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ORIGINAL PAPER

On Neutrosophic Normal Soft Groups

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Abstract The concepts of neutrosophic normal soft group, neutrosophic soft cosets, neutrosophic soft homomorphism are introduced and illustrated by suitable examples in this paper. Several related properties and structural characteristics are investigated. Some of their basic theorems are also established.

Keywords Neutrosophic soft set (NSS) · Neutrosophic normal soft group · Neutrosophic soft cosets · Neutrosophic soft homomorphism

Introduction

Classical mathematical tools may not be appropriate in dealing different uncertainties appeared in several real life fields like economics, sociology, medical science, environment etc. While probability theory, theory of fuzzy set [1], intuitionistic fuzzy set [2] and other mathematical tools are well known and often useful approaches to describe uncertainty, each of these theories has it's inherent difficulties as pointed out by Molodtsov [3]. In 1999, Molodtsov [3] introduced a novel concept of soft set theory which is free from the parametrization inadequacy syndrome of different theories dealing with uncertainty. This makes the theory very convenient and easy to apply in practice. The classical group theory was extended over fuzzy set, intuitionistic fuzzy set and soft set by Rosenfeld [4], Mukherjee and Bhattacharya [5], Sharma [6], Aktas et al. [7] and many others. In accordance of this, several authors applied the theory of fuzzy soft sets, intuitionistic fuzzy soft sets to different algebraic structures, for instance, Maji et al. [8–10], Dinda and Samanta [11], Ghosh et al.

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[12], Mondal and Roy [13], Chetia and Das [14], Basu et al. [15], Augunoglu and Aygun [16], Yaqoob et al. [17], Varol et al. [18], Zhang [19].

As a generalisation of fuzzy set and intuitionistic fuzzy set theory, the neutrosophic set theory makes description of the objective world more realistic, practical and very promising in nature. The concept of neutrosophic set (NS) was first introduced by Smarandache [20]. Later, Maji [21] has combined this concept with the soft set theory. Consequently, the notion of Neutrosophic soft set (NSS) theory has been innovated. Using this concept, several mathematicians have produced their research works in different mathematical structures for instance Sahin et al. [22], Broumi [23], Bera and Mahapatra [24], Maji [25], Broumi et al. [26–33]. But, this concept has been redefined by Deli and Broumi [34]. Accordingly, Bera and Mahapatra [35–38] have studied some algebraic structures upon this concept.

The motivation of the present paper is to extend the notion of neutrosophic soft groups [35] along with investigation of some related properties and theorems. Section "Preliminaries" gives some preliminary useful definitions related to it. In section "Neutrosophic Normal Soft Groups", the notion of neutrosophic normal soft groups (NNSG) is introduced. Section "Neutrosophic Soft Cosets" deals with the neutrosophic soft cosets. Finally in section "Neutrosophic Soft Homomorphism", there has been studied about neutrosophic soft homomorphism.

Preliminaries

We recall some basic definitions related to fuzzy set, soft set, neutrosophic soft set, neutrosophic soft groups for the sake of completeness.

Definition 2.1 [39] **1.** A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative.
(ii) $*$ is continuous.
(iii) $a * 1 = 1 * a = a, \ \forall a \in [0, 1]$.
(iv) $a * b \leq c * d$ if $a \leq c, \ b \leq d$ with $a, b, c, d \in [0, 1]$.

A few examples of continuous t-norm are $a * b = ab, \ a * b = min\{a, b\}, \ a * b = max\{a + b - 1, 0\}$.

2. A binary operation $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm (s-norm) if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative and associative.
(ii) $\diamond$ is continuous.
(iii) $a \diamond 0 = 0 \diamond a = a, \ \forall a \in [0, 1]$.
(iv) $a \diamond b \leq c \diamond d$ if $a \leq c, \ b \leq d$ with $a, b, c, d \in [0, 1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab, \ a \diamond b = max\{a, b\}, \ a \diamond b = min\{a + b, 1\}$.

If for all $a \in [0, 1], \ a * a = a$ and $a \diamond a = a$, then $*$ is called an idempotent t-norm and $\diamond$ is called an idempotent s-norm. If $*$ and $\diamond$ are continuous t-norm and s-norm, respectively, then for $a, b, c \in [0, 1]$,

(i) $(a * b) \wedge (a * c) = a * (b \wedge c), \quad (a * b) \vee (a * c) = a * (b \vee c)$
(ii) $(a \diamond b) \wedge (a \diamond c) = a \diamond (b \wedge c), \quad (a \diamond b) \vee (a \diamond c) = a \diamond (b \vee c)$

Definition 2.2 [20] A neutrosophic set (NS) on the universe of discourse U is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\},$$

where $T, I, F : U \to]^{-}0, 1^{+}[$ and $^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

From philosophical point of view, the neutrosophic set (NS) takes the value from real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. But in real life application in scientific and engineering problems, it is difficult to use NS with value from real standard or nonstandard subset of $]^{-}0, 1^{+}[$. Hence we consider the NS which takes the value from the subset of [0, 1].

Definition 2.3 [3] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

Definition 2.4 [21] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \to P(U)$ is a mapping.

This concept has been modified by Deli and Broumi [34] as given below.

Definition 2.5 [34] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \to P(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(U)$ and therefore it can be written as a set of ordered pairs,

$$N = \{(e, \{\langle x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x)\rangle : x \in U\}) : e \in E\}$$

where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

Example 2.5.1 Let U = $\{h_1, h_2, h_3\}$ be a set of houses and E = $\{e_1$(beautiful), e_2(wooden), e_3(costly)$\}$ be a set of parameters with respect to which the nature of houses are described. Let

$$f_N(e_1) = \{\langle h_1, (0.5, 0.6, 0.3)\rangle, \langle h_2, (0.4, 0.7, 0.6)\rangle, \langle h_3, (0.6, 0.2, 0.3)\rangle\};$$
$$f_N(e_2) = \{\langle h_1, (0.6, 0.3, 0.5)\rangle, \langle h_2, (0.7, 0.4, 0.3)\rangle, \langle h_3, (0.8, 0.1, 0.2)\rangle\};$$
$$f_N(e_3) = \{\langle h_1, (0.7, 0.4, 0.3)\rangle, \langle h_2, (0.6, 0.7, 0.2)\rangle, \langle h_3, (0.7, 0.2, 0.5)\rangle\};$$

Then $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)]\}$ is an NSS over (U, E). The tabular representation of the NSS N is as (Table 1):

Table 1 Tabular form of NSS N

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)

Definition 2.5.2 [34] The complement of a neutrosophic soft set N is denoted by N^c and is defined by:

$$N^c = \left\{\left(e, \left\{\langle x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x)\rangle : x \in U\right\}\right) : e \in E\right\}$$

Definition 2.5.3 [34] Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),\ F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x);\ \forall e \in E \text{ and } x \in U.$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

Definition 2.5.4 [34]

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined by:

$$N_3 = \left\{\left(e, \left\{\langle x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x)\rangle : x \in U\right\}\right) : e \in E\right\}$$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x),\ I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x),$

$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their intersection is denoted by $N_1 \cap N_2 = N_3$ and is defined by:

$$N_3 = \left\{\left(e, \left\{\langle x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x)\rangle : x \in U\right\}\right) : e \in E\right\}$$

where $T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x),\ I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x),$

$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x);$$

Definition 2.6 [35] **1.** Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'AND' operation is denoted by $N_1 \wedge N_2 = N_3$ and is defined by:

$$N_3 = \left\{\left[(a, b), \left\{\langle x, T_{f_{N_3}(a,b)}(x), I_{f_{N_3}(a,b)}(x), F_{f_{N_3}(a,b)}(x)\rangle : x \in U\right\}\right] : (a, b) \in E \times E\right\}$$

where $T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x),\ I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x),$

$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'OR' operation is denoted by $N_1 \vee N_2 = N_3$ and is defined by:

$$N_3 = \{[(a, b), \{\langle x, T_{f_{N_3}(a,b)}(x), I_{f_{N_3}(a,b)}(x), F_{f_{N_3}(a,b)}(x)\rangle : x \in U\}] : (a, b) \in E \times E\}$$

where $T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) \diamond T_{f_{N_2}(b)}(x),\ I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) * I_{f_{N_2}(b)}(x),$

$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) * F_{f_{N_2}(b)}(x);$$

Definition 2.7 [7] Let (F, A) be a soft set over the group G. Then (F, A) is called a soft group over G if $F(a)$ is a subgroup of G, $\forall a \in A$.

Table 2 Tabular form of neutrosophic soft group N

	$f_N(\alpha)$	$f_N(\beta)$	$f_N(\gamma)$	$f_N(\delta)$
e	(0.65, 0.34, 0.14)	(0.88, 0.12, 0.72)	(0.72, 0.21, 0.16)	(0.69, 0.31, 0.32)
a	(0.71, 0.22, 0.78)	(0.71, 0.19, 0.44)	(0.84, 0.16, 0.25)	(0.62, 0.32, 0.42)
b	(0.75, 0.25, 0.52)	(0.83, 0.11, 0.28)	(0.69, 0.31, 0.39)	(0.58, 0.41, 0.66)
c	(0.67, 0.32, 0.29)	(0.75, 0.21, 0.19)	(0.79, 0.19, 0.41)	(0.71, 0.27, 0.53)

Definition 2.8 [35] A neutrosophic set $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in G\}$ over a group (G, o) is called a neutrosophic subgroup of (G, o) if

$$(i)\quad \begin{cases} T_A(xoy) \geq T_A(x) * T_A(y) \\ I_A(xoy) \leq I_A(x) \diamond I_A(y) \\ F_A(xoy) \leq F_A(x) \diamond F_A(y); \text{ for } x, y \in G. \end{cases}$$

$$(ii)\quad \begin{cases} T_A(x^{-1}) \geq T_A(x) \\ I_A(x^{-1}) \leq I_A(x) \\ F_A(x^{-1}) \leq F_A(x); \text{ for } x \in G. \end{cases}$$

An NSS N over a group (G, o) is called a neutrosophic soft group if $f_N(e)$ is a neutrosophic subgroup of (G, o) for each $e \in E$.

Example 2.8.1 Let us consider the Klein's -4 group $V = \{e, a, b, c\}$ and $\mathrm{E} = \{\alpha, \beta, \gamma, \delta\}$ be the set of parameters. We define $f_N(\alpha), f_N(\beta), f_N(\gamma), f_N(\delta)$ as given by Table 2.

The t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = max\{a + b - 1, 0\}$, $a \diamond b = min\{a + b, 1\}$; Then, N forms a neutrosophic soft group over (V, E).

Proposition 2.8.2 [35] *An NSS N over the group (G, o) is called a neutrosophic soft group iff followings hold on the assumption that truth membership (T), indeterministic membership (I) and falsity membership (F) functions of an NSS obey the idempotent t-norm and idempotent s-norm disciplines.*

$$\begin{aligned} T_{f_N(e)}(xoy^{-1}) &\geq T_{f_N(e)}(x) * T_{f_N(e)}(y), \\ I_{f_N(e)}(xoy^{-1}) &\leq I_{f_N(e)}(x) \diamond I_{f_N(e)}(y), \\ F_{f_N(e)}(xoy^{-1}) &\leq F_{f_N(e)}(x) \diamond F_{f_N(e)}(y)); \ \forall x, y \in G, \forall e \in E. \end{aligned}$$

Proposition 2.8.3 [35] *Let N be a neutrosophic soft group over the group G. Then for each $x \in G$, followings hold.*

(i) $T_{f_N(e)}(x^{-1}) = T_{f_N(e)}(x),\ I_{f_N(e)}(x^{-1}) = I_{f_N(e)}(x),\ F_{f_N(e)}(x^{-1}) = F_{f_N(e)}(x)$;
(ii) $T_{f_N(e)}(e_G) \geq T_{f_N(e)}(x),\ I_{f_N(e)}(e_G) \leq I_{f_N(e)}(x),\ F_{f_N(e)}(e_G) \leq F_{f_N(e)}(x)$;

if T follows the idempotent t-norm and I, F follow the idempotent s-norm disciplines, respectively. (e_G being the identity element of G.)

Definition 2.9 [35] Let g be a mapping from a set X to a set Y. If M and N are two neutrosophic soft sets over X and Y, respectively, then the image of M under g is defined as a neutrosophic soft set $g(M) = \{[e, f_{g(M)}(e)] : e \in E\}$ over Y, where $T_{f_{g(M)}(e)}(y) = T_{f_M(e)}[g^{-1}(y)]$, $I_{f_{g(M)}(e)}(y) = I_{f_M(e)}[g^{-1}(y)]$, $F_{f_{g(M)}(e)}(y) = F_{f_M(e)}[g^{-1}(y)]$; $\forall y \in Y$.

The pre-image of N under g is defined as a neutrosophic soft set $g^{-1}(N) = \{[e, f_{g^{-1}(N)}(e)] : e \in E\}$ over X, where $T_{f_{g^{-1}(N)}(e)}(x) = T_{f_N(e)}[g(x)]$, $I_{f_{g^{-1}(N)}(e)}(x) = I_{f_N(e)}[g(x)]$, $F_{f_{g^{-1}(N)}(e)}(x) = F_{f_N(e)}[g(x)]$; $\forall x \in X$.

Neutrosophic Normal Soft Groups

In this section, we have defined the neutrosophic normal soft groups and some basic properties related to it.

Unless otherwise stated, E is treated as the parametric set through out this paper and $e \in E$, an arbitrary parameter.

Definition 3.1 A neutrosophic soft group N over the group (G, o) is called neutrosophic normal soft group (briefly, NNSG) if $f_N(e)$ is a neutrosophic normal subgroup of (G, o) for each $e \in E$ i.e.,

$$\begin{cases} T_{f_N(e)}(yoxoy^{-1}) \geq T_{f_N(e)}(x) \\ I_{f_N(e)}(yoxoy^{-1}) \leq I_{f_N(e)}(x) \\ F_{f_N(e)}(yoxoy^{-1}) \leq F_{f_N(e)}(x) \text{ for } x \in f_N(e), \;\; y \in G. \end{cases}$$

Definition 3.1.1 A neutrosophic soft group N over the group G is called abelian neutrosophic soft group if $\forall x, y \in G$, $\forall e \in E$, the following triplet hold.

$$\begin{cases} T_{f_N(e)}(xoy) = T_{f_N(e)}(yox) \\ I_{f_N(e)}(xoy) = I_{f_N(e)}(yox) \\ F_{f_N(e)}(xoy) = F_{f_N(e)}(yox). \end{cases}$$

Example 3.1.2 Define a mapping $f_M : \mathbf{N} \to NS(\mathbf{Z})$, where $\mathbf{N}$ be the set of natural number, $\mathbf{Z}$ be the set of all integers and for any $n \in \mathbf{N}$, as:

$$T_{f_M(n)}(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ \frac{2}{n} & \text{if } x \text{ is even.} \end{cases}$$

$$I_{f_M(n)}(x) = \begin{cases} \frac{1}{n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even.} \end{cases}$$

$$F_{f_M(n)}(x) = \begin{cases} 1 - \frac{3}{n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even.} \end{cases}$$

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = min\{a, b\}$, $a \diamond b = max\{a, b\}$; Then, M forms a neutrosophic normal soft group over $[(\mathbf{Z}, +), \mathbf{N}]$.

Proposition 3.2 *Let N be an NNSG over a group G. Then $\forall x, y \in G$, $\forall e \in E$,*

(i) $T_{f_N(e)}(yoxoy^{-1}) = T_{f_N(e)}(x), I_{f_N(e)}(yoxoy^{-1}) = I_{f_N(e)}(x), F_{f_N(e)}(yoxoy^{-1}) = F_{f_N(e)}(x)$;

(ii) *N is an abelian neutrosophic soft group over G.*

Proof

$$\begin{aligned} \textbf{(i)} \;\; T_{f_N(e)}(x) &= T_{f_N(e)}[(y^{-1}oy)oxo(y^{-1}oy)] \\ &= T_{f_N(e)}[y^{-1}o(yoxoy^{-1})oy)] \\ &= T_{f_N(e)}[y^{-1}o(yoxoy^{-1})o(y^{-1})^{-1}] \\ &\geq T_{f_N(e)}(yoxoy^{-1}), \text{ by definition.} \end{aligned}$$

Now from definition, $T_{f_N(e)}(yoxoy^{-1}) = T_{f_N(e)}(x)$;

The other two results hold in similar fashion.

$$
\begin{aligned}
\textbf{(ii)} \qquad & T_{f_N(e)}(x) = T_{f_N(e)}(yoxoy^{-1}) \\
\Rightarrow\ & T_{f_N(e)}(xoy) = T_{f_N(e)}(yo(xoy)oy^{-1}), \text{ replacing } x \text{ by } xoy. \\
\Rightarrow\ & T_{f_N(e)}(xoy) = T_{f_N(e)}(yox)
\end{aligned}
$$

Similarly, $I_{f_N(e)}(xoy) = I_{f_N(e)}(yox)$, $F_{f_N(e)}(xoy) = F_{f_N(e)}(yox)$;

Hence, N is an abelian neutrosophic soft group over G. □

Theorem 3.3 *Let N_1 and N_2 be two NNSGs over the group X. Then,*

(i) *$N_1 \cap N_2$ is also NNSG over X.*
(ii) *$N_1 \wedge N_2$ is also NNSG over X.*

Proof **(i)** Let $N_3 = N_1 \cap N_2$. Then for $x, y \in X$ and $e \in E$,

$$
\begin{aligned}
T_{f_{N_3}(e)}(yoxoy^{-1}) &= T_{f_{N_1}(e)}(yoxoy^{-1}) * T_{f_{N_2}(e)}(yoxoy^{-1}) \\
&\geq T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x) \\
&= T_{f_{N_3}(e)}(x) \\
I_{f_{N_3}(e)}(yoxoy^{-1}) &= I_{f_{N_1}(e)}(yoxoy^{-1}) \diamond I_{f_{N_2}(e)}(yoxoy^{-1}) \\
&\leq I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x) \\
&= I_{f_{N_3}(e)}(x)
\end{aligned}
$$

Similarly, $F_{f_{N_3}(e)}(yoxoy^{-1}) \leq F_{f_{N_3}(e)}(x)$;

Hence, the 1st part is completed.

(ii) Let $N_3 = N_1 \wedge N_2$. Then for $x, y \in X$ and $(a, b) \in E \times E$;

$$
\begin{aligned}
T_{f_{N_3}(a,b)}(yoxoy^{-1}) &= T_{f_{N_1}(a)}(yoxoy^{-1}) * T_{f_{N_2}(b)}(yoxoy^{-1}) \\
&\geq T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x) \\
&= T_{f_{N_3}(a,b)}(x) \\
I_{f_{N_3}(a,b)}(yoxoy^{-1}) &= I_{f_{N_1}(a)}(yoxoy^{-1}) \diamond I_{f_{N_2}(b)}(yoxoy^{-1}) \\
&\leq I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x) \\
&= I_{f_{N_3}(a,b)}(x)
\end{aligned}
$$

Similarly, $F_{f_{N_3}(a,b)}(yoxoy^{-1}) \leq F_{f_{N_3}(a,b)}(x)$;

This completes the final part. □

Remark 3.3.1 Generally, union of two neutrosophic normal soft groups is not so. It is possible if any one is contained in other.

For example, let $G = (\mathbf{Z}, +)$, $E = 3\mathbf{Z}$. Consider two neutrosophic soft groups N_1 and N_2 over G as following. For $x, n \in \mathbf{Z}$

$$
T_{f_{N_1}(3n)}(x) = \begin{cases} \frac{1}{2} & \text{if } x = 6kn,\ \exists k \in \mathbf{Z} \\ 0 & \text{others.} \end{cases}
$$

$$
I_{f_{N_1}(3n)}(x) = \begin{cases} 0 & \text{if } x = 6kn,\ \exists k \in \mathbf{Z} \\ \frac{1}{5} & \text{others.} \end{cases}
$$

$$
F_{f_{N_1}(3n)}(x) = \begin{cases} 0 & \text{if } x = 6kn,\ \exists k \in \mathbf{Z} \\ \frac{1}{4} & \text{others.} \end{cases}
$$

and

$$T_{f_{N_2}(3n)}(x) = \begin{cases} \frac{2}{3} & \text{if } x = 9kn,\ \exists k \in \mathbf{Z} \\ 0 & \text{others.} \end{cases}$$

$$I_{f_{N_2}(3n)}(x) = \begin{cases} 0 & \text{if } x = 9kn,\ \exists k \in \mathbf{Z} \\ \frac{1}{6} & \text{others.} \end{cases}$$

$$F_{f_{N_2}(3n)}(x) = \begin{cases} \frac{1}{2} & \text{if } x = 9kn,\ \exists k \in \mathbf{Z} \\ 1 & \text{others.} \end{cases}$$

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = min\{a, b\}$, $a \diamond b = max\{a, b\}$. Then, obviously N_1, N_2 are normal over G.

Let $N_1 \cap N_2 = N_3$; Then for $n = 1,\ x = 6,\ y = 9$ we have,

$$T_{f_{N_3}(3)}(6-9) = T_{f_{N_1}(3)}(-3) \diamond T_{f_{N_2}(1)}(-3) = max\{0, 0\} = 0$$

and

$$\begin{aligned} &T_{f_{N_3}(3)}(6) * T_{f_{N_3}(3)}(9) \\ &= \{T_{f_{N_1}(3)}(6) \diamond T_{f_{N_2}(3)}(6)\} * \{T_{f_{N_1}(3)}(9) \diamond T_{f_{N_2}(3)}(9)\} \\ &= min\left[max\left\{\frac{1}{2}, 0\right\}, max\left\{0, \frac{2}{3}\right\}\right] \\ &= min\left(\frac{1}{2}, \frac{2}{3}\right) \\ &= \frac{1}{2} \end{aligned}$$

Hence $T_{f_{N_3}(3)}(6-9) < T_{f_{N_3}(3)}(6) * T_{f_{N_3}(3)}(9)$
i.e., $N_1 \cup N_2$ is not a neutrosophic soft group, here.
Now, if we define N_2 over G as following:

$$T_{f_{N_2}(3n)}(x) = \begin{cases} \frac{1}{4} & \text{if } x = 12kn,\ \exists k \in \mathbf{Z} \\ 0 & \text{others.} \end{cases}$$

$$I_{f_{N_2}(3n)}(x) = \begin{cases} \frac{1}{2} & \text{if } x = 12kn,\ \exists k \in \mathbf{Z} \\ 1 & \text{others.} \end{cases}$$

$$F_{f_{N_2}(3n)}(x) = \begin{cases} 0 & \text{if } x = 12kn,\ \exists k \in \mathbf{Z} \\ \frac{2}{5} & \text{others.} \end{cases}$$

Then, it can be easily verified that $N_2 \subseteq N_1$ and $N_1 \cup N_2$ is a neutrosophic normal soft group over G.

Theorem 3.4 *Let N be an NNSG over the group G. Suppose, $N|_{e_G} = \{f_N(e)|_{e_G} : e \in E\}$ where, $f_N(e)|_{e_G} = \{x \in G : T_{f_{N(e)}}(x) = T_{f_{N(e)}}(e_G), I_{f_{N(e)}}(x) = I_{f_{N(e)}}(e_G), F_{f_{N(e)}}(x) = F_{f_{N(e)}}(e_G)\}$, e_G being the unit element of G. Then $N|_{e_G}$ is a normal soft group over G, on the assumption that truth membership (T), indeterministic membership (I) and falsity membership (F) functions obey the t-norm and s-norm disciplines.*

Proof Since N is a neutrosophic soft group over G, then for $x, y \in f_N(e)|_{e_G}$;

$$\begin{aligned} T_{f_N(e)}(xoy^{-1}) &\geq T_{f_N(e)}(x) * T_{f_N(e)}(y) \\ &= T_{f_N(e)}(e_G) * T_{f_N(e)}(e_G) \\ &= T_{f_N(e)}(e_G) \\ \text{Further,} \quad T_{f_N(e)}(e_G) &= T_{f_N(e)}\{(xoy^{-1})o(xoy^{-1})^{-1}\} \\ &\geq T_{f_N(e)}(xoy^{-1}) * T_{f_N(e)}(xoy^{-1}) \\ &= T_{f_N(e)}(xoy^{-1}) \end{aligned}$$

Hence $T_{f_N(e)}(xoy^{-1}) = T_{f_N(e)}(e_G)$;

Similar conclusion can be drawn in favour of indeterminacy(I) and falsity membership(F) function.

Therefore, $xoy^{-1} \in f_N(e)|_{e_G}$ and so $f_N(e)|_{e_G}$ is a subgroup of G in classical sense for each $e \in E$. Thus, $N|_{e_G}$ is a soft group over G.

Next, since N is an NNSG over G then for each $e \in E$, $f_N(e)$ is a neutrosophic normal subgroup over G.

Let $x \in f_N(e)|_{e_G}$ and $y \in G$. Then,

$$\begin{aligned} T_{f_N(e)}(yoxoy^{-1}) &= T_{f_N(e)}(x) = T_{f_N(e)}(e_G) \\ I_{f_N(e)}(yoxoy^{-1}) &= I_{f_N(e)}(x) = I_{f_N(e)}(e_G) \\ F_{f_N(e)}(yoxoy^{-1}) &= F_{f_N(e)}(x) = F_{f_N(e)}(e_G) \end{aligned}$$

This shows that $yoxoy^{-1} \in f_N(e)|_{e_G}$ for $x \in f_N(e)|_{e_G}$ and $y \in G$.

Hence, $f_N(e)|_{e_G}$ is a normal subgroup of G in classical sense for each $e \in E$ and so $N|_{e_G}$ is a normal soft group over G in combination of both.

Theorem 3.5 *Let $g : X \to Y$ be an isomorphism in classical sense. If N is a normal neutrosophic soft group over X, then $g(N)$ is so over Y.*

Proof Let for $z_2 \in f_{g(N)}(e)$, $y \in Y$ there be exist $z_1 \in f_N(e)$, $x \in X$ so that $y = g(x)$, $z_2 = g(z_1)$. Now,

$$\begin{aligned} T_{f_{g(N)}(e)}(yoz_2oy^{-1}) &= T_{f_N(e)}[g^{-1}(yoz_2oy^{-1})] \\ &= T_{f_N(e)}[g^{-1}(y)og^{-1}(z_2)og^{-1}(y^{-1})], \text{ as } g^{-1} \text{ is homomorphism.} \\ &= T_{f_N(e)}[g^{-1}(y)og^{-1}(z_2)o(g^{-1}(y))^{-1}], \text{ as } g^{-1} \text{ is homomorphism.} \\ &= T_{f_N(e)}(xoz_1ox^{-1}) \\ &\geq T_{f_N(e)}(z_1) \\ &= T_{f_N(e)}[g^{-1}(z_2)] \\ &= T_{f_{g(N)}(e)}(z_2) \\ I_{f_{g(N)}(e)}(yoz_2oy^{-1}) &= I_{f_N(e)}[g^{-1}(yoz_2oy^{-1})] \\ &= I_{f_N(e)}[g^{-1}(y)og^{-1}(z_2)og^{-1}(y^{-1})], \text{ as } g^{-1} \text{ is homomorphism.} \\ &= I_{f_N(e)}[g^{-1}(y)og^{-1}(z_2)o(g^{-1}(y))^{-1}], \text{ as } g^{-1} \text{ is homomorphism.} \\ &= I_{f_N(e)}(xoz_1ox^{-1}) \\ &\leq I_{f_N(e)}(z_1) \\ &= I_{f_N(e)}[g^{-1}(z_2)] \\ &= I_{f_{g(N)}(e)}(z_2) \end{aligned}$$

Similarly, $F_{f_{g(N)}(e)}(yoz_2oy^{-1}) \leq F_{f_{g(N)}(e)}(z_2)$;

This completes the proof. □

Neutrosophic Soft Cosets

Definition 4.1 Let N be a neutrosophic soft group over the group G and $x \in G$ be a fixed element.

Then the set $xoN = \{xof_N(e) : \forall e \in E\}$ where,

$$\begin{aligned} xof_N(e) &= \{\langle g, T_{xof_N(e)}(g), I_{xof_N(e)}(g), F_{xof_N(e)}(g)\rangle : \forall g \in G\} \\ &= \{\langle g, T_{f_N(e)}(x^{-1}og), I_{f_N(e)}(x^{-1}og), F_{f_N(e)}(x^{-1}og)\rangle : \forall g \in G\} \end{aligned}$$

is called left neutrosophic soft coset of N in G.

Similarly, the right neutrosophic soft coset of N in G is

$$\begin{aligned} &Nox = \{f_N(e)ox : \forall e \in E\} \text{ where,} \\ &f_N(e)ox = \{\langle g, T_{f_N(e)}(gox^{-1}), I_{f_N(e)}(gox^{-1}), F_{f_N(e)}(gox^{-1})\rangle : \forall g \in G\} \end{aligned}$$

Proposition 4.1.1 *N is NNSG over G $\Leftrightarrow$ left and right neutrosophic soft cosets are equal.*

Proof First suppose that N is an NNSG over G. Then,

$$\begin{aligned} xof_N(e) &= \{\langle g, T_{xof_N(e)}(g), I_{xof_N(e)}(g), F_{xof_N(e)}(g)\rangle : \forall g \in G\} \\ &= \{\langle g, T_{f_N(e)}(x^{-1}og), I_{f_N(e)}(x^{-1}og), F_{f_N(e)}(x^{-1}og)\rangle : \forall g \in G\} \\ &= \{\langle g, T_{f_N(e)}(gox^{-1}), I_{f_N(e)}(gox^{-1}), F_{f_N(e)}(gox^{-1})\rangle : \forall g \in G\} \\ &= \{\langle g, T_{f_N(e)ox}(g), I_{f_N(e)ox}(g), F_{f_N(e)ox}(g)\rangle : \forall g \in G\} \\ &= f_N(e)ox \end{aligned}$$

Now, $xoN = \{xof_N(e) : \forall e \in E\} = \{f_N(e)ox : \forall e \in E\} = Nox$

Next suppose that $xoN = Nox$. Then,

$$\begin{aligned} &T_{xof_N(e)}(g) = T_{f_N(e)ox}(g),\ \ I_{xof_N(e)}(g) = I_{f_N(e)ox}(g),\ \ F_{xof_N(e)}(g) = F_{f_N(e)ox}(g) \\ &\Rightarrow T_{f_N(e)}(x^{-1}og) = T_{f_N(e)}(gox^{-1}),\ \ I_{f_N(e)}(x^{-1}og) = I_{f_N(e)}(gox^{-1}),\ \ F_{f_N(e)}(x^{-1}og) \\ &= F_{f_N(e)}(gox^{-1}) \\ &\Rightarrow T_{f_N(e)}(gox^{-1}) = T_{f_N(e)}(x^{-1}og),\ \ I_{f_N(e)}(gox^{-1}) = I_{f_N(e)}(x^{-1}og),\ \ F_{f_N(e)}(gox^{-1}) \\ &= F_{f_N(e)}(x^{-1}og) \\ &\Rightarrow T_{f_N(e)}(xogox^{-1}) = T_{f_N(e)}(g),\ \ I_{f_N(e)}(xogox^{-1}) = I_{f_N(e)}(g),\ \ F_{f_N(e)}(xogox^{-1}) \\ &= F_{f_N(e)}(g) \end{aligned}$$

This shows that N is an NNSG over G.

Thus if N is NNSG over G then left and right neutrosophic soft cosets coincide. In that case, we call only neutrosophic soft coset instead of left or right neutrosophic soft coset, separately. □

Example 4.1.2 Let G be a classical group. Then $N = \{\langle e, f_N(e)\rangle : \forall e \in E\}$ where, $f_N(e) = \{\langle x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x)\rangle : \forall x \in G\}$ with $T_{f_N(e)}(x) = T_{f_N(e)}(e_G)$, $I_{f_N(e)}(x) = I_{f_N(e)}(e_G)$, $F_{f_N(e)}(x) = F_{f_N(e)}(e_G)$; ($e_G$ being identity element in G) is an NNSG of G.

In that case, we can get a neutrosophic soft coset.

For the sake of convenience, we use multiplication as a binary composition in rest of this paper unless otherwise stated e.g., $xoy = xy$.

Theorem 4.2 *Let N be an NNSG over the group G. $\Im$ be the collection of all distinct neutrosophic soft cosets of N in G. Then $\Im$ is a group in classical sense under the law of composition $xNyN = (xy)N$ $\forall x, y \in G$.*

Proof First we show that the composition is well defined in the sense that if $xN = x'N$ and $yN = y'N$ then $xNyN = (x'y')N$ for $x, y, x', y' \in G$.

Now, $xN = x'N$ implies $x^{-1}x' = f_N(e_1), e_1 \in E$ and $yN = y'N$ implies $y^{-1}y' = f_N(e_2), e_2 \in E$.

We show, $(xy)N = (x'y')N$ i,e $(xy)^{-1}(x'y') \in N$.

$$\begin{aligned}\text{Now, } (xy)^{-1}(x'y') &= y^{-1}x^{-1}x'y' \\ &= y^{-1}f_N(e_1)y' \\ &= y^{-1}y'f_N(e_1) \ [as\ y'N = Ny'] \\ &= f_N(e_2)f_N(e_1) \\ &= f_N(e_3) \in N, e_3 \in E;\end{aligned}$$

Hence, composition is well defined. Now,

(i) Clearly, closure axiom is satisfied.
(ii) $xN[yNzN] = xN(yz)N = x(yz)N$ and $[xNyN]zN = (xy)NzN = (xy)zN$ for $x, y, z \in G$.

Now $x(yz) = (xy)z$, since G is a group and so (.) is associative.
(iii) $e_GNxN = (e_Gx)N = xN$ and $xNe_GN = (xe_G)N = xN$, for e_G being unity in G.
(iv) Finally, $x^{-1}NxN = (x^{-1}x)N = e_GN = N$ and

$$xNx^{-1}N = (xx^{-1})N = e_GN = N.$$

□

Thus $\Im$ is a group. This group is said to be the quotient group (or the factor group)of G by N and is denoted by G/N.

Definition 4.3 Let G be a groupoid and N_1, N_2 be two NSSs over G. Then the neutrosophic soft product of N_1 and N_2 is denoted by N_1N_2 and is defined as $N_1N_2 = N_3$ where, for $(a, b) \in E \times E$ and $x \in G$,

$$T_{f_{N_3}(a,b)}(x) = \begin{cases} max_{x=yz}[T_{f_{N_1}(a)}(y) * T_{f_{N_2}(b)}(z)] \\ 0 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

$$I_{f_{N_3}(a,b)}(x) = \begin{cases} min_{x=yz}[I_{f_{N_1}(a)}(y) \diamond I_{f_{N_2}(b)}(z)] \\ 1 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

$$F_{f_{N_3}(a,b)}(x) = \begin{cases} min_{x=yz}[F_{f_{N_1}(a)}(y) \diamond F_{f_{N_2}(b)}(z)] \\ 1 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

Theorem 4.4 *Let N be an NNSG over the group G. Then there exists a natural homomorphism $\phi : G \to G/N$ defined by $\phi(g) = gN$, $\forall g \in G$ in classical sense.*

Proof Let $\phi : G \to G/N$ be given by $\phi(g) = gf_N(e), \forall e \in E$.
We show that ϕ is homomorphism i.e., $\phi(gh) = \phi(g)\phi(h),\ \forall g, h \in G$.
i.e., $(gh)f_N(e) = (gf_N(e))(hf_N(e))$; Now for $x \in G$,

$$\begin{aligned}
(gf_N(e))(x) &= \langle T_{gf_N(e)}(x), I_{gf_N(e)}(x), F_{gf_N(e)}(x)\rangle \\
&= \langle T_{f_N(e)}(g^{-1}x), I_{f_N(e)}(g^{-1}x), F_{f_N(e)}(g^{-1}x)\rangle; \\
(hf_N(e))(x) &= \langle T_{f_N(e)}(h^{-1}x), I_{f_N(e)}(h^{-1}x), F_{f_N(e)}(h^{-1}x)\rangle; \\
((gh)f_N(e))(x) &= \langle T_{f_N(e)}((gh)^{-1}x), I_{f_N(e)}((gh)^{-1}x), F_{f_N(e)}((gh)^{-1}x)\rangle;
\end{aligned}$$

Then,

$$\begin{aligned}
[(g(f_N(e)))(h(f_N(e)))](x) &= \langle max_{x=rs}[T_{g(f_N(e))}(r) * T_{h(f_N(e))}(s)], \\
&\quad min_{x=rs}[I_{g(f_N(e))}(r) \diamond I_{h(f_N(e))}(s)], \\
&\quad min_{x=rs}[F_{g(f_N(e))}(r) \diamond F_{h(f_N(e))}(s)]\rangle \\
&= \langle max_{x=rs}[T_{f_N(e)}(g^{-1}r) * T_{f_N(e)}(h^{-1}s)], \\
&\quad min_{x=rs}[I_{f_N(e)}(g^{-1}r) \diamond I_{f_N(e)}(h^{-1}s)], \\
&\quad min_{x=rs}[F_{f_N(e)}(g^{-1}r) \diamond F_{f_N(e)}(h^{-1}s)]\rangle
\end{aligned}$$

$$\begin{aligned}
\text{Further, } T_{f_N(e)}((gh)^{-1}x) &= T_{f_N(e)}(h^{-1}g^{-1}x) \\
&= T_{f_N(e)}(h^{-1}g^{-1}rs),\ \text{[putting } x = rs] \\
&= T_{f_N(e)}(h^{-1}(g^{-1}rsh^{-1})h) \\
&= T_{f_N(e)}(g^{-1}rsh^{-1}),\ \text{[as } N \text{ is NNSG]} \\
&\geq T_{f_N(e)}(g^{-1}r) * T_{f_N(e)}(sh^{-1}) \\
\text{Hence, } T_{f_N(e)}((gh)^{-1}x) &= max_{x=rs}[T_{f_N(e)}(g^{-1}r) * T_{f_N(e)}(h^{-1}s)]; \\
\text{Similarly, } I_{f_N(e)}((gh)^{-1}x) &= min_{x=rs}[I_{f_N(e)}(g^{-1}r) \diamond I_{f_N(e)}(h^{-1}s)]; \\
F_{f_N(e)}((gh)^{-1}x) &= min_{x=rs}[F_{f_N(e)}(g^{-1}r) \diamond F_{f_N(e)}(h^{-1}s)];
\end{aligned}$$

This shows that, $[(gh)f_N(e)](x) = [(gf_N(e))(hf_N(e))](x) \Rightarrow \phi(gh) = \phi(g)\phi(h)$. □

Lemma 4.5 *Let N be a neutrosophic soft group over a finite group G. Define, $\forall e \in E$,*

$$H = \{g \in G : T_{f_N(e)}(g) = T_{f_N(e)}(e_G);\ I_{f_N(e)}(g) = I_{f_N(e)}(e_G);\ F_{f_N(e)}(g) = F_{f_N(e)}(e_G)\}$$

$K = \{x \in G : Nx = Ne_G\}$ i,e $\{x \in G : T_{f_N(e)x}(g) = T_{f_N(e)e_G}(g);\ I_{f_N(e)x}(g) = I_{f_N(e)e_G}(g);\ F_{f_N(e)x}(g) = F_{f_N(e)e_G}(g);\ \forall g \in G\}$ where e_G is the unity in G.

If $*$ is idempotent t-norm and $\diamond$ is idempotent s-norm, then H and K are subgroups of G. Further $H = K$.

Proof Let $g, h \in H$. Then,

$$\begin{aligned}
T_{f_N(e)}(gh) &\geq T_{f_N(e)}(g) * T_{f_N(e)}(h) \\
&= T_{f_N(e)}(e_G) * T_{f_N(e)}(e_G) \\
&= T_{f_N(e)}(e_G)
\end{aligned}$$

Hence, $T_{f_N(e)}(gh) \geq T_{f_N(e)}(e_G)$.

$$\text{Further, } T_{f_N(e)}(gg^{-1}) \geq T_{f_N(e)}(g) * T_{f_N(e)}(g)$$
$$\text{i.e., } T_{f_N(e)}(e_G) \geq T_{f_N(e)}(g)$$

Thus, $T_{f_N(e)}(e_G) \geq T_{f_N(e)}(g)$. Putting '$gh$' instead of g, $T_{f_N(e)}(e_G) \geq T_{f_N(e)}(gh)$.

Therefore,

$$T_{f_N(e)}(gh) = T_{f_N(e)}(e_G) \tag{1}$$

$$\begin{aligned} \text{Next, } I_{f_N(e)}(gh) &\leq I_{f_N(e)}(g) \diamond I_{f_N(e)}(h) \\ &= I_{f_N(e)}(e_G) \diamond I_{f_N(e)}(e_G) \\ &= I_{f_N(e)}(e_G) \end{aligned}$$

Hence, $I_{f_N(e)}(gh) \leq I_{f_N(e)}(e_G)$

$$\text{Further, } I_{f_N(e)}(gg^{-1}) \leq I_{f_N(e)}(g) \diamond I_{f_N(e)}(g)$$
$$\text{i.e., } I_{f_N(e)}(e_G) \leq I_{f_N(e)}(g)$$

Thus, $I_{f_N(e)}(e_G) \leq I_{f_N(e)}(g)$. Putting '$gh$'instead of g, $I_{f_N(e)}(e_G) \leq I_{f_N(e)}(gh)$.

Hence,

$$I_{f_N(e)}(gh) = I_{f_N(e)}(e_G) \tag{2}$$

In a similar way,

$$F_{f_N(e)}(gh) = F_{f_N(e)}(e_G) \tag{3}$$

So by (1), (2), (3) it follows that, $gh \in H$ for $g, h \in H$.

Now since G is finite, so H is a subgroup of G.

We finally show that $H = K$. Let $k \in K$. Then for $g \in G$,

$$\begin{aligned} &T_{f_N(e)k}(g) = T_{f_N(e)e_G}(g),\ I_{f_N(e)k}(g) = I_{f_N(e)e_G}(g),\ F_{f_N(e)k}(g) = F_{f_N(e)e_G}(g); \\ &\Rightarrow T_{f_N(e)}(gk^{-1}) = T_{f_N(e)}(ge_G{}^{-1}),\ I_{f_N(e)}(gk^{-1}) = I_{f_N(e)}(ge_G{}^{-1}),\ F_{f_N(e)}(gk^{-1}) \\ &= F_{f_N(e)}(ge_G{}^{-1}); \\ &\Rightarrow T_{f_N(e)}(gk^{-1}) = T_{f_N(e)}(g),\ I_{f_N(e)}(gk^{-1}) = I_{f_N(e)}(g),\ F_{f_N(e)}(gk^{-1}) = F_{f_N(e)}(g); \\ &\Rightarrow T_{f_N(e)}(k^{-1}) = T_{f_N(e)}(e_G),\ I_{f_N(e)}(k^{-1}) = I_{f_N(e)}(e_G),\ F_{f_N(e)}(k^{-1}) = F_{f_N(e)}(e_G); \\ &\quad \text{(putting } g = e_G) \end{aligned}$$

This shows that $k^{-1} \in H \Rightarrow k \in H$ as H is a subgroup of G.

Hence,

$$K \subseteq H \tag{4}$$

Next, let $h \in H$.

Then $\forall g \in G$, $T_{f_N(e)h}(g) = T_{f_N(e)}(gh^{-1})$ and $T_{f_N(e)e_G}(g) = T_{f_N(e)}(ge_G{}^{-1}) = T_{f_N(e)}(g)$.

$$\begin{aligned} \text{Now, } T_{f_N(e)}(gh^{-1}) &\geq T_{f_N(e)}(g) * T_{f_N(e)}(h), \text{ by proposition (2.8.3)} \\ &= T_{f_N(e)}(g) * T_{f_N(e)}(e_G) \\ &\geq T_{f_N(e)}(g) * T_{f_N(e)}(g), \text{ by proposition (2.8.3)} \\ &= T_{f_N(e)}(g) \end{aligned}$$

$$\begin{aligned}\text{Also, } T_{f_N(e)}(g) &= T_{f_N(e)}(gh^{-1}h)\\ &\geq T_{f_N(e)}(gh^{-1}) * T_{f_N(e)}(h)\\ &= T_{f_N(e)}(gh^{-1}) * T_{f_N(e)}(e_G)\\ &\geq T_{f_N(e)}(gh^{-1}) * T_{f_N(e)}(gh^{-1}), \text{ by proposition (2.8.3)}\\ &= T_{f_N(e)}(gh^{-1})\end{aligned}$$

This shows that $T_{f_N(e)}(gh^{-1}) = T_{f_N(e)}(g)$. Similar conclusion can be drawn in case of indeterminacy(I) and falsity(F) membership functions.

Thus, $h \in K$ and so

$$H \subseteq K \tag{5}$$

Hence by (4) and (5), $H = K$ and so K is also a subgroup of G. □

Theorem 4.5.1 *Let N be an NNSG over the group G. Let $\theta : G \to G$ be a homomorphism. Then θ leaves invariant the set*

$$H = \left\{x \in G : T_{f_N(e)}(x) = T_{f_N(e)}(e);\ I_{f_N(e)}(x) = I_{f_N(e)}(e);\ F_{f_N(e)}(x) = F_{f_N(e)}(e)\right\}.$$

Also θ induces a homomorphism $\overline{\theta}$ of the neutrosophic soft coset of N defined by $\overline{\theta}(Nx) = N\theta(x)$, on the assumption that $*$ is idempotent t-norm and $\diamond$ is idempotent s-norm.

Proof By lemma (4.5), H is a subgroup of G in classical sense.

First we show that θ is well defined.

For $x, y \in G$, let $Nx = Ny$. We are to prove that $N\theta(x) = N\theta(y)$.

$$\begin{aligned}\text{Then, } \quad & T_{f_N(e)x}(x) = T_{f_N(e)y}(x);\quad T_{f_N(e)x}(y) = T_{f_N(e)y}(y),\ \forall e \in E.\\ \Rightarrow\ & T_{f_N(e)}(e_G) = T_{f_N(e)}(xy^{-1}) = T_{f_N(e)}(yx^{-1})\end{aligned}$$

Similar result can be brought in case of indeterminacy(I) and falsity(F) membership functions. Thus $xy^{-1},\ yx^{-1} \in H$.

Again, since $\theta(H) = H$ so $\theta(xy^{-1}),\ \theta(yx^{-1}) \in H$.

This implies $T_{f_N(e)}(\theta(xy^{-1})) = T_{f_N(e)}(\theta(yx^{-1})) = T_{f_N(e)}(e_G)$ and so on for $I,\ F$.

$$\begin{aligned}\text{Now, } T_{f_N(e)\theta(x)}(g) &= T_{f_N(e)}(g\theta(x^{-1})), \quad \text{for } g \in G\\ &= T_{f_N(e)}(g\theta(y^{-1}yx^{-1}))\\ &= T_{f_N(e)}(g\theta(y^{-1})\theta(yx^{-1}))\\ &\geq T_{f_N(e)}(g\theta(y^{-1})) * T_{f_N(e)}(\theta(yx^{-1}))\\ &= T_{f_N(e)}(g\theta(y^{-1})) * T_{f_N(e)}(e_G)\\ &\geq T_{f_N(e)}(g\theta(y^{-1})) * T_{f_N(e)}(g\theta(y^{-1})), \text{ by proposition (2.8.3)}\\ &= T_{f_N(e)}(g\theta(y^{-1}))\\ &= T_{f_N(e)\theta(y)}(g)\end{aligned}$$

Hence, $T_{f_N(e)\theta(x)}(g) \geq T_{f_N(e)\theta(y)}(g)$. Similarly, $T_{f_N(e)\theta(x)}(g) \leq T_{f_N(e)\theta(y)}(g)$ holds good.

Thus,

$$T_{f_N(e)\theta(x)}(g) = T_{f_N(e)\theta(y)}(g) \tag{6}$$

$$\begin{aligned}
\text{Next, } I_{f_N(e)\theta(x)}(g) &= I_{f_N(e)}(g\theta(x^{-1})) \;\; for\ g \in G \\
&= I_{f_N(e)}(g\theta(y^{-1}yx^{-1})) \\
&= I_{f_N(e)}(g\theta(y^{-1})\theta(yx^{-1})) \\
&\leq I_{f_N(e)}(g\theta(y^{-1})) \diamond I_{f_N(e)}(\theta(yx^{-1})) \\
&= I_{f_N(e)}(g\theta(y^{-1})) \diamond I_{f_N(e)}(e_G) \\
&\leq I_{f_N(e)}(g\theta(y^{-1})) \diamond I_{f_N(e)}(g\theta(y^{-1})) \\
&\quad [\text{by proposition (2.8.3)}, I_{f_N(e)}(e_G) \leq I_{f_N(e)}(g\theta(y^{-1}))] \\
&= I_{f_N(e)}(g\theta(y^{-1})) \\
&= I_{f_N(e)\theta(y)}(g)
\end{aligned}$$

Thus, $I_{f_N(e)\theta(x)}(g) \leq I_{f_N(e)\theta(y)}(g)$. Similarly, $I_{f_N(e)\theta(x)}(g) \geq I_{f_N(e)\theta(y)}(g)$ holds good.

Thus,

$$I_{f_N(e)\theta(x)}(g) = I_{f_N(e)\theta(y)}(g) \tag{7}$$

Similarly, also

$$F_{f_N(e)\theta(x)}(g) = F_{f_N(e)\theta(y)}(g) \tag{8}$$

Therefore, $f_N(e)\theta(x) = f_N(e)\theta(y), \forall e \in E \;\Rightarrow\; N\theta(x) = N\theta(y)$.
Hence, θ is well defined.
Next we show $\overline{\theta}$ is a homomorphism. For $x, y \in G$ we are to prove that,

$$\begin{aligned}
&\overline{\theta}(Nx)(Ny) = \overline{\theta}(Nx)\overline{\theta}(Ny) \\
\Leftrightarrow\ &\overline{\theta}(Nxy) = \overline{\theta}(Nx)\overline{\theta}(Ny) \\
\Leftrightarrow\ &N\theta(xy) = N\theta(x)N\theta(y) \\
\Leftrightarrow\ &N\theta(xy) = N\theta(x)\theta(y)
\end{aligned} \tag{9}$$

As θ is homomorphism, so $\theta(xy) = \theta(x)\theta(y)$ holds $\Rightarrow$ (9) holds $\Rightarrow$ $\overline{\theta}$ is a homomorphism. □

Corollary 4.5.2 *Thus $\overline{\theta}$ defined above is an automorphism if θ is an automorphism and G is finite.*

Proof Since G is finite, it is easy to verify that θ is of finite order. Let the order of θ is k. Then $\theta^k = I$, the identity mapping.

We now prove that θ is one-to-one. For $x, y \in G$ suppose $N\theta(x) = N\theta(y)$. We like to bring $N(x) = N(y)$.

$$\begin{aligned}
\text{Now,} \quad & \overline{\theta}[N\theta(x)] = \overline{\theta}[N\theta(y)] \\
&\Rightarrow N\theta(\theta(x)) = N\theta(\theta(y)) \\
&\Rightarrow N\theta^2(x) = N\theta^2(y) \\
\text{Iterating this,} \quad & N\theta^k(x) = N\theta^k(y) \\
&\Rightarrow N(x) = N(y), \quad \text{as } \theta^k = I
\end{aligned}$$

Hence, θ is one-to-one i.e., $\overline{\theta}$ is automorphism. □

Corollary 4.5.3 *With the hypothesis as in theorem (4.5.1), θ is an automorphism of G if $\overline{\theta}$ is an automorphism and $H = \{e_G\}$ in classical sense.*

Proof Let $\theta(x) = \theta(y)$ for $x, y \in G$. We show that $x = y$.

$$
\begin{aligned}
\text{Now,}\quad & N\theta(x) = N\theta(y)\\
& \Rightarrow \overline{\theta}(Nx) = \overline{\theta}(Ny)\\
& \Rightarrow Nx = Ny, \text{ as } \overline{\theta} \text{ is one-one.}\\
& \Rightarrow T_{f_{N(e)x}}(y) = T_{f_{N(e)y}}(y);\; I_{f_{N(e)x}}(y) = I_{f_{N(e)y}}(y);\; F_{f_{N(e)x}}(y) = F_{f_{N(e)y}}(y)\\
& \Rightarrow T_{f_{N(e)}}(yx^{-1}) = T_{f_{N(e)}}(e_G);\; I_{f_{N(e)}}(yx^{-1}) = I_{f_{N(e)}}(e_G);\; I_{f_{N(e)}}(yx^{-1})\\
& \qquad = I_{f_{N(e)}}(e_G)\\
& \Rightarrow yx^{-1} \in H = \{e_G\}\\
& \Rightarrow yx^{-1} = e_G\\
& \Rightarrow x = y
\end{aligned}
$$

□

Neutrosophic Soft Homomorphism

In this section, first we define an NSS function, then define image and pre-image of an NSS under an NSS function. In continuation, we introduce the notion of neutrosophic soft homomorphism along with some of it's properties.

If M be an NSS over U with respect to the parameter set E, we write (M, E), an NSS over U.

Definition 5.1 Let $\varphi : U \to V$ and $\psi : E \to E$ be two functions where E is the parameter set for each of the crisp sets U and V. Then the pair (φ, ψ) is called an NSS function from U to V. We write, $(\varphi, \psi) : U \to V$.

Definition 5.2 Let (M, E), (N, E) be two NSSs defined over U, V, respectively and (φ, ψ) be an NSS function from U to V. Then,

(1) The image of (M, E) under (φ, ψ), denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined by:

$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{\langle \psi(a), f_{\varphi(M)}\rangle : a \in E\}$ where $\forall b \in \psi(E), \forall y \in V$,

$$
T_{f_{\varphi(M)}(b)}(y) = \begin{cases} max_{\varphi(x)=y}\, max_{\psi(a)=b}\,[T_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y)\\ 0, & \text{otherwise.}\end{cases}
$$

$$
I_{f_{\varphi(M)}(b)}(y) = \begin{cases} min_{\varphi(x)=y}\, min_{\psi(a)=b}\,[I_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y)\\ 1. & \text{otherwise.}\end{cases}
$$

$$
F_{f_{\varphi(M)}(b)}(y) = \begin{cases} min_{\varphi(x)=y}\, min_{\psi(a)=b}\,[F_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y)\\ 1, & \text{otherwise.}\end{cases}
$$

(2) The pre-image of (N, E) under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by:

$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where $\forall a \in \psi^{-1}(E), \forall x \in U$,

$$
\begin{aligned}
T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N[\psi(a)]}(\varphi(x))\\
I_{f_{\varphi^{-1}(N)}(a)}(x) &= I_{f_N[\psi(a)]}(\varphi(x))\\
F_{f_{\varphi^{-1}(N)}(a)}(x) &= F_{f_N[\psi(a)]}(\varphi(x))
\end{aligned}
$$

If ψ and φ is injective (surjective), then (φ, ψ) is injective (surjective).

Example 5.2.1 Let $E = \mathbf{N}$ (the set of natural no.) be the parametric set and $G = (\mathbf{Z}, +)$ be the group of all integers. Define a mapping $f_M : \mathbf{N} \to NS(\mathbf{Z})$ where, for any $n \in \mathbf{N}$ and $x \in \mathbf{Z}$,

$$T_{f_M(n)}(x) = \begin{cases} 0 & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ \frac{1}{n} & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

$$I_{f_M(n)}(x) = \begin{cases} \frac{1}{2n} & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

$$F_{f_M(n)}(x) = \begin{cases} 1-\frac{1}{n} & \text{if } x = 2k-1,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 2k,\ k \in \mathbf{Z}. \end{cases}$$

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a*b = min\{a, b\}, a\diamond b = max\{a, b\}$; Then, $(M, \mathbf{N})$ forms a neutrosophic soft group over $(\mathbf{Z}, +)$.

Now, let $\varphi(x) = 3x + 1$ and $\psi(x) = x^2$ be two functions defined on $\mathbf{Z}$. Then, we have an NSS function $(\varphi, \psi)(M, \mathbf{N}) = (\varphi(M), \psi(\mathbf{N})) = (\varphi(M), \mathbf{N}^2)$ as follows:

For any $a \in \mathbf{N}^2$, $y \in 3\mathbf{Z} + 1$, we have

$$T_{f_M(a)}(x) = \begin{cases} 0 & \text{if } x = 6k-2,\ k \in \mathbf{Z} \\ \frac{1}{\sqrt{a}} & \text{if } x = 6k+1,\ k \in \mathbf{Z}. \end{cases}$$

$$I_{f_M(a)}(x) = \begin{cases} \frac{1}{\sqrt{2a}} & \text{if } x = 6k-2,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 6k+1,\ k \in \mathbf{Z}. \end{cases}$$

$$F_{f_M(a)}(x) = \begin{cases} 1-\frac{1}{\sqrt{a}} & \text{if } x = 6k-2,\ k \in \mathbf{Z} \\ 0 & \text{if } x = 6k+1,\ k \in \mathbf{Z}. \end{cases}$$

Theorem 5.3 *Let (N, E) be a neutrosophic soft group over a group G_1 and (φ, ψ) be a neutrosophic soft homomorphism from G_1 to G_2. Then $(\varphi, \psi)(N, E)$ is a neutrosophic soft group over G_2.*

Proof Let $b \in \psi(E)$ and $y_1, y_2 \in G_2$. For $\varphi^{-1}(y_1) = \phi$ or $\varphi^{-1}(y_2) = \phi$, the proof is straight forward.

So, we assume that there exists $x_1, x_2 \in G_1$ such that $\varphi(x_1) = y_1, \varphi(x_2) = y_2$. Then,

$$\begin{aligned} T_{f_{\varphi(N)}(b)}(y_1y_2) &= max_{\varphi(x)=y_1y_2}\, max_{\psi(a)=b}\, [T_{f_N(a)}(x)] \\ &\geq max_{\psi(a)=b}\, [T_{f_N(a)}(x_1x_2)] \\ &\geq max_{\psi(a)=b}\, [T_{f_N(a)}(x_1) * T_{f_N(a)}(x_2)] \\ &= max_{\psi(a)=b}\, [T_{f_N(a)}(x_1)] * max_{\psi(a)=b}\, [T_{f_N(a)}(x_2)] \end{aligned}$$

$$\begin{aligned} T_{f_{\varphi(N)}(b)}(y_1^{-1}) &= max_{\varphi(x)=y_1^{-1}}\, max_{\psi(a)=b}\, [T_{f_N(a)}(x)] \\ &\geq max_{\psi(a)=b}\, [T_{f_N(a)}(x_1^{-1})] \\ &\geq max_{\psi(a)=b}\, [T_{f_N(a)}(x_1)] \end{aligned}$$

Since, this inequality is satisfied for each $x_1, x_2 \in G_1$ satisfying $\varphi(x_1) = y_1, \varphi(x_2) = y_2$ so we have,

$$\begin{aligned} &T_{f_{\varphi(N)}(b)}(y_1y_2) \\ &\geq (max_{\varphi(x_1)=y_1} max_{\psi(a)=b}\, [T_{f_N(a)}(x_1)]) * (max_{\varphi(x_2)=y_2} max_{\psi(a)=b}\, [T_{f_N(a)}(x_2)]) \\ &= T_{f_{\varphi(N)}(b)}(y_1) * T_{f_{\varphi(N)}(b)}(y_2) \end{aligned}$$

Also, $T_{f_{\varphi(N)}(b)}(y_1^{-1}) \geq (max_{\varphi(x_1)=y_1} max_{\psi(a)=b}[T_{f_N(a)}(x_1)]) = T_{f_{\varphi(N)}(b)}(y_1)$

Similarly, we can show that

$$I_{f_{\varphi(N)}(b)}(y_1 y_2) \leq I_{f_{\varphi(N)}(b)}(y_1) \diamond I_{f_{\varphi(N)}(b)}(y_2), \quad I_{f_{\varphi(N)}(b)}(y_1^{-1}) \geq I_{f_{\varphi(N)}(b)}(y_1);$$
$$F_{f_{\varphi(N)}(b)}(y_1 y_2) \leq F_{f_{\varphi(N)}(b)}(y_1) \diamond F_{f_{\varphi(N)}(b)}(y_2), \quad F_{f_{\varphi(N)}(b)}(y_1^{-1}) \geq F_{f_{\varphi(N)}(b)}(y_1);$$

This completes the proof. □

Theorem 5.4 *Let (M, E) be a neutrosophic soft group over a group G_2 and (φ, ψ) be a neutrosophic soft homomorphism from G_1 to G_2. Then $(\varphi, \psi)^{-1}(M, E)$ is a neutrosophic soft group over G_1.*

Proof For $a \in \psi^{-1}(E)$ and $x_1, x_2 \in G_1$, we have,

$$\begin{aligned}
T_{f_{\varphi^{-1}(M)}(a)}(x_1 x_2) &= T_{f_{M[\psi(a)]}}(\varphi(x_1 x_2)) \\
&= T_{f_{M[\psi(a)]}}(\varphi(x_1)\varphi(x_2)) \\
&\geq T_{f_{M[\psi(a)]}}(\varphi(x_1)) * T_{f_{M[\psi(a)]}}(\varphi(x_2)) \\
&= T_{f_{\varphi^{-1}(M)}(a)}(x_1) * T_{f_{\varphi^{-1}(M)}(a)}(x_2) \\
T_{f_{\varphi^{-1}(M)}(a)}(x_1^{-1}) &= T_{f_{M[\psi(a)]}}\left(\varphi\left(x_1^{-1}\right)\right) \\
&= T_{f_{M[\psi(a)]}}\left(\varphi(x_1)^{-1}\right) \\
&\geq T_{f_{M[\psi(a)]}}(\varphi(x_1)) \\
&= T_{f_{\varphi^{-1}(M)}(a)}(x_1)
\end{aligned}$$

In a similar fashion, the following inequalities also hold.

$$\begin{aligned}
I_{f_{\varphi^{-1}(M)}(a)}(x_1 x_2) &\leq I_{f_{\varphi^{-1}(M)}(a)}(x_1) \diamond I_{f_{\varphi^{-1}(M)}(a)}(x_2), \\
I_{f_{\varphi^{-1}(M)}(a)}\left(x_1^{-1}\right) &\leq I_{f_{\varphi^{-1}(M)}(a)}(x_1); \\
F_{f_{\varphi^{-1}(M)}(a)}(x_1 x_2) &\leq F_{f_{\varphi^{-1}(M)}(a)}(x_1) \diamond F_{f_{\varphi^{-1}(M)}(a)}(x_2), \\
F_{f_{\varphi^{-1}(M)}(a)}\left(x_1^{-1}\right) &\leq F_{f_{\varphi^{-1}(M)}(a)}(x_1);
\end{aligned}$$

Thus, the theorem is completed. □

Theorem 5.5 *Let (N, E) be a normal neutrosophic soft group over a group X and (φ, ψ) be a neutrosophic soft epimorphism from X to Y. Then $(\varphi, \psi)(N, E)$ is a normal neutrosophic soft group over Y.*

Proof It is similar to the proof of theorem (5.3). □

Theorem 5.6 *Let (M, E) be a normal neutrosophic soft group over a group Y and (φ, ψ) be a neutrosophic soft homomorphism from G_1 to G_2. Then $(\varphi, \psi)^{-1}(M, E)$ is a normal neutrosophic soft group over X.*

Proof It is similar to the proof of theorem (5.4). □

Conclusion

Here, the theoretical point of view of normal neutrosophic soft group has been discussed. Along with, we also have defined the neutrosophic soft cosets and neutrosophic soft homomorphism. These are illustrated by proper examples and some related theorems have been developed in each part. These concept will bring a new opportunity in research and development of NSS theory.

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