Inversions And Invariants Of Space And Time

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Die Mathematik ist es, die uns vor dem Trug der Sinne schützt
und uns den Unterschied zwischen Schein und Wahrheit kennen lehrt.

Leonhard Euler

Abstract. This paper is on the mathematical structure of space, time, and gravity. It is shown that electrodynamics is neither charge inversion invariant, nor is it time inversion invariant.

1. Introduction

Since Emmy Noether in the early 20^{th} century, it is common sense that every symmetry can be and should be identified with and described by a unique invariance. Then energy conservation corresponds to time inversion invariance, and momentum conservation would correspond to space reflection invariance, alias parity.
This was not so evident, before:

A century before, physics was ruled by the mathematics of Leonhard Euler, and it pays out to go back that time:
A mechanical system is to be described by a system of equations

$$\frac{df_l}{ds} = \sum_{1\leq k\leq n} \big(\frac{\partial f_l}{\partial q_k}\frac{\partial H}{\partial p_k} - \frac{\partial f_l}{\partial p_k}\frac{\partial H}{\partial q_k}\big) - \frac{\partial f_l}{\partial s}, \quad (l = 1, .., m),$$

where the q_k are generalized coordinates, p_k generalized momenta, s a generalized time parameter, H a motion generating Hamiltonian function, and the parameter curves $f_l = q_l(s, q_1, \ldots, q_n, p_1, \ldots, p_n)$ are the quantities to solve the equations for.

The term in the round brackets above, now called Poisson bracket, then was unknown to Euler. It is speculative what Euler would have identified as the invariants; I bet, it would have been that the product of the inversions of generalized location and momentum coordinates was to be unity and therefore invariant:

$$I_{p_k} I_{q_k} = \mathbb{I}, \quad (k = 1, \ldots, n).$$

Because the q_k can be either location coodinate or time coordinate and the p_k either momentum coordinate or energy coordinate, the equivalent is:

$$I_P I_Q = \mathbb{I}, \;\; I_E I_T = \mathbb{I},$$

where I_Q is mostly denoted as $\mathcal{P}$, and I_T as $\mathcal{T}$. Does it mean that $\mathcal{P}$ and $\mathcal{T}$ are symmetries? - Not at all: Not only since Euler one knew that $-q$ would be different from $+q$, and an object of energy $+E$ should be different from one with energy $-E$. But it was Euler himself, who showed us how to convert any inversion into a symmetry: By making it part of a phase symmetry:

$$t \equiv e^{i\phi}t, (\phi \in \mathbb{R}), \; and \; q \equiv e^{i\phi}q, \; (\phi \in \mathbb{R}).$$

These equations might raise sorrow over the quantum theoretical minds, still they are the outcome of nothing but mathematics. To the relief of quantum theory, let's add the corresponding equations:

$$E \equiv e^{i\phi}E, (\phi \in \mathbb{R}), \; and \; P \equiv e^{i\phi}P, \; (\phi \in \mathbb{R}),$$

which in turn will be well-received. These are the two sides of the medal: One cannot ask for a symmetry of inversion of a quantity without getting that of its "canonically conjugated" quantity altogether - that's the way mathematics goes!

It's not everything, we can tell provenly as to the invariants:
Classical non-relativistic mechanics demands that the energy of a particle in a conservative (alias closed) system be given by

$$E = V + T := V(x_1, \ldots, x_3) + (1/2) \sum_{1 \leq k \leq 3} m(\frac{dx_1}{dt}^2 + \cdots \frac{dx_3}{dt}^2),$$

where V and T are potential and kinetic energy, t is time, and $x1, x_2, x_3$ are the location coordinates.

So, we get another invariance:
The inversion of E composed by inversion of T, the kinetic energy, followed by inversion of V is equal to identity:

$$I_E I_T I_V = \mathbb{I}.$$

Now we should have at hand a "canonical conjugate" of the inversion I_V within the location and time coodinates, but we havn't, sofar. So, let us seek within relativistic mechanics:
We have $E^2 = m_0^2 c^4 + p^2 c^2$, i.e.: $\gamma_0 E = m_0 c + \gamma_1 p_1 c + \cdots + \gamma_3 c_3$, where the γ_μ are the Dirac matrices. That might by too complicated for now, so let's crudely approximate that as Einstein did:

$$E \approx m_0 c^2 + \frac{1}{2} m\big(\frac{dx}{dt}\big)^2.$$

And let's follow Einstein: We have $c^2 t^2 = \tau^2 + |x|^2$, hence $\gamma_0 tc = \tau + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3$, and as an approximation:

$$ct = \tau + (1/2)|x|,$$

where τ is the eigentime. Hence, the conjugated inversion of V is the inversion of eigentime, I_τ. Now, what will cause alarm to physics, is that m_0 is the rest mass, something which never ever will have to be inverted! Is is however mathematically correct to invert rest mass:

To explain: $V(x)$ in relativistic mechanics is energy, and it is at rest, because it does not dependend on time. So, it is rest energy, alias rest mass. Therefore, the inversion of V is equivalent to the inversion of (rest) mass, and we get the identities

$$\mathcal{PTM}_0 = \mathbb{I},$$

where $\mathcal{P}$ is parity, $\mathcal{T}$ is time inversion, $\mathcal{M}_0$ is eigentime inversion, accompanied by

$$I_E I_V I_p = \mathbb{I}.$$

You might say, oh, there is no problem, if only I let $\mathcal{M}_0$ be the charge inversion $\mathcal{C}$, because not only that'll give the $\mathcal{PCT}$ theorem, but also we know that it will make no problem to invert a positive one into its negative. However, nature is electrically neutral overall. So, be prepared...

2. Problem Statement

The mathematically proven invariants of a dynamical, closed system are the products of the inversions

$$I_E I_t = \mathbb{I},\ I_p I_x = \mathbb{I},\ I_E I_p I_{M/V} = \mathbb{I},\ I_x I_t I_\tau = \mathbb{I},$$

where the last two equations may be abstracted to $\mathcal{PCT} = \mathbb{I}$, applicable to both, (E, p) and their conjugated time and space coordinates (t, x) with $\mathcal{C}$ being ambivalent, meaning either charge or mass inversion on (E, p) and eigentime inversion on (t, x). Mathematically, the first two above invariants correspond to complex phase symmetries, implying that overall, the absolute value $|E|$ will be constant over time (by closedness of the system) as well as $|p|$ being constant over time, if isotropy of the system is given.

Now the problem:
Can we ascertain the separate symmetry of $\mathcal{P}$, $\mathcal{C}$, and $\mathcal{T}$ in the (equations of) dynamical systems?

3. Examination

Let's begin with the least suspicious symmetry, the charge inversion $\mathcal{C}$:
You might say, that's easy: given any neutral chunk of matter, then if the charge is inverted, the same neutral thing will come out. And true, inverting all charges of the universe, we shouldn't see any difference.
There is however some poison in the above invariants: these are identities, and they hold universally, in particular they hold locally, down to the very atom! If we invert charges of an atom, we get negatively charged nucleons surrounded by, positrons. And this is not what we ever see on earth and elsewhere: in fact, through ionization of all kinds of matter we are very sure that

the light particles surrounding the nucleusses all repel eachother, so have the same sign of charge, which we take to be negative. There is no way around: charge inversion cannot be a symmetry: nucleusses are positively charged (or neutral), and the the light, outer particles are negatively charged.
But then, since $\mathcal{PTC}$ is the identity, either of $\mathcal{P}$ or $\mathcal{T}$ must be broken.
It is easy to tell $\mathcal{T}$ to be the broken counterpart: as far as we know, the atoms do not prefer one spin-orientation over the other, which makes them invariant as to parity, and $\mathcal{P}$ drops out.

In [1] it was already shown that the electrcromagnetic radiation itself breaks $\mathcal{T}$. Let's add more to it:

Throughout classical mechanics, we are allowed to replace any of the four space time coordinates $t = x_0, x_1 \dots, x_3$ by an equivalent one. These four coordinates are the Euclidean coordinates relative to an observer, according to which the 4-vector potential $(A_\mu(x))_{0\leq\mu\leq 3}$ is expressed. Then we can substitute the time coordinate t by the (observer's) eigentime τ, and the covariant Maxwell equations become $\frac{\partial^2 A_\mu(\tau,x_1,x_2,x_3)}{\partial\tau^2} = j_\mu(\tau,x_1,x_2,x_3)$. Because τ is independent from the location coordinates, we have a one-dimensional second order differential equation and do need special hyperbolic calculative gymnastics: The fundamental solution of this equation has a Green's function given by the Fourier inverse of $\mathbb{R} \ni \omega \mapsto -1/\omega^2$:

$$G : \tau \mapsto G(\tau) := \frac{-1}{2\pi} \int e^{i\tau\omega} \omega^{-2} d\omega.$$

Not only is this solution explicitly depending on eigentime (hence also explicitly depending on Euclidean time), so that the potential defines a dynamical system, that function explicitly depends on the absolute value of eigentime: it converges zero as $|\tau| \to \infty$, and it increases in absolute value as $|\tau| \to 0$!

What that means is that, given the correctness of electrodynamics, a dynamical system of charges will be in the stable state of minimal action, when the charges and their velocities are distributed evenly within the container volume: which is exactly the hypothesis of Maxwell and Boltzman, i.e.: the 2nd law of thermodynamics is already baked into classical electrodynamics!

Remark 3.1. To give an example, consider two balls consiting of the identical metallic element in vacuum. Let both balls be initially at different temperature. Then the charges in one ball are moving more rapidly than in the other, and the mutual impact of radiation of one ball on the other will not sum to zero. If we measure the mass before and after some period of time, we should find that the mass of the ball with the initially higher temperature has decreased, and the other should be massier in the opposite rate: Why? - Because the an increase of temperature must be equivalent to an increase of rest mass. Hence, simply through radiation, the two balls interchange rest mass, and we don't get back to the initial state without the expense of additional energy.

As a result of $\mathcal{PTC} = \mathbb{I}$, reversing time, we should still see the atoms' outer shell be negatively charged, which we sure do.
Hence, our absolute knowledge of the sign of charge is equivalent to the irreversibility of at least electrodynamics, if not all dynamical systems.

And now it gets interesting: what about the neutral mass, disregarding the charges: Again, it appears, parity is conserved: the definition of inert mass is that of its resistence as to acceleration in either direction. That makes inert mass parity invariant from scratch, and at large distances, no un-isotropies have been observed, yet. So, if we disregard the charges, the system would be even $\mathcal{C}$-invariant. Then, as neutral mass would be $\mathcal{C}$- and $\mathcal{P}$-invariant, it would follow that gravitation by itself (presuming equivalence of gravitational and inert mass) should be $\mathcal{T}$-invariant, either.

What makes this question so intriguing is that it is intimately tied to the problem of gravitational radiation: If gravity was to be caused by radiation, then by analogy with electromagnetic radiation, it should be causing irreversibilty within gravity. These effects sure would be hard to detect at long distances, because of the decrease of interaction to zero for eigentime $|\tau| \to \infty$. But that effect would be huge at small distances, that is: right within our earthly reach! ... where currently we cannot spot irreversibility anywhere within classical mechanics...

But then, gravity would appear at least by principle, to be a secondary effect, derivable from electrodynamics.

One could check that through an experiment, which examines gravitational effects between neutral composites of particles and their antiparticle composites.

It appears that such an experiment is already scheduled to take place at CERN (see: [2]).

References

[1] H. D. Hüttenbach, *A Short Note on Time Inversion Symmetry*, `http://vixra.org/abs/1602.0122`, 2016.

[2] Claude Amsler, *Antigravity matters at WAG 2013* `http://cerncourier.com/cws/article/cern/56224`, 2014

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