lower bound to Graviton mass. We will in the future add more structure to this calculation so as to confirm via a precise calculation that the lower bound to the graviton mass, is about 10^{-70} grams. Our lower bound is a dimensional approximation so far. We will make it exact.

2. The flow of energy in a quantum process

Following [4], we start with a quantum system described by a wave function, $\psi(x,t)$, the time evolution of which is given by the Schrödinger equation. The Born probability rule is then used to calculate the probability $P(x',t')$ of finding the system at x' at a later time t'. Thus $P(x',t') = |\psi(x',t')|^2 = R^2(x',t)$ where $R(x',t)$ is the amplitude of the field. So our final result depends only on one of the pair of real numbers in $\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar}$. The information as to how the phase evolves in time is, as it were, 'hidden' in the evolution of the complex wave function $\psi(x,t)$. It would perhaps be revealing to have a pair of equations showing explicitly the evolution of the two real fields $R(x,t)$ and $S(x,t)$.

The simplest way to arrive at the equations containing R and S is to substitute $\psi = Re^{iS/\hbar}$ into the Schrödinger equation and separate the resulting equation into its real and imaginary parts. The imaginary part can be written in the form

$$\frac{\partial \rho}{\partial t} + \nabla . \left(\rho \frac{\nabla S}{m} \right) = 0, \tag{7}$$

where we have written $\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar}$ with $R^2 = \rho$.

Since, at this stage, we are simply analysing the Schrödinger equation, equation (7) provides an expression for the conservation of probability $P(x,t)$. The real part takes the form[1]

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} - \frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) + V = 0. \tag{8}$$

We will call this the quantum Hamilton-Jacobi equation [QHJ] for reasons that we will bring out as we go along.

These two equations must have the same content as the Schrödinger equation and it would surely be of interest to see if they can give a different insight into the evolution of quantum systems. Note we are not departing from the usual interpretation yet, we are merely drawing attention to an alternative form of the mathematical structure. Already one sees that there is a disadvantage of using these two equations as they are no longer linear and therefore more difficult to analyse. Nevertheless as we will show, we can obtain new information about energy flow using equation (8), in spite of Bohr's insistence that you can talk either about an evolution in space-time or about a causal (i.e. momentum-energy) evolution, never both together.

Although the splitting of an equation into its real and imaginary parts is a standard mathematical practice, we will re-derive these two equations again, starting from Heisenberg's expression [5] for the Lagrangian of the Schrödinger field [5] and applying the standard Euler-Lagrange equations, treating $R(x,t)$ and $S(x,t)$ as independent fields. This procedure will enable us to find the components of the energy-momentum tensor, thus allowing us to investigate the energy and momentum flows involved in the quantum process. In this way we are able show that equation (8) is an expression for the local conservation of energy in this evolving quantum process.

This result should not be too surprising since, as is well known, the Schrödinger equation must describe the evolution of the energy involved in the process. Why? Because the expression of the classical dynamical energy, the Hamiltonian, albeit written in operator form, is at the heart of the equation. However by focussing on the complex form of the wave function, we do not explicitly see how this energy flows in the evolving process. The wave function then appears, as it were, 'disembodied' from the energy, so that it then seems to take on, physically, the air of some ghostly shadow of the evolving system, allowing only probability outcomes to be discussed.

We then find that the wave function, with its deterministic equation, can be treated as an entity in its own right giving the probability of finding a particular result. Its role in accounting for the energy flow is then forgotten. In consequence we feel free to add wave functions and to collapse wave functions with no concern as to the energy involved, hoping that it will be taken care of by the Schrödinger equation.

However a realisation that both the addition of wave functions and the collapse of wave functions oc-

[1] We are well aware that we here in this segment are treating S as a Torsor. For a physical treatment of S, see page 95 of Mackey [5].

cur outside of the Schrödinger equation, should be a cause for concern since, unless care is taken with such addition and collapse, any such move could contradict the conservation of energy[2]. The purpose of this paper is find a way to discuss the flow of energy in a quantum process rather than relying *only* on the $\psi(x,t)$ and the Schrödinger equation.

3. Kuzmichevs quantum constraint equations

In this section we follow Kuzmichevs paper [157] and consider the homogeneous, isotropic and spatially closed quantum cosmological system (universe). The geometry of such a universe is described by the Robertson-Walker metric. This metric has a maximally symmetric three-dimensional subspace of a four-dimensional space-time. Since we consider the spatially closed Universe, then the geometry of the space-time depends on a single cosmological parameter, namely the cosmic scale factor a which describes the overall expansion or contraction of the Universe [158]. The scale factor is a field variable which determines gravity in the formalism under consideration. We assume that from the beginning the Universe is filled with matter in the form of the uniform scalar field ϕ, the state of which is given by some Hermitian Hamiltonian, $H_\phi = H_\phi^\dagger$. This Hamiltonian is defined in a curved space-time, and therefore, in the general case, it depends on a scale factor a as a parameter, $H_\phi = H_\phi(a)$. In addition, it will be accepted that the Universe is filled with a perfect fluid in the form of relativistic matter (further referred as radiation) with the proper energy $M_\gamma = \frac{E}{2a}$ in the comoving volume $\frac{1}{2}a^3$, where E is a real constant proportional to the number of particles of the perfect fluid. The perfect fluid defines a material reference frame [159, 160].

The restrictions in the form of the first-class constraint equations are imposed on the state vector of the quantum Universe $\Psi = \langle a, \phi | \Psi(T) \rangle$, where T is a time parameter. These constraints can be reduced to two equations [160–162],

$$\left(-i\partial_T - \frac{2}{3}E\right)\Psi = 0, \tag{9}$$

$$\left(-\partial_a^2 + a^2 - 2aH_\phi - E\right)\Psi = 0, \tag{10}$$

[2] We are talking about energy non-conservation outside the limits imposed by the energy-time uncertainty principle.

where Eq. (9) describes the time evolution of Ψ, when the number of particles of the perfect fluid conserves, while Eq. (10) determines the quantum states of the Universe at some fixed instant of time $T = T_0$, T_0 is an arbitrary constant taken as a time reference point. The coefficient $\frac{2}{3}$ in Eq. (9) is caused by the choice of the parameter T as the time variable. This time variable is connected with the proper time τ by the differential equation $d\tau = a dT$. Following the ADM formalism [163, 164], one can extract the so-called lapse function N, that specifies the time reference scale, from the total differential dT: $dT = N d\eta$, where η is the "arc time" [165, 166].

The quantum constraints (9) and (10) can be rewritten in the form of the time-dependent Schrödinger-type equation

$$-i\partial_T \Psi = \frac{2}{3}\mathcal{H}\Psi, \tag{11}$$

where

$$\mathcal{H} = -\partial_a^2 + a^2 - 2aH_\phi. \tag{12}$$

The minus sign before the partial derivative ∂_T is stipulated by the specific character of the cosmological problem, namely that the classical momentum conjugate to the variable a is defined with the minus sign [167, 168].

The partial solution of Eqs. (9) and (10) has a form

$$\Psi(T) = e^{i\frac{2}{3}E(T-T_0)}\Psi(T_0), \tag{13}$$

where the vector $\Psi(T_0) \equiv \langle a, \phi | \psi \rangle$ satisfies the stationary equation

$$\mathcal{H}|\psi\rangle = E|\psi\rangle. \tag{14}$$

From the condition

$$\begin{aligned} 0 &= \frac{d}{dT}\int D[a,\phi]\,|\Psi|^2 \\ &= -i\frac{2}{3}\int D[a,\phi]\,\Psi^*\left[\mathcal{H}^\dagger - \mathcal{H}\right]\Psi, \end{aligned} \tag{15}$$

where $D[a,\phi]$ is the measure of integration with respect to the fields a and ϕ chosen in an appropriate way, it follows that the operator (12) is Hermitian: $\mathcal{H} = \mathcal{H}^\dagger$.

4. Non zero scale factor, initially and what this is telling us physically. Starting with a configuration from Unruh

Begin with the starting point of [112]

$$\Delta l \cdot \Delta p \geq \frac{\hbar}{2} \tag{16}$$

We will be using the approximation given by Unruh [112], of a generalization we will write as

$$(\Delta l)_{ij} = \frac{\delta g_{ij}}{g_{ij}} \cdot \frac{l}{2}, \quad (\Delta p)_{ij} = \Delta T_{ij} \cdot \delta t \cdot \Delta A. \tag{17}$$

If we use the following, from the Roberson-Walker metric [140].

$$\begin{array}{l} g_{tt} = 1, \quad g_{rr} = \frac{-a^2(t)}{1-k\cdot r^2}, \quad g_{\theta\theta} = -a^2(t)\cdot r^2, \\ g_{\phi\phi} = -a^2(t)\cdot \sin^2\theta \cdot d\phi^2. \end{array} \tag{18}$$

Following Unruh [112], write then, an uncertainty of metric tensor as, with the following inputs

$$a^2(t) \sim 10^{-110}, r \equiv l_P \sim 10^{-35} meters. \tag{19}$$

Then, the surviving version of Eq. (16) and Eq. (17) is, then, if $\Delta T_{tt} \sim \Delta\rho$

$$\begin{array}{l} V^{(4)} = \delta t \cdot \Delta A \cdot r, \\ \delta g_{tt} \cdot \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot \frac{r}{2} \geq \frac{\hbar}{2} \\ \Leftrightarrow \delta g_{tt} \cdot \Delta T_{tt} \geq \frac{\hbar}{V^{(4)}}. \end{array} \tag{20}$$

This Eq. (20) is such that we can extract, up to a point the HUP principle for uncertainty in time and energy, with one very large caveat added, namely if we use the fluid approximation of space-time [140]

$$T_{ii} = diag(\rho, -p, -p, -p). \tag{21}$$

Then

$$\Delta T_{tt} \sim \Delta\rho \sim \frac{\Delta E}{V^{(3)}.} \tag{22}$$

Then, Eq.(20) and Eq. (21) and Eq. (22) together yield

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2}, \quad Unless\ \delta g_{tt} \sim O(1). \tag{23}$$

How likely is $\delta g_{tt} \sim O(1)$? Not going to happen. Why? The homogeneity of the early Universe will keep

$$\delta g_{tt} \neq g_{tt} = 1. \tag{24}$$

In fact, we have that from Giovannini [140], that if ϕ is a scalar function, and $a^2(t) \sim 10^{-110}$, then if

$$\delta g_{tt} \sim a^2(t) \cdot \phi << 1. \tag{25}$$

Then, there is no way that Eq. (23) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$. Hence, the Mukhanov suggestion as will be discussed toward the end of this article, is not feasible. Finally, we will discuss a lower bound to the mass of the graviton.

5. How we can justifying writing very small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ values

To begin this process, we will break it down into the following coordinates:
In the $rr, \theta\theta$, and $\phi\phi$ coordinates, we will use the Fluid approximation, $T_{ii} = diag(\rho, -p, -p, -p)$ [170] with

$$\begin{array}{l} \delta g_{rr} T_{rr} \geq -\left|\frac{\hbar\cdot a^2(t)\cdot r^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0, \\ \delta g_{\theta\theta} T_{\theta\theta} \geq -\left|\frac{\hbar\cdot a^2(t)}{V^{(4)}(1-k\cdot r^2)}\right| \xrightarrow[a\to 0]{} 0, \\ \delta g_{\phi\phi} T_{\phi\phi} \geq -\left|\frac{\hbar\cdot a^2(t)\cdot \sin^2\theta\cdot d\phi^2}{V^{(4)}}\right| \xrightarrow[a\to 0]{} 0. \end{array} \tag{26}$$

If as an example, we have negative pressure, with T_{rr}, $T_{\theta\theta}$, and $T_{\phi\phi} < 0$, and $p = -\rho$, then the only choice we have, then is to set $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$, **since there is no way that** $p = -\rho$ is zero valued. Having said this, the value of δg_{tt} being non zero, will be part of how we will be looking at a lower bound to the graviton mass which is not zero.

In our analysis of Pre Planckian space-time according to the HUP which is written in this paper in terms of a reduction of contributions of all but the time component of the metric tensor, we face the problem of arguing how fluctuations drop off, unless they are directly connected to the time component. Which makes sense, since if there is a nonsingular start to the universe, as given by [150,171], the Pre planckian space-time regime is part and parcel of an emergent space-time which would place a premium upon non spatial metric tensor fluctuations. Hence, we will delineate reasons for why the metric tensor fluctuations are restricted to the time components only.

6. Lower bound to the graviton mass using Barbour's emergent time

In order to start this approximation, we will be using Barbour's value of emergent time [115,116] restricted to the Plank spatial interval and massive gravitons, with a massive graviton [117]

$$(\delta t)^2_{emergent} = \frac{\sum_i m_i l_i \cdot l_i}{2 \cdot (E-V)} \to \frac{m_{graviton} l_P \cdot l_P}{2 \cdot (E-V)}. \quad (27)$$

Initially, as postulated by Babour [115, 116], this set of masses, given in the emergent time structure could be for say the planetary masses of each contribution of the solar system. Our identification is to have an initial mass value, at the start of creation, for an individual graviton.
If $(\delta t)^2_{emergent} = \delta t^2$ in Eq.(20), using Eq.(20) and Eq. (27) we can arrive at the identification of

$$m_{graviton} \geq \frac{2\hbar^2}{(\delta g_{tt})^2 l_P^2} \cdot \frac{(E-V)}{\Delta T_{tt}^2}. \quad (28)$$

Key to Eq. (28) will be identification of the kinetic energy which is written as $E - V$. This identification will be the key point raised in this manuscript. Note that it raises the distinct possibility of an initial state, just before the 'big bang' of a kinetic energy dominated 'pre inflationary' universe. I.e. in terms of an inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [170]. The key finding which is in [118] is, that, if the kinetic energy is dominated by the 'inflaton' that

$$K.E. \sim (E-V) \sim \dot{\phi}^2 \propto a^{-6}. \quad (29)$$

This is done with the proviso that $w < -1$, where w= *pressure /density* [172]. I.e., the convention referred to is of avoiding *density*= - *pressure*, which is used frequently. In effect, what we are saying is that during the period of the 'Planckian regime' we can seriously consider an initial density proportional to Kinetic energy, and call this K.E. as proportional to [170]

$$\rho_w \propto a^{-3(1-w)}. \quad (30)$$

If we are where we are in a very small Planckian regime of space-time, we could, then say write Eq. (30) as proportional to $g^* T^4$ [170], with g^* initial degrees of freedom, and T the initial temperature as low just before the onset of inflation. The question to ask, then is, what is the value of the initial degrees of freedom, and what is the temperature, T, at the start of expansion? For what it is worth, the starting supposition, is that there would then be a likelihood for an initial low temperature regime.

7. Metric uncertainty principle as inter relationship of general relativity and quantum geomerodynamics

We will be using, the inputs from Section 3 extensively as a way to intertwine the predictions as to a HUP connected with the metric tensor of space-time and the resulting initial conditions for space-time according to Geometrodynamics. The end result will be that we are supplying initial conditions which cannot be obtained, by other means. We also will quantify via a version of dust dynamics, how this affects candidate DM and possibly DE contributions to initial cosmological conditions. To do this, we will review the concepts used in both the Heisenberg Uncertainty principle, for metric tensors, and the Geometrodynamics equations used. The conclusion of what we are talking about is use of the HUP, for metric tensors to form bounds on the Getrodynamics equations in the pre Planckian space-time era.

7.1. Application of the HUP to metric tensors

We will be examining a Friedmann equation for the evolution of the scale fa tor, using explicitly two cases, one case being when the acceleration of expansion of the scale factor is kept in, another when it is out, and the intermediate cases of when the acceleration factor, and the scale factor is important but not dominant. In doing so we will be tying it in our discussion with the earlier work done on the HUP but from the context of how the acceleration term will affect the HUP, and making sense of why our generalized uncertainty principle, as given in the beginning of Eq. (31) is from [3, 112, 150] leading to a restriction of the metric tensor fluctuation to being the time component only, in the denominator of the modified HUP expression. [3] gives us the initial generalized HUP, and [112, 150] express the fluctuation restricted to

$$\begin{gathered}\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}_{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V^2_{\text{volume}}} \\ \xrightarrow[uv \to tt]{} \left\langle \left(\delta g_{tt}\right)^2 \left(\hat{T}_{tt}\right)^2 \right\rangle \geq \frac{\hbar^2}{V^2_{\text{volume}}} \\ \& \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+ \ . \end{gathered} \quad (31)$$

Namely we will be working with

$$\delta t\Delta E=\tfrac{\hbar}{\delta g_{tt}}\equiv\tfrac{\hbar}{a^2(t)\cdot\phi}\ll\hbar \tag{32}$$

$$\Leftrightarrow S_{\text{initial}}(\text{with}[\delta g_{tt}])=(\delta g_{tt})^{-3}S_{\text{initial}}(\text{without}[\delta g_{tt}])$$
$$\gg S_{\text{initial}}(\text{without}[\delta g_{tt}])$$

i.e. the fluctuation $\delta g_{tt}\ll 1$ dramatically boost initial entropy. Not what it would be if $\delta g_{tt}\approx 1$. The next question to ask would be how could one actually have

$$\delta g_{tt}\sim a^2(t)\cdot\phi\ \xrightarrow[\textit{Very Large}]{}\ 1 \tag{33}$$

Furthermore, we have that Eq. (31) has an explicit restriction of the modified HUP: to be influenced by only the time fluctuation of the metric tensor, which is given by , and this in the denominator of the modified HUP is $\ll 1$. Eq. (32) is highlighted by the term $\ll 1$ in the denominator of the modified HUP leading to specific entropy generation. As is expected, in the Pre Planckian to Planckian transition, referred to in Eq. (32), second line, delineates if $\ll 1$ that the entropy generation is very different, than when approaches 1, which is after the Pre Planckian to Planckian emergent physics regime. In addition, Eq. (33) specifically alludes to if approaches 1, marking the transition to the Planckian regime and beyond, and this is due to the inflaton growing extremely large.

In short, we would require an enormous "inflaton" style ϕ valued scalar function, and $a^2(t)\sim 10^{-110}$. How could ϕ be initially quite large? Within Planck time the following for mass holds, as a lower bound

$$m_{\text{gravitation}}\geq\frac{2\hbar^2}{(\delta g_{tt})^2 l_p^2}\cdot\frac{(E-V)}{\Delta T_{tt}^2}. \tag{34}$$

Here, we are using the following approximation as to Kinetic energy in the beginning of the expansion of the Universe.

$$K.E.\sim(E-V)\sim\dot{\phi}^2\infty a^{-6}. \tag{35}$$

Then, up to first order, we could approximate, with $H.O.T.$ being higher order terms

$$\dot{\phi}\sim a^{-3}\Leftrightarrow\phi\approx t\cdot a^{-3}+H.O.T\ . \tag{36}$$

This Eq.(36) will be considerably refined in the subsequent document.

7.2. Metric uncertainty principle and its applications in Geometrodynamics

From Eq. (10) we have

$$\langle u_k|H_\phi|u_{k'}\rangle=M_k(a)\cdot\delta_{k,k'}$$
$$\xrightarrow[V(\phi)-\lambda_\alpha\phi^\alpha\&\ k-k']{}\varepsilon_k\cdot\left(\tfrac{\lambda_\alpha}{2}\right)^{\left(\frac{2}{2+\alpha}\right)}a^{\left(\frac{3(2-\alpha)}{2+\alpha}\right)}$$
$$\xrightarrow[\alpha-2]{}\langle u_k|H_\phi|u_{k'}\rangle=M_k(\alpha)\cdot\delta_{k,k'}$$
$$\xrightarrow[V(\phi)-\lambda_\alpha\phi^\alpha\&\ k-k']{}\varepsilon_k\cdot\left(\tfrac{\lambda_\alpha}{2}\right)^{\left(\frac{2}{2+\alpha}\right)}a^{\left(\frac{3(2-\alpha)}{2+\alpha}\right)}$$
$$\xrightarrow[\alpha-2]{}\sqrt{2\lambda_2}\cdot(k+1/2)=M_2(a). \tag{37}$$

Here, we can also assign a density functional and then a change of energy as given by $\Delta E=2\cdot 10^{-\gamma}\cdot l_p^3 M_2(a)/a^3$. So then, that one will have

$$\rho_m=2M_2(a)/a^3=\sqrt{2^3\cdot\lambda_2}\cdot a^3\cdot(k+1/2)\,,$$
$$\Delta E=2\cdot 10^{-\gamma}\cdot l_p^3 M_2(a)/a^3=$$
$$=10^{-\gamma}\cdot l_p^3\cdot\sqrt{2^3\cdot\lambda_2}\cdot a^{-3}\cdot(k+1/2). \tag{38}$$

Here the subscript k, as in Eq. (38) is a "particle count" and we will refer to this heavily in the rest of this paper. If we have Eq. (38) we will, if we have an emergent field reference using a change in energy, in the Pre Planckian domain as

$$\delta g_{tt}\approx\frac{\hbar}{\delta t\Delta E}= \tag{39}$$
$$=(\delta t)^{-1}\cdot\frac{\hbar}{10^{-\gamma}\cdot l_p^3\cdot\sqrt{2^3\cdot\lambda_2}\cdot a^{-3}\cdot(k+1/2)}\,.$$

Or, if the inequality is strictly adhered to

$$\delta g_{tt}\geq\frac{\hbar}{\delta t\Delta E}= \tag{40}$$
$$=(\delta t)^{-1}\cdot\frac{\hbar}{10^{-\gamma}\cdot l_p^3\cdot\sqrt{2^3\cdot\lambda_2}\cdot a^{-3}\cdot(k+1/2)}\,.$$

The smallness of the initial scale factor would be of the order of $a^{-3}\sim 10^{165}$, and we have that $k\sim 10^{20}$, initially, and that $l_p^3\sim 10^{-105}$, and we pick $\hbar=1$ dimensionally, so then if $\delta t\sim 10^{-44}$, we have if we use Eq. (39) as an estimator, that the following has to be done to insure in pre Planckian space time, for the following to hold:

$$\begin{aligned}
&\lambda_2 \leq 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} \leq 1 \\
&\qquad \Leftrightarrow \delta t \Delta E \geq 1 \\
\& \ \ &\lambda > 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} > 1 \\
&\qquad \Leftrightarrow \delta t \delta E < 1.
\end{aligned} \tag{41}$$

I.e. the violation of an uncertainty principle for commences for any situation which implies restraints on $\lambda_2 \leq 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} \leq 1 \Leftrightarrow \delta t \Delta E \geq 1$ when

$$\begin{aligned}
&\lambda_2 > 10^{-74+2\gamma} \Leftrightarrow \delta g_{tt} > 1 \\
&\qquad \Leftrightarrow \delta t \Delta E < 1.
\end{aligned} \tag{42}$$

For the problem represented by Eq. (42) to hold it would mean that the following Pre-Planckian Potential energy would be then small when the following Potential energy as given in Eq. (43) is much smaller than the Kinetic energy given in Eq. (32)

$$V(\phi) = \lambda_\alpha \phi^\alpha = \lambda_2 \phi^2. \tag{43}$$

From inspection, for Eq. (43) to hold, for our physical system we would want Eq. (41) to hold which would mean an extremely small Potential energy, as opposed to the large value of the Kinetic energy given in Eq. (34). Hence the role of Geometrodynamics given in Eqs. (37) and (38), will in the case of a quartic potential imply that Eq. (43) as Potential energy is much smaller than the kinetic energy as represented for Pre Planckian space-time physics.

8. Discussion and conclusions

A way to rewrite the approach given here in terms of the early Universe theory is to refer to Einstein spaces [121] as well as to make certain of the terms and components of the stress energy tensor [122] as we can write it as a modified Einstein field equation. With, then N as a constant.

$$R_{ij} = \mathrm{N} g_{ij} \tag{44}$$

Here, the term in the Left hand side of the metric tensor is a constant, so then if we write, with R also a constant [122]

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ij}} = -\frac{1}{8\pi} \cdot [\mathrm{N} - R + \Lambda] \cdot g_{ij} \tag{45}$$

The terms, if we use the fluid approximation given by Eq. (21) as well as the metric given in Eq. (18) will then tend to a constant energy term on the RHS of Eq. (45) as well as restricting i, and j, to, correspodingly, t and t.
So as to recover, via the Einstein spaces, the seemingly heuristic argument given above. Furthermore when we refer to the Kinetic energy space as an inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [170], we can also then utilize the following operator equation for the generation of an 'inflaton field' given by the following set of equations

$$\begin{array}{l}
\phi\,(t,\cdot) = \cos(t\sqrt{K}) f + \frac{\sin(t\sqrt{K})}{\sqrt{K}} g \\
f(x) = \phi(0,x) \\
g(x) = \frac{\partial \phi(0,x)}{\partial t} \\
-\frac{\partial^2 \phi}{\partial t^2} = K\phi
\end{array} \tag{46}$$

In the case of the general elliptic operator K if we are using the Fulling reference [123] in the case of the above Roberson-Walker metric, with the results that the elliptic operator, in this case become,

$$\begin{array}{l}
K = -\nabla^2 + (m^2 + \xi R) \\
= -\sum_{i,j} \frac{\partial_i \left(g^{ij} \sqrt{|\det g|} \partial_j \right)}{\sqrt{|\det g|}} + (m^2 + \xi R) \\
\xrightarrow[i,j \to t,t]{} -\frac{\partial^2}{\partial t^2} + (m^2 + \xi R)
\end{array} \tag{47}$$

Then, according to [123], if R above, in Eq. (47) is initially a constant, we will see then, if m is the inflation mass, that

$$\begin{array}{l}
\phi\,(t,\cdot) = \cos(t\sqrt{K}) f \\
-\frac{\partial^2}{\partial t^2} \to \omega^2 \\
\Leftrightarrow \phi\,(t,\cdot) = \cos(t\sqrt{\omega^2 + (m^2 + \xi R)})
\end{array} \tag{48}$$

Then c_1 as an unspecified, for now constant will lead to a first approximation of a Kinetic energy dominated initial configuration, with details to be gleaned from [123–125] to give more details to the following equation, R here is linked to curvature of spacetime, and m is an inflaton mass, connected with the field $\phi\,(t,\cdot) = \cos(t\sqrt{K}) f$ with the result that

$$\dot{\phi}^2\,(t,\cdot) \approx \left[\omega^2 + (m^2 + \xi R)\right] \cdot c_1 >> V(\phi) \tag{49}$$

If the frequency, of say, Gravitons is of the order of Planck frequency, then this term, would likely dominate Eq.(49). More of the details of this will be worked out, and also candidates for the $V(\phi)$ will be ascertained, most likely, we will be looking the Rindler Vacuum as specified in [126] as well as also details of what is relevant to maintain local covariance in the initial space-time fields as given in [127].

Why is a refinement of Eq. (49) necessary?

The details of the elliptic operator K will be cleaned from [123–125] whereas the details of inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [170] are important to get a refinement on the lower mass of the graviton. The mass, m, in Eq.(47) for the inflaton, not the Graviton, so as to have links to the beginning of the expansion of the Universe. We look to what Corda did, in [128] for guidance as to picking values of m relevant to early universe conditions.

Finally, as far as Eq. (49) is concerned, there is one serious linkage issue to classical and quantum mechanics, which should be the bridge between classical and quantum regimes, as far as space time applicability. Namely, from Wald (28), if we look at first of all arbitrary operators, A and B

$$(\Delta A)^2 \cdot (\Delta B)^2 \geq \left(\frac{1}{2i} \langle [A, B] \rangle \right). \tag{50}$$

As we can anticipate, the Pre Planckian regime may the place to use classical mechanics, and then to bridge that to the Planckian regime, which would be quantum mechanical. Taking [126] again, this would lead to a sympletic structure via the following modification of the Hamilton equations of motion, namely we will from (28) get the following rewrite,

$$\begin{array}{l}
\frac{dq_\mu}{dt} = \frac{\partial H}{\partial p_\mu}, \ \frac{dp_\mu}{dt} = -\frac{\partial H}{\partial q_\mu} \\
H = H(q_{1,......,}q_n; p_{1,......,}p_n) \\
y = (q_{1,......,}q_n; p_{1,......,}p_n) \\
\Omega^{\mu\nu} = 1, if \nu = \mu + n \\
\Omega^{\mu\nu} = 0, otherwise \\
\frac{dy^\mu}{dt} = \sum_{\nu=1}^{n} \Omega^{\mu\nu} \frac{\partial H}{\partial y^\nu}
\end{array} \tag{51}$$

Then there exists a re formulation of the Poisson brackets, as seen by

$$\{f, g\} = \Omega^{\mu\nu} \nabla_\mu f \nabla_\nu g. \tag{52}$$

So, then the following, for classical observables, f, and g, we could write, by [126]

$$\begin{array}{l}
\wedge : \Theta \to \hat{\Theta} \\
\Theta = classical - observable \\
\hat{\Theta} = quantum - observable \\
\hbar = 1 \\
\left[\hat{f}, \hat{g}\right] = i \cdot \left(\overset{\wedge}{\{f, g\}}\right)
\end{array} \tag{53}$$

Then, we could write, say Eq. (50) and Eq.(53) as

$$\begin{array}{l}
\left[\hat{f}, \hat{g}\right] = i \cdot \left(\overset{\wedge}{\{f, g\}}\right) \\
f = classical - observable \\
\hat{f} = quantum - observable \\
\left(\Delta \hat{f}\right)^2 \cdot (\Delta \hat{g})^2 \geq \left(\frac{1}{2i} \left(\left\langle \left[\hat{f}, \hat{g}\right] \right\rangle \right) \right) = \left(\frac{1}{2} \left(\left(\overset{\wedge}{\{f, g\}} \right) \right) \right)
\end{array} \tag{54}$$

If so, then we can set, in the interconnection between the Planck regime, and just before the Planck regime, say, by setting classical variables, as given by

$$\begin{array}{l}
f = -\frac{[\aleph - R + \Lambda] \cdot g_{tt}}{8\pi} \\
g = \delta g_{tt}
\end{array} \tag{55}$$

Then by utilization of Eq.(54) we may be able to effect more precision in our early Universe derivation, especially making use of derivational work, in addition as to what is given here, as to understand how to construct a very early universe partition function Z based upon the inter relationship between Eq.(54) and Eq.(55) so as to write up an entropy based upon, as given in [126]

$$S(entropy) = \ln Z + \beta E. \tag{56}$$

If this program were affected, with a first principle construction of a partition function, we may be able to answer if Entropy were zero in the Planck regime, or something else, which would give us more motivation to examine the sort of partition functions as stated in [129, 130]. See **appendix A** as to possible scenarios. Here keep in mind that in the Planck regime we have non standard physics. Appendix A indicates that due to the variation we have worked out in the Planckian regime of space-time that the initial entropy is not zero. The consequences of this show up in this paper's **Appendix B**, as to a specific formulation of the Ricci scalar. The consequences of Appendix A and **Appendix B** may be for a small cosmological constant, and large "Hubble expansion" that there would be an initially large magnitude of cosmological pressure, even if negative, which would give credence to a non zero cosmological entropy, that if large negative pressure, even in the Pre Planckian regime will lead to a large ΔT_{tt} terms which would show up in Eq. (1A), even if we used a partition function based upon Lattice Hamiltonians, as on page 135 of [130] which would usually in a lattice gauge arrangement would have considerably smaller contributions than ΔT_{tt}. Note the conditions of flat space, are that Eq.(B9) almost vanishes due to the behavior of the numerator, no matter how small $a^2_{initial}$is. The supposition is that the numerator becomes far smaller than $a^2_{initial}$. The initiation of conditions of flat space, is also the regime in which we think that non zero entropy is started, and **Appendix C** gives an initial estimate of what we think Entropy would be in the aftermath of the uncertainty relationship we have outlined in this paper, i.e. to first order, $S_{initial(graviton)} \sim 10^{37}$. We finalize our treatment as of space-time fluctuations and geometry by considering the applications of **Appendix D** to graviton mass, and **Appendix E** to the Riemann-Penrose inequality for conditions as to a minimum frequency, as a consequence of cosmological evolution, and what it portrays as consequences for Electromagnetic fields. Appendix D and E give varying initial graviton masses as a starting point, with Appendix D giving a higher initial graviton mass than what is assumed as of today. Finally, **Appendix F** states a pre Planckian kinetic energy so the inflaton $\dot{\phi}^2 >> (P.E \sim V)$ [170]. This last step, so important to our development will be considerably refined in a future paper.

What we are doing now is confirming the material given in this paper as well as giving an explanation for our future research activity. The quartic potential, we used above, is the simplest version of the potential systems in this paper and the cases of non quartic potential should be examined fully, as part of a comprehensive study. This will be part of the research project which the authors will initiate in future publications. We should keep this discussion and the discussion of scalar fields separate from the ideas given in inflation, namely of the fluctuations not necessarily having an upper bound of

$$\tilde{\tilde{\phi}} > \sqrt{\frac{60}{2\pi}}\, M_p \approx 3.1 M_p \equiv 3.1. \tag{57}$$

Since our modeling is not predicated upon the inflationary model of cosmology but which is addressing the issue brought up in [147], which is the contribution of Pre Planckian space time to cosmological evolution we wish to adhere to non inflationary treatments as to Eq. (43) and Eq. (57) but will adhere to the questions poised at the beginning of this document. Furthermore we will adhere to, in future papers in delineating a departure from the standard treatment of the evolution of the scalar field, as given in conventional inflation cosmology as the following

$$\frac{d\phi}{dt} = -\frac{V'(\phi)}{3H(\phi)} + \frac{H^{3/2}(\phi)}{2\pi} \cdot \xi(t). \tag{58}$$

This has a quasi "quantum mechanical" effective white noise introduced term $\xi(t)$, and is similar to $\xi(t)$ in a first order differential equation being a "driving" term to a quasi chaotic oscillatory behavior to the scalar field. We argue that this Eq. (58) in [148] is wrong, albeit well motivated by conventional inflationary cosmology and part of our future discussion will be in, for the Pre Planckian regime of space time as partly brought up in [149] discussing what we are putting in instead as a replacement. This Eq. (58) contravenes our description of Kinetic energy as the dominant term in Pre Planckian space-time physics which deserves future developments for establishing experimental measurements.

Appendix A. Scenarios as to the value of entropy in the beginning of space-time nucleation

We will be looking at inputs from page 290 of [23] so that if $E \sim M \sim \Delta T_{tt} \cdot \delta t_{time} \cdot \Delta A \cdot l_P$
$S(entropy) = \ln Z + \frac{(E \sim \Delta T_{tt} \cdot \delta t \cdot \Delta A \cdot l_P)}{k_B T_{temperature}}$ (1A)
And using Ng's infinite quantum statistics, we have to first approximation [131, 132]

$$S(entropy) \sim \ln Z + \frac{((E \sim \Delta T_{tt}) \cdot \delta t \cdot \Delta A \cdot l_P)}{k_B T_{temperature}}$$
$$\sim \ln Z + \left(\frac{\hbar}{k_B T_{temperature}\delta g_{tt}}\right)$$
$$\xrightarrow[T_{temperature} \to \#anything]{} [S(entropy) \sim n_{count}] \neq 0$$
(2A)

This is due to a very small but non vanishing δg_{tt} with the partition functions covered by [130], and also due to [131, 132] with n_{count}a non zero number of initial ‘particle’ or information states, about the Planck regime of space-time, so that the initial entropy is non zero.

Appendix B. Calculation of the Ricci Tensor for a Roberson-Walker space-time, with its effect upon the measurement of if or not a space time, is open, closed or flat

We begin with Kolb and Turner [170] discussion of the Roberson-Walker metric, say page 49 with, if R is the Ricci scalar, and k the measurement of if we have a close, open, or flat universe, that if

$a = a_{initial} \cdot \exp(H \cdot t)$ (B1)

Then by [170]

$H^2 = -\frac{k}{a^2} + \frac{8\pi G\rho}{3}$ (B2)

$3H^2 + \left[\frac{2k}{a^2} + \frac{R}{6}\right] = 0$ (B3)

Leading to

$a^2 = \frac{1}{k} \cdot \left[\frac{R}{6} + 8\pi G\rho\right]$ (B4)

If $\rho = -p$ [7], then with a bit of algebra

$|p| = \frac{1}{8\pi G} \cdot \left[\frac{R}{6} + (a_{initial})^2 \cdot \exp\left[\sqrt{\frac{4\Lambda}{3}} \cdot t_{time}\right]\right]$ (B5)

Next, using [134], on page 47, at the boundary between Pre Planckian to Planckian space-time we will find

$R = 8\pi \cdot \left(T_0^0 + T_1^1 + T_2^2 + T_3^3\right) + 4\Lambda \xrightarrow[\Pr e-Planckian-Conditions]{} 8\pi \cdot \left(T_0^0\right) + 4\Lambda$ (B6)

Then, we can obtain right at the start of the Planckian era,

$|p|_{Planckian} \sim \frac{1}{8\pi G} \cdot \left[\frac{8\pi \cdot \left(T_0^0+T_1^1+T_2^2+T_3^3\right)+4\Lambda}{6}\right]$ (B7)

The consequences of this would be that right after the entry into Planckian space time, that there would be the following change of pressure

$$|p|_{\Pr e-Planckian} = \frac{1}{8\pi G} \cdot \left[\frac{8\pi \cdot \left(T_0^0\right)+4\Lambda}{6} + (a_{initial})^2\right] \times$$
$$\times \left[\exp\left[\sqrt{\frac{4\Lambda}{3}} \cdot t_{time}\right]\right]$$
$$\Rightarrow |p|_{\Pr e-Planckian} \sim \frac{1}{8\pi G} \cdot \left|\frac{8\pi \cdot \left(T_0^0\right)+4\Lambda}{6} + 0^+\right|$$
$$|p|_{Planckian} \sim \frac{1}{8\pi G} \cdot \left|\frac{8\pi \cdot \left(T_0^0+T_1^1+T_2^2+T_3^3\right)+4\Lambda}{6}\right|$$
$$\Delta P = |p|_{Planckian} - |p|_{\Pr e-Planckian} \sim \left[\frac{\left(T_1^1+T_2^2+T_3^3\right)}{6G}\right]$$

(B8)

Then, the change in the k term would be like, say, from Pre Planckian to Planckian space time

$\Delta k = \frac{1}{a_{initial}^2} \cdot [8\pi G(\rho - \Delta P)]$ (B9)

This goes almost to zero if the numerator shrinks far more than the denominator, even if the initial scale factor is of the order of 10^{-110} or so.

Appendix C. Initial entropy, from first principles

We are making use of the Padmanabhan publication of [135, 136] where we will make use of

$\rho_\Lambda \approx \frac{GE_{system}^6}{c^8\hbar^4} \Leftrightarrow \Lambda \approx \frac{1}{l_{Planck}^2} \cdot (E_{system}/E_{Planck})^6$ (C1)

Then, if E_{system} is for the energy of the Universe after the initiation of Eq.(20) as a bridge between Pre Planckian, to Planckian physics regimes we could write, then

$$E_{system} \propto n_{gravitons} \cdot m_{graviton}$$
$$\Lambda \approx \frac{1}{l_{Radius-Universe-today}^2}$$
$$\Leftrightarrow m_{graviton} \sim 10^{-62} grams \Rightarrow n_{gravitons} \sim 10^{37}$$
$$\Rightarrow S_{initial(graviton)} \sim 10^{37} at - Planck - time$$

(C2)

The value of initial entropy, $S_{initial(graviton)} \sim 10^{37}$ should be contrasted with the entropy for the entire Universe as given in [137] below.

Appendix D. Information flow, Gravitons, and also upper bounds to Graviton mass

Here we can view the possibility of considering the following, namely [138] is extended by [139], so we can we make the following identification

$N = N_{graviton}|_{r_H} = \frac{c^3}{G \cdot \hbar} \cdot \frac{1}{\Lambda} \approx \frac{1}{\Lambda}$ (D1)

Should the N above, be related to entropy, and Eq. (17)? This supposition has to be balanced against the following identification, namely, as given by T. Padmanabhan [135, 136]

$\Lambda_{Einstein-Const.Padmanabhan} = 1/l_{Planck}^2 \cdot (E/E_{Planck})^6$. (D2)

But should the energy in the numerator in Eq. (D2) be given as say by (C2), of Appendix C, we have defacto quinessence. then there would have been defacto quintessence, i.e. variation in the “Einstein constant”, which would have a large impact upon mass of the graviton, with a sharp decrease in g_* being consistent with an evolution to the ultra light value of the Graviton, with initial frequencies of the order of say for wavelength values initially the size of an atom,

$\omega_{initial}|_{r_H \sim atomic-size} \sim 10^{21} Hz$ (D3)

The final value of the frequency would be of a magnitude smaller than one Hertz, so as to have value of the mass of the graviton would be then of the order of $10^{-62} grams$ [117], due to Eq.(D2) approaching [138] below, namely

$\Lambda_{Einstein-Const.} = 1/l^2_{Radius-Universe}$. (D4)

Leading to the upper bound of the Graviton mass of about $10^{-62} grams$ [138, 139] in the present era

$m_{graviton} = \frac{\hbar}{c} \cdot \sqrt{\frac{(2\Lambda)}{3}} \approx \sqrt{\frac{(2\Lambda)}{3}}$. (D5)

Eq. (D5) has a different value if the entropy / particle count is lower, as has been postulated in this note. But the value of Eq.(D5) becomes the Graviton mass of about $10^{-62} grams$ [117] in the present era which is in line with the entropy being far larger in the present era [137].

Appendix E. Applying the Riemannian Penrose Inequality with applications in our fluctuation

$\delta g_{tt} \sim a^2(t) \cdot \phi << 1$ (E1)

Refining the inputs from Eq. (E1) means more study as to the possibility of a non zero minimum scale factor [171], as well as the nature of ϕ as specified by Giovannini [140]. We hope that this can be done as to give quantifiable estimates and may link the non zero initial entropy to either Loop quantum gravity "quantum bounce" considerations [142] and/or other models which may presage modification of the sort of initial singularities of the sort given in [1]. Furthermore if the non zero scale factor is correct, it may give us opportunities as to fine tune the parameters given in [171] below

$$\alpha_0 = \sqrt{\frac{4\pi G}{3\mu_0 c}} B_0$$
$$\widehat{\lambda}\ (defined) = \Lambda c^2/3$$

$$a_{\min} = a_0 \cdot \left[\frac{\alpha_0}{2\widehat{\lambda}\,(defined)} \right] \times$$
$$\times \left[\left(\sqrt{\alpha_0^2 + 32\ \widehat{\lambda}\ (defined) \cdot \mu_0 \omega \cdot B_0^2} - \alpha_0 \right) \right]^{1/4}$$

(E2)

Where the following is possibly linkable to minimum frequencies linked to E and M fields [171], and possibly relic Gravitons

$B > \frac{1}{2 \cdot \sqrt{10 \mu_0 \cdot \omega}}$ (E3)

So, now we investigate the question of applicability of the Riemann Penrose inequality which is [143], p. 431, which is stated as

Riemann Penrose Inequality: Let (**M, g**) be a complete, asymptotically flat 3- manifold with Non negative-scalar curvature, and total mass m, whose outermost horizon Σ has total surface area A. Then

$m_{total-mass} \geq \sqrt{\frac{A_{surface-Area}}{16\pi}}$ (E4)

And the equality holds, iff (**M, g**) is isometric to the spatial isometric spatial Schwartzshield manifold **M** of mass **m** outside their respective horizons.

Assume that the frequency, say using the frequency of Eq.(E3) , and $A \approx A_{\min}$of Eq.(E4) is employed. So then say we have , if we use dimensional analysis appropriately, that

$$(v = velocity \equiv c) = f(frequency) \times \lambda(wavelength)$$
$$\Rightarrow \omega \approx \omega_{initial} \sim \frac{c}{d_{\min}} \sim \left. \frac{1}{d_{\min}} \right|_{c \equiv 1} \& d_{\min} \sim A^{1/3} \propto a_{\min}$$

(E5)

Assume that we also set the input frequency as to Eq. (E3) as according to $10 < \zeta \leq 37$ i.e. does

$$\left(m_{total-mass} \sim 10^{\zeta} \cdot m_{graviton} \right)^2 \propto a^3_{\min}/16\pi$$
$$\Leftrightarrow \omega \approx \omega_{initial} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta} \cdot m_{graviton} \right)^{-2/3}$$

(E6)

Our supposition is that Eq.(E6) should give the same frequency as of Eq. (D3) above. So if we have in doing this, this is a frequency input into Eq. (E3) above where we are safely assuming a graviton mass of about [117]

$$\begin{array}{l} m_{total-mass} \sim 10^{37} \cdot m_{graviton} \\ m_{graviton} \sim 10^{-62} grams \end{array} \quad (E7)$$

Does the following make sense? I.e. look at it, when $10 < \zeta \leq 37$

$$\left(m_{total-mass} \sim 10^{\zeta} \cdot m_{graviton} \right)^2 \propto a^3_{\min}/16\pi$$
$$\Leftrightarrow \omega \approx \omega_{initial} \sim \frac{1}{d_{\min}} \sim \left(16\pi \times 10^{\zeta} \cdot m_{graviton} \right)^{-2/3}$$

(E8)

We claim that if this is an initial frequency and that it is connected with relic graviton production, that the minimum frequency would be relevant to Eq. (E3), and may play a part as to admissible B fields.

Note, if Appendix D is used, this makes a re do of Eq. (E8) which is a way of saying that the graviton mass given by [117] no longer holds.

In either case, Eq. (E8) and Eq. (E3) in some configuration may argue for implementation of work, it was done in reference [144], as to relic cylindrical GW, i.e. their allowed frequency and magnitude, so considered.

Appendix F. First principle treatment of pre Planckian kinetic energy so the Inflaton $\dot{\phi}^2 >> (P.E \sim V)$

We give this as a plausibility argument which undoubtedly will be considerably refined, but its importance cannot be overstated. I.e. this is for Pre inflationary, Pre Planckian physics, so as to get a lower bound to the Graviton mass. To do this, we look at what [170] is saying and also we will be enlisting a new reference, [145], by Bojowald, and also T. Padmanbhan [146] as to details to put in, so as to confirm a dominance of Kinetic energy. Start with a Friedman equation of

$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k_{curvature}}{a^2} = \frac{4\pi G}{3} \cdot \frac{p_\phi^2}{a^6} + \Lambda$ (F1)

We will treat, then the Hubble parameter, as

$\left(\frac{\dot{a}}{a}\right) = H_{initial} \equiv \frac{2}{t\cdot\left(1+\frac{P}{\rho}\right)} \xrightarrow[P=-\rho+\varepsilon^+]{} \frac{2}{t\cdot\left(\frac{\varepsilon^+}{\rho}\right)} \xrightarrow[t\to t_P]{} \frac{2\rho}{t_P\cdot\varepsilon^+}$ (F2)

Now from Padmanabhan [146], we can write density, in terms of flux according to

$\frac{d\rho}{dt} = \frac{1}{V^{(3)}=Volume} \cdot (A = Area) \cdot (\Im = Flux) \sim \frac{(\Im=Flux)}{l_P}$ (F3)

Then using p. 463 of [146], if T is temperature, here, then if N is the particle count in the flux region per unit time (say Planck time), as well as using the 'ideal gas law' approximation, for superhot conditions

$\frac{d\rho}{dt} = \frac{1}{V^{(3)}=Volume} \cdot (A = Area) \cdot (\Im = Flux) \sim \frac{(\Im=Flux)}{l_P}$

$\rho \sim \frac{(\Im=Flux)}{c}$

$\Rightarrow H = \frac{N}{\varepsilon^+} \cdot \frac{1}{V^{(4)}=4-Dim\ Volume} \cdot \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_B T}{m_{flux-particle}}}$

(F4)

Next, according to [145] we can make the following substitution.

$p_\phi = a^3 \cdot \dot{\phi}$ (F5)

Therefore, if

$\dot{\phi}^2 \approx a^{-6} \cdot (12\pi G) \cdot V^{(4)} \cdot \left(H^2 + |\Lambda|\right) \approx a^{-6} \cdot (12\pi G) \cdot V^{(4)}$
$\left(\left[\frac{N}{\varepsilon^+} \cdot \frac{1}{V^{(4)}=4-Dim\ Volume} \cdot \sqrt{\frac{8}{\pi}}\sqrt{\frac{k_B T}{m_{flux-particle}}}\right]^2 + |\Lambda|\right)$

(F6)

If the scale factor is very small, say of the order of $a = a_{initial} \sim 10^{-55}$, then no matter how fall the initial volume is, in four space (it cancels out in the first part of the brackets), its easy to see then that $\dot{\phi}^2 >> (P.E \sim V)$ [170].

We will in the future add more structure to this calculation so as to confirm via a precise calculation that the lower bound to the graviton mass, is about $10^{-70} grams$.

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1. C. Will, in: *General Relativity and Gravitation, A Centennial Perspective*, Edited by A. Ashtekar, B. Berger, J. Isenberg, M. MacCallum, Cambridge University Press, Cambridge, 2015.
2. C. Will, *The Confrontation between General Relativity and Experiment*, http://relativity.livingreviews.org/Articles/lrr-2014-4/download/lrr-2014-4Color.pdf
3. T. G. Downes, G. J. Milburn, *Optimal Quantum Estimation for Gravitation*, 2012, *arXiv gr-qc/1108.5220.*
4. B. J. Hiley, D. Robson, *Quantum Processes and Energy-Momentum Flow*, 2014, *arXiv quant-ph/1411.7826.*
5. George W. Mackey, *Mathematical Foundations of Quantum Mechanics*, Dover, New York, 1963.
6. ' t Hooft, G., *Dimensional reduction in quantum gravity*, 1993, *arXiv gr-qc/9310026.*
7. A.D.K. Plato, C.N. Hughes, M.S. Kim, *Contemporary Physics*, 1–19 (2016), *arXiv quant-ph/1602.03878.*
8. R.J. Adler, D.I. Santiago, *Modern Physics Letters A*, **14**, (20), 1371–1381 (1999).
9. S.L. Adler, *Journal of Physics A: Mathematical and Theoretical*, **40**, (4), 755 (2007).
10. A. Albrecht, A. Retzker, M.B. Plenio, *Physical Review A*, **90**, (3), 033834 (2014).
11. D. Amati, M. Ciafaloni, G. Veneziano, *Physics Letters B*, **216**, (1), 41–47 (1989).
12. G. Amelino-Camelia, *Nature*, **398**,(6724), 216(1999).
13. G. Amelino-Camelia, *et al.* , *Physical Review D*, **62**, (2), 024015 (2000).
14. G. Amelino-Camelia, *et al.*, *Physics Letters B*, **477**, (4), 436–450 (2000).
15. G. Amelino-Camelia, *Physics Letters B*, **510**, (1), 255–263 (2001).

16. G. Amelino-Camelia, *International Journal of Modern Physics D*, **11**, (01), 35–59 (2002).
17. G. Amelino-Camelia, *Nature*, **462**,(7271), 291(2009).
18. G. Amelino-Camelia, *Symmetry*, **2**, (1), 230 (2010).
19. G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S. Sarkar, *Nature*, **393**, (6687), 763–765 (1998).
20. G. Amelino-Camelia, L. Smolin, *Physical Review D*, **80**, (8), 084017 (2009).
21. C. Arenz, R. Hillier, M. Fraas, D. Burgarth, *Phys. Rev. A*, **92**, 022102 (2015).
22. C. Bambi, *Classical and Quantum Gravity*, **25**, (10), 105003 (2008).
23. S. Barnett, P. Radmore, *Methods in theoretical quantum optics*, Oxford University Press, Oxford, 1997.
24. A. Bassi, K. Lochan, S. Satin, *et al.*, *Reviews of Modern Physics*, **85**, (2), 471–527 (2013).
25. M. Bawaj, C. Biancofiore, M. Bonaldi, *et al.*, *Nat. Commun.*, **6**, 7503 (2015).
26. J.D.Bekenstein, *Phys.Rev. D*, **86**,(12), 124040(2012).
27. J.D. Bekenstein, *Foundations of Physics*, **44**, (5), 452–462 (2014).
28. S. Benczik, L.N. Chang, D. Minic, T. Takeuchi, *Physical Review A*, **72**, (1), 012104 (2005).
29. S. Bose, K. Jacobs, P. Knight, *Physical Review A*, **56**, (5), 4175 (1997).
30. S. Bose, K. Jacobs, P. Knight, *Physical Review A*, **59**, (5), 3204 (1999).
31. F. Brau, *Journ. of Phys. A: Mathematical and General*, **32**, (44), 7691 (1999).
32. M.P. Bronstein, *Physikalische Zeitschrift der Sowjetunion*, **9**, 140 (1936).
33. P. Busch, T. Heinonen, P. Lahti, *Physics Reports*, **452**, (6), 155–176 (2007).
34. P. Busch, P. Lahti, R.F. Werner, *Physical Review Letters*, **111**, (16), 160405 (2013).
35. A.O. Caldeira, A.J. Leggett, *Physica A: Statistical Mechanics and its Appl.*, **121**, (3), 587–616 (1983).
36. S. Carlip, *Reports on Progress in Physics*, **64**, (8), 885 (2001).
37. L.N. Chang, D. Minic, N. Okamura, T. Takeuchi, *Physical Review D*, **65**, (12), 125028 (2002).
38. J. Christian, in: *Physics meets philosophy at the Planck scale*, **305**, 2001.
39. W. Christiansen, Y.J. Ng, H. Van Dam, *Physical Review Letters*, **96**, (5), 051301 (2006).
40. I. Dadić, L. Jonke, S. Meljanac, *Physical Review D*, **67**, (8), 087701 (2003).
41. L. Diósi, *Physics letters A*, **105**, (4), 199–202 (1984).
42. L. Diósi, *Physics letters A*, **120**, (8), 377–381 (1987).
43. L. Diósi, B. Lukács, *Annalen der Physik*, **499**, (7), 488–492 (1987).
44. L. Diósi, B. Lukacs, *Il Nuovo Cimento B*, **108**, (12), 1419–1421 (1993).
45. F. Dowker, *Contemp. Phys.*, **47**, (1), 1–9 (2006).
46. A. Frenkel, *Found. of Phys.*, **32**, (5), 751–771 (2002).
47. L.J. Garay, *International Journal of Modern Physics A*, **10**, (02), 145–165 (1995).
48. G.C. Ghirardi, A. Rimini, T. Weber, *Physical Review D*, **34**, (2), 470 (1986).
49. N. Gisin, *Helv. Phys. Acta*, **62**, (4), 363 (1989).
50. A. Groardt, J. Bateman, H. Ulbricht, A. Bassi, Optomechanical test of the Schrödinger-Newton equation, 2015, *arXiv:1510.01696*.
51. H. Grote, L.S. Collaboration, *et al.*, *Classical and Quantum Gravity*, **25**, (11), 114043 (2008).
52. W. Heisenberg, *Annalen der Physik*, **424**, (1-2), 20–33 (1938).
53. S. Hossenfelder, *Liv. Rev. Relativ.*, **16**, (2), 23 (2013).
54. S. Hossenfelder, *SIGMA*, **10**, (074), 8 (2014).
55. F.Károlyházy, *Il Nuovo Cim. A*, **42**,(2), 390 (1966).
56. F. Károlyházy, A. Frenkel, B. Lukács, in: *Quantum concepts in space and time*, vol. 1, 109–128 (1986).
57. A. Kempf, *Journ. of Math. Phys.*, **35**, (9), 4483–4496 (1994).
58. A. Kempf, *Journal of Physics A: Mathematical and General*, **30**, (6), 2093 (1997).
59. A. Kempf, G. Mangano, R.B. Mann, *Phys. Rev. D*, **52**, (2), 1108–1118 (1995).
60. E. Kennard, *Zeitschrift für Physik*, **44**, (4-5), 326–352 (1927).
61. J. Kowalski-Glikman, *Phys. Lett. A*, **286**, (6), 391-394 (2001).
62. R. Laughlin, *Intern. Journ. of Mod. Phys. A*, **18**, (06), 831–853 (2003).
63. M. Maggiore, *Phys. Lett. B*, **319**, (1), 83–86 (1993).
64. M. Maggiore, *Phys. Lett. B*, **304**, (1), 65–69 (1993).
65. J. Magueijo, L. Smolin, *Physical Review Letters*, **88**, (19), 190403 (2002).
66. S. Mancini, V. Man'ko, P. Tombesi, *Physical Review A*, **55**, (4), 3042 (1997).
67. W. Marshall, C. Simon, R. Penrose, D. Bouwmeester, *Phys. Rev. Lett.*, **91**, (13), 130401 (2003).
68. C.A. Mead, *Phys. Rev.*, **135**, (3B), B849 (1964).
69. F. Mercati, D. Mazón, G. Amelino-Camelia, J.M. Carmona, J.L. Cortés, J. Induráin, C. Lämmerzahl, G.M. Tino, *Classical and Quantum Gravity*, **27**, (21), 215003 (2010).
70. G. Milburn, G. *Phys. Rev. A*, **44**, (9), 5401 (1991).
71. G. Milburn, *New Journ. of Phys.*, **8**, (6), 96 (2006).
72. C.W. Misner, J.A. Wheeler, *Annals of physics*, **2**, (6), 525–603 (1957).
73. I.M. Moroz, R. Penrose, P. Tod, *Classical and Quantum Gravity*, **15**, (9), 2733 (1998).
74. Y.J. Ng, Limitation to quantum measurements of spacetime distances, 1994, *arXiv hep-th/9406110*.

75. Y.J. Ng, in: *Planck Scale Effects in Astrophysics and Cosmology*, Springer, 321–349 (2005).
132. Y.J. Ng, *Entropy*, **10**, (4), 441–461 (2008).
77. Y.J. Ng, H. Van Dam, *Europhys. Lett.*, **38**, (5), 401 (1997).
78. Y.J. Ng, H. Van Dam, *Foundations of Physics*, **30**, (5), 795–805 (2000).
79. T. Pavlopoulos, *Phys. Rev.*, **159**, (5), 1106 (1967).
80. R. Penrose, in: *Quantum concepts in space and time*, vol. 1, 129–146 (1986).
81. R. Penrose, *General relativity and gravitation*, **28**, (5), 581–600 (1996).
82. R. Penrose, *Philosoph. Trans. of the Royal Soc. of London A: Mathematical, Physical and Engineering Sciences*, **356**, (1743), 1927–1937 (1998).
83. R. Penrose, *Mathematical physics*, 266–282 (2000).
84. R. Penrose, in: *The Ninth Marcel Grossmann Meeting*, **1**, 3–6 (2002).
85. R. Penrose, *The Road to Reality: A Complete Guide to the Physical Universe*, Jonathan Cape, 2004.
86. R. Penrose, in: *Journal of Physics: Conference Series*, IOP Publishing, **174**, 012001 (2009).
87. R. Penrose, *Found. of Phys.*, **44**, (5), 557–575 (2014).
88. B. Pepper, R. Ghobadi, E. Jeffrey, C. Simon, D. Bouwmeester, *Physical Review Letters*, **109**, (2), 023601 (2012).
89. A. Peres, *Am. J. Phys*, **48**, (7), 552 (1980).
90. A. Peres, N. Rosen, *Phys. Rev.*, **118**, (1), 335 (1960).
91. I. Pikovski, M.R. Vanner, M. Aspelmeyer, M. Kim, Č. Brukner, *Nature Physics*, **8**, (5), 393–397 (2012).
92. C. Quesne, V. Tkachuk, *Journal of Physics A: Mathematical and General*, **36**, (41), 10373 (2003).
93. C. Quesne, V. Tkachuk, *Journal of Physics A: Mathematical and General*, **37**, (14), 4267 (2004).
94. C. Quesne, V. Tkachuk, *Journal of Physics A: Mathematical and General*, **38**, (8), 1747 (2005).
95. H.P. Robertson, *Phys. Rev.*, **34**, (1), 163–164 (1929).
96. L. Rosenfeld, *Zeitschrift für Physik*, **65**, (9-10), 589–599 (1930).
97. L.Rosenfeld, *Annal. d. Phys.*, **397**, 1,113-152 (1930).
98. L. Rosenfeld, *Nuclear Physics*, **40**, 353–356 (1963).
99. C. Rovelli, *Notes for a brief history of quantum gravity*, (2000) *arXiv:gr-qc/0006061v3.*
100. C. Rovelli, S. Speziale, *Phys. Rev. D*, **67**, (6) (2003).
101. R. Ruffini, S. Bonazzola, *Physical Review*, **187**, (5), 1767 (1969).
102. H. Salecker, E. Wigner, *Physical Review*, **109**, (2), 571 (1958).
103. F. Scardigli, *Phys. Lett. B*, **452**, (1-2), 39–44 (1999).
104. E. Schrödinger, *Ber. Kgl. Akad. Wiss. Berlin*, **24**, 296–303 (1930).
105. J. Stachel, in: *Philosophical problems of the internal and external worlds: Essays on the philosophy of Adolf Grunbaum*, 129–160 (1993).
106. M. Stetsko, *Phys. Rev. A*, **74**, (6), 062105 (2006).
107. M. Stetsko, V. Tkachuk, *Physical Review A*, **74**, (1), 012101 (2006).
108. M. Stetsko, V. Tkachuk, *Physics Letters A*, **372**, (31), 5126–5130 (2008).
109. V. Vasileiou, J. Granot, T. Piran, and G. Amelino-Camelia, “A planck-scale limit on spacetime fuzziness and stochastic lorentz invariance violation”, *Nature Physics.* 2015.
110. J.A. Wheeler, *Physical Review*, **97**, (2), 511 (1955).
111. J.A. Wheeler, *Geometrodynamics*, vol. 1, Acad. Press, 1962.
112. W. G. Unruh, *Canadian Journal of Physics,* **64**, No. 2: 128-130 (1986); (doi: 10.1139/p86-019)
140. M. Giovannini, *A Primer on the Physics of the Cosmic Microwave Background*, World Press Scientific, Hackensack, 2008.
170. E. W. Kolb, M. S. Turner, *The Early Universe*, Addison-Wesley Publ. Comp., Redwood, 1990.
115. J. Barbour, *The Nature of Time*, 2009; http://arxiv.org/pdf/0903.3489.pdf
116. J. Barbour, in: *Quantum Field Theory and Gravity, Conceptual and Mathematical Advances in the Search for a Unified Framework*, edited by F. Finster, O. Muller, M. Nardmann, J. Tolksdorf, E. Zeidler, Birkhauser, London, 2010.
117. A. Goldhaber, M. Nieto, *Rev. Mod. Phys.* **82**, 939-979, (2010); http://arxiv.org/abs/0809.1003
118. W. J. Handley, S. D. Brechet, A. N. Lasenby, and M. P. Hobson, *Phys. Rev.* D **89**, 063505 (2014). http://arxiv.org/pdf/1401.2253v2.pdf
119. J. D. Walecka, *Introduction to Modern Physics, Theoretical Foundations,* World Press Scientific, Singapore, 2008.
120. R. Penrose, *Cycles of Time: An Extraordinary New View of the Universe*, The Bodley Head, London, 2010.
121. A.Z. Petrov, *Einstein Spaces*, Pergamon Press, Oxford London, 1969.
122. D. Gorbunov, V. Rubakov, *Introduction to the Theory of the Early Universe, Cosmological Perturbations and Inflationary Theory*, World Scientific Publishing, Singapore, 2011.
123. S. A. Fulling *Aspects of Quantum Field Theory in Curved Space- Time* , Cambridge University Press, Cambridge, 1991.
124. H. Gutfreund, J. Renn, *The Road To Relativity, The history and Meaning of Einstein's “ The Foundation of General Relativity”*, Princeton University Press, Princeton, 2015.
125. J. Griffiths, J. Podolsky, *Exact Space-Times in Einstein's General Relativity*, Cambridge University

Press, Cambridge, 2009.
126. R. M. Wald, *Quantum Field theory in Curved Spacetime and Black Hole Thermodynamics*, University of Chicago Press, USA, 1994.
127. K. Fredenhagen, and K. Rejzner, in: *Quantum Field Theory and Gravity, Conceptual and Mathematical Advances in the Search for a Unified Framework*, edited by F. Finster, O. Muller, M. Nardmann, J. Tolksdorf, E. Zeidler, Birkhauser, London, 2010.
128. C. Corda, *Primordial gravity's breath*, 2012; http://arxiv.org/abs/1110.1772
129. S. Gilen and D. Oriti, in: *Quantum Field Theory and Gravity, Conceptual and Mathematical Advances in the Search for a Unified Framework*, edited by F. Finster, O. Muller, M. Nardmann, J. Tolksdorf, E. Zeidler, Birkhauser, London, 2010.
130. N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge Univ. Press, London, 1982.
131. Y. Jack Ng, *Phys. Lett.* B, **657**, 10-14, (2007).
132. Y. Jack Ng *Entropy* **10**, 441-461 (2008); DOI: 10.3390/e10040441
133. H. Hambler, *Quantum Gravitation, The Feynman Path Integral Approach*, Springer Verl., Berlin, 2009.
134. I. Ciufolini and J. Wheeler, *Gravitation and Inertia,* Princeton University Press, Princeton, 1995.
135. T. Padmanabhan, *Cosmological constant – the weight of the vacuum*, (2003); arXiv:hep-th/0212290v2
136. T. Padmanabhan, *A brief history of cosmological constant*, http://ned.ipac.caltech.edu/level5/Sept02/Padmanabhan/Pad1_2.html
137. C. Egan, C. H. Lineweaver, *Astrophys. Journ.* **710**, 1825–1834, (2010); http://www.mso.anu.edu.au/~charley/papers/EganLineweaverApJOnline.pdf
138. A. F. Ali, S. Das, *Phys. Lett.* B, **741**, 276-279 (2015).
139. I. Haranas and I. Gkigkitzis, in: *International Scholarly Research Notices*, Volume 2014, (2014), http://www.hindawi.com/journals/isrn/2014/718251/
140. M. Giovannini, *A Primer on the Physics of the Cosmic Microwave Background*, World Press Scientific, Hackensack, New Jersey, USA, 2008.
171. C.S. Camara, M.R. de Garcia Maia, J.C. Carvalho, J.A.S. Lima, *Nonsingular FRW cosmology and Non Linear dynamics*, 2004; Arxiv astro-ph/0402311.
142. C. Rovelli, and F. Vidotto, *Covariant Loop Quantum Gravity, An Elementary Introduction to Quantu Gravity and Spinfoam Theory*, Cambridge University Press, Cambridge, UK, 2015
143. G. Galloway, P. Miao, and R. Schoen, in: *General Relativity and Gravitation, A centennial Perspective*, editors A. Ashtekar (Editor in Chief), B. Berger, J.Isenberg and M. MacCallum , Cambridge University Press, Cambridge, UK, 2015.
144. H. Wen, F.Y. Li, Z.Y. Fang, *High-frequency gravitational waves from magnetars and gamma-ray bursts*, 2016; http://arXiv:1608.03186v.
145. M. Bojowald, in: *The Arrows of Time, a Debate in Cosmology*, editors Laura Mersini, Rudy Vaas, Springer Verlag, Berlin, 2012; http://arxiv.org/pdf/0910.3200.pdf
146. T. Padmanabhan, *Gravitation, Foundations and Frontiers*, Cambridge Univ. Press, Cambridge, 2010.
147. A. Beckwith, *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 19-24 (2016). doi: 10.4236/jhepgc.2016.21002
148. A. Meszhlumian, in: *Texas/PASCOS 92: Relativivistic Astrophysics and Particle Cosmology*, editors C. Akerlof, and M. Srednicki, Annals of the New York Academy of Sciences, volume 688, 1992.
149. A. Beckwith, *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 75-82 (2016). doi: 10.4236/jhepgc.2016.21008.
150. A. Beckwith, *Journal of High Energy Physics, Gravitation and Cosmology*, **2**, 106-124 (2016). doi: 10.4236/jhepgc.2016.21012..
151. L.D. Landau, E.M. Lifshitz *Quantum Mechanics. Course of Theoretical Physics, Vol. 2,* Pergamon Press, Oxford 1965.
152. V.E. Kuzmichev, V.V. Kuzmichev, *Acta Phys. Pol. B* **40**, 2877 (2009) [arXiv:0905.4142 [gr-qc]]
153. V.E. Kuzmichev, V.V. Kuzmichev, *Ukr. J. Phys.* **55**, 626 (2010).
154. J.B. Hartle, S.W. Hawking, *Phys. Rev.* **D28**, 2960 (1983).
155. G.W. Gibbons, J.B. Hartle, *Phys. Rev.* **D42**, 2458 (1990).
156. R. Bousso, S.W. Hawking, *Grav. Cosmol. Suppl.* **4**, 28 (1998) [gr-qc/9608009]
157. V.E. Kuzmichev, V.V. Kuzmichev, *Ukr. J. Phys.* **61**, (5) 449–458 (2016).
158. K.A. Olive *et al.* (Particle Data Group), *Chin. Phys. C* **38**, 090001 (2014).
159. J.D. Brown, D. Marolf, *Phys. Rev.* **D53**, 1835 (1996) [gr-qc/9509026].
160. V.E. Kuzmichev, V.V. Kuzmichev, *Acta Phys. Pol. B* **39**, 979 (2008) [arXiv:0712.0464 [gr-qc]]
161. V.E. Kuzmichev, V.V. Kuzmichev, *Acta Phys. Pol. B* **44**, 2051 (2013) [arXiv:1307.2383 [gr-qc]]
162. V.E. Kuzmichev, V.V. Kuzmichev, *Ukr. J. Phys.* **60**, 664 (2015).
163. P.A.M. Dirac, Proc. Roy. Soc. **A246**, 333 (1958).
164. R. Arnowitt, S. Deser, C.M. Misner, in: *Gravitation: An Introduction to Current Research*, edit. L. Witten, Wiley, New York, 1962 [arXiv:gr-qc/0405109].
165. C.M. Misner, K.S. Thorne, J.A. Wheeler, *Gravita-*

tion, Freeman, San Francisco 1973.
166. L.D. Landau, E.M. Lifshitz, *The Classical Theory of Fields. Course of Theoretical Physics, Vol. 2*, Butterworth-Heinemann, Amsterdam 1975.
167. A.D. Linde, *Elamentary Particle Physics and Inflationary Cosmology*, Harwood, Chur 1990.
168. V.E. Kuzmichev, V.V. Kuzmichev, *Eur. Phys. J.* **C23**, 337 (2002) [astro-ph/0111438]
169. B. Novosyadlyj, in: *Dark Energy: Observational Evidence And Theoretical Models*, edited by V. Shulga (Akademperiodyka, Kyiv, 2013).
170. E.W. Kolb, M.S. Turner, *The Early Universe*, Addison-Wesley, Redwood City (1990).
171. C.S. Camara, M.R. de Garcia Maia, J.C. Carvalho, J.A.S. Lima, *Nonsingular FRW Cosmology and Non Linear Dynamics*, 2004, *arXiv astro-ph/0402311.*
172. Matts Roos, *Introduction to Cosmology*, 3rd edition, John Wiley and Sons, Ltd, West Sussex, UK (2003).
173. I. Dymnikova, M. Fil'chenkov, *Phys. Lett.* **B545**, 214 (2002).
174. M. Rowan-Robinson, *Cosmology*, Clarendon Press, Oxford 2004.

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УЗАГАЛЬНЕНИЙ ПРИНЦИП НЕВИЗНАЧЕНОСТІ ГАЙЗЕНБЕРГА В КВАНТОВІЙ ГЕОМЕТРОДИНАМІЦІ ТА ЗТВ

Р е з ю м е

В даній роботі розглядається потік енергії у Всесвіті на основі простої квантової системи, що описується нелінійним рівнянням Гамільтона-Якобі, яке виникає в рамках стандартного квантового формалізму рівняння Шредінґера. Розглядаються також випадки домінування випромінювання, баротропної рідини та квантової матерії-енергії. В результаті, сформульовано узагальнений принцип невизначеності Гайзенберга (УПНГ) для метричного тензора та, на основі формалізму Кузьмичових для квантової геометродинаміки, встановлено внутрішній взаємозв'язок між УПНГ для метричного тензора та умовами, які постулюються у випадку стану, в якому домінуючою є баротропна рідина у формі пилу.