A Thing Exists If It Is A Grouping Defining What Is Contained Within: Application to The Russell Paradox and Godel's Incompleteness Theorem

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Abstract

The Russell Paradox (1) considers the set, R, of all sets that are not members of themselves. On its surface, it seems like R belongs to itself only if it doesn't belong to itself. This is where the paradox come from. Here, a solution is proposed that is similar to Russell's method based on his theory of types (1,2) but is instead based on the definition of why things exist as described in previous work (3). In that work, it was proposed that a thing exists if it is a grouping defining what is contained within. A corollary is that a thing, such as a set, does not exist until what is contained within is defined. A second corollary is that after a grouping defining what is contained within is present, and the thing exists, if one then alters the definition of what is contained within, the first existent entity is destroyed and a different existent entity is created. Based on this, set R of the Russell Paradox does not even exist until after the list of the elements it contains (e.g. the list of all sets that aren't members of themselves) is defined. Once this list of elements is completely defined, R then springs into existence. Therefore, because it doesn't exist until after its list of elements is defined, R obviously can't be in this list of elements and, thus, cannot be a member of itself; so, the paradox is resolved. Additionally, one can't then put R back into its list of elements after the fact because if this were done, it would be a different list of elements, and it would no longer be the original set R, but some new set. This same type of reasoning is then applied to the Godel Incompleteness Theorem, which roughly states that there will always be some statements within a formal system of arithmetic (system P) that are true but that can't be proven to be true. Briefly, this reasoning suggests that arguments such as the Godel sentence and diagonalization arguments confuse references to future, not yet existent statements with a current and existent statement saying that the future statements are unprovable. Current and existent statements are different existent entities than future, not yet existent statements and should not be conflated. In conclusion, a new resolution of the Russell Paradox and some issues with the Godel Incompleteness Theorem are described.

The Russell Paradox

The Russell Paradox (1) considers the set, R, of all sets that are not members of themselves. On its surface, R seems to belong to itself only if it doesn't belong to itself. This is where the paradox comes from. Several ways to resolve this paradox have been proposed over the years starting with Russell's own method based on the theory of types (1,2). Here, a solution is proposed that is similar to that based on the theory of types but is instead based on the definition of why things exist as described previously by this author (3). In that work, it was proposed that a thing exists if it is a grouping defining what is contained within. This grouping, or definition, is equivalent to a boundary or edge defining what is contained within and giving substance and existence to the thing. A corollary to this is that a thing does not exist until what is contained within is defined. For instance, a set does not exist until after its list of elements is defined. The set's complete definition and subsequent existence is then denoted by the curly braces around the list of elements. A second corollary is that after a grouping defining what is contained within is present, and the thing exists, if one then alters the definition of what is contained within, the first existent entity is destroyed and a different existent entity is created. Now, in regard to the Russell Paradox, suppose one is writing a list of all the sets that are not members of themselves. When you're done writing the complete list, you wish to call that grouping of sets, set R. Based on this, set R does not even exist until after the list of the elements it contains (e.g. the list of all sets that aren't members of themselves) is defined. Once this list of elements is completely defined, R then springs into existence. Therefore, because it doesn't exist until after its list of elements is defined, R obviously can't be a member of itself; so, the paradox is resolved. Additionally, you can't then put R back into its list of elements after the fact because if you did this, it would be a different list of elements, and it would no longer be the original set R, but some new set. Overall, because set R can't be a member of itself, there is no paradox.

Another way of looking at this is via the idea of perspective, or reference frames. When writing the list of elements in a set, the mathematician's perspective is that of being inside the set. However, once the membership list is fully defined, the set springs into existence, and the mathematician's vantage point now shifts to the "outside" of the set, to a different reference frame, where he can see the set as a whole and as being in existence. The existent set, as a whole, cannot then be pushed back into the original membership list because this list is in a different reference frame, internal to the set. This suggests that the way we perceive sets, and, in general, things that exist, depends on the perspective, or reference frame, we're observing them from. This idea was discussed in another paper by the author (5).

Godel's Incompleteness Theorem

This same reasoning applies to Godel's Incompletness Theorems (6), the first of which very roughly states that there will always be some statements within a formal system of arithmetic, P, that are true but that can't be proven to be true based on system P. Two informal explanations of the proof of this theorem make use of the ideas of a Godel sentence (4) and diagonalization (7). The Godel sentence summarizes the incompleteness theorem in a more natural language-type format and can be paraphrased (4) as:

1. "System P will never say that this sentence is true."
2. Now, call this sentence G for Godel.
3. Note that G is equivalent to: 'System P will never say G is true' ".

But G, which is defined as the whole sentence "System P will never say that this sentence is true." is only defined and, therefore, only exists after there is a whole sentence. So, the words "this sentence" in "System P will never say that this sentence is true." are only a reference to the future, not yet existent, whole sentence G; they are not the whole sentence G itself. This means that at the time of utterance of statement 1, G does not yet exist, and, therefore, that it is incorrect to go back in after the fact and replace "this sentence" with "G". This is similar to what happened with the Russell Paradox except here one is incorrectly equating a reference to a future, not yet existent, entity (whole sentence G) with that entity itself. Stated differently, one might say that when reading statement 1, the mind of the reader, or observer, is in the reference frame that is inside that sentence and that the sentence is referring to a future, not yet existent, reference frame in which the whole sentence actually exists. Then, when statement 1 is completed, the mind of the observer jumps to the outside of the sentence and can see it as a whole and call it sentence G. But, once that happens, the external observer can't then stick the sentence as a whole, G, back inside the internal reference frame after the fact. This would be changing the initial statement 1. This jumping back and forth between perspectives, or reference frames, is where the problem lies. In sum, the use of the Godel sentence in explaining the Incompleteness theorem is suspect.

Next, an informal paraphrasing of the diagonalization argument for Godel's theorem, as explained in Wikipedia (7), uses the following puzzle-like sentence:

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable.

and describes the argument as:

> "This sentence does not directly refer to itself, but when the stated transformation is made the original sentence is obtained as a result, and thus this sentence asserts its own unprovability. The proof of the diagonal lemma employs a similar method."

The problem with this argument is as follows. Suppose the above sentence is called G. The structure of sentence G is:

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", when preceded by itself in quotes, is unprovable.",   when preceded by itself in quotes,   is   unprovable.
|___________________________________________||____________________________| |_|  |_________|
                 Subject part 1                          Subject part 2              Verb   Object
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This sentence says that when one does a transformation as suggested by the Subject parts 1 and 2, which is to

A. Take the phrase inside the quotes:

,when preceded by itself in quotes, is unprovable.

B. Make a copy of it, put quotes around the copy and put the quoted copy before the phrase itself to get a new sentence, H:

", when preceded by itself in quotes, is unprovable.", when preceded by itself in quotes, is unprovable.

the result is an unprovable statement. So, sentence G is a reference to a future, not yet existent sentence, call it H, that has the structure:

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", when preceded by itself in quotes, is unprovable.",   when preceded by itself in quotes, is unprovable.
|___________________________________________||__________________________________________|
      Transformation from Subject part 2                  Stuff inside quotes of Subject part 1

                                            or

              "Subject part 1"                                  Subject part 1
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Sentences G and H look the same, but they are different in structure and meaning. G is a reference to a future, not yet existent sentence, H, that will come into existence once the transformation in the subject of G is done. But G and H are different existent entities. Once H is created and exists, one can't then go back to sentence G and give it a different structure as "Subject part 1" Subject part 1. Just as with the Godel sentence, jumping back and forth between internal reference frames (reading sentence G) and external reference frames (sentence H) causes problems in the reasoning.

Taken together, the above reasoning suggests that these two informal proofs of Godel's Incompleteness Theorems are problematic. This doesn't mean that the theorems are necessarily incorrect; it just means that there are problems with these particular pieces of evidence.

Conclusions

In conclusion, in an accompanying paper (3), a thing was defined as existing if it were a grouping or relationship defining what is contained within. Until this grouping defining what is contained within is present, the thing does not exist. After the grouping defining what is contained within is present, and the thing exists, if one then alters the definition of what is contained within, the first existent entity is destroyed and a different existent entity is created. Using this definition of an existent entity, a new resolution of the Russell Paradox and some concerns with the informal proofs of the Godel Incompleteness Theorem are presented.

References

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