Introducing Marxian General Equilibrium Economics

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Abstraction：The quantitative Marxism function system is developed on the basis of the labor theory of value as the micro foundation including Marx labour value function, Marx surplus value function, Marx production function. The heterogeneous aggregation problem is overcome by using matrix analysis of the macro input-output data resulting price eigenvalues and product value thus the details about an economic system such as the rate of profit, the surplus rate of value, the elasticity of capital output. The falling tendency of the rate of profit may not be true if the economy undergoes an general equilibrium.

Key words: transformation problem, roundaboutness, Solow residue, Okishio theorem， value theory, Marx production function, Marx General Equilibrium, Marx GE eigenvalue, Marx price eigenvector, division of labour, turnpike growth, Verdoorn's theory, Kaldor steady state, ecological Marxism, aggregation problem, two Cambridge debate

JEL: E11, O47

This research attempts to place the formal Sraffian model with linear production sets into a general equilibrium framework and to derive a quantitative transformation theorem about Marxian theory of labor value and production price. Marxian reproduction solution established a dynamic general economic equilibrium which can be characterized by input-output ratio, namely, the reduced Organic Composite of Capital divided by the total productivity rate. The labor value thus the value rate of profit (ROP) can be determined from the production price by the use of the input-output matrix analysis. The increased value ROP and the decreased price ROP of USA around 2006/2007 revealed that there was an OCC reduction. Under the framework of the dynamic Marxian general equilibrium, it is possible to undergo an optimal planning about an economic system by the regulation of the government input, entrepreneur taxation, and minimal wage rate. In the first part of this paper, a neoclassical framework is proposed which places the Marxian conceptions of both Constant Capital and Variable Capital into a Cobb-Douglas production function like model in order to obtain the mathematical formulations of $Q = B_0 e^{ft} C^{\beta} V^{1-\beta}$ and $M = b_0 e^{pt} C^{\beta} V^{1-\beta}$ as well as $Y = a_0 e^{Ft} C^{\alpha} V^{1-\alpha}$, which leads to the Marxian 1st theorem about technical progress: $m = \alpha\dot{n} + (1-\alpha)\dot{w} + p\alpha / \beta$. In the second part, the general equilibrium properties of the quantitative Marxian productivity theories are investigated by using variation method. The Marxian 2nd theorem about dynamic equilibrium asserts, there is a input-output equilibrium existed in the reproduction process between Two Departments $\dot{Y}^* = \dot{C}^*$; The Marxian 3rd theorem states that only equilibrium growth leads to the positive value of the productivity parameter which is defined as the product of the change rate of the organic composite of capital with the labor output elasticity of Cobb-Douglas production function[$F = (1-\alpha)\dot{g}^*$], as well as the rising rate of profit. The present paper is also a generalization of the precise conditions under which the profit rate rises or falls. Only when an economic system achieves the Marxian equilibrium including its each production Department, there would be no business cycle; otherwise there exists some potential crisis. At last, an ecological Marxism model is proposed as a criterion for a regional optimal economic growth.

I. Marx Productivity Economics[1]

Marx's labor theory of value[2] pointed out that the value of each commodity (Q) contained three sources: the first part is “constant capital (C)”, representing the value transferred from raw materials and machinery used up, the second part is “variable capital (V)” replacing the value of the labor power, and the third part is the surplus value (M) including net profit (P) and taxation (T). Therefore, the total value Q is expressed as a linear production function:

$$Q = C + V + M = C + V + p'(C+V) = P'(C+V) = P'Cv$$
$$M = P + T = p'(C+V) = p'Cv = m'V$$
$$Cv = C + V, P' = \frac{Q}{C+V}, p' = \frac{M}{Cv} = \frac{m'}{1+g}$$

Where

C = nK (K: capital, n: capital turnover rate), constant capital;

V = wL (L: labor, w: per-capita wages), variable capital

P': the productivity rate, $P' = Q/(C+V) = p'+1$

M: the surplus value

p': the rate of profit, $p' = M/(C+V) = m'/(g+1) = P'-1$

m': the rate of surplus value, m'= M/V

g: the organic composition of capital (OCC): $g = C/V = nK/(wL) = nk/w$

Differentiating Q with respect to time t yields:

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1 曾尔曼.《马克思生产力经济学导引》[M]，厦门大学出版社，2016.01.

2 马克思.《资本论》I [M]. 北京：人民出版社，1975.

$$\frac{dQ}{Q}=\frac{dP'}{P'}+\frac{dCv}{Cv}=\frac{dP'}{P'}+\frac{C}{Cv}\frac{dC}{C}+\frac{V}{Cv}\frac{dV}{V}$$

$$\frac{dQ}{Qdt}=\frac{dP'}{P'dt}+\beta\frac{dC}{Cdt}+(1-\beta)\frac{dV}{Vdt}=f+\beta\frac{dC}{Cdt}+(1-\beta)\frac{dV}{Vdt}$$

$$\beta\equiv\frac{C}{Cv}=\frac{g}{g+1},1-\beta=\frac{V}{Cv}=\frac{1}{g+1},\frac{dP'}{P'dt}\equiv f$$

re-integration gives the Labor Value Function[3] Q as:

$$Q=B_0e^{ft}C^{\beta}V^{1-\beta}$$

f: the productivity growth rate,

$$f=\dot{Q}-\dot{C}+(1-\beta)\dot{g}=(1-\beta)\dot{g}^*,(\dot{Q}=\dot{C})$$

and the cost function is: $Cv=C+V=c_0C^{\beta}V^{1-\beta}$.

Similarly, differentiates M with time t: $M=p'Cv=p'(C+V)$

$$\frac{dM}{M}=\frac{dp'}{p'}+\frac{dCv}{Cv}=\frac{dp'}{p'}+\frac{C}{Cv}\frac{dC}{C}+\frac{V}{Cv}\frac{dV}{V}$$

$$\frac{dM}{Mdt}=\frac{dp'}{p'dt}+\beta\frac{dC}{Cdt}+(1-\beta)\frac{dV}{Vdt}=p+\beta\frac{dC}{Cdt}+(1-\beta)\frac{dV}{Vdt},\frac{dp'}{p'dt}\equiv p=f\gamma$$

After integration, Marx surplus value function[4] M is obtained as:

$$M=b_0e^{pt}C^{\beta}V^{1-\beta},$$

$$m'=b_0e^{pt}g^{\beta},$$

$$p'=\frac{M}{Cv}=\frac{m'}{g+1}=(1-\beta)m'=b_0e^{pt}\frac{g^{\beta}}{g+1}=b\frac{g^{\beta}}{g+1}$$

As well as the function of the rate of surplus value m', p is the growth rate of profit.

Using the similar mathematical process, the Cobb-Douglas production function[5] can be rewritten as:

$$Y=V+M=V+p'Cv=p'(C+\frac{P'}{p'}V)\equiv p'(C+V')=p'Cv'$$

$$\frac{dY}{dtY}=\frac{dp'}{p'dt}+\frac{dCv'}{Cv'dt}=\frac{dp'}{p'dt}+\frac{C}{Cv'}\frac{dC}{Cdt}+\frac{V'}{Cv'}\frac{dV'}{V'dt}=p+\alpha\frac{dC}{Cdt}+(1-\alpha)\frac{dV'}{V'dt},$$

$$V'=\gamma wL，\ \gamma=\frac{P'}{p'}=1+\frac{1}{p'}，\ \alpha\equiv\frac{C}{Cv'}=\frac{g}{g+\gamma}<\beta,1-\alpha=\frac{\gamma}{g+\gamma}>1-\beta$$

After integration, the production function became:

3 曾尔曼.《厦门科技》2014（3）31-35.

4 曾尔曼.《厦门科技》2015（2）27-29;第四届中国经济学年会会议论文.

5 Cobb C.W., Douglas P.H. “A Theory of Production”, Amer. Econ. Rev. 1928,8(1), Spp1.139-165.

$$Y = a_0 e^{pt} C^{\alpha} V'^{1-\alpha} = a_0 e^{pt} C^{\alpha} V^{1-\alpha} \gamma^{1-\alpha} = a_0 e^{pt} (nK)^{\alpha} (\gamma w L)^{1-\alpha},$$
$$Y = AK^{\alpha} L^{1-\alpha} = A_0 e^{mt} K^{\alpha} L^{1-\alpha}$$
$$\Rightarrow p = m - \alpha\dot{n} - (1-\alpha)\dot{w}' = m - \alpha\dot{n} - (1-\alpha)(\dot{w} + \dot{\gamma}),$$
$$p = f\gamma, \dot{\gamma} = \dot{P}' - \dot{p}' = f - p$$
$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + [1 - (1-\alpha)(1+p')^{-1}]p = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{p'+\alpha}{p'+1} p$$

Therefore, it's obvious that the rate of technical change (m) could be characterized as the linear combination of the growth rate of profit (p'), the wage(w), and the capital circulation (n) with respect to the labor output elasticity of C-D production function(1-α). Technological progress ought to improve the rate of profit, wage rates and cash flow. Technical change stems from the division of labor, exacerbated by the division of labor. The rate of technological progress (Solow residue[6]) is proportional to the growth rate of profit p (thus productivity growth rate f).

Okishio Theorem[7] asserts that if real wages remain unchanged, the rate of profit necessarily rises in consequence of an cost-saving technology innovation. Then after transformation, the relationship between the profit rate and the organic composition of capital can be obtained:

$$p = \frac{m - \alpha\dot{n} - (1-\alpha)\dot{w}}{(1-\alpha+\alpha\gamma)/\gamma} = \frac{\dot{y} - \alpha\dot{g} - \dot{w}}{(1-\alpha+\alpha\gamma)/\gamma} = (\dot{y} - \alpha\dot{g} - \dot{w})\frac{p'+1}{p'+\alpha}$$

the growth rate of profit is determined by the growth rate of labor productivity (y) positively only; it seems that Marx was right about that the rate of profit tends to fall due to the rise of the OCC (g).

Differentiates p' with t:

$$p' = \frac{M}{Cv} = \frac{m'}{g+1} = \frac{Y-V}{C+V} = \frac{y-w}{nk+w} = \frac{\frac{y}{w} - 1}{g+1}$$

[6] Solow RM. Technical Change and the Aggregate Production Function [J]. The Review of Economics and Statistics, 1957, 39(3): 312-320.
[7] Okishio, N. "Technical Change and the Rate of Profit", Kobe Univ. Econ. Review, 7, 1961, pp. 85–99.

$$p \equiv \frac{dp'}{p'dt} = \frac{d(y/w-1)}{(y/w-1)dt} - \frac{d(1+g)}{(1+g)dt} = \frac{\dot{y}-\dot{w}}{1-w/y} - \beta\dot{g} = (\dot{y}-\dot{w}-\alpha\dot{g})\frac{p'+1}{p'+\alpha}$$

$$\Rightarrow \frac{\beta}{\alpha} = \frac{p'+1}{p'+\alpha} = \frac{1}{1-w/y} = \frac{Y}{M},$$

$$\frac{1-\alpha}{1-\beta} = \frac{Q}{Y},$$

$$1-\beta = \frac{p'}{m'},$$

$$1-\alpha = \frac{p'+1}{m'+1}$$

$$p' = \frac{\alpha(1-\beta)}{\beta-\alpha} = \frac{\frac{1}{\beta}-1}{\frac{1}{\alpha}-\frac{1}{\beta}},$$

$$P' = p'+1 = \frac{\beta(1-\alpha)}{\beta-\alpha} = \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-\frac{1}{\beta}}$$

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{p'+\alpha}{p'+1}p \Leftrightarrow$$

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta}p$$

And the following relationships about the equilibrium state are obtained:

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta}p = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{1-\alpha}{1-\beta}f$$

$$= (1-\alpha)(\frac{f}{1-\beta} + \dot{w} - \dot{n}) + \dot{n}$$

$$= (1-\alpha)\dot{k} + (\dot{y}-\dot{k}) \Rightarrow$$

$$\frac{f}{1-\beta} + \dot{w} - \dot{n} = \dot{k}$$

$$\dot{y}* = \dot{n}* + \dot{k}*,$$

$$f = (1-\beta)(\dot{k} + \dot{n} - \dot{w}) = (1-\beta)\dot{g}*$$

$$m = \dot{n}* + (1-\alpha)\dot{k}*$$

Therefore, the Marx production function is obtained as:

$$Y = A_0 e^{mt} K^\alpha L^{1-\alpha} = \frac{A_0}{n_0^\alpha w_0^{1-\alpha}} e^{[\alpha\dot{n}+(1-\alpha)\dot{w}+\frac{\alpha}{\beta}p]t} K^\alpha n_0^\alpha w_0^{1-\alpha} L^{1-\alpha}$$
$$= a_0 e^{\frac{\alpha}{\beta}pt} C^\alpha V^{1-\alpha} = a_0 e^{\frac{1-\alpha}{1-\beta}ft} C^\alpha V^{1-\alpha} (a_0 \equiv \frac{A_0}{n_0^\alpha w_0^{1-\alpha}})$$
$$= a_0 e^{(1-\alpha)\dot{g}^* t} C^\alpha V^{1-\alpha}$$
$$\equiv a_0 e^{Ft} C^\alpha V^{1-\alpha}$$

Since the above derivation is based on the equilibrium condition, the symbol "*" is put as a label, and the productivity development parameter (F) is characterized as:

$$F = (1-\alpha)\dot{g}*$$

Thus， the Solow residue combined with Okishio theorem could be rewritten as Marxian 1st theorem about technical change, which depends upon the combination of the growth rates of capital circulating, the wage, and the profit rate:

$$m = \alpha\dot{n} + (1-\alpha)\dot{w} + \frac{\alpha}{\beta} p = \alpha\dot{n} + (1-\alpha)\dot{w} + F$$

Under C-D production function situation where only K and L are taken into consideration, there is:
m=F.

The division of labor by Adam Smith[8] can be characterized as the labor output elasticity $(1-\alpha)$ of the Cobb-Douglas production function:

$$d_L \equiv 1-\alpha = \frac{\gamma}{g+\gamma} = \frac{p'+1}{m'+1};$$

similarly, the degree of the roundabout production by Allyn Young[9] can be characterized as the variable capital output elasticity $(1-\beta)$ of the Marx production function：

$$d_R \equiv 1-\beta = \frac{1}{1+g} = \frac{p'}{m'},$$

$$\gamma_\downarrow = 1 + \frac{1}{p'^\uparrow} = \frac{\beta}{1-\beta}\frac{1-\alpha}{\alpha} = \frac{\frac{1}{(1-\beta)^\uparrow} - 1}{\frac{1}{1-\alpha^\uparrow} - 1},$$

8 亚当·斯密著，郭大力 王亚南译. 国民财富的性质和原因的研究[M]. 北京：商务印书馆，1972.
9 Young AA. Increasing Returns and Economic Progress [J]. The Economic Journal, 1928, 38: 527-42.

$$\dot{\gamma} = \frac{d(P'/p')}{(P'/p')dt} = f - p = f(1-\gamma) = (1-\beta)\dot{g}[1 - \frac{\beta(1-\alpha)}{(1-\beta)\alpha}] = \dot{g}(1-\frac{\beta}{\alpha}) < 0$$

$$\dot{d}_L = \dot{\gamma} - \frac{d(g+\gamma)}{(g+\gamma)dt} = \dot{\gamma} - \frac{g}{g+\gamma}\dot{g} - \frac{\gamma}{g+\gamma}\dot{\gamma} = \frac{g}{g+\gamma}(\dot{\gamma} - \dot{g}) = -\beta\dot{g}* + \alpha(\dot{g}* - \dot{g})$$

$$\dot{d}_R = -\frac{d(1+g)}{(1+g)dt} = -\frac{g}{1+g}\dot{g} = -\beta\dot{g}$$

$$\dot{d}_L - \dot{d}_R = (\alpha - \beta)(\dot{g}* - \dot{g})$$

$$p = \frac{1-\alpha}{\alpha}\beta\dot{g}* = \frac{\beta}{\alpha}F = \gamma f$$

$$p - \dot{\alpha} = (1-\alpha)(\dot{g}* - \dot{g}) = \dot{Y} - \dot{C}, p \cong \dot{\alpha}$$

$$f - \dot{\beta} = (1-\beta)(\dot{g}* - \dot{g}) = \dot{Q} - \dot{C}, f \cong \dot{\beta}$$

Schumpeterian innovation[10] function or degree of innovation can be characterized by $\frac{\gamma}{g}$ since $\frac{1}{g} = \frac{1-\beta}{\beta}$ and the labor division coefficient d_L has the same change rate as the degree of the roundabout production d_R:

$$S := \frac{\gamma}{g} = \frac{1-\alpha}{\alpha} = \frac{Q}{M}\frac{V}{C} = \frac{B_0 e^{ft} g^{\beta-1}}{b_0 e^{pt} g^{\beta}} = \frac{B_0 e^{(f-p)t}}{b_0 g} = \frac{B_0}{b_0} e^{(1-\frac{\beta}{\alpha})\dot{g}*t} g^{-1}$$

$$\dot{S} = \dot{\gamma} - \dot{g} = (1-\frac{\beta}{\alpha})\dot{g}* - \dot{g} = (\dot{g}* - \dot{g}) - \frac{\beta}{\alpha}\dot{g}* \cong -\frac{\beta}{\alpha}\dot{g}*$$

$$\dot{m}' = p + \beta\dot{g} = \frac{\beta}{\alpha}\dot{g}* - \beta(\dot{g}* - \dot{g}) \cong \frac{\beta}{\alpha}\dot{g}*$$

$$S^{\uparrow} = \frac{Q/C}{M/V} = \frac{\mathrm{Y}^{\uparrow} + \mathrm{C}}{\mathrm{C}}\frac{\mathrm{Y}^{\uparrow} - \mathrm{M}}{\mathrm{M}} = \frac{(Y+C)/C}{(Y-V)/V} = \frac{\frac{Y}{C}+1}{\frac{Y}{V}-1} = \frac{\frac{1}{C_{\downarrow}} + \frac{1}{Y}}{\frac{1}{V^{\uparrow}} - \frac{1}{Y}}$$

$$\dot{\mathrm{S}}^{\uparrow} = \dot{\mathrm{Q}}^{\uparrow} - \dot{\mathrm{M}}^{\downarrow} - \dot{\mathrm{g}}^{\downarrow} = \dot{\mathrm{Q}}^{\uparrow} + \dot{\mathrm{V}}^{\uparrow} - \dot{\mathrm{C}}^{\downarrow} - \dot{\mathrm{M}}$$

$$\frac{\partial S}{\partial C} = \frac{-C^{-2}}{\frac{1}{V} - \frac{1}{Y}} < 0,$$

$$\frac{\partial S}{\partial V} = (\frac{1}{C} + \frac{1}{Y})(-)(\frac{1}{V} - \frac{1}{Y})^{-2}(-)V^{-2} > 0,$$

$$\frac{\partial S}{\partial Y} = \frac{V}{CM}\frac{Q}{CM} > 0$$

All the equations are:

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[10] Schumpeter, J.A. *The theory of economic development: an inquiry into profits, capital, credit, interest, and the business cycle* translated from the German by Redvers Opie (1961) New York: OUP

LTV: $Q = C + V + M = C + Y = B_0 e^{ft} C^{\beta} V^{1-\beta} = B_0 e^{(1-\beta)\dot{g}^* t} C^{\beta} V^{1-\beta}$,

MPF: $Y = M + V = a_0 e^{Ft} C^{\alpha} V^{1-\alpha} = a_0 e^{(1-\alpha)\dot{g}^* t} C^{\alpha} V^{1-\alpha}, \alpha p = \beta F = \alpha \gamma f, p > F > f$

STV: $M = b_0 e^{pt} C^{\beta} V^{1-\beta} = b_0 e^{\frac{1-\alpha}{\alpha}\beta \dot{g}^* t} C^{\beta} V^{1-\beta}$

$CostFun.: Cv = C + V = c_0 C^{\beta} V^{1-\beta}, g + 1 = c_0 g^{\beta}$

Productivity: $P' = Q / Cv = B_0 c_0^{-1} e^{ft} = B_0 c_0^{-1} e^{(1-\beta)\dot{g}^* t}$

$SuplusValueRate$: $m' = M / V = b_0 e^{pt} g^{\beta} = b_0 e^{\frac{\beta}{\alpha} Ft} g^{\beta} = b_0 e^{\frac{1-\alpha}{\alpha}\beta \dot{g}^* t} g^{\beta}$

Profit Rate: $p' = M / Cv = \dfrac{m'}{g+1} = m'(1-\beta) = b_0 e^{pt} \dfrac{g^{\beta}}{g+1}$

II. Transformation Problem[11]

If there is no currency inflation, and the values of commodities keep invariant, then we have:

C+V=Cv=N=const.,

C'+V'=P_1C+P_2V,

dN=0=dCv=d(C'+V')

dC=dV=0

Total value (C+V) equal total production price (C'+V'):

$$\begin{aligned}
dN &= d(P_1 C + P_2 V) = C dP_1 + P_1 dC + P_2 dV + V dP_2 = C dP_1 + V dP_2 \\
&= Cv[\beta dP_1 + (1-\beta) dP_2] = Cv[\beta \delta P_1 + (1-\beta)\delta P_2] \\
&\cong Cv[\beta \ln(1+\delta P_1) + (1-\beta)\ln(1+\delta P_2)] = Cv[\beta \ln P_1 + (1-\beta)\ln P_2] \\
&= Cv \ln(P_1^{\beta} P_2^{1-\beta}) = 0 \Rightarrow
\end{aligned}$$

$$P_1^{\beta} P_2^{1-\beta} = 1 \Leftrightarrow P^{\beta} P_2 = 1 \Rightarrow$$

$$\beta \dot{P}_1 + (1-\beta)\dot{P}_2 = 0, or:$$

$$\beta \delta P_1 + (1-\beta)\delta P_2 = 0$$

let δI denotes the inflation index and

$$\delta P_1 := PPI - 1, \delta P_2 := CPI - 1$$

$$\beta = \frac{\delta P_2}{\delta P_2 - \delta P_1} = \frac{CPI - 1}{CPI - PPI},$$

$$Q' = C' + V' + M' = P_1 C + P_2 V + P_3 M = B(P_1 C)^{\beta} (P_2 V)^{1-\beta} = Q P_1^{\beta} P_2^{1-\beta} = Q$$

$$M' = P_3 M = b(P_1 C)^{\beta} (P_2 V)^{1-\beta} = M P_1^{\beta} P_2^{1-\beta} = M, P_3 = 1;$$

$$P_1 C + P_2 V = C + V;$$

$$M' = Q' - (P_1 C + P_2 V) = P_3 M = M = p'(C+V) = r'(P_1 C + P_2 V)$$

Total profit equals total surplus values.

For a Marxian two production departments system:

11 Samuelson,PA.: Understanding the Marxian notion of exploitation: a summary of the so-called transformation problem between Marxian values and competitive prices, *Jour. Econ. Liter.* 1971, 9, 399-431

$$C_1+V_1+M_1=C_1+Y_1=Q_1=C_1+C_2=C,(Y_1=C_2:Simple\mathrm{Re}production)$$
$$C_2+V_2+M_2=C_2+Y_2=Q_2=Y_1+Y_2=Y$$
$$Q_1+Q_2=Q$$
$$P_1C_1+P_2Y_1=P_1Q_1$$
$$P_1C_2+P_2Y_2=P_2Q_2$$
$$\begin{pmatrix}C_1 & Y_1\\ C_2 & Y_2\end{pmatrix}\begin{pmatrix}P_1\\ P_2\end{pmatrix}=\begin{pmatrix}Q_1 & 0\\ 0 & Q_2\end{pmatrix}\begin{pmatrix}P_1\\ P_2\end{pmatrix}$$
$$\begin{pmatrix}\frac{C_1}{Q_1} & \frac{Y_1}{Q_1}\\ \frac{C_2}{Q_2} & \frac{Y_2}{Q_2}\end{pmatrix}\begin{pmatrix}P_1\\ P_2\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}P_1\\ P_2\end{pmatrix}=\lambda\begin{pmatrix}P_1\\ P_2\end{pmatrix}$$
$$\lambda_1=1,\lambda_2=\frac{C_1}{Q_1}+\frac{Y_2}{Q_2}-1=\frac{C_1}{Q_1}\frac{Y_2}{Q_2}-\frac{C_2}{Q_2}\frac{Y_1}{Q_1}$$
λ: Marx eigenvalue,
P_i: Marx price eigenvector
Or in a three production departments system described by J. Winternitz[12]:
$$(1):P_1C_1+P_2V_1+P_3M_1=P_1Q_1=P_1C=P_1(C_1+C_2+C_3)$$
$$(2):P_1C_2+P_2V_2+P_3M_2=P_2Q_2=P_2V=P_2(V_1+V_2+V_3)$$
$$(3):P_1C_3+P_2V_3+P_3M_3=P_3Q_3=P_3M=P_3(M_1+M_2+M_3)$$
$$\begin{pmatrix}C_1 & V_1 & M_1\\ C_2 & V_2 & M_2\\ C_3 & V_3 & M_3\end{pmatrix}\begin{pmatrix}P_1\\ P_2\\ P_3\end{pmatrix}=\begin{pmatrix}Q_1 & 0 & 0\\ 0 & Q_2 & 0\\ 0 & 0 & Q_3\end{pmatrix}\begin{pmatrix}P_1\\ P_2\\ P_3\end{pmatrix}$$
$$\begin{pmatrix}\frac{C_1}{Q_1} & \frac{V_1}{Q_1} & \frac{M_1}{Q_1}\\ \frac{C_2}{Q_2} & \frac{V_2}{Q_2} & \frac{M_2}{Q_2}\\ \frac{C_3}{Q_3} & \frac{V_3}{Q_3} & \frac{M_3}{Q_3}\end{pmatrix}\begin{pmatrix}P_1\\ P_2\\ P_3\end{pmatrix}=\lambda\begin{pmatrix}P_1\\ P_2\\ P_3\end{pmatrix},$$
$$\lambda_1=1,$$
$$\lambda_2+\lambda_3=\frac{C_1}{Q_1}+\frac{V_2}{Q_2}+\frac{M_3}{Q_3}-1,$$
$$\lambda_2*\lambda_3=\det\begin{vmatrix}\frac{C_1}{Q_1} & \frac{V_1}{Q_1} & \frac{M_1}{Q_1}\\ \frac{C_2}{Q_2} & \frac{V_2}{Q_2} & \frac{M_2}{Q_2}\\ \frac{C_3}{Q_3} & \frac{V_3}{Q_3} & \frac{M_3}{Q_3}\end{vmatrix}$$

[12] ‘Values and Prices: a solution to the so-called transformation problem’, Econ. Jour. 1948, 58, 276-280.

$$P_1^{\beta}P_2^{1-\beta}=I \Leftrightarrow \ln P_2=\ln I+\beta\ln\frac{P_2}{P_1}, \beta\dot{P}_1+(1-\beta)\dot{P}_2=\dot{I}, or: \beta\delta P_1+(1-\beta)\delta P_2=\delta I$$

1. For I≠1, using WolframAlpha[13]:

P^β=(1-β+P*β)*I			
β =		0.6	0.7
I =	1.05	0.796±0.583i	0.757±0.606i
	1.10	0.624±0.749i	0.561±0.758i
	1.15	0.477±0.837i	0.400±0.826i

2. For I=1:

$$P^{\beta}=\frac{1}{P_2}\xrightarrow{P_2=\frac{P_1}{p}=\frac{C+V}{PC+V}}P^{\beta}=P\beta+1-\beta$$

$$1.e^{\beta\ln P}=1+\beta\ln P+\frac{\beta^2\ln^2 P}{2}+\ldots=1+\beta(P-1)\xrightarrow{\ln P\equiv y}$$

$$(a)\frac{\beta}{2}y^2=e^y-1-y=\frac{y^2}{2}+\frac{y^3}{6}, y=3(\beta-1)=\ln P, P=e^{-3(1-\beta)},$$

$$(b)\frac{\beta}{2}y^2+\frac{\beta^2y^3}{6}=\frac{y^2}{2}+\frac{y^3}{6}, y=\frac{-3}{1+\beta}\Rightarrow P=e^{\frac{-3}{1+\beta}},$$

$$(c)\frac{\beta}{2}y^2+\frac{\beta^2y^3}{6}+\frac{\beta^3y^4}{24}=\frac{y^2}{2}+\frac{y^3}{6}\Rightarrow P=e^{\frac{2}{\beta^3}(1-\beta^2\pm\sqrt{(1-\beta)(1+\beta-\beta^2+2\beta^3)})}$$

$$(d)\frac{\beta}{2}y^2+\frac{\beta^2y^3}{6}+\frac{\beta^3y^4}{24}=\frac{y^2}{2}+\frac{y^3}{6}+\frac{y^4}{24}\Rightarrow P=e^{\frac{-2(1+\beta)\pm i2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2}}$$

$$\Leftrightarrow P=e^{\frac{-2(1+\beta)}{1+\beta+\beta^2}}\left(\cos\frac{2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2}\pm i\sin\frac{2\sqrt{2+\beta+2\beta^2}}{1+\beta+\beta^2}\right),\cdots$$

$$2:\frac{d}{d\beta}(P^{\beta})=\frac{d}{d\beta}(e^{\beta\ln P})=e^{\beta\ln P}\ln P=\frac{d}{d\beta}(P\beta+1-\beta)=P-1$$

$$\xrightarrow{\ln P\equiv x}\beta=\frac{\ln(\frac{e^x-1}{x})}{x}\approx\frac{1}{x}(\frac{e^x-1}{x}-1)=\frac{e^x-1-x}{x^2}\approx\frac{1}{2}+\frac{x}{6}+\frac{x^2}{24}\Rightarrow$$

$$x=-2\pm2\sqrt{6\beta-2}, \beta>1/3, P=e^{-2\pm2\sqrt{6\beta-2}};$$

$$if: \beta<1/3, x=-2\pm i2\sqrt{-6\beta+2},$$

$$P=e^{-2\pm i2\sqrt{-6\beta+2}}=e^{-2}[\cos(2\sqrt{-6\beta+2})\pm i\sin(2\sqrt{-6\beta+2})]$$

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[13] http://www.wolframalpha.com/

Taking into consideration of the prices, the change rate of OCC would be:

$$g_P = \frac{P_1 C}{P_2 V} = Pg \Rightarrow \dot{g}_P = \dot{P} + \dot{g}$$

$$\dot{P} = \frac{dP}{Pdt} = \frac{d\ln Pd\beta}{d\beta dt} = \pm\frac{6\beta}{\sqrt{6\beta - 2}}\dot{\beta} = \pm\frac{6\beta(1-\beta)}{\sqrt{6\beta-2}}\dot{g}; \dot{g}_P = (1 \pm \frac{6\beta(1-\beta)}{\sqrt{6\beta-2}})\dot{g}$$

III. Economic crisis theory

In *Das Kapital,* Marx[14] defined the reproduction schemes as abstract, two-sector models of the production and circulation of capital. Department one produces means of production, the value of its output(Q_1) is made up of $C_1 + V_1 + M_1 = Q_1$; where C_1 is the constant capital and V_1 the variable capital used up in production, M_1 is the surplus value produced. Department two produces means of consumption and the value of its output (Q_2) is likewise made up of $C_2 + V_2 + M_2 = Q_2$. Simple reproduction requires that capitalists in Department two acquire means of production to the value C_2 from Department one in order to be able to produce again, namely：$C_2 = V_1 + M_1 = Y_1$， therefor---the Marxian 1st theorem: $\dot{C}_2 = \dot{Y}_1$; Then, Marx's theory about crisis is:

$$Y_1 = Y_1^0 e^{\dot{Y}_1 t} = C_2 = C_2^0 e^{\dot{C}_2 t}$$

$$\Rightarrow t = \frac{\ln Y_1^0 - \ln C_2^0}{\dot{C}_2 - \dot{Y}_1} = \begin{cases} \infty, \dot{Y}_1 = \dot{C}_2; \\ 0, Y_1^0 = C_2^0 \end{cases}$$

Namely, if an economic system achieves the Marxian equilibrium including its each production Department ($\dot{Y} = \dot{Y}_1 = \dot{C}_1 = \dot{Y}_2 = \dot{C}_2 = \dot{C}$), there would be no business cycle; otherwise there exists some potential crisis:

[14] 马克思.《资本论》I [M]. 北京：人民出版社，1975.

$$\dot{\beta} \cong f = \frac{\beta'}{\beta} \Rightarrow \dot{f} = \ddot{\beta} = \frac{d\beta'}{\beta' dt} - \dot{\beta} = \frac{\beta''}{\beta f} - f = \dot{d}_R \Rightarrow$$

$$\beta'' = (f - \beta\dot{g})\beta f = \beta(1-\beta)(1-2\beta)\dot{g}^2 = -\beta\frac{(g-1)\dot{g}^2}{(1+g)^2} = -\beta\omega^2,$$

$$\omega \equiv \frac{\sqrt{g-1}\dot{g}}{1+g} = \sqrt{(2\beta-1)(1-\beta)}\dot{g}$$

$$\Rightarrow \beta_t = A_\beta \sin(\omega t) + B_\beta \cos(\omega t) = \beta_0 \sin(\omega t + \theta),$$

$$\beta_0 \equiv \sqrt{A_\beta^2 + B_\beta^2} \le 1, \theta \equiv arctg\frac{A_\beta}{B_\beta}$$

$$\Rightarrow T = \frac{2\pi}{\dot{g}}\frac{g+1}{\sqrt{g-1}} = \frac{2\pi}{\dot{g}}(\sqrt{g-1} + \frac{2}{\sqrt{g-1}}) \ge \frac{4\sqrt{2}\pi}{\dot{g}};$$

$$\dot{\beta}_t = \frac{d\beta_t}{\beta_t dt} = \frac{\omega}{tg(\omega t + \theta)} = f = (1-\beta_t)\dot{g} = \frac{\sqrt{(2\beta-1)(1-\beta)}\dot{g}}{tg(\omega t + \theta)}$$

$$\Rightarrow tg(\omega t + \theta) = \sqrt{\frac{2\beta-1}{1-\beta}} = \sqrt{g-1}, \sin(\omega t + \theta) = \sqrt{\frac{g-1}{g}} = \sqrt{\frac{2\beta-1}{\beta}}$$

$$p' = \frac{\frac{1}{\beta} - 1}{\frac{1}{\alpha} - \frac{1}{\beta}} = \frac{\frac{1}{1-\alpha} - 1}{\frac{1}{1-\beta} - \frac{1}{1-\alpha}}$$

$$\dot{\alpha} \cong p = \frac{\alpha'}{\alpha} \Rightarrow \dot{p} = \ddot{\alpha} = \frac{d\alpha'}{\alpha' dt} - \dot{\alpha} = \frac{\alpha''}{\alpha p} - p = \dot{S} + \dot{\beta} + \ddot{g},$$

$$\Rightarrow \alpha'' = (p - \frac{\beta}{\alpha} + 1 - \beta)\dot{g}p\alpha = -(2\beta-1)S\beta\dot{g}^2\alpha = -(2\beta-1)\gamma(1-\beta)\dot{g}^2\alpha = -\varpi^2\alpha,$$

$$\varpi \equiv \sqrt{(2\beta-1)S\beta}\dot{g} = \dot{g}\sqrt{(2\beta-1)\gamma(1-\beta)} = \omega\sqrt{\gamma}$$

$$\Rightarrow \alpha_t = \alpha_0 \sin(\varpi t + \vartheta) < \beta_t, \alpha_0 \equiv \sqrt{A_\alpha^2 + B_\alpha^2} \le 1, \vartheta \equiv arctg\frac{A_\alpha}{B_\alpha}$$

$$\Rightarrow T' = \frac{2\pi}{\dot{g}\sqrt{(2\beta-1)S\beta}} = \frac{2\pi}{\dot{g}\sqrt{(2\beta-1)\gamma(1-\beta)}} \ge \frac{4\sqrt{2}\pi}{\dot{g}\sqrt{\gamma}}$$

$$\dot{Y} = \frac{Y'}{Y}, \ddot{Y} = \frac{dY'}{Y'dt} - \dot{Y}, Y'' = (\ddot{Y} + \dot{Y})Y' = 0$$

$$\Rightarrow \ddot{Y} + \dot{Y} = 0 \Rightarrow \ddot{Y} = -\dot{Y} = -\dot{C}$$

$$Y'' = (\dot{Y} - \dot{C})Y' = -Y\dot{Y}(\dot{C} - \dot{Y})$$

$$\Rightarrow Y = A_0 \sin(\omega t) + B_0 \cos(\omega t), \omega = \sqrt{\dot{Y}(\dot{C} - \dot{Y})};$$

$$or: Y'' = (\dot{C} - \dot{Y})Y', \omega = \sqrt{\dot{Y}(\dot{Y} - \dot{C})}$$

Moreover, under the situation of simple production:

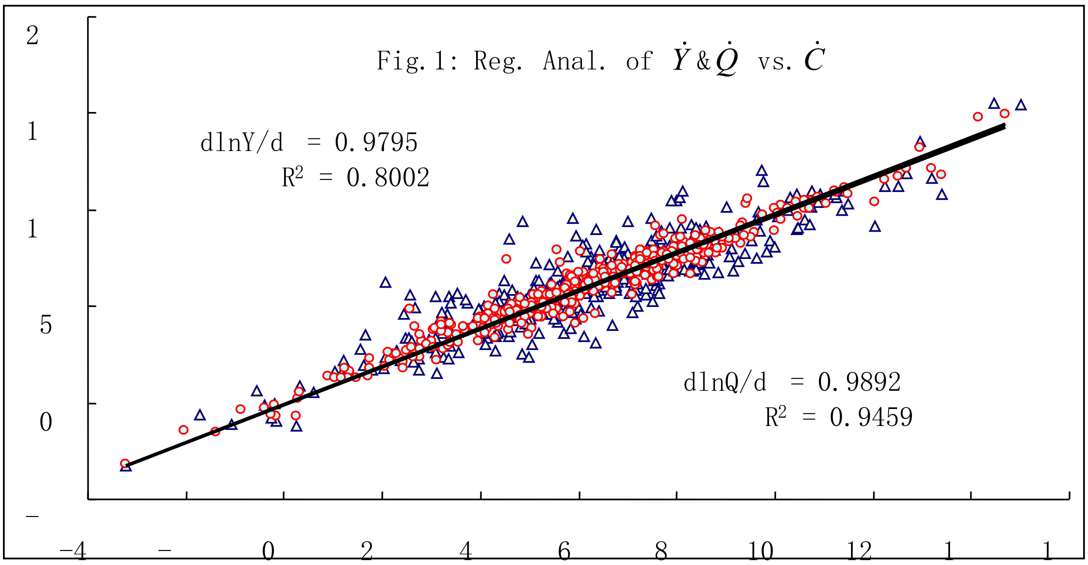
$$Q_1 = C_1 + V_1 + M_1 = C_1 + Y_1 = C_1 + C_2 = C$$
$$Q_2 = C_2 + V_2 + M_2 = C_2 + Y_2 = Y_2 + Y_1 = Y$$
$$Q = Q_1 + Q_2 = C + Y \Leftrightarrow Q\dot{Q} = C\dot{C} + Y\dot{Y} \Leftrightarrow (1-\alpha)\dot{Q} = (\beta-\alpha)\dot{C} + (1-\beta)\dot{Y}$$

according to Leontief's input-output theory[15] and the steady state analysis of the CD production function[16], there should be a dynamic input-output equilibrium between Department I (input: C) and Department II (output: Y=V+M)

$$\dot{y}^{SS} = \dot{k}^{SS}, \dot{n}^{SS} = 0 \Leftrightarrow \dot{Y}^{SS} = \dot{k}^{SS} + \dot{L}^{SS} + \dot{n}^{SS} = \dot{C}^{SS} = \dot{Q}^{SS};$$

$$Y/C = b_0 e^{(1-\alpha)\dot{g}^* t} g^{\alpha-1}, \dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha-1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g})$$
$$Q/C = B_0 e^{(1-\beta)\dot{g}^* t} g^{\beta-1}, \dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta-1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g})$$
$$\therefore \dot{Y}^* = \dot{C}^* = \dot{Q}^* \Leftrightarrow \dot{g}^* = \dot{g}$$

Marxian 2nd theorem about reproducibility states: there is a dynamic input-output equilibrium inside an economic system. The regression analysis of the USA manufacture industry data[17] from 1958-1996 supported the above results：



The Marxian 3rd theorem about productivity development asserts: only Marxian equilibrium leads to productivity development and a rising profit rate.

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[15] Wassily Leontief.Conference on Research in Income and Wealth, 1955. "Input-Output Analysis: An Appraisal," NBER Books, National Bureau of Economic Research, Inc, number 2864.

[16] Solow R.M. A Contribution to the Theory of Economic Growth [J]. The Quarterly Journal of Economics, 1956, (1), 65-94

[17] NBER-CES Manufacturing Industry Database [EB/OL].(2011-02-02)[2012-10-11]. www.nber.org.

$$F = \dot{Y} - \alpha\dot{C} - (1-\alpha)\dot{V} = (1-\alpha)\dot{g}^* > 0 \Leftrightarrow \dot{Y} = \dot{C}, \dot{g}^* > 0;$$
$$f = \dot{Q} - \beta\dot{C} - (1-\beta)\dot{V} = (1-\beta)\dot{g}^* > 0 \Leftrightarrow \dot{Q} = \dot{C}, \dot{g}^* > 0$$
$$\because \dot{g}^* = \dot{g} = \frac{f}{1-\beta} = \frac{\dot{\beta}}{1-\beta}, \beta = \frac{g}{g+1} = \frac{C}{Cv}, \dot{\beta} = \dot{C} - \dot{C}v = \dot{Q} - \dot{C}v$$
$$Q = C + V + M = Cv + M$$
$$\dot{Q} - \dot{C}v = \frac{M(\dot{M} - \dot{Q})}{Cv} = p'(-\dot{\gamma}) > 0$$
$$\therefore \dot{g}^* = \dot{g} = \frac{f}{1-\beta} = \frac{\dot{\beta}}{1-\beta} > 0$$
$$p = \frac{1-\alpha}{\alpha}\beta\dot{g}^* = \frac{\beta}{\alpha}F = \gamma f > 0 \Leftrightarrow F > 0 \Leftrightarrow f > 0$$

IV. Marxian Equilibrium Growth

The tendency of the rate of profit depends on the OCC(g) and the output elasticity of the constant capital(β), similar to the conclusion obtained by D.H. Dickinson[18]:

$$p' = b_0 e^{pt}\frac{g^\beta}{g+1} = b\frac{g^\beta}{g+1}$$
$$\frac{\partial p'}{\partial g} = \frac{p'}{g}(\beta - \frac{g}{g+1}) \geq 0$$
$$\Leftrightarrow \beta \geq \frac{g}{g+1},$$
$$\frac{\partial^2 p'}{\partial g^2} = -\frac{p'\beta}{g^2(g+1)} < 0$$
$$p'_{\max} = b\beta^\beta(1-\beta)^{1-\beta} \Rightarrow (\beta \to 0 or 1) b > p'_{\max} \geq \frac{b}{2}(\beta = \frac{1}{2})$$

By means of variation, Marx was right about the falling rate of the profit only under the competitive equilibrium situation with constant division of labour, otherwise the rate of profit would not fall：

$$p' = \frac{M}{Cv} = \frac{b_0 e^{pt} C^\beta V^{1-\beta}}{C+V} = \frac{b_0 e^{(1-\alpha)\frac{\beta}{\alpha}\dot{g}t} C^\beta V^{1-\beta}}{a_0 C^\beta V^{1-\beta}} = \frac{b_0}{a_0} e^{\frac{\beta}{\alpha}(1-\alpha)\frac{g'}{g}t}$$
$$\frac{\partial p'}{\partial g} = p'\frac{-\beta g' t}{\alpha g^2}(1-\alpha) = -\frac{p'}{g}\frac{1-\alpha}{\alpha}\beta\dot{g}t$$
$$\frac{d}{dt}(\frac{\partial p'}{\partial g'}) = \frac{d}{dt}[p'\frac{\beta t}{\alpha g}(1-\alpha)] = \frac{p'}{g}\frac{1-\alpha}{\alpha}\beta(tp + 1 - t\dot{g})$$
$$\Rightarrow pt = -1 < 0$$
$$\Rightarrow p' = p'_0 t^{-1}$$

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18 “The falling rate of profit in Marxian economics ”, Rev. Econ. Stud., 1975, 24, 120-130

$$if: \frac{d}{dt}(\frac{\partial p'}{\partial g'}) = \frac{d}{dt}(p'\frac{\beta t}{g}S) = \frac{p'}{g}S\beta(tp + 1 + \dot{S}t + \dot{\beta}t - t\dot{g}) = -\frac{p'}{g}S\beta\dot{g}t \Rightarrow$$

$$tp + \dot{S}t + \dot{\beta}t = -1 = tp + \dot{S}t + ft = t(\frac{1-\alpha}{\alpha}\beta - \frac{\beta}{\alpha} + 1 - \beta)\dot{g} = t(1-2\beta)\dot{g} \Rightarrow$$

$$\dot{g}t = \frac{1}{2\beta - 1} > 0 \Leftrightarrow \beta > \frac{1}{2};$$

$$or:$$

$$pt = \frac{1-\alpha}{\alpha}\beta\dot{g}t = \frac{S\beta}{2\beta-1} = \frac{\gamma}{g-1} > 0 \Leftrightarrow g > 1 \Leftrightarrow C > V$$

The exploitation rate will decrease under the equilibrium state:

$$m' = \frac{M}{V} = a_0 e^{pt} g^{\beta} = a_0 e^{\beta\frac{1-\alpha}{\alpha}\dot{g}t} g^{\beta} = a_0 g^{\beta + \frac{\beta(1-\alpha)\dot{g}t}{\alpha \ln g}}, \dot{g} = \frac{dg}{gdt} = \frac{g'}{g}, S := \frac{1-\alpha}{\alpha}$$

$$\frac{\partial m'}{\partial g} = a_0 e^{\frac{1-\alpha}{\alpha}\beta\frac{g'}{g}t}[\frac{1-\alpha}{\alpha}\beta\frac{(-)g'}{g^2}tg^{\beta} + \beta g^{\beta-1}] = m'\frac{\beta}{g}(1 - \frac{1-\alpha}{\alpha}\frac{g'}{g}t) = \frac{M}{C}\beta(1 - S\dot{g}t)$$

$$\frac{\partial m'}{\partial g'} = a_0 e^{\frac{1-\alpha}{\alpha}\beta\frac{g'}{g}t}(\frac{1-\alpha}{\alpha}\beta\frac{t}{g})g^{\beta} = m'\frac{\beta}{g}t\frac{1-\alpha}{\alpha} = \beta\frac{1-\alpha}{\alpha}\frac{M}{C}t = \beta S\frac{m'}{g}t$$

$$(1): \frac{d}{dt}(\frac{\partial m'}{\partial g'}) = \beta\frac{1-\alpha}{\alpha}(\frac{M'}{C}t - \frac{Mt}{C^2}C' + \frac{M}{C}) = \beta\frac{1-\alpha}{\alpha}\frac{M}{C}(\dot{M}t - \dot{C}t + 1)$$

$$= \frac{\partial m'}{\partial g} = \frac{M}{C}\beta(1 - \frac{1-\alpha}{\alpha}\dot{g}t) \Rightarrow \dot{M}t - \dot{C}t + 1 = \frac{\alpha}{1-\alpha} - \dot{g}t$$

$$\Leftrightarrow \dot{m}'t = \frac{\alpha}{1-\alpha} - 1 = S^{-1} - 1 = -\dot{S}t = \frac{\beta}{\alpha}\dot{g}t < 0 \Rightarrow m' = m'_0 t^{S^{-1}-1}, \alpha < \frac{1}{2}$$

$$(2): \frac{d}{dt}(\frac{\partial m'}{\partial g'}) = \beta S\frac{m'}{g}[(\dot{\beta} + \dot{S} + \dot{m}' - \dot{g})t + 1] = \frac{M}{C}\beta(1 - \frac{1-\alpha}{\alpha}\dot{g}t) \Rightarrow ft = S^{-1} - 1$$

$$\Rightarrow \dot{m}'t = \frac{\beta}{\alpha}\dot{g}t = \frac{\beta(S^{-1}-1)}{(1-\beta)\alpha} = (\frac{1}{1-\alpha} - \frac{1}{\alpha})\frac{\beta}{1-\beta} < 0 \Leftrightarrow \alpha < \frac{1}{2}$$

Moreover， the rate of variable capital accumulation will increase under equilibrium state：

$$V = \frac{b_0}{B_0} e^{(\frac{\beta}{\alpha}-1)\frac{g'}{g}t} CS$$

$$\frac{\partial V}{\partial g} = -(\frac{\beta}{\alpha}-1)\frac{g'}{g^2}tV = (1-\frac{\beta}{\alpha})\frac{\dot{g}}{g}tV$$

$$\frac{d}{dt}(\frac{\partial V}{\partial g'}) = \frac{d}{dt}[V(\frac{\beta}{\alpha}-1)\frac{t}{g}] = (\frac{\beta}{\alpha}-1)(\dot{V}\frac{tV}{g} - \frac{tV\beta}{g\alpha}\dot{g} + \frac{V}{g} - V\frac{t\dot{g}}{g}) \Rightarrow$$

$$\dot{V}t = \frac{\beta}{\alpha}\dot{g}t - 1 = \dot{m}'t - 1 > 0 \xrightarrow{ft=S^{-1}-1} \frac{1}{1-\alpha} - \frac{1}{\alpha} > \frac{1-\beta}{\beta} \Leftrightarrow \frac{g+\gamma}{\gamma} - \frac{g+\gamma}{g} > \frac{1}{g}$$

$$\Leftrightarrow g^2 > (1+\gamma)\gamma > 2 \Rightarrow g > \sqrt{2} \approx 1.414 > 1;$$

$$1-\beta = \frac{1}{1+g} < \sqrt{2}-1 \approx 0.414, (2\alpha >)\beta > 0.586;$$

$$\alpha^2 + (2g-1)\alpha - g > 0 \Rightarrow (\beta >)\alpha > \frac{1+\sqrt{4g^2+1}-2g}{2} > 0.5$$

A maximum production would achieve under equilibrium state together with a minimum input requirement under equilibrium state:

$$Y = A_0 e^{\frac{\alpha}{\beta}pt} C^{\alpha} V^{1-\alpha} = A_0 e^{\frac{1-\alpha}{1-\beta}ft} g^{\alpha} V = A_0 e^{\frac{1-\alpha}{g}g't + \alpha \ln g} V$$

$$\frac{\partial Y}{\partial g} = Y(\frac{\alpha}{g} - \frac{1-\alpha}{g^2}g't) = \frac{Y}{g}[\alpha - (1-\alpha)\dot{g}t]$$

$$\frac{d}{dt}(\frac{\partial Y}{\partial g'}) = \frac{d}{dt}(Y\frac{1-\alpha}{g}t) = Y\frac{1-\alpha}{g}(\dot{Y}t + \dot{d}_L t + 1 - \dot{g}t)$$

$$\Rightarrow \dot{Y}t = S^{-1} - 1 > 0 \Leftrightarrow \alpha > 0.5, S < 1, Y = Y_0 t^{S^{-1}-1};$$

$$or: (\dot{Y} - \beta\dot{g})t = S^{-1} - 1 \equiv b \xrightarrow{\beta\dot{g}\equiv a} \ln Y = at + b\ln t + c \geq 0$$

$$\Leftrightarrow \dot{Y}t = \beta\dot{g}t + S^{-1} - 1 \geq 0 \Leftrightarrow S^{-1} = \frac{\alpha}{1-\alpha} \geq 1 - \beta\dot{g}t \Leftrightarrow \alpha \geq \frac{1-\beta\dot{g}t}{2-\beta\dot{g}t}$$

$$Cv = a_0 C^{\beta} V^{1-\beta} = a_0 g^{\beta} V \xrightarrow{1-\frac{f}{\dot{g}}=\beta} Cv = a_0 e^{(1-\frac{f}{\dot{g}})\ln g} V = a_0 e^{(1-\frac{fg}{g'})\ln g} V$$

$$\frac{\partial Cv}{\partial g} = Cv[(-\frac{f}{g'})\ln g + \frac{1}{g} - \frac{f}{g'}] = \frac{Cv}{g}[1-(1-\beta)(1+\ln g)] = \frac{V}{g}(g - \ln g)$$

$$\frac{d}{dt}(\frac{\partial Cv}{\partial g'}) = \frac{d}{dt}[Cv\ln g(\frac{fg}{g'^2})] = \frac{d}{dt}(Cv\frac{1-\beta}{g\dot{g}}\ln g) = \frac{d}{dt}(\frac{V}{g\dot{g}}\ln g) = \frac{V}{g}(\frac{\ln g\dot{V}}{\dot{g}} + 1 - \ln g)$$

$$\Rightarrow \frac{\dot{V}}{\dot{g}} = \frac{g-1}{\ln g} \Leftrightarrow \frac{\dot{V}}{\dot{C}} = \frac{g-1}{g-1+\ln g} \Leftrightarrow \dot{g} = \frac{\ln g}{g-1+\ln g}\dot{C} = \frac{1}{1+\frac{g-1}{\ln g}}\dot{Y} \approx \frac{\dot{Y}}{2}(g \approx 1)$$

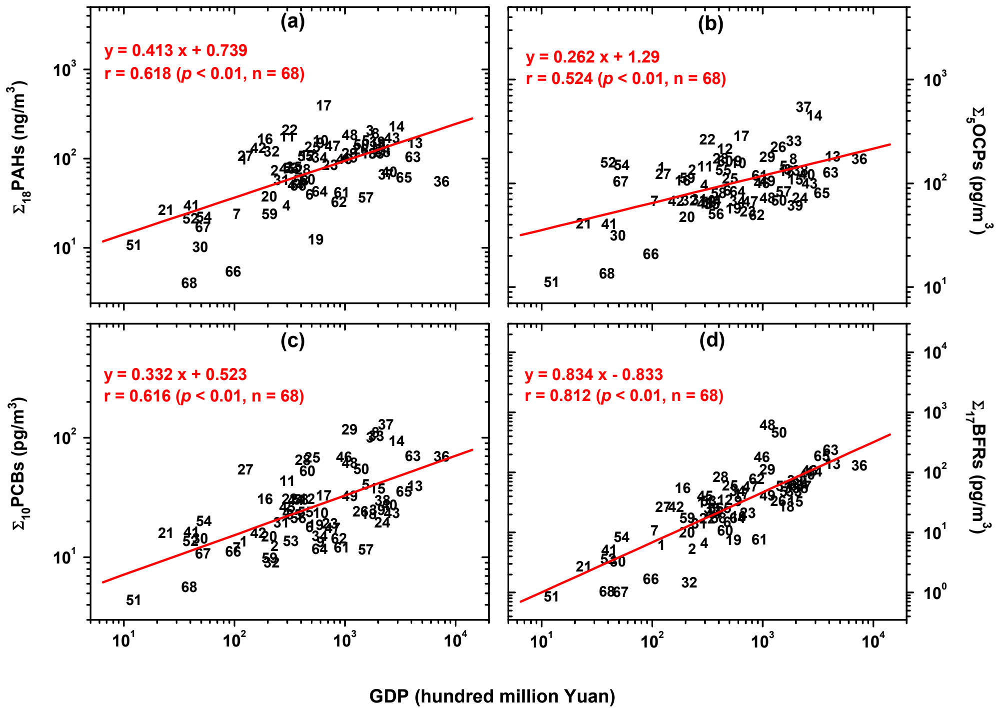
$$f = (1-\beta)\dot{g} = \frac{1-\beta}{1+\frac{g-1}{\ln g}}\dot{Y} \propto (1-\beta)\dot{Y} = (1-\beta)(\dot{y} + \dot{I} + \dot{E})$$

$$or: \frac{d}{dt}(\frac{\dot{V}}{\dot{g}}\ln g) = \frac{d}{dt}(g-1) \Rightarrow \frac{\dot{V}}{\dot{g}} = g, \frac{\dot{V}}{\dot{C}} = \frac{g}{g+1} = \beta, \dot{g} = (1-\beta)\dot{C}, f = (1-\beta)^2\dot{C}$$

$$\dot{V}^* = \beta\dot{C}^* \Leftrightarrow \dot{w}^* = \beta\dot{Y}^* - \dot{L}^*$$
$$\dot{g}^* = (1-\beta)\dot{C}^* = (1-\beta)\dot{Y}^*$$
$$f = (1-\beta)\dot{g}^* = (1-\beta)^2\dot{C}^* = (1-\beta)^2\dot{Y}^*$$
$$p = \frac{1-\alpha}{\alpha}\beta\dot{g}^* = S\beta\dot{g}^* = S\beta(1-\beta)\dot{Y}^* \le \frac{S\dot{Y}^*}{4} \approx 1\% : S = 0.5, \dot{Y} = 8\%; S = 2, \dot{Y} = 2\%$$
$$\dot{S} = -\frac{\beta}{\alpha}\dot{g}^* = -\frac{\beta}{\alpha}(1-\beta)\dot{Y}^*$$
$$\dot{f} = -\beta\dot{g}^* = \dot{d}_L^* = \dot{d}_R^* = \dot{F} = -\frac{g^*}{g^*+1}\frac{dg^*}{g^*dt} = -\frac{d(g^*+1)}{(g^*+1)dt} = -(\dot{C}v^* - \dot{V}^*) = \dot{V}^* - \dot{C}v^*$$
$$F = (1-\alpha)\dot{g}^* = (1-\alpha)(1-\beta)\dot{Y}^* = d_L d_R\dot{Y}^*$$

V. The ecological Marxism

The first-rate reaction of natural decomposition of some Persistent Organic Pollutants is： $C = C_0 e^{-k_i t}$ ， k: reaction rate, C: concentration of certain POPs； the economic growth could be related to the amount of the POPs[19] as： $\ln Y = a + b\ln C$ ，



a、b are both positive coefficients； therefore, the total change of the POPs is：

$$\dot{C} = \frac{d[(\ln Y - a)/b]}{dt} - k = \frac{d\ln Y}{bdt} - k = \frac{\dot{Y}}{b} - k$$

The environment couldn’t be worse meaning the amount of POPs wouldn’t increase:

[19] Private communication with Prof. QQ WANG (Chem. Coll., XiaMen Univ.)

$$\dot{C} \le 0 \Leftrightarrow \frac{\dot{Y}}{b} \le k \Leftrightarrow \dot{Y} \le bk < k (if : b < 1)$$

namely, the growth rate of GDP should not be over a critic value.

VI. Theory of productivity development[20]

From the Marxian value theory equations system derived formerly,

$$LabourValueTheoryEquantion:$$
$$Q = C + \mathrm{V} + \mathrm{M} = \mathrm{C} + \mathrm{Y} = B_0 e^{ft} C^{\beta} V^{1-\beta},$$
$$Q : TotalValue, C : ConstantCapital, V : VariableCapital, M : SurplusValue;$$
$$g \equiv \frac{C}{V} : \underline{O}rganic\underline{C}apital\underline{C}omposite; \beta \equiv \frac{g}{g+1} : reducedOCC;$$
$$1-\beta \equiv d_R, RoundaboutProductionDegree; f := (1-\beta)\dot{g}^* = \dot{\beta}^*, ProductivityGrowthRate$$

$$CostFunction : Cv \equiv C + V = c_0 C^{\beta} V^{1-\beta}, g + 1 = c_0 g^{\beta}$$

$$MarxProductionFunction:$$
$$Y = M + V = a_0 e^{Ft} C^{\alpha} V^{1-\alpha},$$
$$F := (1-\alpha)\dot{g}^*, ProductionDevelopmentCoefficient$$
$$\alpha \equiv \frac{g}{g+\gamma}, 1-\alpha \equiv d_L : LaborDivisionDegree, \gamma \equiv \frac{P'}{p'} = 1 + \frac{1}{p'}$$

the Marxian general equilibrium is obtained as a macro dynamic process:

$$Y / C = b_0 e^{Ft} g^{\alpha-1} \Rightarrow$$
$$\dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha-1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g});$$
$$Q / C = B_0 e^{ft} g^{\beta-1} \Rightarrow$$
$$\dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta-1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g});$$
$$\therefore \dot{g} = \dot{g}^* \Rightarrow \dot{Y}^* = \dot{C}^* = \dot{Q}^*$$

and also, the Marxian productivity development models:

(a) turnpike growth:

$$\dot{Y}^* = \dot{V}^* + \frac{F}{1-\alpha},$$
$$\dot{y}^* = \dot{w}^* + \frac{F}{1-\alpha}, \qquad ;$$
$$F = (1-\alpha)(\dot{y}^* - \dot{w}^*)$$

(b) steady state growth:

[20] 曾尔曼，《马克思生产力经济学导引》，厦门大学出版社,(Erman ZENG: Introducing Marxian Productivity Economics, Xiamen Univ. Press), p.123, 2016

$$\dot{Y}^* = \dot{C}^* = \dot{Q}^* = \dot{V}^* = \dot{M}^* \Leftrightarrow$$
$$\dot{y}^* = \dot{n}^* + \dot{k}^* = \dot{w}^*, m = \dot{n}^* + (1-\alpha)\dot{k}^*;$$
$$F = \dot{g}^* = p = f = 0$$

(c) optimal growth:

$$\dot{Y}^* = \dot{C}^* = \dot{Q}^* = \frac{\dot{V}^*}{\beta} = \frac{F}{(1-\alpha)(1-\beta)}, \quad .$$
$$F = (1-\alpha)(1-\beta)\dot{Y}^* = d_L d_R \dot{Y}^*$$

In fact,

$$F = \dot{Y} - \dot{V} - \alpha\dot{g} = \frac{M}{V}(\dot{M} - \dot{Y}) - \alpha\dot{g} = m'(\dot{\alpha} - \dot{\beta}) - \alpha\dot{g} \cong -\alpha\dot{g}$$
$$= -\frac{g}{g+\gamma}\frac{dg}{gdt} = -\frac{\gamma}{g+\gamma}\frac{dg}{\gamma dt} \approx -(1-\alpha)\frac{d(g/\gamma)}{dt} = -(1-\alpha)\frac{d}{dt}[\frac{\alpha}{1-\alpha}]$$
$$= -\alpha[\dot{\alpha} - \dot{d}_L] = \alpha(\dot{\gamma} - \dot{g}) = \frac{d(1-\alpha)}{(1-\alpha)dt}$$
$$= \dot{d}_L$$
$$\frac{m}{\dot{y}} = 1 - \alpha'\frac{\dot{k}}{\dot{y}} = 1 - \frac{rK}{Y}\frac{ydk/dt}{kdy/dt} = 1 - \frac{rC}{nY}\frac{Y\Delta k}{K\Delta y} = 1 - \frac{r\alpha}{np'}\frac{Y\Delta C}{C\Delta Y} = 1 - \frac{r\alpha}{np'}\frac{\dot{C}}{\dot{Y}} \approx 1 - \alpha = d_L$$
$$\frac{Y}{C} = \frac{p'}{\alpha} = \frac{p'}{1-(1-\alpha)} = \frac{p'}{1-d_L}$$

Verdoorn's Law:

$$\dot{Q} = f + \beta\dot{C} + (1-\beta)\dot{V} = \dot{C} + (1-\beta)(\dot{g}^* - \dot{g}) = \dot{Q}_I + f - \dot{\beta}$$
$$\dot{Y} = F + \alpha\dot{C} + (1-\alpha)\dot{V} = \dot{C} + (1-\alpha)(\dot{g}^* - \dot{g}) = \dot{y} + \dot{L} = \dot{Q}_{II}$$
$$\dot{y} = F + \alpha\dot{Q}_I + (1-\alpha)\dot{V} - \dot{L}$$
$$= \alpha\dot{Q}_I + F + (1-\alpha)\dot{w} - \alpha\dot{L} = \alpha[\dot{Q} - (f - \dot{\beta})] + F + (1-\alpha)\dot{w} - \alpha\dot{L}$$
$$= \alpha(\dot{Q} - \dot{L}) + F + \alpha(\dot{\beta} - f) + (1-\alpha)\dot{w}$$

$$\dot{Y} = \dot{y} + \dot{L} = \frac{1}{Y}(Y_1\dot{Y}_1 + Y_2\dot{Y}_2) = \frac{1}{Y}(Y_1\dot{y}_1 + Y_2\dot{y}_2) + \frac{1}{Y}(Y_1\dot{L}_1 + Y_2\dot{L}_2)$$
$$\dot{y} = \frac{Y_1}{Y}\dot{y}_1 + (1 - \frac{Y_1}{Y})\dot{y}_2 + \frac{Y_1}{Y}\dot{L}_1 + (1 - \frac{Y_1}{Y})\dot{L}_2 - \dot{L}$$

Robinson[21] argues that there is a contradiction between the first and second volumes of *Capital*: in *Capital, Volume I*, Marx assumes that a rising labor

[21] *An Essay on Marxian Economics* (1942), Second Edition (1966) (The Macmillan Press Ltd, ISBN 0-333-05800-3)

productivity leads to a rising rate of exploitation, whereas in *Capital, Volume III* he assumes that rising labor productivity could lead, through a stable rate of exploitation, to a rising rate of real wages and a declining rate of profit.

$$\frac{Y}{M}=\frac{\beta}{\alpha}=\frac{Y/V}{m'}=\frac{y/w}{m'}$$

$$\dot{y}=\dot{w}+\dot{m}'+\dot{\beta}-\dot{\alpha}=\dot{w}+p+\beta\dot{g}+\dot{\beta}-\dot{\alpha}=\dot{w}+F+\alpha\dot{g}$$

$$if:\dot{m}'=p+\beta\dot{g}=\frac{\beta}{\alpha}\dot{g}^{*}-\beta(\dot{g}^{*}-\dot{g})=0$$

$$\Rightarrow F=-\alpha\dot{g}=\frac{\alpha}{\beta}p,\dot{g}^{*}=\frac{-\alpha\dot{g}}{1-\alpha}$$

$$then:\dot{\beta}-\dot{\alpha}=(1-\frac{\beta}{\alpha})\dot{g}^{*}+(\beta-\alpha)(\dot{g}^{*}-\dot{g})=0,$$

$$\dot{y}=\dot{w},$$

$$p=-\beta\dot{g}>0$$

$$if:0>\dot{g}=\dot{n}+\dot{k}-\dot{w}\Leftrightarrow\dot{n}+\dot{k}<\dot{w}=\dot{y}$$

$$\Leftrightarrow\dot{C}<\dot{Y}=\dot{V}$$

$$\Rightarrow F<0,p<0$$

$$However,\dot{C}=\dot{Y}(g.e.):$$

$$\dot{y}^{\uparrow}=\dot{w}+F+\alpha\dot{g}=\dot{w}+\frac{F}{1-\alpha}$$

$$=\dot{w}^{\uparrow}+\frac{\alpha/\beta}{1-\alpha}p^{\uparrow}$$

Besides, from

$$SurplusValueTheoryEquantion:$$

$$M=b_0e^{pt}C^{\beta}V^{1-\beta},$$

$$p:=\frac{1-\alpha}{\alpha}\beta\dot{g}^{*},\Pr ofitGrowthRate;$$

$$\alpha p=\beta F=\alpha\gamma f,p>F>f;$$

$$SuplusValueRate:\ m'=M/V=b_0e^{pt}g^{\beta}$$

Profit Rate: $p'=M/Cv=\frac{m'}{g+1}=m'(1-\beta)=m'd_R=b_0e^{pt}\frac{g^{\beta}}{g+1}=c_0^{-1}b_0e^{pt}$

Productivity: $P'=Q/Cv=B_0c_0^{-1}e^{ft}=p'+1=(1-\alpha)(m'+1)=(m'+1)d_L$

we see that the rate of profit (p’) in terms of value might not tend to fall as long as either the rate of surplus value (m’) in terms of value or the degree of the roundabout production (1-β) increases, so does the productivity (P’) rate. So far, we are talking about the value system all the time, however, it is the production price system existed in reality; we cannot assure that the price ROP (r’) behave the same way as the value ROP (p’):

$$r'=\frac{P_3 M}{P_1 C+P_2 V}=\frac{P_3}{P_2}\frac{m'}{gP+1}=p'\frac{P_3}{P_2}\frac{g+1}{g'+1}, g'\equiv gP=\frac{P_1 C}{P_2 V}$$

VII. Characterization of Kaldor's facts[22]

If there is no inflation, the total values of input commodities represented by the production prices remain unchanged:

$$\mathrm{dN}=\mathrm{d}(\mathrm{P}_1\mathrm{C}+\mathrm{P}_2\mathrm{V})=\mathrm{CdP}_1+\mathrm{P}_1\mathrm{dC}+\mathrm{P}_2\mathrm{dV}+\mathrm{VdP}_2=\mathrm{CdP}_1+\mathrm{VdP}_2$$
$$=\mathrm{Cv}[\beta\mathrm{dP}_1+(1-\beta)\mathrm{dP}_2]=\mathrm{Cv}[\beta\delta\mathrm{P}_1+(1-\beta)\delta\mathrm{P}_2]$$
$$\cong\mathrm{Cv}[\beta\ln(1+\delta\mathrm{P}_1)+(1-\beta)\ln(1+\delta\mathrm{P}_2)]=\mathrm{Cv}[\beta\ln\mathrm{P}_1+(1-\beta)\ln\mathrm{P}_2]$$
$$=Cv\ln(P_1^{\beta}P_2^{1-\beta})=0$$
$$\Rightarrow P_1^{\beta}P_2^{1-\beta}=1,$$
$$P^{\beta}=1/P_2, P\equiv P_1/P_2$$
$$\beta\dot{P}_1+(1-\beta)\dot{P}_2=0, or: \beta\delta\mathrm{P}_1+(1-\beta)\delta\mathrm{P}_2=0$$

$$M'=P_3 M=b(P_1 C)^{\beta}(P_2 V)^{1-\beta}=MP_1^{\beta}P_2^{1-\beta},$$
$$\Rightarrow P_3=P_1^{\beta}P_2^{1-\beta}=1;$$
$$Q'=P_1 C+P_2 V+P_3 M=B(P_1 C)^{\beta}(P_2 V)^{1-\beta}=QP_1^{\beta}P_2^{1-\beta}$$
$$\Rightarrow Q'=Q=C+V+M$$
$$\Rightarrow P_1 C+P_2 V=C+V;$$
$$M'=Q'-(P_1 C+P_2 V)=P_3 M=M=p'(C+V)=r'(P_1 C+P_2 V), P_3=1$$
$$\Rightarrow p'=r'$$

therefore, total production prices equal to total values of labor and commodities; total profits equal to total surplus values; the price rate of profit (r') equals the value rate of profit (p'). When there is some inflation:

$$P_1^{\beta}P_2^{1-\beta}=I=P^{\beta}P_2=P_3\neq 1, P\equiv\frac{P_1}{P_2}$$
$$r'=\frac{P_3 M}{P_1 C+P_2 V}=\frac{P_3}{P_2}\frac{m'}{gP+1}=p'P^{\beta}\frac{g+1}{g'+1}, g'\equiv gP=\frac{P_1 C}{P_2 V}$$
$$r\equiv\frac{dr'}{r'dt}=p+\beta\dot{P}+\beta\dot{g}-\beta'\dot{g}'=p+(\beta-\beta')\dot{g}'=\dot{m}'+\beta\dot{P}-\beta'\dot{g}', \beta'\equiv\frac{g'}{g'+1}$$
$$r<0\Leftrightarrow p<(\beta'-\beta)\dot{g}';$$
$$\beta'-\beta=\frac{g'}{g'+1}-\frac{g}{g+1}=\beta\frac{P-1}{g'+1}=P_1\left(\frac{1}{P_2}-\frac{1}{P_1}\right)\beta(1-\beta')<0$$
$$\Leftrightarrow p=\frac{1-\alpha}{\alpha}\beta\dot{g}^*<P_1\left(\frac{1}{P_2}-\frac{1}{P_1}\right)\beta(1-\beta')\dot{g}'$$
$$\Leftrightarrow \dot{g}^*<\frac{\alpha}{1-\alpha}P_1\left(\frac{1}{P_2}-\frac{1}{P_1}\right)(1-\beta')\dot{g}'=\frac{\alpha}{1-\alpha}(P-1)(1-\beta')(\dot{g}+\dot{P})$$

[22] Kaldor, Nicholas (1957). "A Model of Economic Growth". *The Economic Journal*. **67** (268): 591–624.

Marx is right about the falling tendency of the price rate of profit, but not the value rate of profit.

At the same time, the labor value can be determined by the transformation model; the companying eigenvectors of a 3x3 matrix are the coefficients of price-value ratio of an economic system, which is divided into three production sectors, namely, first the production of the means of production (Department I), second the production of articles of consumption (Department II), and third the production of capital goods (Department III):

$$C_1P_1 + V_1P_2 + M_1P_3 = Q_1P_1$$
$$C_2P_1 + V_2P_2 + M_2P_3 = Q_2P_2$$
$$C_3P_1 + V_3P_2 + M_3P_3 = Q_3P_3$$

so, the mean labor value of the three production departments can be calculated as following:

$$w_i = \frac{V'_i}{P_2 L_i} = \frac{w'_i}{P_2}$$

In Marxian models, competition among firms may lead to an "equilibrium" characterized by equal profit rates or exploitation/surplus rates in all sectors. But here is no such guarantee that a dynamic economic process of price adjustment will converge to a uniform rate of surplus value or equal-profit-rate[23]. However, the Marxian general equilibrium state could be characterized by the input/(total) output ratio with the reduced organic composite of capital β divided by the productivity P':

$$Q = C + V + M = (C+V)(1+p') = C(1+\frac{1}{g})(1+p') = C\frac{1+p'}{\beta} = \frac{P'}{\beta}C$$

$$\Rightarrow C = \zeta Q, \zeta \equiv \frac{\beta}{P'} = \frac{\beta}{1+p'} = \frac{C}{Q} = \frac{C}{C+Y} = (\frac{Y}{C}+1)^{-1}$$

$$\frac{C}{Q} = \frac{Q-Y}{Q} = 1 - \frac{1-\beta}{1-\alpha} = \frac{\beta-\alpha}{1-\alpha} < \frac{C}{Y} = \frac{Q-Y}{Y} = \frac{1-\alpha}{1-\beta} - 1 = \frac{\beta-\alpha}{1-\beta}$$

$$\dot{\zeta} = \dot{C} - \dot{Q} = \dot{\beta} - \dot{P}' = (1-\beta)\dot{g} - f = \dot{\beta} - \dot{\beta}^* = (1-\beta)(\dot{g} - \dot{g}^*)$$

$$\left|\sum_j \frac{C'_{ij}}{P_i}\right| = \frac{\beta_i}{1+p'_i}\left|\frac{Q'_i}{P_i}\right| = \lambda\left|\frac{Q'_i}{P_i}\right| \Leftrightarrow \sum_j \left|\frac{C'_{ij}}{Q'_i}\right|\left|\frac{1}{P_i}\right| = \frac{\beta_i}{1+p'_i}\left|\frac{1}{P_i}\right| = \lambda\left|\frac{1}{P_i}\right|$$

Here we see ζ is much more suitable than the surplus value rate m' or the profit rate p' to characterize the economic equilibrium state since we can take advantage of the intermediate input coefficient matrix to calculate the eigenvalue ζ directly, besides when approaching the Marxian general equilibrium state, the value of ζ changed little.

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23 Nikaido, H: "Refutation of the dynamic equalization of profit rates in Marx's scheme of reproduction" 1978, Univ. South. Cal.;

$$\frac{1}{m'}=\frac{V}{M}=\frac{Y}{M}-1=\frac{\beta}{\alpha}-1=\frac{1}{p'(g+1)}=\frac{1-\beta}{p'};$$

$$\frac{C}{Y}=\frac{C}{V+M}=\frac{g}{1+m'}=\frac{g}{1+p'(1+g)}=\frac{\alpha}{p'}$$

$$Q=C+V+M=(C+V)(1+p')=V(g+1+m')=Vm'\gamma\Rightarrow$$

$$1)\frac{1}{1+p'}=\frac{1}{P'}=\frac{C+V}{Q}=\frac{Cv}{Q}>\frac{C}{Q},-\dot{P}'=-f=-\dot{\beta}^*=\dot{C}v-\dot{Q}=(\beta-1)\dot{g}^*;$$

$$2)\frac{1}{m'}=\frac{V\gamma}{Q}=\frac{Y-M}{M}=\frac{\beta}{\alpha}-1=\frac{\beta-\alpha}{\alpha},-\dot{m}'=-(p+\beta\dot{g})$$

By using the Matrix analysis technique upon the Input-Output Table, the eigenvalue of the intermediate input coefficients matrix could be obtained, and the reduced OCC β thus the degree of roundabout production(1-β)[24], as well as the value rate of profit p'. Similarly,

$$\frac{C}{Y}=\frac{\alpha}{p'}\rightarrow\left|\sum_j\frac{C'_{ij}}{P_i}\right|=\frac{\alpha_i}{p'_i}\left|\frac{Q'_i-C'_i}{P_i}\right|=\frac{\alpha_i}{p'_i}\left|\frac{V'_i+M'_i}{P_i}\right|=\xi\left|\frac{Y'_i}{P_i}\right|$$

$$\sum_j\left|\frac{C'_{ij}}{Y'_i}\right|\left|\frac{1}{P_i}\right|=\frac{\alpha_i}{p'_i}\left|\frac{1}{P_i}\right|=\xi\left|\frac{1}{P_i}\right|$$

from the eigenvalue of the intermediate input coefficients matrix, the elasticity of the capital production α thus the degree of labor division[25] could also be obtained, and so on as well as OCC and ROP:

$$\frac{C}{M}=\frac{\beta}{p'}\rightarrow\left|\sum_j\frac{C'_{ij}}{P_i}\right|=\frac{\beta_i}{p'_i}\left|\frac{M'_i}{P_i}\right|\Leftrightarrow\sum_j\left|\frac{C'_{ij}}{M'_i}\right|\left|\frac{1}{P_i}\right|=\frac{\beta_i}{p'_i}\left|\frac{1}{P_i}\right|=\mu\left|\frac{1}{P_i}\right|$$

$$\frac{C}{V}=g=\frac{\beta}{1-\beta}\rightarrow\left|\sum_j\frac{C'_{ij}}{P_i}\right|=\frac{\beta_i}{1-\beta_i}\left|\frac{V'_i}{P_i}\right|\Leftrightarrow\sum_j\left|\frac{C'_{ij}}{V'_i}\right|\left|\frac{1}{P_i}\right|=\frac{\beta_i}{1-\beta_i}\left|\frac{1}{P_i}\right|=\upsilon\left|\frac{1}{P_i}\right|;$$

$$or:\frac{C}{C+V}=\frac{C}{Q-M}=\beta$$

$$Q=C+V+M=(C+V)(1+p')=V(1+g)(1+p')=V\frac{1+p'}{1-\beta}\Rightarrow$$

$$\frac{V'_i}{P_{2i}}=\frac{1-\beta_i}{1+p_i'}\frac{Q'_i}{P_i}=(\frac{1}{1+p_i'}-\zeta)\frac{Q'_i}{P_i};$$

$$\sum_j\frac{C'_{ij}}{P_j}+\frac{V'_i}{P_{2i}}=\frac{1}{1+p_i'}\frac{Q'_i}{P_i}=\frac{1}{p_i'}\frac{M'_i}{P_{3i}};$$

$$\sum_j\frac{C'_{ij}}{P_j}+\frac{V'_i}{P_{2i}}+\frac{M'_i}{P_{3i}}=\frac{Q'_i}{P_i}=\frac{M'_i}{P_{3i}}\frac{1+p_i'}{p_i'}.$$

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24 Young AA. Increasing Returns and Economic Progress [J]. The Economic Journal, 1928, 38: 527-42.

25 Adam Smith : The Wealth of Nations (Bantam Classics),2003

According to the Germany 2000 input-output matrix[26] (7-sectoral aggregation, Table 1), the eigenvalue ζ equals 0.510，the companying (value-price transformation) eigenvector is：[0. 345, 0. 638, 0. 474, 0. 391, 0. 189, 0. 196, 0. 149]^T.

Table 1. (Germany)	1	2	3	4	5	6	7
1, Agri.	0. 028	0	0. 045	0	0	0. 002	0. 002
2, ManuExpo	0. 09	0. 282	0. 05	0. 022	0. 003	0. 008	0. 011
3, OthManu	0. 142	0. 232	0. 324	0. 287	0. 03	0. 055	0. 065
4, Cnst.	0. 007	0. 003	0. 006	0. 017	0. 006	0. 028	0. 016
5, BizSer	0. 142	0. 121	0. 14	0. 107	0. 332	0. 134	0. 096
6, CnsmSer	0. 036	0. 053	0. 051	0. 108	0. 072	0. 152	0. 049
7, SocialSe	0. 031	0. 006	0. 011	0. 007	0. 007	0. 013	0. 024

According to the UK 2000 input-output data[27] (change from 123x123 to 3x3, Table 2）, the eigenvalue of ζ equals to 0.537, the companying (value-price transformation) eigenvector is：[0. 8906 0. 3504 0. 2898]^T, the value rate of profit (ROP) p'=8.55%, the price ROP r'=20.7%，β=ζ*(1+p')=0.537*1.0855=0.583, α=λ*p'=1.1899*0.0855=0.102:

Table 2. (UK)	Ic	IIv	IIIm			
Ic	10289. 57	2095. 598	1683. 799	0. 016655	0. 103059	0. 001421
IIv	291327. 8	5920. 842	137581. 3	0. 471564	0. 291179	0. 116074
IIIm	77415. 61	3516. 56	455519. 9	0. 125311	0. 17294	0. 38431
V	140424	3177	381289	0. 61353	0. 567178	0. 501805
M	98333	5624	209218	0. 22730	0. 156241	0. 321684
Q	617790	20334	1185292	0. 15917	0. 276581	0. 176512

According to the PRC 2000 input-output direct consumption coefficient data[29](6x6 to 3x3, Table 3):

Table 3 (PRC)	Agr	Ind	Con	T	Bus	Other
Agric	0. 1525828	0. 0577784	0. 0038685	0. 0011993	0. 0537537	0. 0073763
Indus	0. 2047609	0. 5685721	0. 5426612	0. 3459779	0. 2759972	0. 229536
Const	0. 002155	0. 0009704	0. 0005986	0. 0196918	0. 0043322	0. 0282213
T&T	0. 0139507	0. 0243611	0. 0694626	0. 0390754	0. 0314647	0. 0656174
Busi	0. 0190034	0. 0405975	0. 0650467	0. 019845	0. 0863221	0. 0394385
Other	0. 0292091	0. 0258501	0. 0501379	0. 0589961	0. 1138938	0. 1140228

the eigenvalue of ζ equals to 0.658, changed to a 3x3 table:

[26] Flaschel,P: 《Topics in Classical Micro- and Macroeconomics: Elements of a Critique of Neoricardian Theory》，p.64，Springer，2010
[27] https://data.gov.uk/dataset/input-output_supply_and_use_tables
[29] 刘起运等编著，《投入产出分析》，人大版，2006，p.163

0.7696	0.5042	0.6025
0.1206	0.4024	0.2815
0.1098	0.0933	0.1160

the companying (value-price transformation) eigenvector is: $(0.954, 0.261, 0.146)^T$, the value rate of profit p'=2.20%， the price rate of profit r'=12.9%，β=ζ*(1+p')= 0.673.

According to the USA 1997-2014 input-output data[30] (15x15 to 3x3, Table 4), in the year 2000 the eigenvalue of ζ equals to 0.489:

Table 4.		11	21	22	23	31G	42	44RT	48TW	51	FIRE	PROF	6	7	81	G
	λ	Agric	Minin	Utili	Const	Manuf	Whole	Retai	Trans	Infor	Finan	Profe	Educa	Arts,	Other	Govern.
2014	**0.485**	0.4364	0.1773	0.1735	0.2956	**0.4964**	0.1413	0.1544	0.3348	0.2270	**0.1185**	0.1586	0.1909	0.2433	0.1786	0.2000
2013	**0.477**	0.4171	0.1636	0.1727	0.3065	**0.4977**	0.1414	0.1605	0.3389	0.2283	**0.1221**	0.1606	0.1946	0.2455	0.1791	0.2102
2012	**0.481**	0.4807	0.1558	0.1500	0.3017	**0.4942**	0.1327	0.1467	0.3235	0.2247	**0.1045**	0.1469	0.1834	0.2355	0.1662	0.2083
2011	**0.478**	0.4452	0.1551	0.1567	0.3110	**0.4967**	0.1417	0.1527	0.3291	0.2192	**0.1160**	0.1526	0.1927	0.2482	0.1723	0.2117
2010	**0.463**	0.4784	0.1548	0.1738	0.3136	**0.4851**	0.1306	0.1546	0.3007	0.2034	**0.1255**	0.1530	0.1926	0.2471	0.1717	0.2092
2009	**0.447**	0.5229	0.1260	0.1628	0.3206	**0.4783**	0.1000	0.1361	0.2798	0.2027	**0.1293**	0.1510	0.1890	0.2444	0.1622	0.2085
2008	**0.492**	-0.4683	-0.1664	-0.2360	-0.3257	**-0.482**	-0.1294	-0.1367	-0.3143	*-0.1738*	**-0.132**	*-0.1353*	-0.1836	-0.2335	-0.1672	-0.2044
2007	**0.49**	-0.4614	-0.1786	-0.2261	-0.3201	**-0.494**	-0.1262	-0.1378	-0.3116	-0.1845	**-0.131**	-0.1437	-0.1865	-0.2293	-0.1600	-0.2008
2006	**0.484**	0.4518	0.2100	0.2120	0.3321	**0.4931**	0.1296	0.1350	0.2896	0.2007	**0.1371**	0.1420	0.1913	0.2301	0.1579	0.2020
2005	**0.493**	0.4320	0.2351	0.2509	0.3280	**0.4815**	0.1367	0.1401	0.2848	0.1790	**0.1519**	0.1463	0.2016	0.2350	0.1591	0.1964
2004	**0.473**	-0.4090	-0.2505	-0.2055	-0.3496	**-0.501**	-0.1344	-0.1435	-0.2692	-0.1964	**-0.138**	-0.1435	-0.1982	-0.2410	-0.1633	-0.2003
2003	**0.466**	0.4398	0.2467	0.2125	0.3430	**0.4957**	0.1275	0.1304	0.2499	0.2176	**0.1209**	0.1381	0.1992	0.2404	0.1582	0.1946
2002	**0.467**	-0.4634	-0.2187	-0.1925	-0.3412	**-0.505**	-0.1324	-0.1277	-0.2439	-0.2236	**-0.109**	-0.1345	-0.1986	-0.2391	-0.1484	-0.1915
2001	**0.477**	-0.4479	-0.2341	-0.2544	-0.3252	**-0.493**	-0.1165	-0.1196	-0.2293	-0.2476	**-0.110**	-0.1391	-0.1973	-0.2406	-0.1553	-0.1909
2000	**0.489**	-0.4301	-0.2612	-0.2337	-0.3268	**-0.492**	-0.1226	-0.1312	-0.2481	-0.2556	**-0.123**	-0.1462	-0.1955	-0.2373	-0.1354	-0.1854
1999	**0.486**	-0.4651	-0.2341	-0.1781	-0.3439	**-0.510**	-0.1184	-0.1259	-0.2355	-0.2102	**-0.111**	-0.1455	-0.1949	-0.2532	-0.1409	-0.1811
1998	**0.487**	-0.4431	-0.2400	-0.1565	-0.3518	**-0.523**	-0.1064	-0.1147	-0.2250	-0.2066	**-0.110**	-0.1470	-0.2020	-0.2792	-0.1440	-0.1819
1997	**0.489**	-0.4310	-0.2342	-0.1379	-0.3607	**-0.523**	-0.1119	-0.1220	-0.2564	-0.2022	**-0.103**	-0.1345	-0.1971	-0.2869	-0.1404	-0.1818

the value ROP p'=15.8% the price ROP r'=28.8%， β=ζ*(1+p')= 0.566. Detailed analyses reveal that, around year 2007/2008, the price ROP and the value ROP underwent a little different trajectory, theoretical derivation explains that USA was in the transition of economic structure transformation:

$$r \equiv \frac{dr'}{r'dt} = p + \beta\dot{P} + \beta\dot{g} - \beta'\dot{g}' = p + (\beta - \beta')\dot{g}' = \dot{m}' + \beta\dot{P} - \beta'\dot{g}', \beta' \equiv \frac{g'}{g'+1}$$

$$if: r < 0, \& p = \dot{m}' - \beta\dot{g} > 0 \Leftrightarrow \beta'\dot{g}' - \beta\dot{P} > \dot{m}' > \beta\dot{g} \Leftrightarrow (\beta - \beta')\dot{g}' < 0;$$

$$\because \beta - \beta' = \frac{g}{g+1} - \frac{g'}{g'+1} = \beta\frac{1-P}{g'+1} = P_1(\frac{1}{P_1} - \frac{1}{P_2})\beta(1-\beta') > 0$$

$$\therefore \dot{g}' = \dot{g} + \dot{P} < 0; if: \dot{P} = 0, \dot{g} < 0$$

30 http://www.bea.gov/industry/io_annual.htm

Table 5	1/Pc	1/Pv	1/Pm	p' (%)	r'	β	I	g	g'
1997	0.8245	0.4009	0.3994	19.2	31.2	0.582	1.639	1.418	2.917
1998	0.8299	0.4194	0.3679	16.9	30.3	0.569	1.617	1.383	2.737
1999	0.8356	0.4108	0.3647	16.7	30.1	0.567	1.627	1.390	2.828
2000	0.8562	0.3724	0.3581	15.8	28.8	0.566	1.680	1.412	3.247
2001	0.8113	0.3914	0.4343	20.2	29.5	0.573	1.683	1.353	2.804
2002	0.8527	0.3799	0.3585	17.2	**31.1**	0.547	1.690	1.333	2.992
2003	0.8074	0.4018	**0.4320**	**21.5**	**31.6**	0.567	1.675	1.345	2.703
2004	0.8113	0.3914	**0.4343**	**21.5**	**31.5**	0.575	1.680	1.388	2.877
2005	0.8150	0.3713	**0.4448**	**22.2**	**31.6**	0.602	1.678	1.460	3.205
2006	0.7914	0.3841	**0.4755**	**24**	**31.6**	0.6	1.687	1.460	3.008
2007	**0.7820**	0.3851	**0.4900**	**24.5**	**31.1**	**0.61**	**1.686**	1.466	2.978
2008	0.8661	0.3372	0.3689	17.2	**30.5**	0.577	**1.721**	1.497	3.844
2009	0.8311	0.3723	0.4131	22.4	34.4	0.588	1.680	1.313	2.932
2010	0.7477	0.3976	0.5319	30.2	34.1	0.603	1.719	1.386	2.607
2011	-0.8407	-0.3383	0.4229	22.2	33.3	0.583	-1.739	1.452	3.608
2012	-0.8515	-0.3372	0.4016	20.8	33.3	0.581	-1.731	1.451	3.665
2013	-0.8373	-0.3412	0.4273	22.5	33.4	0.588	-1.730	1.459	3.580
2014	-0.8446	-0.3418	0.4121	21.3	33.1	0.588	-1.720	1.454	3.593

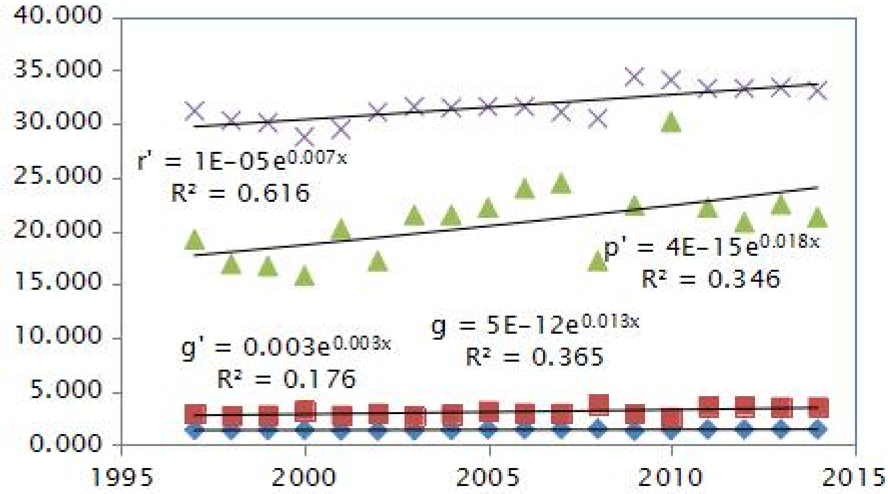


Fig.1 Analysis of USA's price ROP r' vs. value ROP p'

In 2007/2008, there is a changing of the price OCC (organic composite of capital) in the states, the price ROP fell while the value ROP rose, which coincides with the results about the productivity analysis[31]:

$$F = \dot{Y} - \dot{C} - (1-\alpha)\dot{g} = \dot{Y} - \dot{C} - \frac{p'+1}{m'+1}\dot{g}$$

[31] 曾尔曼，《马克思生产力经济学导引》，p.169, 厦门大学出版社，2016

Table6 USA	m'	P/T	p'	(p')net	g	F
1997	0.78	5.34	0.32	0.27	1.46	0
1998	0.75	5.24	0.31	0.26	1.43	-0.014
1999	0.75	5.36	0.31	0.26	1.43	-0.002
2000	0.72	5.27	0.29	0.25	1.44	-0.018
2001	0.72	5.43	0.3	0.25	1.39	0.01
2002	0.74	5.27	0.32	0.27	1.35	0.019
2003	0.74	5.27	0.32	0.27	1.36	0
2004	0.77	5.31	0.32	0.27	1.41	0.006
2005	0.79	5.39	0.32	0.27	1.47	-0.004
2006	0.79	5.3	0.32	0.27	1.48	0
2007	0.78	5.34	0.31	0.26	1.5	**-0.005**
2008	0.77	5.3	0.31	0.26	1.52	**-0.012**
2009	0.78	5.39	0.33	0.28	1.36	0.036
2010	0.82	5.57	0.34	0.29	1.41	0.01
2000-10:		β=0.467	f=0.0025	α=0.038	F=0.0045	p=0.010

VIII. Optimal Economic Planning

According to Marxian general equilibrium, which is indeed of macro dynamic, the growth rates of both departments should equal:

$$Q_1 = C_1 + \underline{V_1 + M_1} = C_1 + \underline{Y_1} = C_1 + \underline{C_2} = C$$

$$Q_2 = Y = \underline{C_2} + V_2 + M_2 = \underline{C_2} + Y_2 = \underline{Y_1} + Y_2 = Y$$

$$Q = C + V + M = C + Y = Q_1 + Q_2$$

$$Y / C = b_0 e^{Ft} g^{\alpha - 1} \Rightarrow$$

$$\dot{Y} - \dot{C} = (1-\alpha)\dot{g}^* + (\alpha - 1)\dot{g} = (1-\alpha)(\dot{g}^* - \dot{g});$$

$$Q / C = B_0 e^{ft} g^{\beta - 1} \Rightarrow$$

$$\dot{Q} - \dot{C} \doteq (1-\beta)\dot{g} + (\beta - 1)\dot{g}^* = (1-\beta)(\dot{g}^* - \dot{g});$$

$$\therefore \dot{g} = \dot{g}^* \Rightarrow \dot{Y}^* = \dot{C}^* = \dot{Q}^*$$

Therefore, there could be a policy regulation among the government input, namely constant capital ($C' = C - \delta_k$), and the income ($Y' = Y + \delta_k$) distribution—the wage rate ($\delta_1 / L = \delta_w$), and the surplus value ($\delta_M = \delta_2 + \delta$) including government taxation δ and entrepreneur profit δ_2:

$$\dot{Y}=\frac{V\dot{V}+M\dot{M}}{Y}=\frac{Y-M}{Y}\dot{V}+\frac{P\dot{P}+T\dot{T}}{Y}, M=P+T=T(\kappa+1), \kappa\equiv P/T$$

$$\dot{Y}=(1-\frac{\alpha}{\beta})\dot{V}+\frac{\alpha}{\beta}(\frac{\dot{P}}{1+\kappa^{-1}}+\frac{\dot{T}}{1+\kappa});$$

$$Q\dot{Q}=C\dot{C}+Y\dot{Y}, C'=C-\delta_k, Y'=Y+\delta_k=V'+M'=V'+P'+T',$$

$$\dot{Q}'=\frac{C'}{Q}\dot{C}+\frac{Y'}{Q}\dot{Y}=\dot{C}'=\dot{Y}'\Rightarrow \dot{Q}'=\frac{C_t-\delta_k}{Q_t}\dot{C}+\frac{Y+\delta_k}{Q_t}\dot{Y}=\dot{Q}+\frac{\dot{Y}-\dot{C}}{Q_t}\delta_k=\dot{C}'=\dot{C}-\frac{\delta_k}{C_t}$$

$$\Rightarrow \delta_k=\frac{\dot{C}-\dot{Q}}{\frac{\dot{Y}-\dot{C}}{Q_t}+\frac{1}{C_t}};$$

$$\dot{Y}'=\frac{\Delta Y_t}{Y_t\Delta t}=(1-\frac{\alpha}{\beta})\frac{\Delta V+\delta_1}{V\Delta t}+\frac{\alpha}{\beta}(\frac{\frac{\Delta P_t+\delta_2}{P_t\Delta t}}{1+\kappa_t^{-1}}+\frac{\frac{\Delta T_t+\delta}{T_t\Delta t}}{1+\kappa_t}),$$

$$\delta_1+\delta_2+\delta=\delta_k$$

$$\frac{\delta}{T_t}=\dot{Y}-\dot{T}+\beta\dot{g}*(\frac{1}{\alpha}-1-\frac{1}{\beta})=\dot{Y}-\dot{T}+p\frac{\alpha}{1-\alpha}(\frac{1}{\alpha}-1-\frac{1}{\beta})$$

$$\rightarrow \delta=T_t[\dot{Y}-\dot{T}+p(1-\frac{\alpha}{\beta(1-\alpha)})](if: \alpha>0.5, \frac{1}{\alpha}-1-\frac{1}{\beta}<0);$$

$$\dot{Y}'\underline{\underline{\Delta t=1}}(1-\frac{\alpha}{\beta})(\frac{\delta_1}{V_t}+\dot{V})+\frac{\alpha}{\beta}(\frac{\dot{P}+\frac{\delta_2}{P_t}}{1+\kappa_t^{-1}}+\frac{\dot{T}+\frac{\delta}{T_t}}{1+\kappa_t})$$

$$=(1-\frac{\alpha}{\beta})(\dot{V}+\frac{\delta_1}{V_t})+\frac{\alpha}{\beta}(\frac{\dot{P}}{1+\kappa_t^{-1}}+\frac{\dot{T}}{1+\kappa_t})+\frac{\alpha}{\beta}(\frac{\delta_2+\delta}{M_t})$$

$$=\dot{Y}+(1-\frac{\alpha}{\beta})\frac{\delta_1}{V_t}+\frac{\alpha}{\beta}(\frac{\delta_k-\delta_1}{M_t})=\dot{C}'=\dot{C}-\frac{\delta_k}{C_t}$$

$$\rightarrow \delta_1=\frac{\dot{C}-\dot{Y}-\delta_k(\frac{1}{C_t}+\frac{\alpha}{\beta}\frac{1}{M_t})}{(1-\frac{\alpha}{\beta})\frac{1}{V_t}-\frac{\alpha}{\beta}(\frac{1}{M_t})}$$

Analysis of PRChina's input-output data (1997-2012)[32] showed: α=0.636, β=0.890, p=-0.00679<0, at year 2010: V_{2010}=1.91^10(k￥), M_{2010}=1.573^10(k￥), T_{2010}=5.99^8, L=7.61^9, κ_{2010}=1.626, δ_1=1854^8>0, δ_w=244，δ_2=1458^8，δ_k=3657^8，

$$\dot{C}=0.152>\dot{Q}=0.148>\dot{Y}=0.137\le\dot{T}=0.138, \dot{M}=0.154, \dot{P}=0.165, \dot{V}=0.126$$ ：

32 曾尔曼：《马克思生产力经济学导引》，2016, p158，厦门大学出版社.

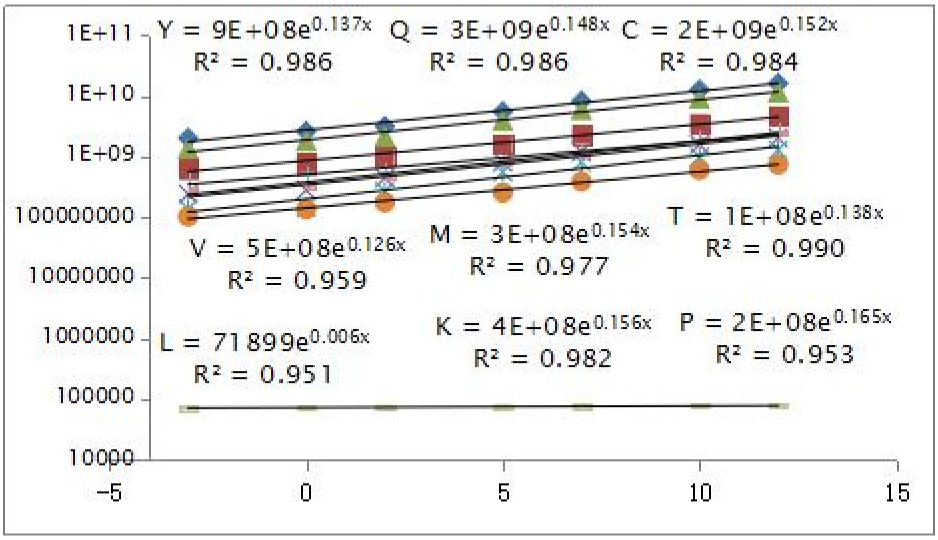


Fig.2 Analysis of PRChina's input-output data (1997-2012)

$\delta_1 > 0$, $\delta_2 > 0$， $\delta = \delta_k - (\delta_1 + \delta_2) = 344$^8(RMB)>0:

Table 7 PRC	δ2	δ(10κ)	δ1	δk	δw
2012	19983573	4226791	21739075	45949439	283.42
2010	14583165	3440343	18543975	36567483	243.66
2007	11710888	2211914	9937937	23860738	131.94
2005	8517318	1454998	5735870	15708186	76.84
2002	5034615	1002757	2460647	8498019	33.579
2000	2636323	770200	3862052	7268575	53.576
1997	2541417	588307	2303198	5432923	32.988

IX. Conclusion

Marxian reproduction solution established an economic equilibrium, which can be characterized by input-(total) output ratio, namely, the reduced Organic Composite of Capital divided by the total productivity. The labor value can be determined from the production price by the use of the input-output matrix analysis. The value ROP and the price ROP analyses of input-output data provide an accurate description on the OCC change, which reflects the industry structure adjustment. Under the framework of the dynamic Marxian general equilibrium, it is possible to undergo an optimization about an economic system by the regulation of the input, taxation, and minimal wage rate, so as to realize the development of the productivity of the society.

In short, this study is aimed to obtain a quantitative description of Marxian capital theory including Marx labour value function and Marx surplus value function as well as Marx production function. The labor output elasticity ($1-\alpha$) of Cobb-Douglas production function is defined as the parameter for the division of labor. The productivity parameter in Marx production function is defined as the product of the change rate of the organic composite of capital with the coefficient of the division of labor. Furthermore, three Marxian theorems are proposed, which assert that there is a dynamic equilibrium existed in reproduction between the Two Departments, only equilibrium growth leads to the positive value of the productivity parameter (Productivity Development Theorem) and also the rate of profit, of which the change rate combined that of the wage and capital circulating with respect to the capital output elasticity of Cobb-Douglas production function characterizes the technological progress rate or as called the Solow residue. By means of variation, the tendency of the profit rate to fall is proved under the situation that the degree of the labor division remains unchanged.

X. P.S.---Analyses of OECD countries

Using the input-output data of some OECD countries[33], the productivity growth rates are obtained[34]:

$$F=\dot{Y}-\alpha\dot{C}-(1-\alpha)\dot{V}=\dot{Y}-\dot{C}+(1-\alpha)\dot{g},$$
$$1-\alpha=\frac{Q}{Y}(1-\beta)=\frac{p'+1}{m'+1},1-\beta=\frac{p'}{m'},\frac{Y}{M}=\frac{\beta}{\alpha}$$
$$F=\dot{Y}-\dot{C}+\frac{p'+1}{m'+1}\dot{g}=\frac{d\ln Y}{dt}-\frac{d\ln C}{dt}+\frac{p'+1}{m'+1}\frac{d\ln g}{dt}$$
$$F_t=\ln(Y/C)_t-\ln(Y/C)_{t-1}+\frac{p_t'+1}{m_t'+1}(\ln g_t-\ln g_{t-1})$$

$$F=\dot{Y}-\dot{M}+p+(\beta-\alpha)\dot{g}=p+(\beta-\alpha)\dot{g}=-\alpha\dot{g}=-\frac{g}{g+\gamma}\frac{dg}{gdt}=-\frac{\gamma}{g+\gamma}\frac{dg}{\gamma dt}$$
$$=-(1-\alpha)\frac{d(g/\gamma)}{dt}=-(1-\alpha)\frac{d}{dt}[\frac{\alpha}{1-\alpha}]=-\alpha[\dot{\alpha}-\dot{d}_L]=\alpha(\dot{\gamma}-\dot{g})=\dot{d}_L$$

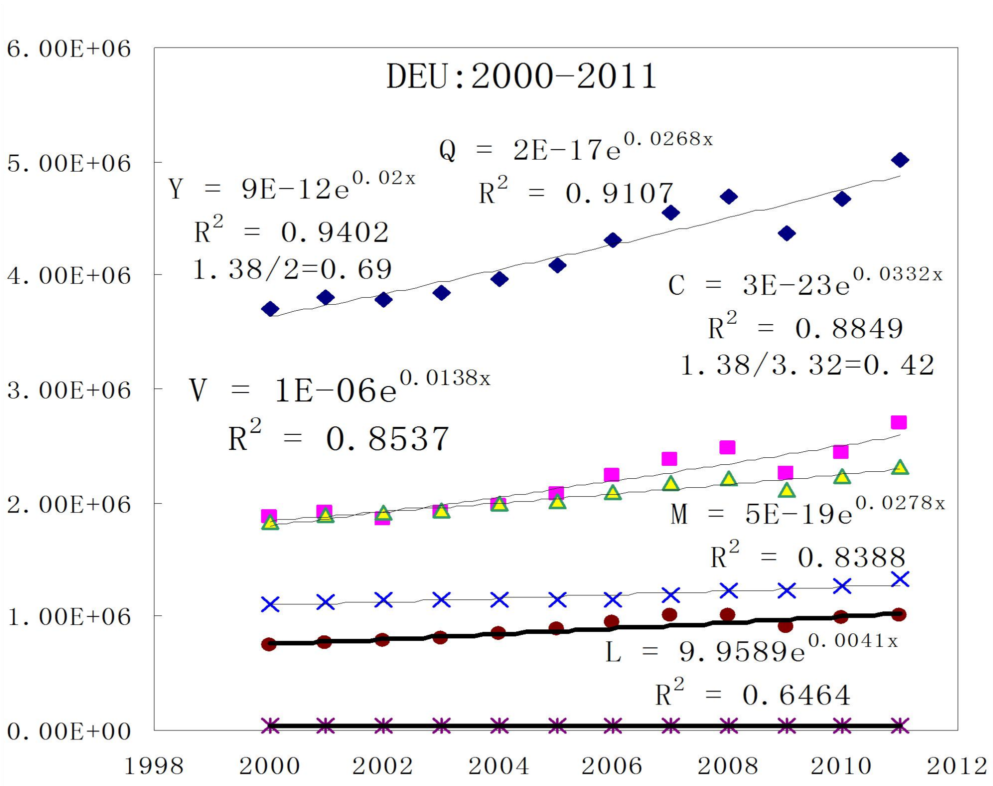
DEU	m'	p'	P/T	g	p'(n)	F
1991	0.62	0.25	1.79	1.53	0.16	
1992	0.60	0.24	1.66	1.47	0.15	-0.002
1993	0.61	0.25	1.54	1.43	0.15	0.010
1994	0.64	0.26	1.68	1.45	0.16	0.015
1995	0.65	0.26	1.71	1.47	0.17	0.002
1996	0.66	0.27	1.71	1.48	0.17	0.004
1997	0.69	0.27	1.73	1.53	0.17	0.009
1998	0.69	0.27	1.74	1.56	0.17	-0.002
1999	0.68	0.26	1.66	1.61	0.16	-0.018
2000	0.65	0.24	1.55	1.67	0.15	-0.023
2001	0.67	0.25	1.55	1.69	0.15	0.007
2002	0.68	0.26	1.56	1.63	0.16	0.019
2003	0.69	0.26	1.56	1.66	0.16	-0.003
2004	0.73	0.27	1.65	1.72	0.17	0.014

33 http://stats.oecd.org

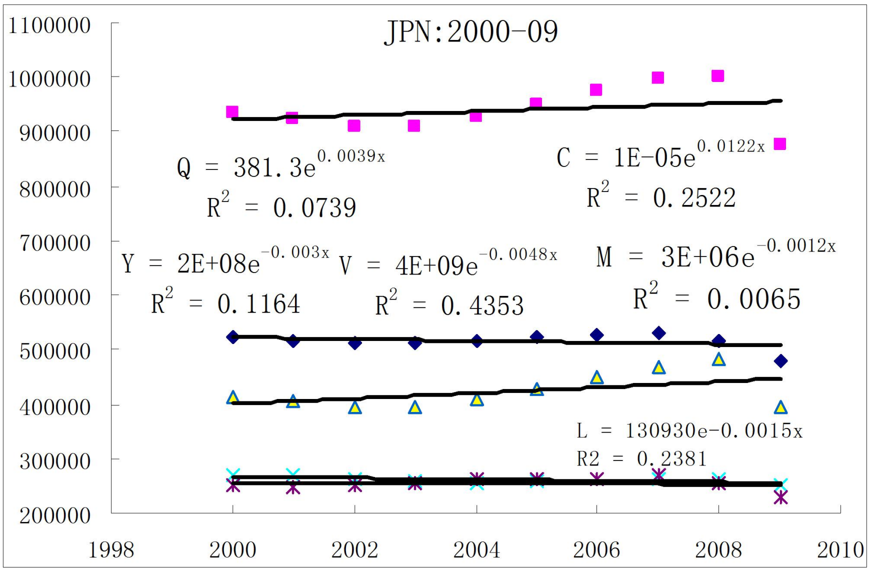
34 曾尔曼：《厦门科技》2015(2)27.

2005	0.76	0.27	1.72	1.83	0.17	0.002
2006	0.80	0.27	1.86	1.93	0.18	0.008
2007	0.83	0.28	1.88	2.01	0.18	0.005
2008	0.80	0.27	1.76	2.02	0.17	-0.018
2009	0.72	0.25	1.44	1.82	0.15	-0.021
2010	0.76	0.26	1.63	1.92	0.16	0.010
2011	0.75	0.25	1.60	2.03	0.15	-0.024

2000-11: β=0.725 f=-0.0011 α=0.440 F=-0.0023 p=–0.0054



$$\dot{C} > \dot{M} > \dot{Q} > \dot{Y} > \dot{V}, p < F < f < 0, \dot{m}' > 0$$

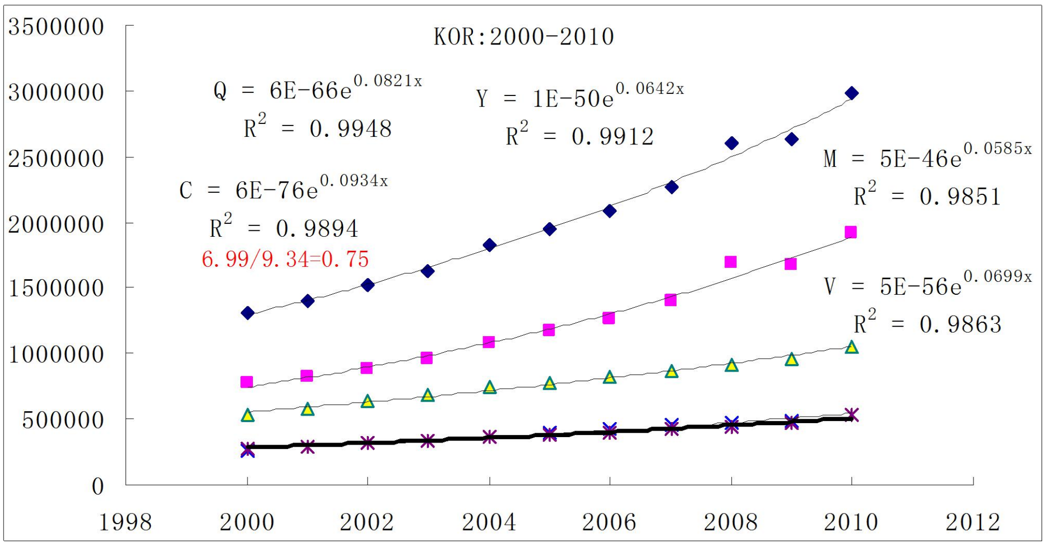


JPN	m'	p'	g	P/T	p' (net)	F
2000	0.93	0.37	1.53	0.84	0.17	
2001	0.92	0.37	1.51	0.79	0.16	-0.002
2002	0.95	0.38	1.51	0.84	0.17	0.019
2003	0.98	0.39	1.53	0.84	0.18	0.008
2004	1.02	0.39	1.60	0.84	0.18	0.006
2005	1.02	0.39	1.65	0.87	0.18	-0.008
2006	0.99	0.37	1.70	0.81	0.16	-0.024
2007	1.02	0.37	1.79	0.84	0.17	0.000
2008	0.97	0.34	1.84	0.75	0.15	-0.036
2009	0.91	0.36	1.56	0.67	0.14	0.017

2000-09: β=0.309 f=0.0066 α=–0.27 F=0.012 p=0.025

–4

$$\dot{C} > \dot{Q} > 0 > \dot{M} > \dot{Y} > \dot{V}, p > F > f > 0, \dot{m}' > 0$$



$$\dot{C} > \dot{Q} > \dot{V} > \dot{Y} > \dot{M}, \mathrm{p} < \mathrm{F} < \mathrm{f} < 0, \dot{\mathrm{m}}' < 0$$

KOR	m'	g	p'	F
1970	1.711	3.497	0.380	
1971	1.732	3.585	0.378	-0.004
1972	1.813	3.673	0.388	0.017
1973	1.789	3.846	0.369	-0.032
1974	1.914	4.436	0.352	-0.033
1975	1.857	4.347	0.347	-0.009
1976	1.753	4.074	0.345	-0.004
1977	1.614	3.708	0.343	-0.006
1978	1.464	3.498	0.325	-0.032
1979	1.368	3.493	0.305	-0.039
1980	1.284	3.825	0.266	-0.077
1981	1.324	3.877	0.272	0.011
1982	1.288	3.739	0.272	0.001
1983	1.212	3.564	0.266	-0.013
1984	1.230	3.542	0.271	0.010
1985	1.263	3.483	0.282	0.022
1986	1.284	3.442	0.289	0.014
1987	1.242	3.423	0.281	-0.016
1988	1.174	3.195	0.280	-0.003
1989	1.071	2.939	0.272	-0.016
1990	1.023	2.789	0.270	-0.004
1991	0.977	2.649	0.268	-0.005
1992	0.981	2.613	0.272	0.007
1993	0.981	2.555	0.276	0.008
1994	0.971	2.521	0.276	0.000
1995	0.940	2.526	0.267	-0.017
1996	0.890	2.496	0.255	-0.022

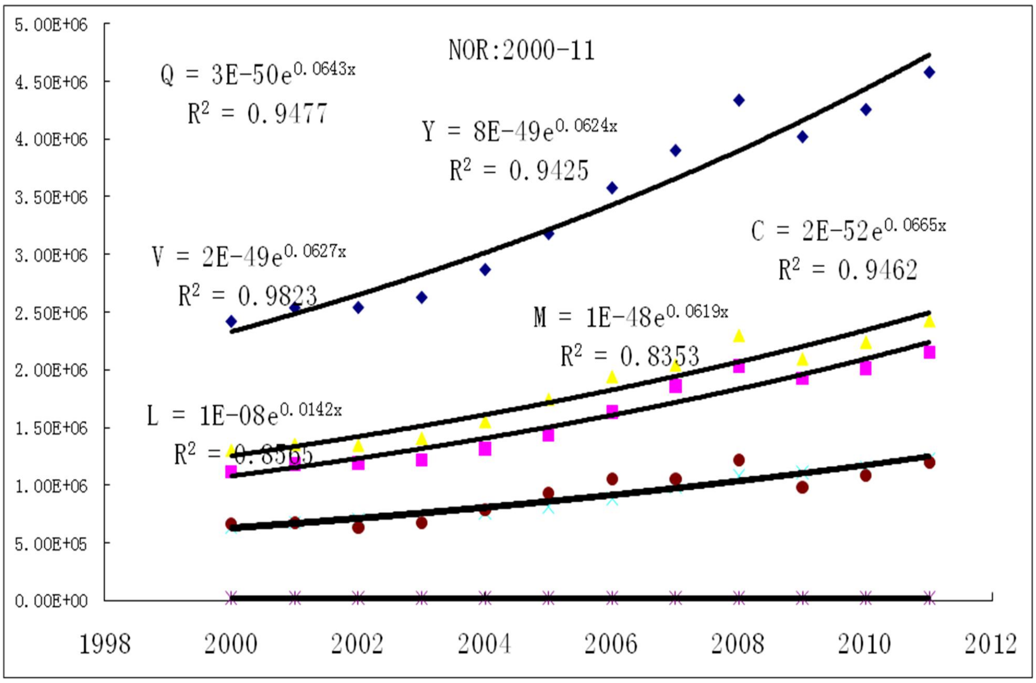
1997	0.951	2.596	0.264	0.018
1998	1.050	2.887	0.270	0.009
1999	1.086	2.936	0.276	0.011
2000	1.080	2.986	0.271	-0.010
2001	1.038	2.873	0.268	-0.006
2002	1.042	2.839	0.271	0.006
2003	1.001	2.794	0.264	-0.014
2004	1.009	2.941	0.256	-0.015
2005	0.958	2.970	0.241	-0.030
2006	0.940	3.014	0.234	-0.014
2007	0.948	3.105	0.231	-0.007
2008	0.936	3.549	0.206	-0.057
2009	0.942	3.399	0.214	0.019
2010	1.003	3.652	0.216	0.002

2001-10: β=0.705 f=-0.0045 α=0.188 F=-0.010 p=–0.020

NOR	P/T	g	m'	p'	p'(n)	F
1970	1.731	1.745	0.815	0.297	0.188	
1971	1.874	1.680	0.729	0.272	0.177	-0.039
1972	1.887	1.601	0.713	0.274	0.179	0.003
1973	2.046	1.607	0.730	0.280	0.188	0.009
1974	2.392	1.757	0.759	0.275	0.194	-0.008
1975	2.067	1.665	0.694	0.260	0.176	-0.024
1976	1.960	1.614	0.653	0.250	0.165	-0.017
1977	1.869	1.600	0.638	0.245	0.160	-0.007
1978	1.771	1.545	0.657	0.258	0.165	0.020
1979	1.856	1.682	0.753	0.281	0.182	0.034
1980	2.301	1.749	0.859	0.312	0.218	0.047
1981	2.328	1.771	0.870	0.314	0.219	0.002
1982	2.307	1.734	0.857	0.313	0.219	-0.001
1983	2.100	1.745	0.898	0.327	0.222	0.020
1984	2.158	1.768	0.958	0.346	0.237	0.027
1985	2.042	1.803	0.932	0.333	0.223	-0.020
1986	1.560	1.720	0.731	0.269	0.164	-0.097
1987	1.432	1.714	0.691	0.254	0.150	-0.023
1988	1.318	1.685	0.684	0.255	0.145	0.000
1989	1.470	1.711	0.794	0.293	0.174	0.059
1990	1.596	1.734	0.836	0.306	0.188	0.019
1991	1.690	1.707	0.844	0.312	0.196	0.009
1992	1.629	1.670	0.817	0.306	0.190	-0.009
1993	1.735	1.697	0.852	0.316	0.200	0.014
1994	1.739	1.734	0.833	0.305	0.193	-0.016
1995	1.854	1.731	0.844	0.309	0.201	0.007

1996	2.030	1.753	0.885	0.321	0.215	0.018
1997	2.060	1.782	0.883	0.317	0.214	-0.006
1998	1.693	1.750	0.737	0.268	0.169	-0.075
1999	1.808	1.735	0.776	0.284	0.183	0.024
2000	2.729	1.753	1.040	0.378	0.276	0.135
2001	2.575	1.749	0.997	0.363	0.261	-0.020
2002	2.191	1.681	0.899	0.335	0.230	-0.039
2003	2.353	1.675	0.928	0.347	0.244	0.016
2004	2.817	1.720	1.028	0.378	0.279	0.042
2005	3.323	1.768	1.145	0.413	0.318	0.047
2006	3.551	1.841	1.185	0.417	0.325	0.005
2007	3.239	1.886	1.069	0.371	0.283	-0.063
2008	3.439	1.882	1.123	0.390	0.302	0.026
2009	2.254	1.729	0.882	0.323	0.224	-0.096
2010	2.350	1.745	0.939	0.342	0.240	0.028
2011	2.367	1.754	0.978	0.355	0.250	0.018

2000-11: β=1 f=-0.0026 α=1 F=-0.0048 p=–0.0096

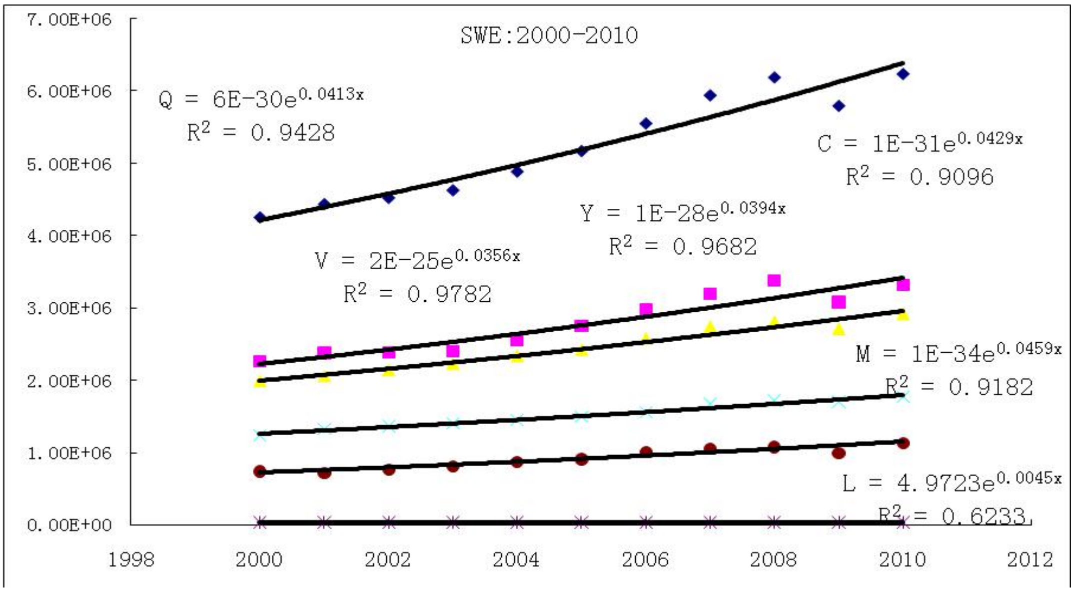


$\dot{C} > \dot{Q} > \dot{V} \geq \dot{Y} \geq \dot{M} > 0, \text{p} < \text{F} < \text{f} < 0, \dot{\text{m}}' \approx 0$

SWE	m'	p'	g	P/T	p'(net)	F
1993	0.56	0.21	1.67	1.79	0.13	
1994	0.59	0.21	1.75	2.17	0.15	0.011

1995	0.66	0.23	1.81	2.27	0.16	0.034
1996	0.61	0.22	1.73	1.74	0.14	-0.022
1997	0.62	0.22	1.78	1.59	0.14	0.003
1998	0.63	0.22	1.80	1.37	0.13	0.000
1999	0.66	0.23	1.83	1.18	0.13	0.016
2000	0.60	0.21	1.83	1.17	0.12	-0.034
2001	0.55	0.20	1.79	0.98	0.10	-0.029
2002	0.56	0.21	1.74	1.00	0.10	0.014
2003	0.58	0.22	1.70	1.07	0.11	0.017
2004	0.61	0.22	1.76	1.17	0.12	0.008
2005	0.61	0.22	1.83	1.22	0.12	-0.007
2006	0.65	0.22	1.91	1.28	0.13	0.015
2007	0.63	0.22	1.91	1.23	0.12	-0.013
2008	0.63	0.21	1.96	0.98	0.11	-0.009
2009	0.59	0.21	1.81	0.76	0.09	-0.007
2010	0.64	0.22	1.87	1.01	0.11	0.024

2000-10: β=0.643 f=0.00126 α=0.238 F=0.0027 p=0.0074

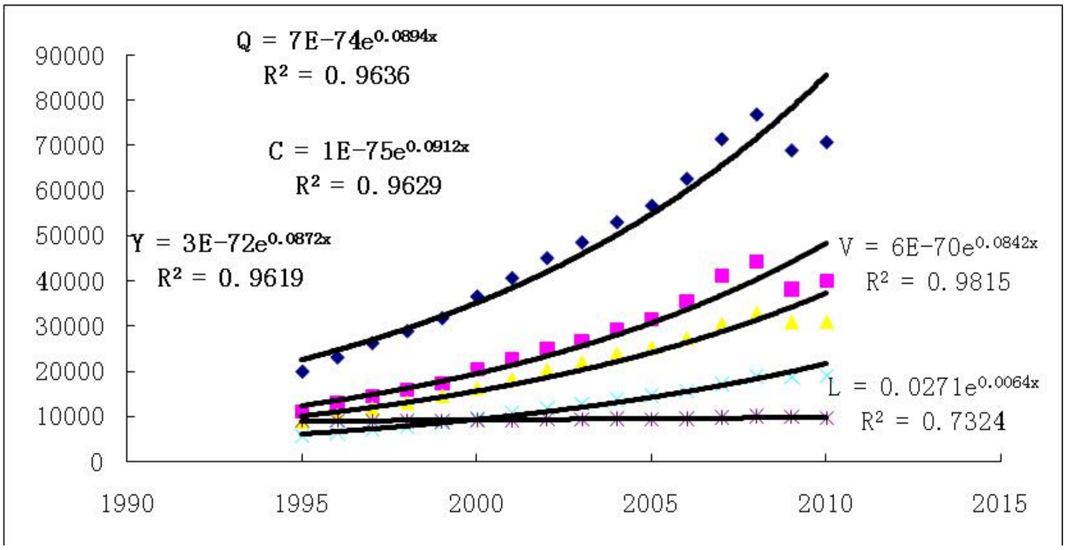


$\dot{M} > \dot{C} \geq \dot{Q} \geq \dot{Y} > \dot{V}, \mathrm{p} > \mathrm{F} > \mathrm{f} > 0, \dot{\mathrm{m}}' > 0$

SVN		p'	P/T	m'	p'(n)	g	F
	1995	0.185	1.002	0.544	0.093	1.933	
	1996	0.198	0.921	0.595	0.095	2.001	0.024
	1997	0.221	1.036	0.675	0.113	2.047	0.043
	1998	0.228	1.017	0.698	0.115	2.067	0.011
	1999	0.238	1.088	0.722	0.124	2.037	0.018
	2000	0.222	0.975	0.696	0.110	2.137	-0.028
	2001	0.223	0.974	0.689	0.110	2.087	0.002

2002	0.229	1.082	0.710	0.119	2.099	0.011
2003	0.235	1.198	0.724	0.128	2.077	0.011
2004	0.232	1.195	0.721	0.126	2.112	-0.007
2005	0.230	1.201	0.723	0.125	2.148	-0.004
2006	0.227	1.380	0.740	0.132	2.255	-0.005
2007	0.226	1.517	0.763	0.136	2.379	-0.003
2008	0.219	1.480	0.726	0.130	2.321	-0.014
2009	0.211	1.237	0.638	0.117	2.022	-0.016
2010	0.201	1.179	0.623	0.109	2.096	-0.019

1995-2010: β=0.792 f=-0.00034 α=0.525 F=-0.00076 p=−0.0017

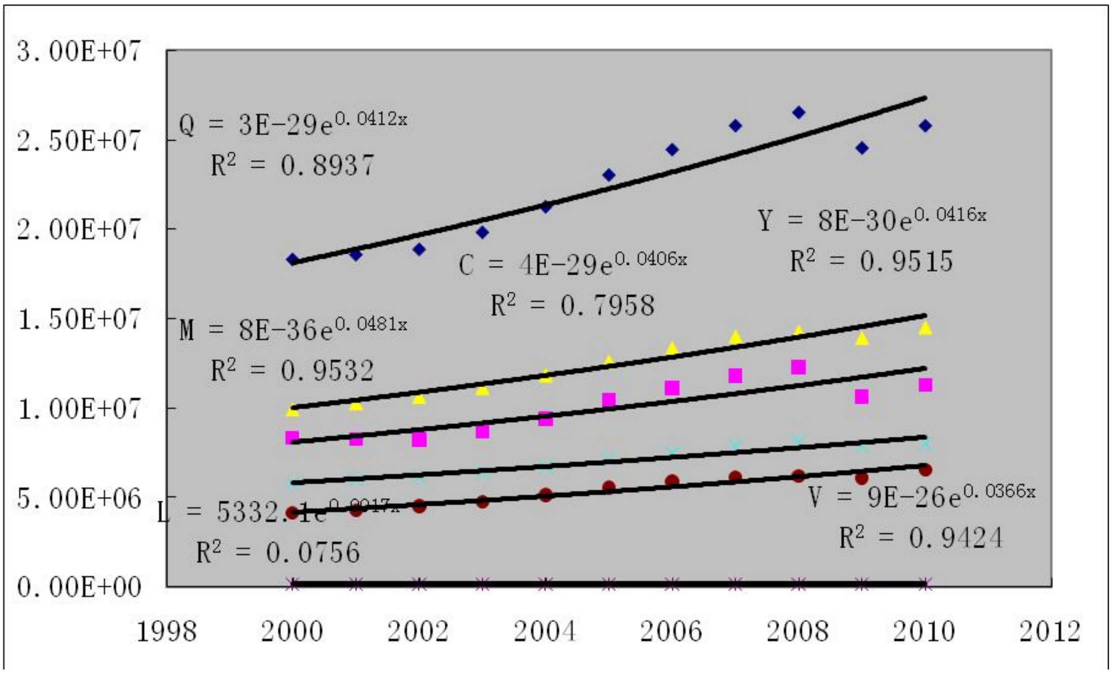


$\dot{C} > \dot{Q} > \dot{Y} > \dot{V}, \mathrm{p} < \mathrm{F} < \mathrm{f} < 0$

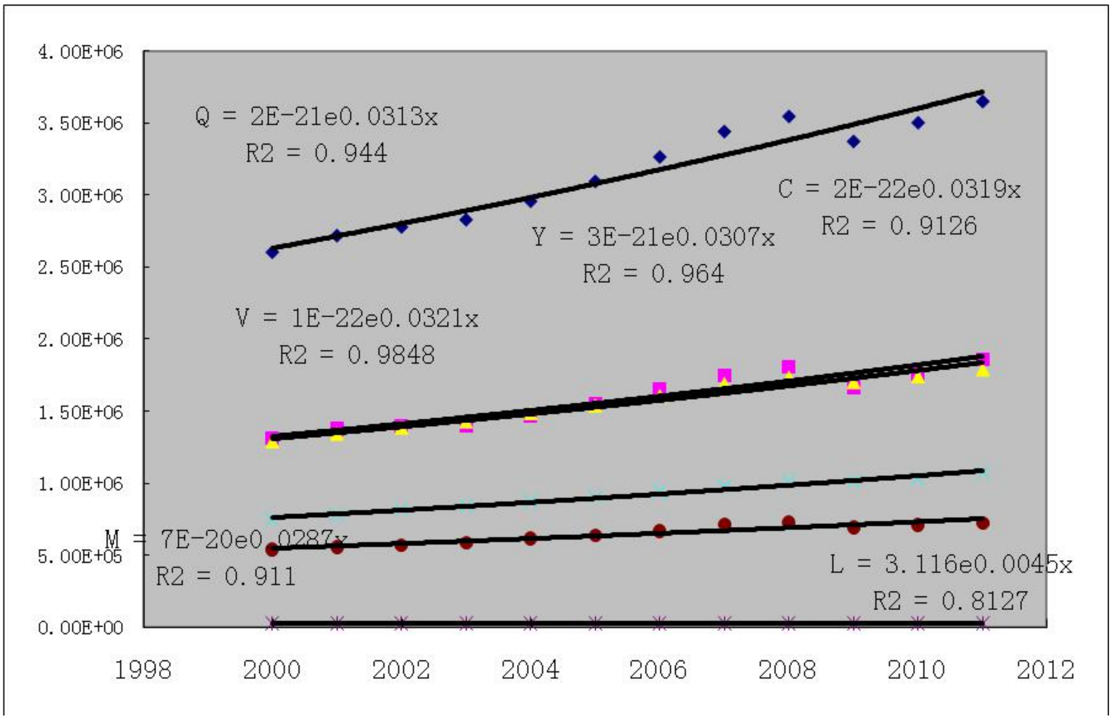
USA	m'	P/T	p'	(p')net	g	F
1987	0.73	5.30	0.30	0.25	1.43	
1988	0.73	5.22	0.30	0.25	1.44	-0.005
1989	0.75	5.33	0.31	0.26	1.43	0.015
1990	0.74	5.21	0.31	0.26	1.42	-0.001
1991	0.74	4.94	0.31	0.26	1.36	0.009
1992	0.75	4.97	0.32	0.26	1.35	0.005
1993	0.75	5.15	0.32	0.27	1.37	0.002
1994	0.77	5.01	0.32	0.27	1.39	0.005
1995	0.77	5.15	0.31	0.26	1.44	-0.011
1996	0.78	5.31	0.32	0.27	1.46	0.006
1997	0.78	5.34	0.32	0.27	1.46	0.000
1998	0.75	5.24	0.31	0.26	1.43	-0.014
1999	0.75	5.36	0.31	0.26	1.43	-0.002
2000	0.72	5.27	0.29	0.25	1.44	-0.018
2001	0.72	5.43	0.30	0.25	1.39	0.010

2002	0.74	5.27	0.32	0.27	1.35	0.019
2003	0.74	5.27	0.32	0.27	1.36	0.000
2004	0.77	5.31	0.32	0.27	1.41	0.006
2005	0.79	5.39	0.32	0.27	1.47	-0.004
2006	0.79	5.30	0.32	0.27	1.48	0.000
2007	0.78	5.34	0.31	0.26	1.50	-0.005
2008	0.77	5.30	0.31	0.26	1.52	-0.012
2009	0.78	5.39	0.33	0.28	1.36	0.036
2010	0.82	5.57	0.34	0.29	1.41	0.010

2000-10: $\beta=0.467$ f=0.0025 $\alpha=0.038$ F=0.0045 p=0.010



$\dot{V} < \dot{C} \le \dot{Q} \le \dot{Y} < \dot{M}, p > F > f > 0, \dot{m} > 0$



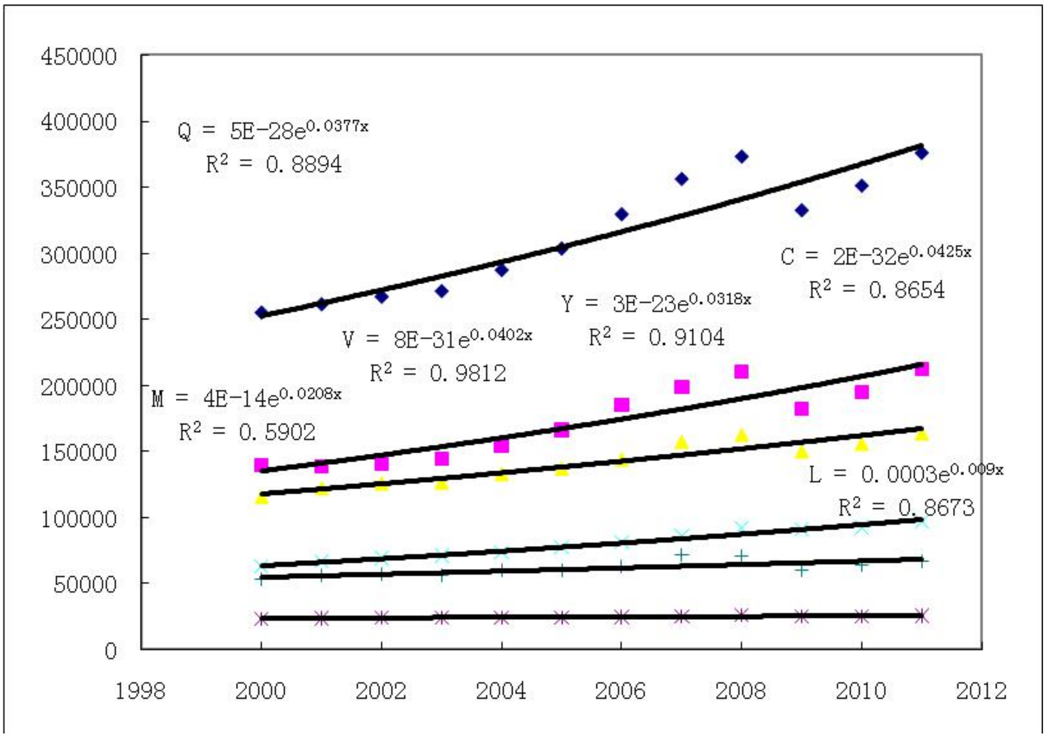
FRA	m'	p'	g	P/T	p'(n)	F
2000	0.720	0.261	1.755	10.532	0.239	
2001	0.714	0.259	1.757	11.344	0.238	-0.0034
2002	0.700	0.259	1.708	11.043	0.237	-0.0009
2003	0.701	0.263	1.670	11.110	0.241	0.0063
2004	0.710	0.264	1.693	10.440	0.241	0.0017
2005	0.710	0.260	1.726	10.020	0.237	-0.0051
2006	0.710	0.257	1.765	11.538	0.236	-0.0057
2007	0.730	0.261	1.791	11.447	0.240	0.0073
2008	0.723	0.258	1.796	11.272	0.237	-0.0048
2009	0.685	0.258	1.653	9.814	0.234	-0.0014
2010	0.686	0.254	1.703	11.711	0.234	-0.0066
2011	0.676	0.247	1.741	10.282	0.225	-0.0118
2000-11:	β=0.622	f=-0.00074	α=0.239	F=-0.0015	p=–0.0037	
2001-08:	β=0.579	f=0.00029	α=0.154	F=0.00056	p=0.0013	

$$\dot{V} \geq \dot{C} \geq \dot{Q} \geq \dot{Y} \geq \dot{M} > 0, \mathrm{p} \approx \mathrm{F} \approx \mathrm{f} \approx \dot{\mathrm{m}}' \approx 0$$

FIN	m'	p'	P/T	g	p'(n)	F
2000	0.846	0.261	1.782	2.237	0.167	
2001	0.835	0.270	1.750	2.091	0.172	0.0151
2002	0.833	0.272	1.772	2.063	0.174	0.0027
2003	0.793	0.260	1.668	2.053	0.162	-0.0208
2004	0.809	0.261	1.688	2.103	0.164	0.0020

2005	0.773	0.245	1.577	2.155	0.150	-0.0275
2006	0.783	0.238	1.663	2.296	0.148	-0.0139
2007	0.838	0.252	1.792	2.324	0.162	0.0268
2008	0.776	0.236	1.591	2.295	0.145	-0.0306
2009	0.655	0.218	1.197	2.011	0.119	-0.0354
2010	0.690	0.221	1.379	2.116	0.128	0.0065
2011	0.696	0.218	1.410	2.199	0.127	-0.0073

2000-11: β=0.734 f=-0.0049 α=0.408 F=-0.011 p=–0.025

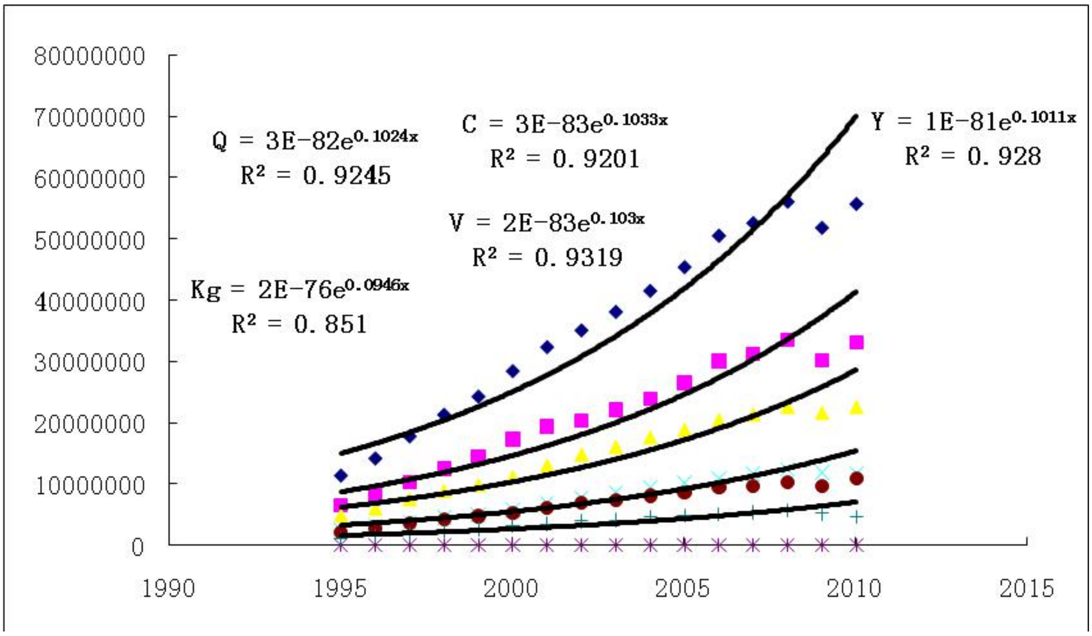


$$\dot{C} > \dot{V} > \dot{Q} > \dot{Y} > \dot{M}, p < F < f < 0, \dot{m} < 0$$

HUN	m'	g	p'(n)	P/T	p'	F
1995	0.831	2.491	0.122	1.045	0.238	
1996	0.884	2.603	0.126	1.059	0.245	0.014
1997	0.928	2.655	0.140	1.223	0.254	0.016
1998	0.935	2.734	0.141	1.289	0.250	-0.006
1999	0.966	2.912	0.137	1.246	0.247	-0.007
2000	0.897	2.953	0.124	1.197	0.227	-0.041
2001	0.905	2.835	0.137	1.374	0.236	0.019
2002	0.902	2.605	0.154	1.611	0.250	0.028
2003	0.851	2.549	0.147	1.578	0.240	-0.020
2004	0.856	2.524	0.155	1.758	0.243	0.006
2005	0.845	2.592	0.151	1.788	0.235	-0.015
2006	0.875	2.755	0.151	1.825	0.233	-0.005
2007	0.830	2.673	0.142	1.689	0.226	-0.014

2008	0.835	2.713	0.143	1.743	0.225	-0.002
2009	0.820	2.540	0.137	1.436	0.232	0.013
2010	0.934	2.835	0.149	1.582	0.244	0.021

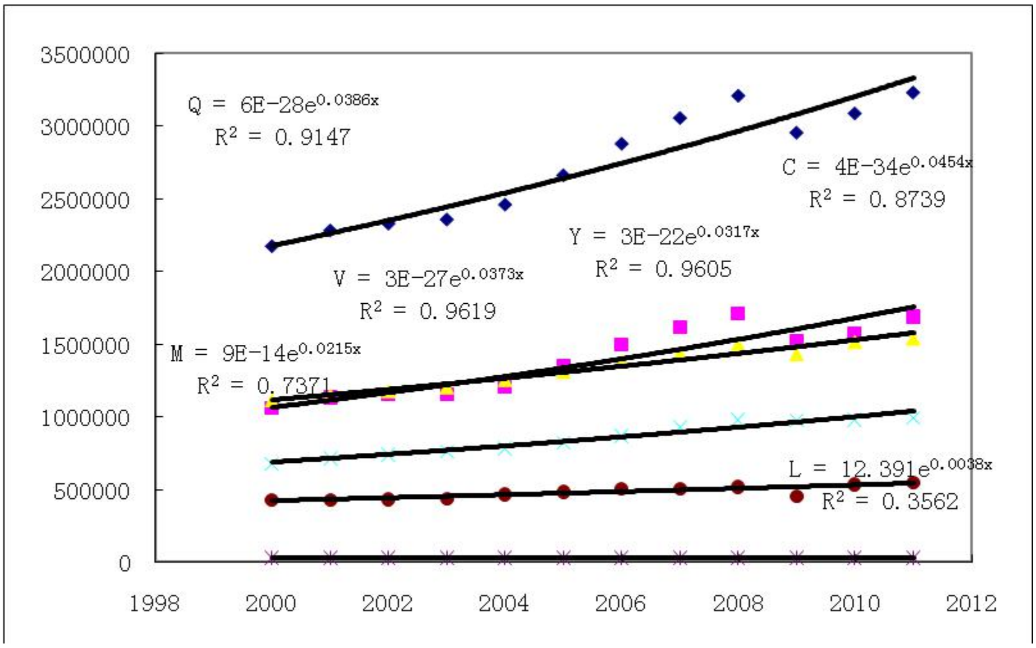
1995-2010: β=0.716 f=-0.00082 α=0.307 F=-0.0020 p=−0.0042



$\dot{C} \geq \dot{V} \geq \dot{Q} \geq \dot{Y}, \mathrm{p} \approx \mathrm{F} \approx \mathrm{f} \approx \dot{\mathrm{m}}' \approx 0$

DNK	P/T	p'	m'	g	p'(n)	F
2000	1.105	0.248	0.633	1.557	0.130	
2001	0.979	0.232	0.599	1.578	0.115	-0.024
2002	0.929	0.229	0.583	1.548	0.110	-0.006
2003	0.882	0.229	0.575	1.512	0.107	0.000
2004	0.926	0.235	0.595	1.532	0.113	0.009
2005	0.996	0.223	0.589	1.638	0.111	-0.019
2006	0.990	0.215	0.583	1.718	0.107	-0.015
2007	0.866	0.199	0.545	1.741	0.092	-0.028
2008	0.730	0.193	0.530	1.744	0.082	-0.010
2009	0.535	0.182	0.465	1.556	0.063	-0.022
2010	0.803	0.211	0.549	1.602	0.094	0.050
2011	0.848	0.205	0.554	1.704	0.094	-0.011

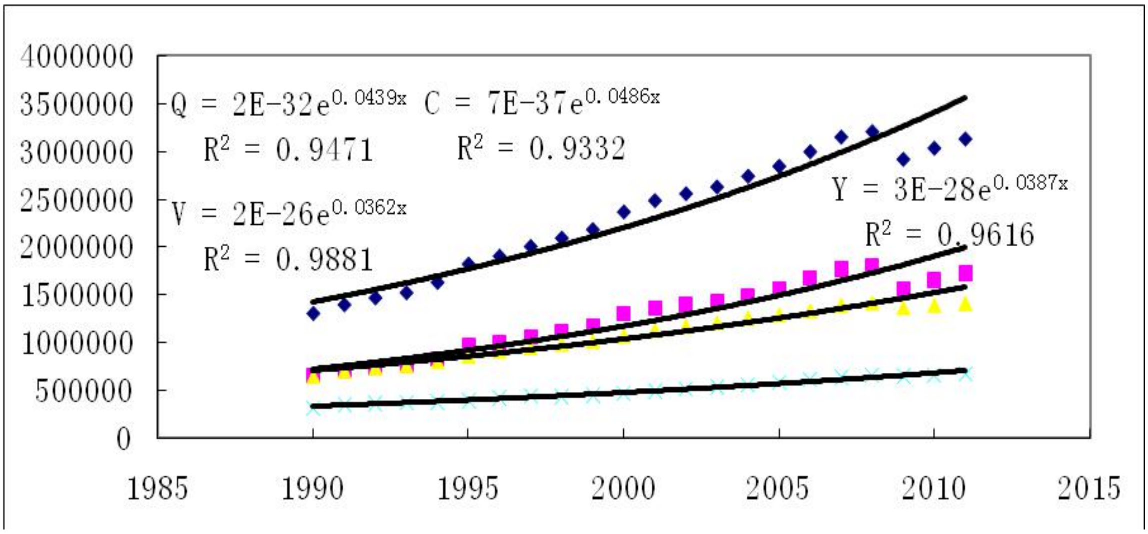
2000-11: β=0.578 f=-0.0031 α=0.135 F=-0.0062 p=−0.018



$\dot{C} > \dot{Q} > \dot{V} > \dot{Y} > \dot{M}, p < F < f < 0, \dot{m} < 0$

ITA	P/T	g	m'	p'	p'(n)	F
1990	2.381	2.086	1.031	0.334	0.235	
1991	2.331	2.010	1.013	0.337	0.236	0.004
1992	2.278	2.023	1.015	0.336	0.233	-0.001
1993	2.045	2.062	1.043	0.341	0.229	0.007
1994	2.122	2.188	1.111	0.349	0.237	0.011
1995	2.243	2.446	1.181	0.343	0.237	-0.010
1996	2.282	2.369	1.181	0.350	0.244	0.012
1997	2.174	2.409	1.159	0.340	0.233	-0.016
1998	1.806	2.545	1.249	0.352	0.227	0.019
1999	1.852	2.599	1.246	0.346	0.225	-0.010
2000	1.875	2.760	1.279	0.340	0.222	-0.010
2001	1.882	2.743	1.283	0.343	0.224	0.004
2002	1.801	2.684	1.265	0.343	0.221	0.001
2003	1.794	2.642	1.250	0.343	0.220	0.000
2004	1.786	2.665	1.258	0.343	0.220	0.000
2005	1.671	2.669	1.216	0.331	0.207	-0.019
2006	1.603	2.732	1.185	0.318	0.196	-0.023
2007	1.603	2.783	1.199	0.317	0.195	-0.001
2008	1.558	2.724	1.154	0.310	0.189	-0.012
2009	1.437	2.384	1.104	0.326	0.192	0.026
2010	1.429	2.504	1.119	0.319	0.188	-0.011
2011	1.369	2.576	1.117	0.312	0.181	-0.012

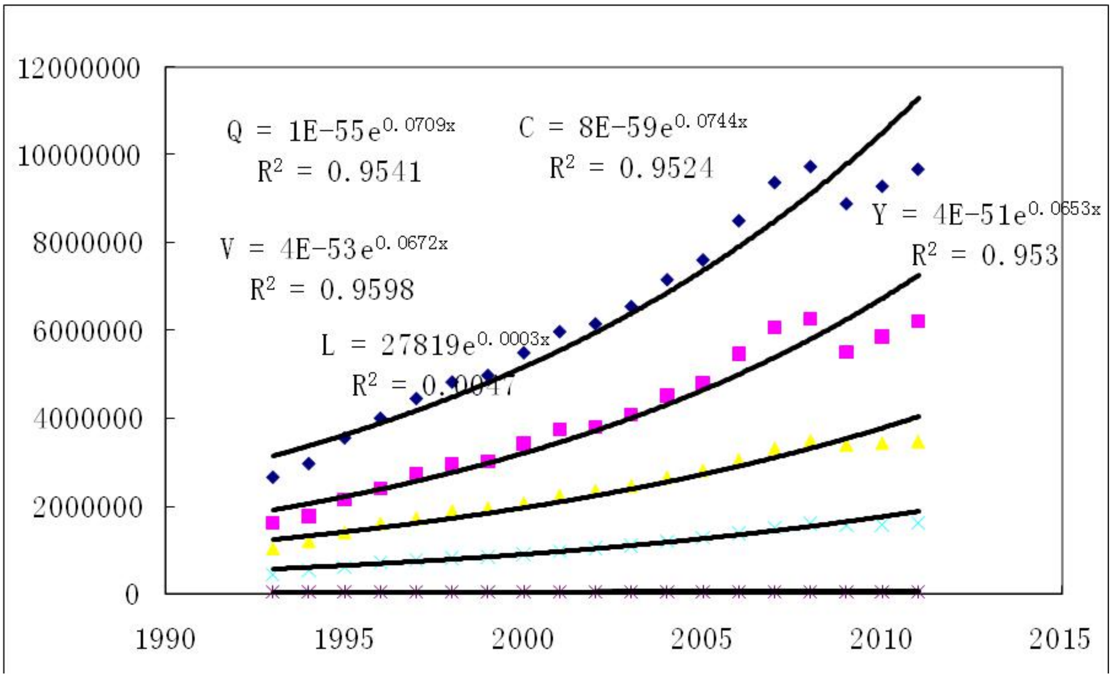
2000-11: $\beta=0.603$ f=-0.0032 $\alpha=0.125$ F=-0.0072 $p=-0.013$



$\dot{C} > \dot{Q} > \dot{Y} > \dot{V}, \mathrm{p} < \mathrm{F} < \mathrm{f} < 0$

CZE	p'	P/T	m'	g	p'(n)	F
1993	0.271	1.529	1.230	3.538	0.164	
1994	0.276	1.562	1.166	3.232	0.168	0.008
1995	0.278	1.591	1.246	3.481	0.171	0.004
1996	0.277	1.613	1.192	3.301	0.171	-0.002
1997	0.260	1.510	1.151	3.430	0.156	-0.035
1998	0.275	1.693	1.243	3.522	0.173	0.030
1999	0.279	1.624	1.261	3.516	0.173	0.009
2000	0.265	1.558	1.257	3.747	0.161	-0.030
2001	0.262	1.646	1.257	3.793	0.163	-0.005
2002	0.263	1.591	1.210	3.593	0.162	0.002
2003	0.256	1.601	1.194	3.659	0.158	-0.015
2004	0.250	1.646	1.183	3.729	0.156	-0.013
2005	0.247	1.737	1.162	3.700	0.157	-0.006
2006	0.239	1.859	1.179	3.925	0.156	-0.018
2007	0.236	1.923	1.184	4.015	0.155	-0.008
2008	0.236	1.880	1.153	3.874	0.154	0.001
2009	0.258	1.736	1.163	3.512	0.164	0.046
2010	0.247	1.690	1.158	3.690	0.155	-0.023
2011	0.234	1.628	1.129	3.825	0.145	-0.028

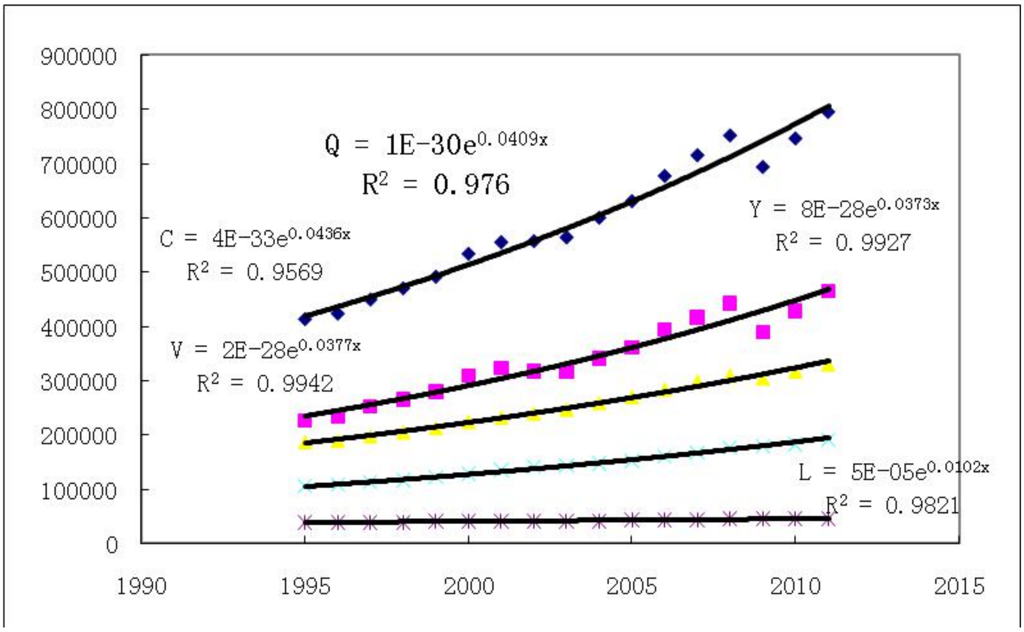
1993-2011: β=0.695 f=-0.0012 α=0.190 F=-0.0033 p=−0.0060



$\dot{C} > \dot{Q} > \dot{V} > \dot{Y}, \mathrm{p} < \mathrm{F} < \mathrm{f} < 0$

BEL	m'	p'	P/T	p'(n)	g	F
1995	0.761	0.243	1.497	0.146	2.126	
1996	0.753	0.238	1.382	0.138	2.162	-0.009
1997	0.758	0.234	1.356	0.135	2.234	-0.007
1998	0.764	0.234	1.369	0.135	2.271	-0.002
1999	0.734	0.224	1.292	0.126	2.273	-0.017
2000	0.750	0.220	1.341	0.126	2.408	-0.008
2001	0.715	0.212	1.268	0.118	2.381	-0.017
2002	0.705	0.217	1.274	0.121	2.253	0.010
2003	0.725	0.226	1.264	0.126	2.210	0.017
2004	0.762	0.230	1.326	0.131	2.314	0.008
2005	0.775	0.230	1.368	0.133	2.368	0.000
2006	0.780	0.225	1.331	0.129	2.465	-0.010
2007	0.781	0.225	1.392	0.131	2.478	-0.001
2008	0.747	0.214	1.286	0.120	2.496	-0.022
2009	0.706	0.222	1.135	0.118	2.179	0.015
2010	0.743	0.222	1.242	0.123	2.352	-0.001
2011	0.735	0.214	1.233	0.118	2.440	-0.016

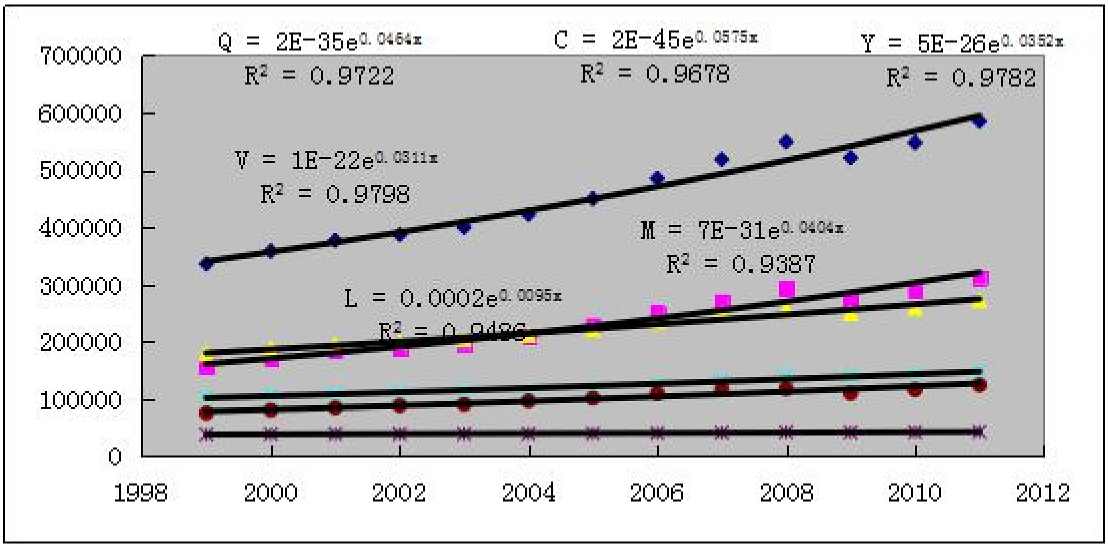
1995-2011: β=0.641 f=-0.00058 α=0.165 F=-0.0013 p=–0.0031



$\dot{C} > \dot{Q} > \dot{V} \geq \dot{Y}, \mathrm{p} < \mathrm{F} < \mathrm{f} < 0, \dot{\mathrm{m}}' \approx 0$

AUT	p'	P/T	m'	g	p'(n)	F
1999	0.289	1.296	0.729	1.521	0.163	
2000	0.290	1.328	0.754	1.600	0.166	0.0011
2001	0.290	1.295	0.779	1.684	0.164	0.0001
2002	0.297	1.310	0.801	1.697	0.168	0.0098
2003	0.293	1.325	0.801	1.737	0.167	-0.0065
2004	0.296	1.432	0.836	1.827	0.174	0.0047
2005	0.291	1.501	0.850	1.915	0.175	-0.0072
2006	0.291	1.587	0.875	2.006	0.178	-0.0010
2007	0.290	1.624	0.886	2.057	0.179	-0.0017
2008	0.273	1.526	0.850	2.118	0.165	-0.0284
2009	0.267	1.297	0.787	1.947	0.151	-0.0104
2010	0.270	1.362	0.816	2.025	0.156	0.0046
2011	0.270	1.426	0.839	2.109	0.159	-0.0001

1999-2011: β=0.726 f=-0.0039 α=0.453 F=-0.0081 p=–0.018



$\dot{C} > \dot{Q} > \dot{M} > \dot{Y} > \dot{V}, p < F < f < 0, \dot{m}' > 0$

Table. Partial OECD countries' Input-Output analysis

2000--2011	Y/C	C/Y	g 增长率
JPN（日本）	1.305	0.766	0.015
USA（美国）	1.233	0.811	0
NOR（挪威）	1.170	0.855	0.004
AUS（奥地利）	1.089	0.918	0.021
DNK（丹麦）	1.045	0.957	0.013
UK(英国）	1.031	0.97	0
DEU（德国）	1.026	0.975	0.013
FRA（法国）	0.993	1.007	0.001
SWE（瑞典）	0.895	1.117	0.003
FIN（芬兰）	0.873	1.146	0.01
ITA（意大利）	0.833	1.20	0
SVN(斯诺文尼亚）	0.816	1.226	0.006
BEL（比利时）	0.746	1.34	0.001
KOR（韩国）	0.746	1.341	0.029
HUN（匈牙利）	0.692	1.446	0
CZE（捷克）	0.601	1.663	0.005