Every even integer greater than six can be expressed as the sum of two co-prime odd integers atleast one of which is a prime

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Abstract: In this paper we prove a simple theorem that is distantly related to the Even Goldbach conjecture and is weaker than Chen's theorem regarding the expression of any even integer as the sum of a prime number and a semiprime number. We show that any even integer greater than six can be written as the sum of two odd integers coprime to one another and atleast one of them is a prime.

Proof:

Consider any even integer $2n>6$.
Let it be $2n=2p_1p_2p_3p_4\ldots\ldots p_k$ where p_1, p_2, p_3, ……..p_k are all the remaining prime factors.

Bertrand's postulate suggests that there must be atleast one positive prime integer between $n= p_1p_2p_3p_4\ldots\ldots p_k$ and $2n=2p_1p_2p_3p_4\ldots\ldots p_k$

Let us call this prime as p and this will be an odd prime integer. This prime integer p will be co-prime to all the factors of 2n.

Therefore there must exist another odd integer x such that :
$p+x=2n$

Since p is coprime to all the prime factors of 2n, x must also be coprime to all the prime factors of 2n and similarly x must be co-prime to p (otherwise common factors would arise for the three integers x,p,2n).

Consistently $n<p<2n$ and
x is an odd integer such that: $3\leq x<n$.
Also note that p, x, n are unequal.