Ultra Scientist Vol. 27(3)A, 229-244 (2015).

Merging of Semilattices and their Special Properties

M. REEHANA PARVEEN[1] and P. SEKAR[2]

(Acceptance Date 23rd November, 2015)

Abstract

In this paper for the first time authors define the new notion of merging of semilattices. The properties of merged semilattices is studied and several interesting results are proved in this direction.

Key words: semilattice, pseudo semilattice, pseudo atoms, pseudo buds, symmetric lattice, n-ary semilattice.

1 Introduction

In this paper for the first time merging of semilattices is defined, described and developed. However merging of lattices is carried out in[5]. This paper has three sections. The first section is introductory in nature and in the section two merging of semilattices is carried out. They have several interesting properties. In section three conclusions based on our study is given.

2 Merging of Semilattices and their Properties:

In this section merging of semilattices is defined for the first time and their properties is carried out.

Definition 2.1: Let $\{S_1, \cup\}$ *and* $\{S_2, \cup\}$ *be any two semilattices. The merging takes place only when the vertex sets of* S_1 *and* S_2 *is non empty.*

This is expressed by the following examples.

Example 2.1: Let $\{S_1, \cup\}$ and $\{S_2, \cup\}$ be two semilattice whose Hasse diagram is given in Figure 2.1.

It is clear both S_1 and S_2 has 1 and a_1 as common vertices.

The merging is done as given in Figure 2.2.

Clearly the merged figure is not a semilattice. It is defined as a pseudo semilattice. However the merged figure is a graph.

It is important to note that in general one can also merge a semilattice $(S, \cup)$ with $(S_1, \cap)$ provided they have some elements in common.

The resulting figure is always graph

but it can also be a lattice at times. To this end some examples are supplied.

Example 2.2. Let $\{S_1, \cup\}$ and $\{S_2, \cap\}$ be two semilattices whose Hasse diagram is given in Figure 2.3.

The merged pseudo semilattice is given in Figure 2.4.

Clearly after merging of these semilattices one gets a lattice evident from the diagram.

For two trees which are semilattices has imbibed the operation $\cup$ and $\cap$ and has becomes in this case a lattice which is also a graph. Thus the pseudo semilattice in this case is a lattice.

Example 2.3. Let $\{S_1, \cup\}$ and $\{S_2, \cup\}$ be two semilattice.

The Hasse diagram is given in Figure 2.5 and Figure 2.6. Let P be the merged pseudo semilattice.

This is only a graph not a semilattice as seen from Figure 2.7.

Example 2.4: Let $(S_1, \cup)$ and $\{S_2, \cap\}$ be two semilattices whose Hasse diagram is given in Figures 2.8 and 2.9.

Merging of $\{S_1, \cup\}$ and $\{S_2, \cap\}$ are given in Figure 2.10.

This pseudo semilattice is a graph not a semilattice for neither the operation $\cup$ nor $\cap$ can be given to the merged structure.

Let us consider the following example where $(S, \cup)$ is a chain lattice and $(S, \cap)$ is a binary semilattice are merged.

Example 2.5: Let $(S_1, \cup)$ and $\{S_2, \cap\}$ be two semilattices given by the following Figures 2.11 and 2.12.

The merged pseudo semilattice is given in Figure 2.13.

This is not a semilattice under '$\cup$' or '$\cap$' it is only a pseudo semilattice hence a graph.

In view of all these the following theorem is proved.

Theorem 2.1: Let $\{S_1, \cup\}$ and $\{S_1, \cup\}$ (or $(S_1, \cap)$) be two semilattices with common vertices.

The pseudo semilattice got by merging the common vertices in general is not a semilattice only a graph.

Proof: Follows from the fact that in general they may not be compatible under $\cup$ or $\cap$ or $\cup$ and $\cap$. However there may be exceptional cases. So pseudo semilattices are in general graphs.

Example 2.6: Let $(S, \cup)$ be a 5-ary semilattice and $(S_1, \cap)$ be a 3-ary semilattice whose Hasse diagram is given in Figures 2.14 and 2.15.

The merged pseudo semilattice is given in Figure 2.16.

Clearly this is not a semilattice only a graph which is well known as pseudo semilattice.

In view of all this one can develop define the notion of merging of special types of lattices.

In the first place the n-ary semilattice this paper defines the notion of layer / depth.

Further if there are two neutrosophic n-ary semilattices $\{P(S), \cup\}$ and $\{P(S), \cap\}$ with same number of pseudo atoms and pseudo buds, both $\{P(S), \cup\}$ and $\{P(S), \cap\}$ have the same length chains that is both are maximum t-lengthened neutrosophic semilattices with the pseudo atoms and pseudo buds are the same then there exists a perfect merging of $\{S, \cup\}$ and $\{S, \cap\}$ neutrosophic n-ary semilattices.[4]

Definition 2.2: Let $\{P(S), \cap\}$ and $\{P(S), \cup\}$ be two n-ary neutrosophic or usual semilattices with uniform t-length. Let $\{s_1, .., s_p\}$ be the pseudo buds of $\{P(S), \cap\}$ and $\{s_1, \ldots, s_p\}$ be the pseudo atoms of $\{P(S), \cup\}$.

Then merging of these two semilattices is a lattice which is defined as the special symmetric lattices merged in the vertices $\{s_1, s_2, \ldots, s_p\}$.

This is illustrated by the following examples.

Example 2.7: Let $\{S_1, \cap\}$ and $\{S_2, \cup\}$ be two binary semilattices of maximum 3-lengthed given by Figure 2.17 and Figure 2.18.

Clearly $\{c_1, c_2, \ldots, c_8\}$ be the pseudo buds of $\{S_1, \cap\}$

Let $\{c_1, c_2, c_3, \ldots, c_8\}$ be the pseudo atoms of $\{S_2, \cup\}$. The merged pseudo lattice has the Figure 2.19.

Clearly

$x_1 \cap x_2 = a_1$, $y_1 \cap y_2 = \{0\}$,
$x_3 \cap x_4 = a_2$, $x_2 \cap x_3 = \{0\}$;
$a_1 \cup a_2 = \{1\}$, $b_1 \cup b_2 = y_1$,
$b_2 \cup b_3 = \{1\}$;

and so on.

Thus the merged pseudo semilattice is symmetric about $\{c_1, c_2, \ldots, c_8\}$. Clearly the merged pseudo semilattice is defined as the symmetric lattice under the inherited operations $\cup$ and $\cap$.

Example 2.8: Let $\{S_1, \cup\}$ and $\{S_2, \cap\}$ be two 3-ary semilattices of uniform 2-lengthed given by the following Figures 2.20 and 2.21.

$\{S_1, \cup\}$ be 3-ary semilattice of uniform length 2 with $\{b_1, b_2, b_3, \ldots, b_9\}$ as the pseudo atoms of $\{S_1, \cup\}$.

Let $\{S_2, \cap\}$ be the 3-ary semilattice with $\{b_1, b_2, \ldots, b_9\}$ as the pseudo buds given by Figure 2.21.

The merged pseudo semilattices is given in Figure 2.22.

This is a symmetric lattice the symmetry is about $\{b_1, b_2, b_3, \ldots, b_9\}$

$a_1 \cap a_2 = \{0\}$; $x_1 \cup x_2 = \{1\}$ and so on.

Thus the examples of merging of n-ary semilattices with $\cup$ and $\cap$ of uniform t-length is a symmetric lattice.

Next the notion of merging of m-ary (S_1, $\cup$} and n-ary {S_2, $\cap$) semilattice but which have the same number of pseudo atoms and pseudo buds respectively which are common described first by examples.

Example 2.9: Let {S_1, $\cup$} and {S_2, $\cap$} be a 8-ary and a 2-ary semilattice of uniform length 1 and 3, respectively.

Let them have the pseudo atom and pseudo buds to be common. They are given by the Fig--ures 2.23 and 2.24.

The merged pseudo semilattice is given in Figure 2.25.

Clearly $c_1 \cup c_2 = \{1\}$; $b_1 \cup b_2 = \{1\}$, $a_2 \cap a_3 = c_1$ and so on.

Clearly the merged pseudo semilattice is a lattice of length four.

However the merged pseudo lattice is not symmetric about $\{a_1, a_2, \ldots, a_8\}$. In view of all these the following theorem are proved.

Theorem 2.2: Let {S_1, $\cup$} and {S_2, $\cap$} be two n-ary semilattices (neutrosophic) or otherwise of uniform length t. Let {x_1, …, x_p} be the pseudo atoms and pseudo buds respectively of {S_1, $\cup$} and {S_2, $\cap$}.
i) The pseudo merged semilattice is a symmetric lattice, symmetric about {x_1, x_2, …, x_p}
ii) Length of the symmetric lattice is 2t.

Proof: Under the conditions of the theorem clearly as both are n-ary semilattices and of same uniform length after merging the resultant pseudo semilattice becomes a symmetric lattice by definition 5.2. Hence (i) is true.

Now for proof of (ii) it clear that as length of both the semilattice is the same and they are uniform length the length of the lattice adds upto 2t as merging of the pseudo buds and pseudo atoms takes place. Hence proof of (ii).

Theorem 2.3: Let {S_1, $\cup$} be the n-ary semilattice which is t-length (uniformly maximum t-length) and {S_2, $\cap$} be a m-ary semilattice which is r-length (uniformly maximum r-length) with {d_1, d_2, …, d_p} as the pseudo atom {S_1, $\cup$} and pseudo buds of {S_2, $\cup$}.
i) The merged pseudo semilattice is a lattice which is not symmetric.
ii) Length of the lattice is t + r.

Proof: The merging of the semilattices have the common coordinates {d_1, d_2, .., d_p} one gets a pseudo semilattice which is a lattice. Clearly since (S_1, $\cup$} and {S_2, $\cap$} are n-ary and m-ary semilattices ($m \neq n$) they cannot yield a symmetric lattice. Further as they are different lengths one sees they cannot symmetric. For some lattice to be symmetric both the semilattice must be of same length. Hence they are not symmetric i) true.

Proof of (ii) The merging takes places by merging the pseudo buds with the pseudo atoms so the length of the lattice cannot be reduced or increased so the length of the lattice is p + r. Hence the proof.

It is important to record that this sort of merging is very special and in general there

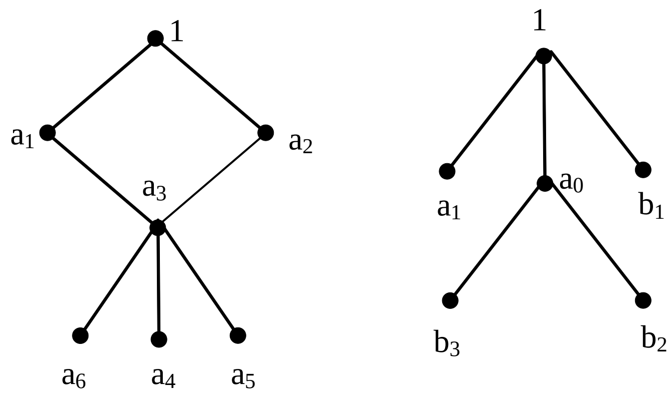


Figure 2.1

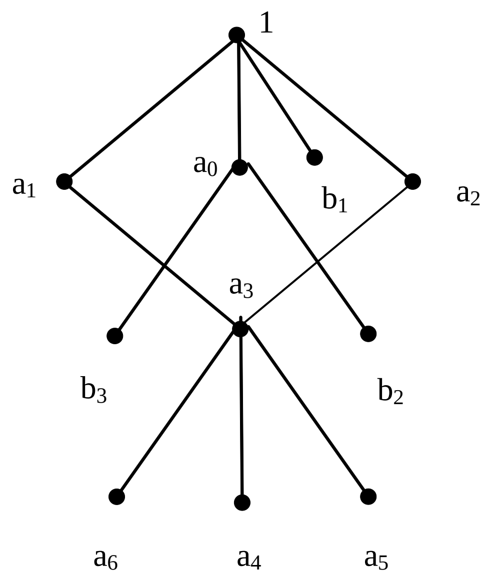


Figure 2.2

$\{S_1, \cup\}$ $(S_2, \cap\}$

Figure 2.3

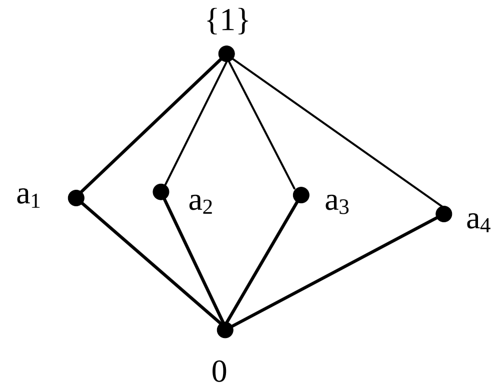


Figure 2.4

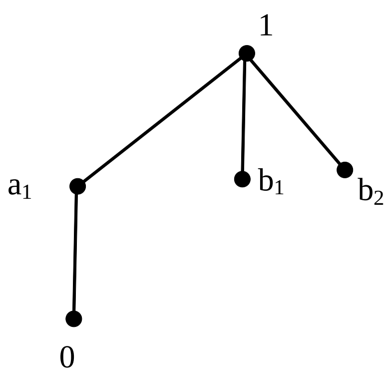


Figure 2.5

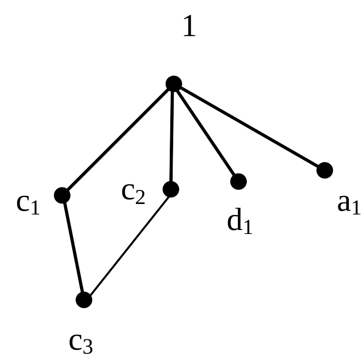


Figure 2.6

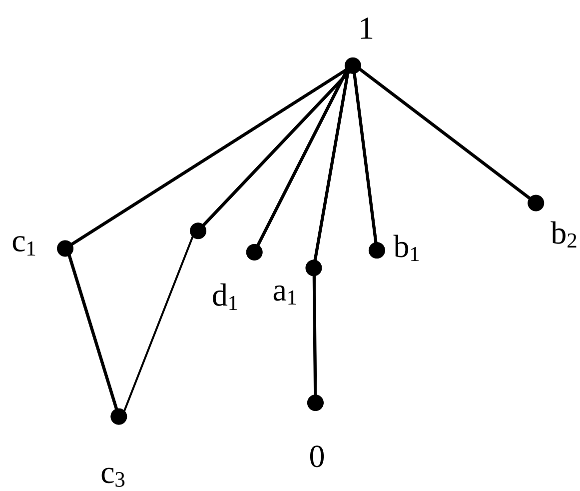


Figure 2.7

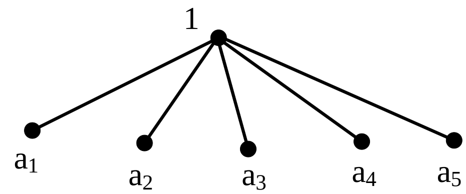


Figure 2.8

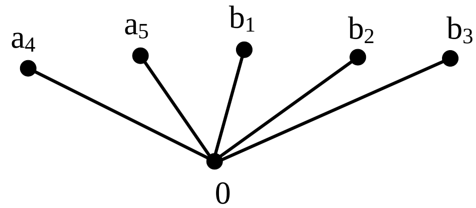


Figure 2.9

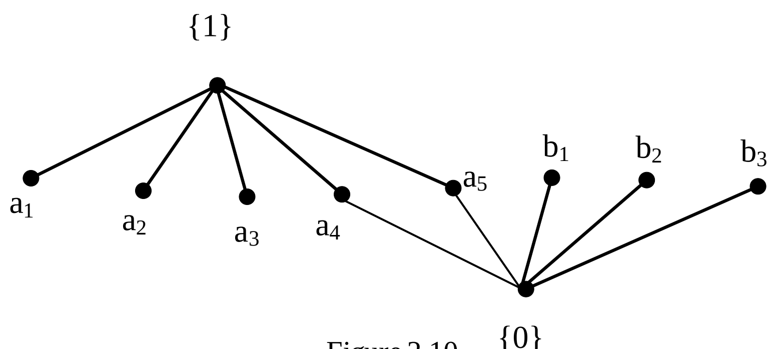


Figure 2.10

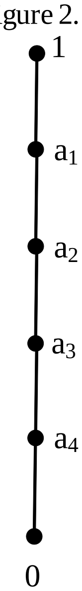


Figure 2.11

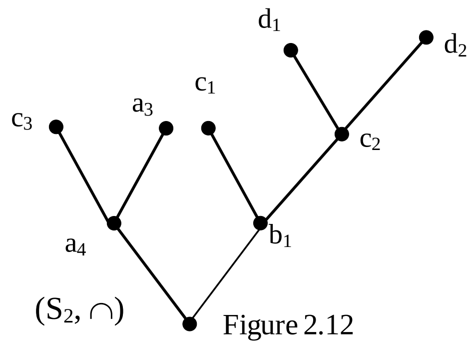


Figure 2.12

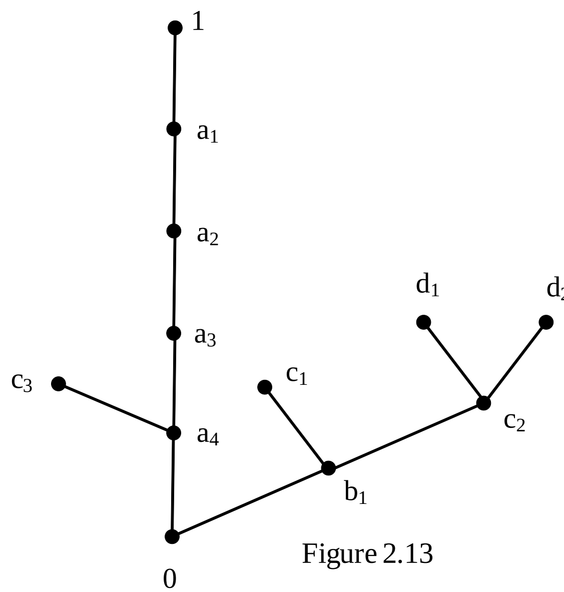


Figure 2.13

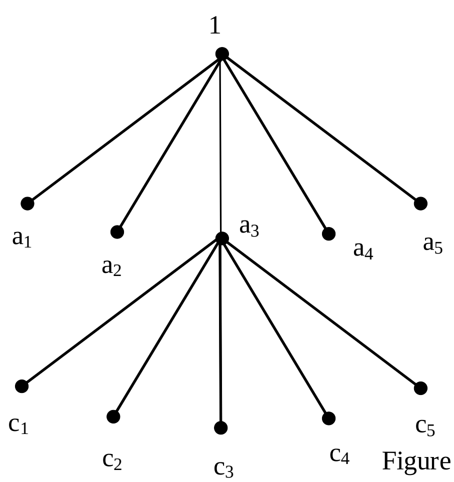
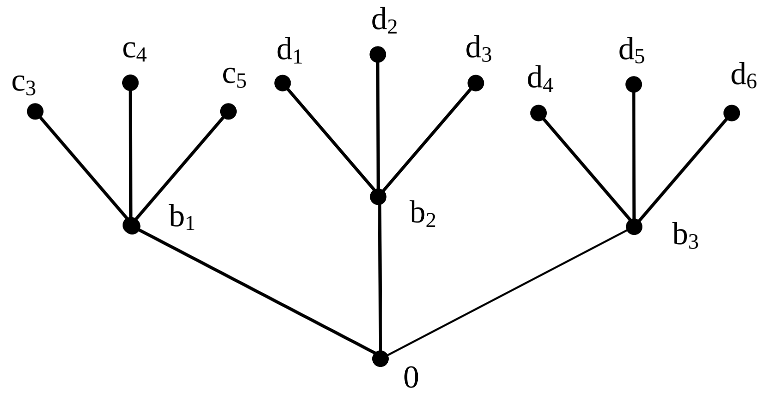


Figure 2.14. $\{S_2, \cup\}$-5-ary semilattice



$\{S_1, \cap\}$ the 3-array semilattice: Figure 2.15

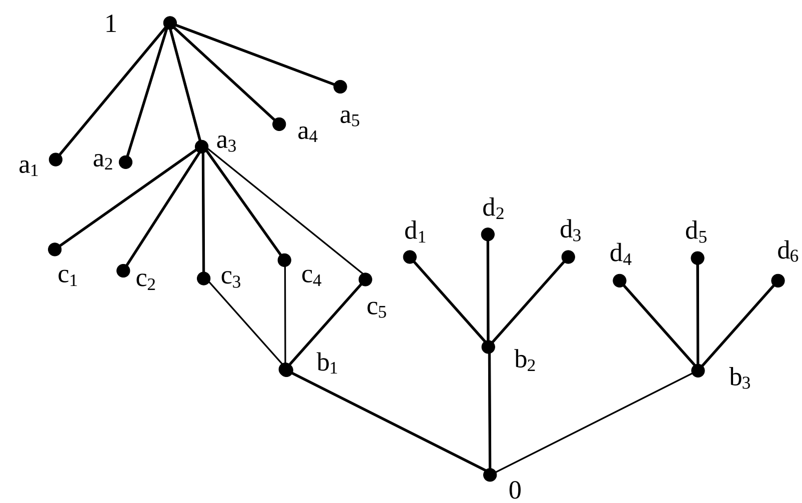


Figure 2.16

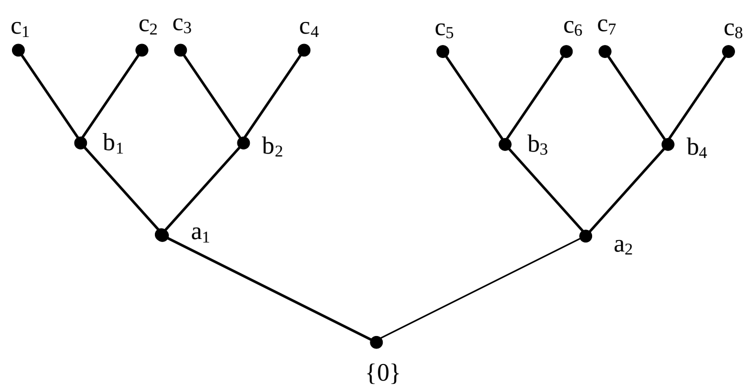


Figure 2.17: $\{S, \cap\}$

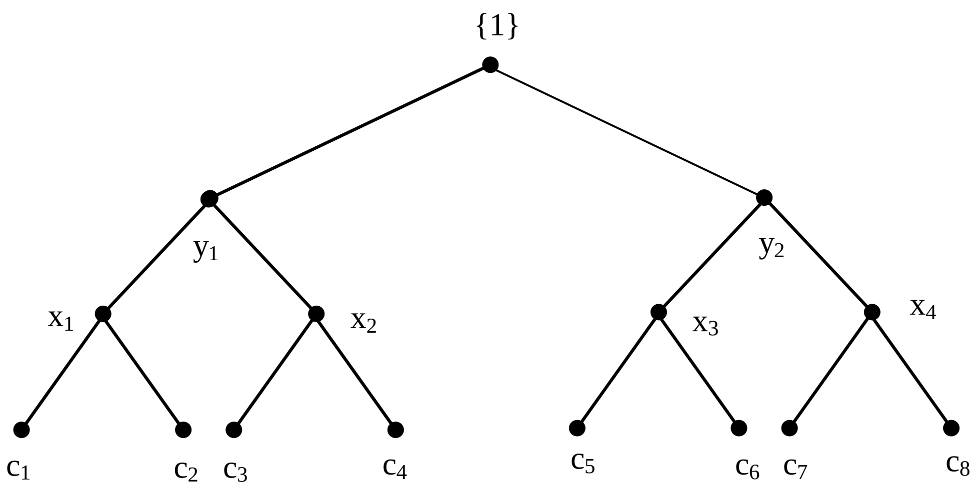


Figure 2.18 $\{S_2, \cup\}$

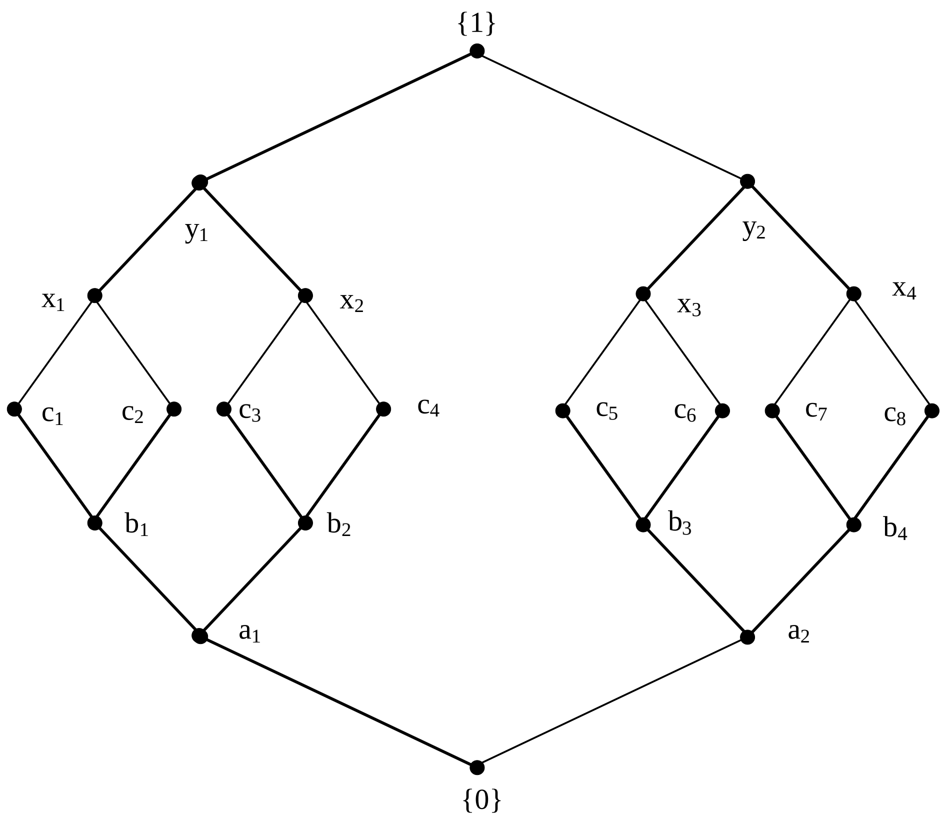


Figure 2.19

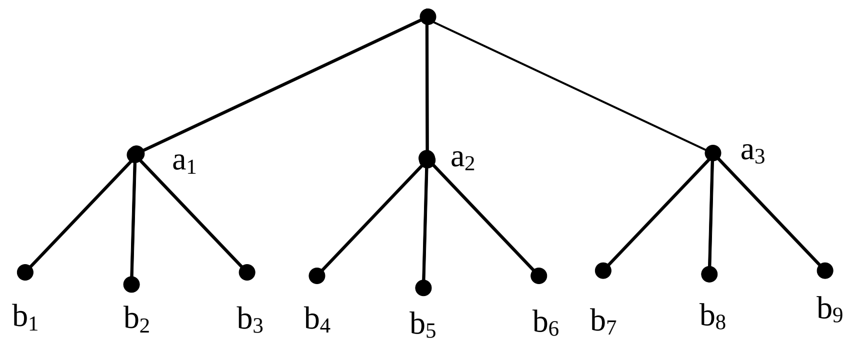


Figure 2.20 $\{S_1, \cup\}$

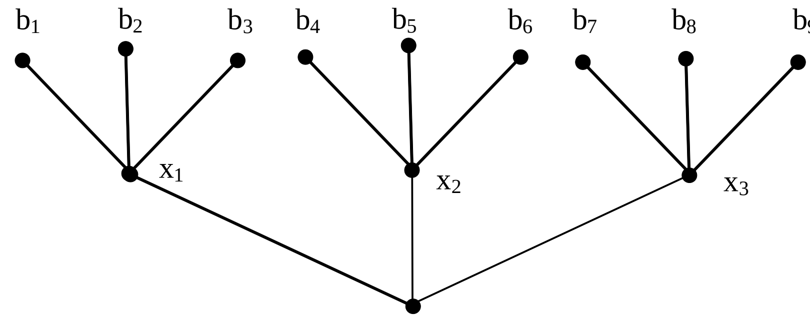


Figure 2.21: $\{S_2, \cap\}$

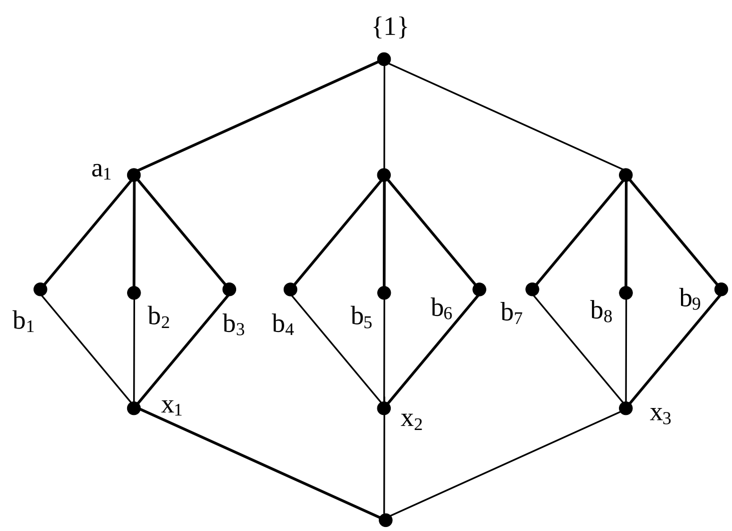


Figure 2.22

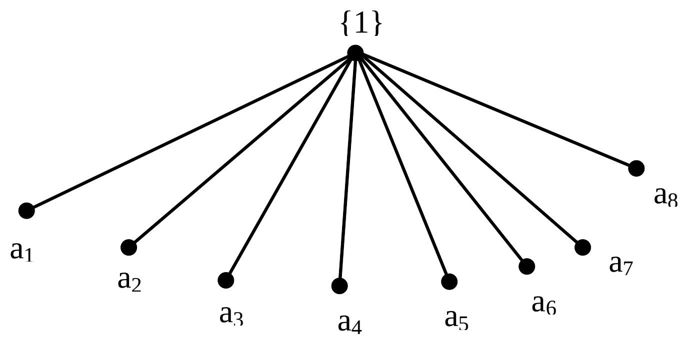


Figure 2.23 $\{S_1, \cup\}$

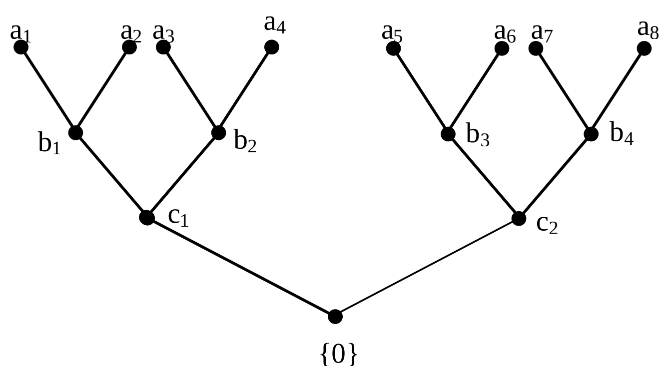


Figure 2.24 $\{S_2, \cap\}$

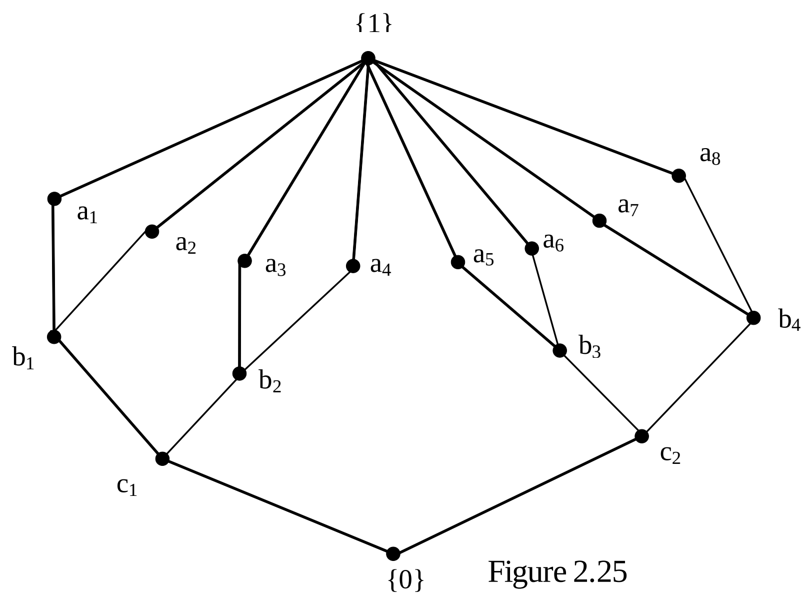


Figure 2.25

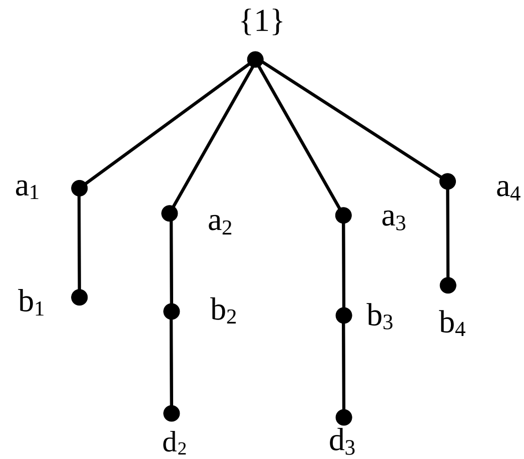


Figure 2.26

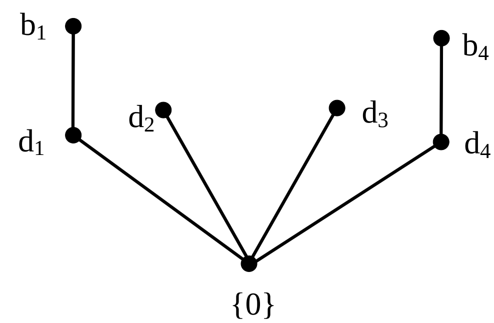


Figure 2.27

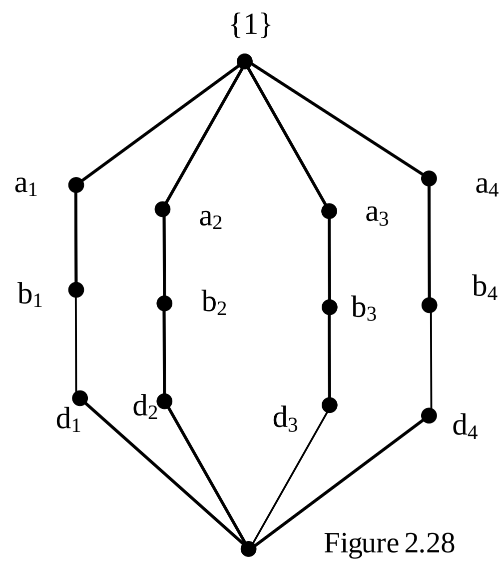


Figure 2.28

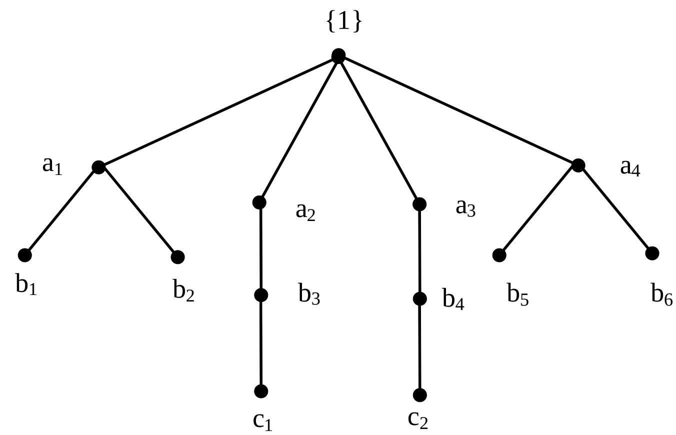


Figure 2.29 $\{S_1, \cup\}$

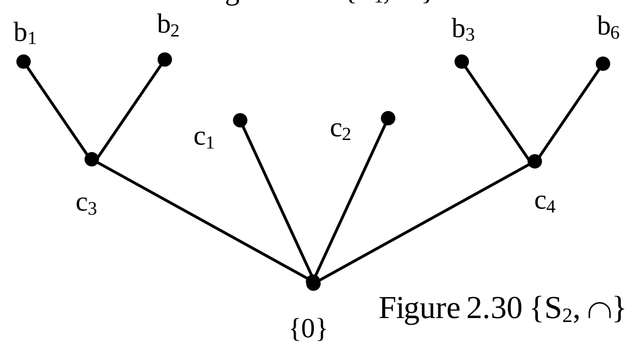


Figure 2.30 $\{S_2, \cap\}$

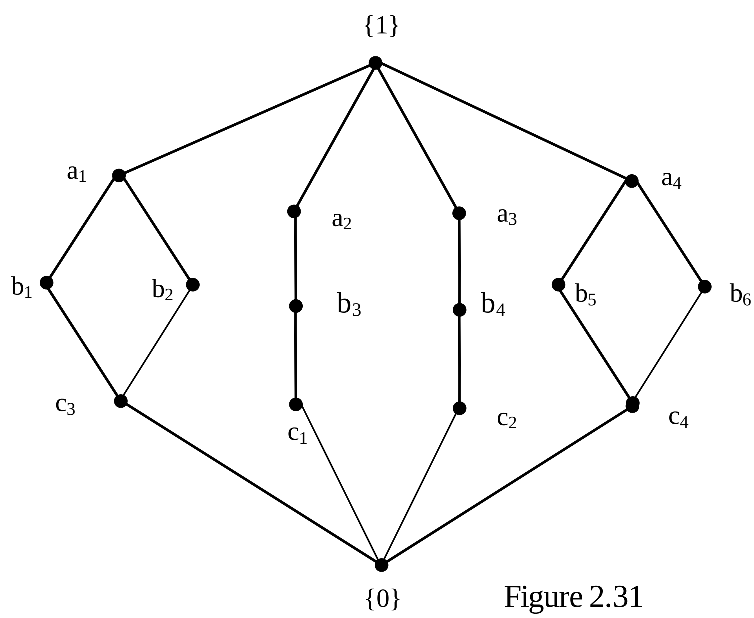


Figure 2.31

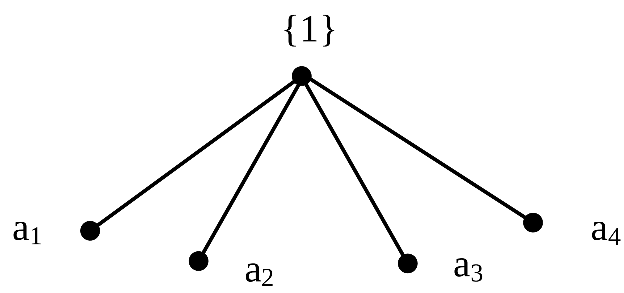


Figure 2.32

$\{S_1, \cup\}$ and $\{S_2, \cap\}$

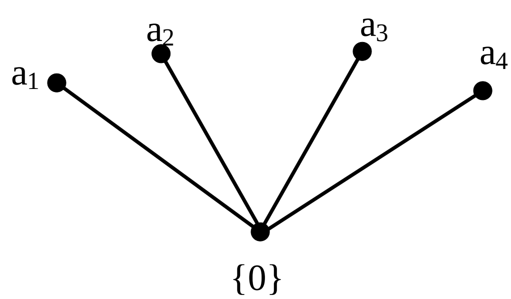


Figure 2.33

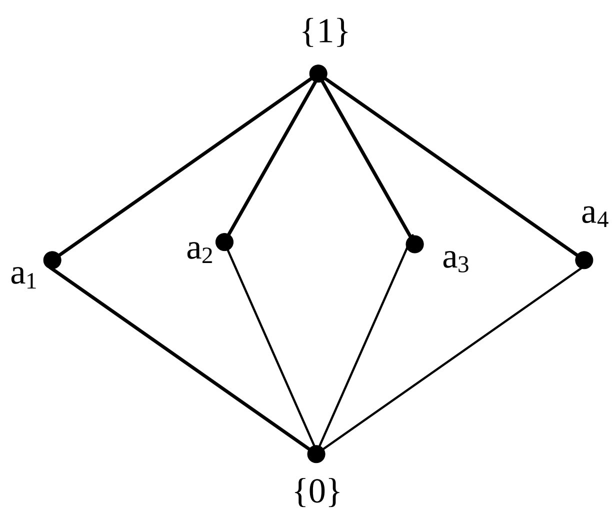


Figure 2.34

are other types of merging also, which will be represented by the following example.

Example 2.10: Let $\{S_1, \cup\}$ and $\{S_2, \cap\}$ be two 4-ary semilattice which are not uniform t-length. They are given by the Figure 2.26 and Figure 2.27. The merging takes places by merging the common vertices $\{d_2, d_3\}$ and b_1 and b_4.

This is described by the Figure 2.28.

$$a_i \cap a_j = \{0\},$$
$$b_j \cap b_i = \{0\}. \; 1 \le i, j \le 4$$
$$c_1 \cup c_2 = \{1\} \text{ and}$$
$$d_i \cup d_j = \{1\}; \; 1 \le i, j \le 4,$$

This is the way operations $\cup$ and $\cap$ are performed on the merged pseudo semilattice which in this case is a lattice.

So any m-ary semilattice $\{S_1, \cup\}$ and $\{S_2, \cap\}$ can after merging give a lattice even if they are not uniform length.

Example 2.11: Let $\{S_1, \cup\}$ and $\{S_2, \cap\}$ be two semilattices given by Figure 2.29 and Figure 2.30. Merging of the semilattices is given as Figure 2.31.

Clearly the merged semilattice is a lattice in this case also.

Finding conditions for the merging of semilattices $\{S_1, \cup\}$ and $\{S_2, \cap\}$ to be lattice is left for future study.

Example 2.12: Let $\{S_1, \cup\}$ and $\{S_2, \cap\}$ be two semilattices given by Figure 2.32 and Figure 3.33.

The merged semilattice is given in Figure 2.34.

This is a modular lattice. However the merged pseudo semilattice is only a graph when both the semilattice are taken under the operation $\cup$ (or $\cap$ intersection, or used in the mutually exclusive sense only).

When merging of semilattices $\{S_1, \cup\}$ an $\{S_2, \cap\}$ takes place and if there are common vertices the resultant merged pseudo semilattice can be a graph or a lattice but never a semilattice as two operations and $\cap$ are to be defined on the merged structure.

4. Conclusions

In this paper for the first time the new concept of merging of semilattice are defined and developed. These under special conditions can give symmetric lattices. In general the merged semilattices are only graphs. Several interesting properties about merging of lattices and semilattices are derived in this paper.

References

1. Birkhoff G., Lattice Theory, 2nd Edition, Amer, Math. Soc, (1948).
2. Gratzer G., General Lattice Theory, Birkhauser VerlageBasel and Stullgart, (1978).
3. Cormen, T.H., *et al.*, Introduction to Algorithms, McGraw Hills, (2001).
4. Reehana Praveen, M., and Sekar, P., New Class of n-ary semilattices and their properties (to appear)
5. Vasantha Kandasamy, W.B., Smarandache, F. and Ilanthenral, K., Pseudo lattice graphs and their applications to Fuzzy and Neutrosophic models, EuropaNova (2014).