VARIATIONS OF THE GRAVITATIONAL CONSTANT WITH TIME IN THE FRAMEWORK OF THE EXPANDING UNIVERSE

BY

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Abstract. This paper is an attempt to find an answer in the matter of expanding universe from the time variance of the gravitational constant point of view. We took as reference the equivalent variation of the gravitational constant of a static universe and report the observational data to it. The equivalent variation of the gravitational constant of a static universe is estimated in a reference universe hypothesis. An expanding force is balanced by an attracting force and this is the basis from which we can establish a formal time variance for the gravitational constant in two cases. The first one correspond to an expanding universe, hence this case can't be a reference for our evaluation. The second case corresponds to a static universe and it is the same as a de Sitter universe. This is the reason why this case can be a reference case. Thus the observational data smaller than our reference theoretical value, are linked to a collapsing universe and the observational data greater than the same reference value are characteristic to an expanding universe.

Key words: expansion of the universe; dark energy; gravitational constant.

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1. Introduction

From a historical perspective the birth of the idea of fundamental physical constants variation was the article published in the journal Nature by the great English physicist P. Dirac in 1937. He suggested the possibility that some fundamental constants may vary, including the universal constant of gravitational attraction, due to age of the universe. According to this idea, in the past the universal attraction constant was higher, its evolution over time is decreasing, up to the present epoch. Teller (1948) came in contradiction with Dirac's hypothesis; he was arguing that it would come in conflict with paleontological evidence. He pointed out that Earth's temperature can depend on the gravitational constant, so that a variation of the latter would lead to a variation of the temperature. By evaluating then the Earth's temperature from 300 million years ago he concluded that it should be more than 20%, close to the boiling water point, in which case the existence of life on our planet would have been unthinkable, assuming a higher gravitational constant with only 10% .

This was the beginning of an amazing career for the idea of universal gravitational constant variation. To present day there have been measured many values of this variation, there were made several assumptions so that it have developed a literature specific to a new sub-sub-field of physics. Table 1 is a selective overview of some of the assumptions made and of the experimental data which support them. What do all these values have in common? Note that there are no large differences between them. The data measured at our solar system distances are close to those measured at cosmic distances, galactic and extragalactic. So they seem somewhat credible in the case of a unitary manifestation of nature at the macroscopic scale or in a unifying hypothesis of all these assumptions. What all these situations have in common? Maybe just, I said a little more before, a unifying hypothesis of their own. And what could be then this assumption? What could have shared the Earth temperature variation, with variation of the radius of the Earth, with the variation of rotation period of the Moon around the Earth, with the changing distances between planets of the solar system, with the change of pulse period of pulsars in binary systems, with changes in stellar evolution, with nucleosynthesis at different ages of the universe with variation of the universal attraction constant? Their point of agreement would seem to be only the expansion of the universe. If we think that all these effects were measurable due to a single cause, that is the expansion of the universe, then we have the general picture of the gravitational constant variation, with close values at all levels where measurements were made.

We might even think a little further than that, namely, by taking into account all these experimental values, assuming that the expansion of the universe has generated all, which could be the state of the universe according to these observational data? Is it expanding forever or not? In this paper we try to answer these questions. We thus will compare values of different variations of the universal gravitational constant with some reference values, which we must find them, and from this comparison to decide which is the state of the universe in terms of expansion or recession. The big problem here seems to be what those reference values are. If we find, for example, a reference value for the universal gravitational constant variation corresponding to a static universe and then another reference value corresponding to an eternal expanding universe, it seems that we solved the problem. Higher values of the universal gravitational constant variation than the reference value for an eternal expanding universe corresponds therefore to a universe in recession, and smaller ones to an expanding universe.

In the following we will detailed these ideas and begin by setting the reference values.

2. The Gravitational Constant Variation in a Reference Universe

In ref. [35] the authors derive Newton's law for a spherical shell, in their attempt to demonstrate the Newton's theorem in a different way. In this connection the gravitational potential Φ(r) must verify the condition:

(1) $$m_{(\alpha)}\Phi_{(r)} + 2\pi\sigma\alpha\gamma_{(\alpha)} = \frac{2\pi\sigma\alpha}{r}\int_{r-\alpha}^{r+\alpha}\beta\Phi_{(\beta)}d\beta$$

where m (α) is the equivalent mass, σ is the thickness of the spherical shell, γ is a constant which can be added to the potential without the law of gravity deduced from it to be altered in any way. They obtained for the potential Φ(r) a first expression:

$$\Phi_{(r)} = \frac{A_1e^{\xi\cdot r} + A_2e^{-\xi\cdot r}}{r} + A_3$$

(2)

where has been noted by A some arbitrary constants. Potential (2) is a Yukawa-type potential.

Much later, the authors of paper [36] derive another expression of the potential Φ(r) which satisfies condition (1):

$$\Phi_{(r)} = \frac{B_1}{r} + B_2r^2 + B_3$$

where B are arbitrary constants, corresponding to algebraic potentials solution. In paper [37] it shows that for:

$$\Phi_{(r)} = \frac{B_1}{r} + B_2r^2$$

(3)

it can be inferred the expression analogous to the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{kc^2}{r^2} + 2B_2$$

that by identifying the first Friedmann equation:

(4) $$H^2 = \frac{8\pi G}{3}\rho + \frac{kc^2}{r^2} + \frac{\Lambda c^2}{3}$$

can give us the Newtonian equivalent for the cosmological constant:

(5) $$2B_2 = \frac{\Lambda c^2}{3}$$

For simplicity we consider $2B_2 = A$.

To validate the solution (5) for the additional part of the potential (3), it is necessary that this potential to be a solution of the Poisson equation. Since the potential (3), with solution (5), is specific to a spherical shell universe, and not to a compact sphere one, it is necessary to do a trick. So we must extend the thickness of the spherical shell near to its center. Under these conditions, solving the Poisson equation is formal. To additional part of the gravitational potential it will corresponds the energy density of the vacuum or the dark energy density. Of course, the latter is the best solution. But has the disadvantage that it is hard verifiable in practice. This is why we propose another solution to the Poisson equation, which has the advantage that it can be easily verified in practice and serves our purpose. What is it? To the additional part of the gravitational potential it will not longer corresponds a density of dark energy but a gravitational constant of the form:

$$G' = G + \frac{Ar^3}{3M}$$

To create a basis of comparison between the literature values obtained for the variation of the gravitational constant presented above and our results we need to transform the previous formula. If we make the derivative with respect to time and divide the result to G we get:

$$\left(\frac{dG/dt}{G}\right) = \frac{Ar^3 H}{3MG}$$

(6)

where it has been taken into account the Hubble's law:

$$\frac{dr}{dt} = Hr$$

Formula (6) can be simplified taking into account (5), the expression of the mass:

$$M = \frac{4\pi\rho r^3}{3}$$

and equation (4), where it was made the assumption that we are in a flat space to be consistent with current observational realities:

$$\frac{8\pi\rho G}{3} = H^2 - \frac{\Lambda c^2}{3}$$

After a simple calculation equation (6) gets a much simpler and easier to implement value:

(7) $$\left(\frac{dG/dt}{G}\right) = \frac{1}{3}\frac{1}{\frac{1}{H} - \frac{H}{\Lambda c^2}}$$

If the value for the Hubble constant is in agreement with current observations, i.e. 70 km / s / MPs and the cosmological constant is $10^{-52} m^{-2}$, expression (7) has the value $5{,}5 \times 10^{-10} yr^{-1}$.

Consider the equation (4), the first Friedmann equation. We apply to this equation the condition written as:

(8) $$GM / r^2 = \Lambda c^2 r / 3$$

If we write the mass M depending on matter density, the condition (8) will appear in a slightly altered form:

(8') $$4\pi G\rho = \Lambda c^2$$

By introducing this condition in equation (4), it will result after some elementary steps the Friedmann equation specific to this case:

(9) $$H^2 = 4\pi G\rho$$

There is an equivalent relation to (9), which is obtained from equation (4) and condition (8), but the expression is in accordance to the cosmological constant. We obtain the formula:

(10) $$H^2 = \Lambda c^2$$

If in expression (7) is taken into account (10), we obtain after some elementary steps the formula:

(11) $$(dG/dt)/G = H/3$$

It is obvious that this formula is valid only if the relation (8) is valid. If we adopt a value for the Hubble constant to be in unanimous acceptance of the international scientific community, namely $2{,}29 \times 10^{-18} s^{-1}$, then we can evaluate the expression (11) as $7{,}25 \times 10^{-11} yr^{-1}$. A value, if we look compared to the experimental values presented in the previous section, much closer to the experimental measurements than we expected.

An important consequence of equations (9) and (10) occur if we evaluate the ratio between material density and critical density from which the universe is flat:

$$\Omega = \rho / \rho_c$$

Taking into account the mentioned equations we can calculate this ratio as:

(12) $$\Omega = 4\pi G\rho / H^2 = \Lambda c^2 / H^2 = 1$$

a limit value, which it make us to conclude that this case, in accordance to the condition (8), corresponds to the case of a static universe. If we actually go a little further with reasoning and calculate the deceleration parameter proper to condition (8):

$$q = -\ddot{a}a / \dot{a}^2$$

we see that to make this assessment it is easier to evaluate the second Friedmann equation specific to this case. It is quite obvious that starting from the general form of the second Friedmann equation:

$$\dot{H} + H^2 = \ddot{a}/a = -(4\pi G/3)(\rho + 3p/c^2) + \Lambda c^2/3$$

where it takes into account the condition (8 '), the intermediate result is obtained:

$$\dot{H} + H^2 = \ddot{a}/a = -4\pi Gp/c^2$$

With this intermediate result we can immediately assess the deceleration parameter:

$$q = -(\ddot{a}/a)(a^2/\dot{a}^2) = p/\rho c^2$$

Now if we consider the state equation of the cosmic fluid:

$$p = \omega\rho c^2$$

and an universe dominated by matter, ω=0, it is obvious that we have the image of a static universe. Using this last result we can answer the question: what kind of universe is described by equation (4)? The ratio between material density and critical density specific to equation (4) is:

(13) $$\Omega' = 8\pi G\rho / 3H^2$$

The ratio of the expressions (12) and (13) can be calculated very simple and it is 3 / 2. Now knowing that the limit value of the expression (12) is one, obviously it can be inferred that the expression (13) is 2 / 3. So a subunit value, which tells us that the universe is expanding forever, in matter domination epoch. Of course this is a purely theoretical rough value, the difference from experimental values are reflected only in the absence of matter and energy in the universe.

3. Discussions

From the previous theoretical considerations we have seen that the evaluation of the expression (11) corresponds to a G variation specific to a static universe and the evaluation of expression (7) corresponds to a variation of the constant G in an expanding universe. We are now able to analyze from this perspective the experimental values listed in Table 1. Lower values of G constant variations than the reference value will be specific to a recessing universe, while higher values of G constant variations than the reference value will be specific to an expanding universe.

The expected results of the analysis are contained in Table 2. We have been expected that these findings to show an expansion of the universe they look exactly in most of the cases. If we not consider the inconclusive and inconsistent to the observed expansion of the universe results, the remaining results are quite encouraging. Only in one case it is clearly a recession, in reference [6]. But even here nothing is clear, because thereafter, [7], the same author reanalyzed the same data and finds contrary results to initial ones.

Where there is a recession and an expansion, this is not due to theoretical ranges than the determinations were made. Universal gravitational constant variance values resulted from the calculation, within a theory or another, from these experimental intervals, also in the form of intervals. Ironically, just these experimental measurements, analyzed by comparison with expression (11), are the cause of equivocal conclusions. We would have been more useful to our approach not some experimental intervals but just simple values.

4. Conclusions

Throughout this paper we tried to answer the question whether or not the universe is in expansion, to be consistent with experimental data, by analyzing some experimental data concerning the variation of universal gravitational constant. We have taken as reference the variation of this constant in a static universe and we report the experimental data to this reference. The universal gravitational constant variation was estimated in a so-called reference universe hypothesis. In this situation there is an expanding force that counterbalances the force of attraction, which is the basis of satisfying the Poisson equation through we have established a formal variation of the constant G, which was done in two cases. In the first case, which corresponds to an expanding universe, the variation of G helped us to relate to the reference in order to be able to establish what kind of universe it is. The second case, corresponding to a static universe is just the searched reference. Thus, the observational data lower than the reference values are related to an universe in recession, while the observational data higher than the reference values will reflect an expanding universe.

The analysis of experimental data led to some expected results. The observational data do meet our expected conclusions. The universe is expanding but and this is reflected in the observational values, all supposed to be the result of expansion, of the universal gravitational constant variation.

hypothesis	values	reference
Earth temperature	$dG/dt/G) < 2\times 10^{-11} yr^{-1}$	[1]
Expanding Earth	$(-dG/dt/G) \leq 8\times 10^{-12} yr^{-1}$	[2]
Earth-Moon system	$(dG/dt/G) = (2{,}6 \pm 15)\times 10^{-11}$	[3]
	$\lvert dG/dt/G \rvert < 2\times 10^{-11} yr^{-1}$	[4]
	$(dG/dt/G) = (-0{,}5 \pm 2)\times 10^{-11}$	[5]
	$(dG/dt/G) = (-8 \pm 5)\times 10^{-11} yr$	[6]
	$(dG/dt/G) = (3{,}2 \pm 1{,}1)\times 10^{-11}$	[7]
Solar System	$\lvert dG/dt/G \rvert < 4\times 10^{-10} yr^{-1}$	[8]
	$\lvert dG/dt/G \rvert < 1{,}5\times 10^{-10} yr^{-1}$	[9]
	$(dG/dt/G) = (-2 \pm 10)\times 10^{-12}$	[10]
	$(dG/dt/G) = (0{,}0 \pm 2{,}0)\times 10^{-12}$	[11]
	$\lvert dG/dt/G \rvert \leq 3\times 10^{-11} yr^{-1}$	[12]
	$\lvert dG/dt/G \rvert < 1{,}04\times 10^{-11} yr^{-1}$	[13]
	$\lvert dG/dt/G \rvert < 6\times 10^{-12} yr^{-1}$	[14]
	$\lvert dG/dt/G \rvert < 8\times 10^{-12} yr^{-1}$	[15]
	$\lvert dG/dt/G \rvert < 10^{-12} yr^{-1}$	[16]
	$\lvert dG/dt/G \rvert = (2 \pm 4)\times 10^{-12} yr^{-1}$	[17]
	$\lvert dG/dt/G \rvert < 3\times 10^{-11} yr^{-1}$	[18]
	$\lvert dG/dt/G \rvert < 10^{-11} yr^{-1}$	[19]
Pulsars	$(dG/dt/G) = (1{,}0 \pm 2{,}3)\times 10^{-11}$	[20]

	$(dG/dt/G) < (1{,}10 \pm 1{,}07) \times 10^{-}$	[21]
	$(dG/dt/G) = (-9 \pm 18) \times 10^{-12}\,y$	[22]
	$\lvert dG/dt/G \rvert < 10^{-10}\,yr^{-1}$	[23]
	$0 \le -(dG/dt/G) < 6{,}8 \times 10^{-11}\,yr$	[24]
	$0 \le -(dG/dt/G) < 5{,}5 \times 10^{-11}\,yr$	[25]
Stellar evolution	$\lvert dG/dt/G \rvert < 2 \times 10^{-11}\,yr^{-1}$	[26]
	$\lvert dG/dt/G \rvert < 1{,}6 \times 10^{-12}\,yr^{-1}$	[27]
	$(dG/dt/G) = (-0{,}6 \pm 4{,}2) \times 10^{-}$	[28]
	$0 \le -(dG/dt/G) < 4 \times 10^{-11}\,yr^{-}$	[29]
	$(dG/dt/G) = (-1{,}4 \pm 2{,}1) \times 10^{-1}$	[30]
	$0 \le -(dG/dt/G) < 4 \times 10^{-11}\,yr^{-}$	[31]
Nucleosynthesis	$\lvert dG/dt/G \rvert < (2 \pm 9{,}3) \times 10^{-12}\,yr$	[32]
	$\lvert dG/dt/G \rvert < 1{,}7 \times 10^{-13}\,yr^{-1}$	[33]
	$\lvert dG/dt/G \rvert < 9 \times 10^{-13}\,yr^{-1}$	[34]

Table 1

Reference	Expansion/Recession
[1]	E
[2]	E
[3]	E+R
[4]	E
[5]	E+R
[6]	R
[7]	E
[8]	E+R
[9]	E+R

[10]	E+R
[11]	E+R
[12]	E
[13]	E
[14]	E
[15]	E
[16]	E
[17]	E+R
[18]	E
[19]	E
[20]	E+R
[21]	E
[22]	E+R
[23]	E+R
[24]	E
[25]	E
[26]	E
[27]	E
[28]	E+R
[29]	E+R
[30]	E+R
[31]	E+R
[32]	E+R
[33]	E
[34]	E

Table 2

R E F E R E N C E S

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