Ramanujan's Radicals , Fractals

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Abstract

In this note we present some Fractals related with the polynomial equation :

$$\alpha^9 + 3\,\alpha^6 + 3\,\alpha^3 - 1 = 0$$

Keywords: Ramanujan's radicals , fractals , number pi

1. Introducción

Recordamos algunas fórmulas de Ramanujan :

$$\sqrt[3]{\sqrt[3]{2}-1} = \sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}} \tag{1}$$

$$\left(\frac{3-2\sqrt[4]{5}}{3+2\sqrt[4]{5}}\right)^{1/4} = \frac{\sqrt[4]{5}-1}{\sqrt[4]{5}+1} \tag{2}$$

$$\left(\sqrt[3]{5}-\sqrt[3]{4}\right)^{1/2} = \frac{1}{3}\left(\sqrt[3]{2}+\sqrt[3]{20}-\sqrt[3]{25}\right) \tag{3}$$

$$\left(\sqrt[3]{28}-\sqrt[3]{27}\right)^{1/2} = \frac{1}{3}\left(\sqrt[3]{98}-\sqrt[3]{28}-1\right) \tag{4}$$

El radical $x = \sqrt[3]{\sqrt[3]{2}-1}$ es solución de la ecuación : $x^9 + 3\,x^6 + 3\,x^3 - 1 = 0$.

2. Los Radicales : α , β

Sean α , β , los radicales definidos como :

$$\alpha = \sqrt[3]{\sqrt[3]{2} - 1} = \sqrt[3]{\frac{1}{9}} - \sqrt[3]{\frac{2}{9}} + \sqrt[3]{\frac{4}{9}} \tag{5}$$

$$\beta = \frac{1-\alpha}{1+\alpha} = -1 + \sqrt[3]{4}\left(1 - \sqrt[3]{\sqrt[3]{2} - 1} + \sqrt[3]{\left(\sqrt[3]{2} - 1\right)^2}\right) \tag{6}$$

Una fórmula con la constante pi : $\pi = 3.141592$ ...,

$$\pi = 4\tan^{-1}(\alpha) + 4\tan^{-1}(\beta) \tag{7}$$

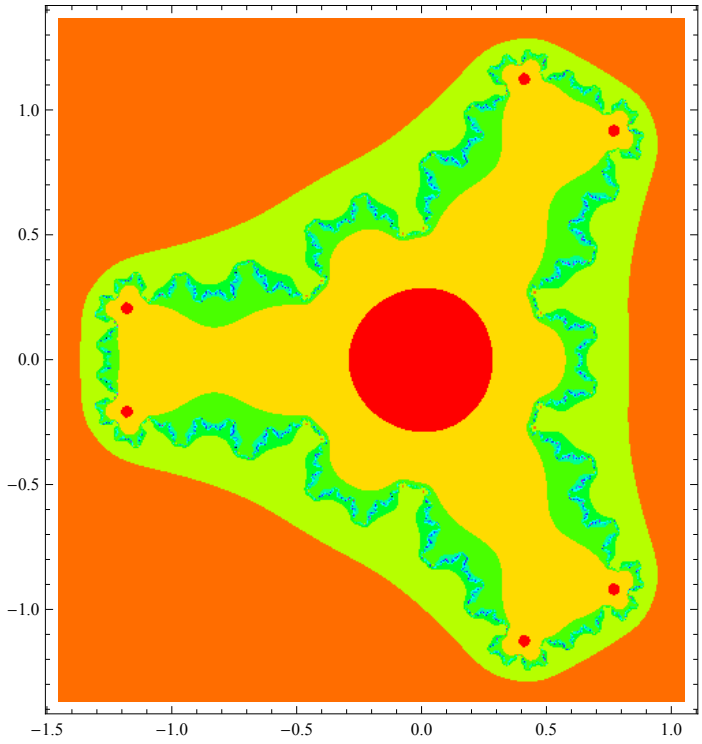
Los números α , β satisfacen las ecuaciones siguientes :

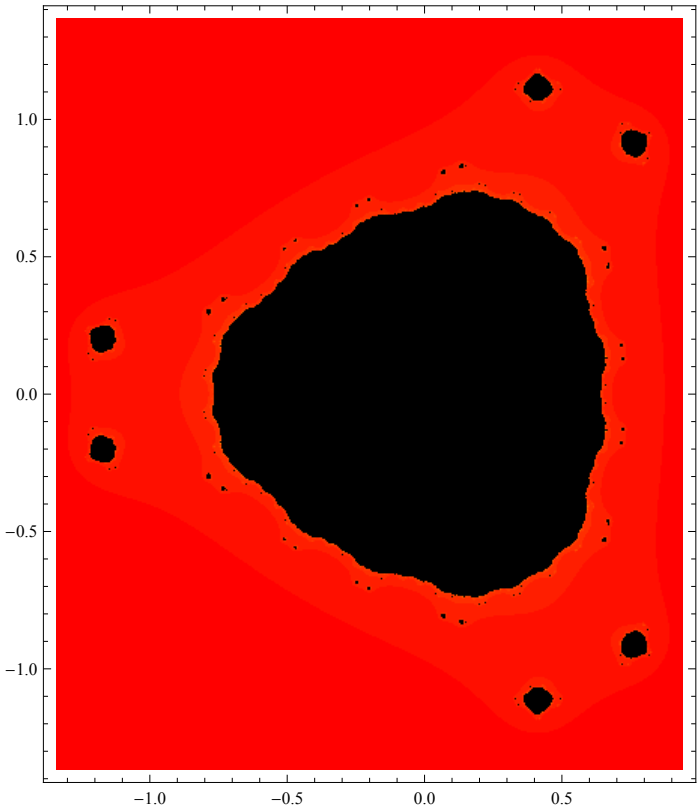
$$\alpha^9 + 3\alpha^6 + 3\alpha^3 - 1 = 0 \tag{8}$$

$$\beta^9 + 9\beta^8 + 36\beta^7 - 24\beta^6 + 126\beta^5 + 18\beta^4 + 84\beta^3 + 9\beta - 3 = 0 \tag{9}$$

Podemos poner (8) *y* (9) en la forma : $z = F(z)$ *y* obtener fractales asociados *a* $F(z)$.

3. Algunos fractales asociados a (8)

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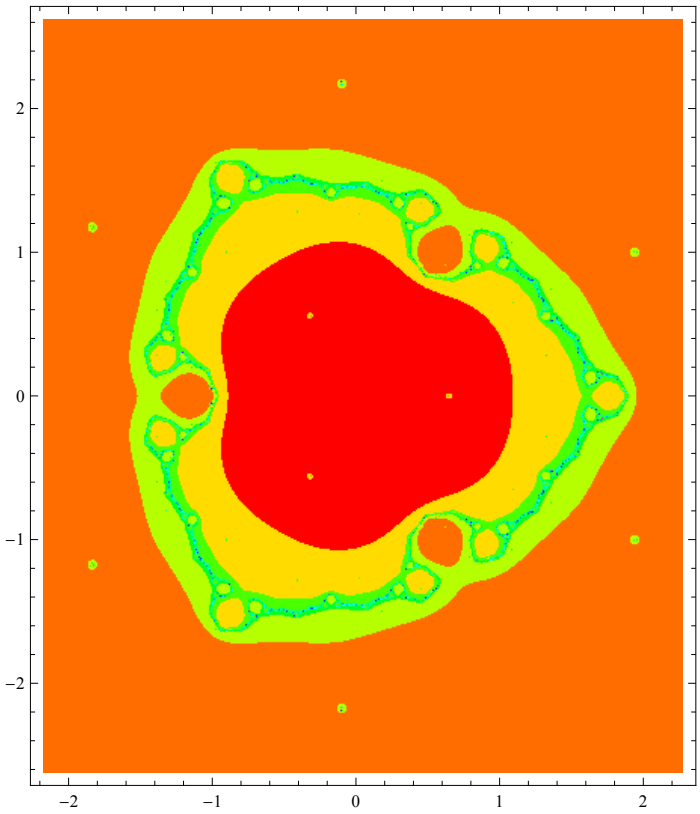
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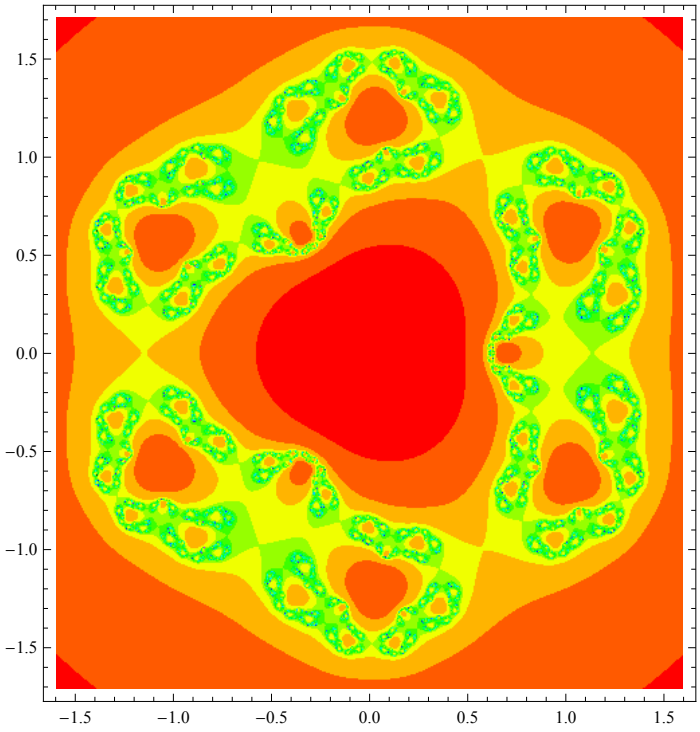
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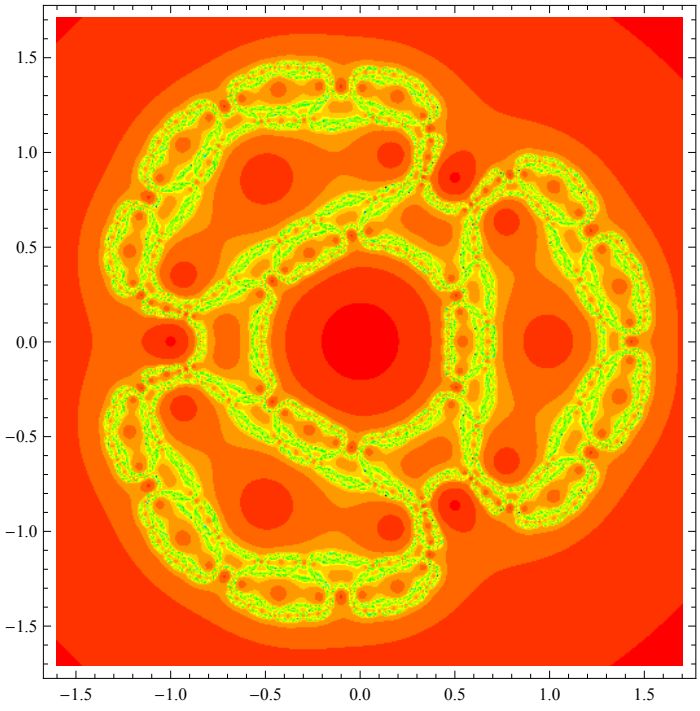
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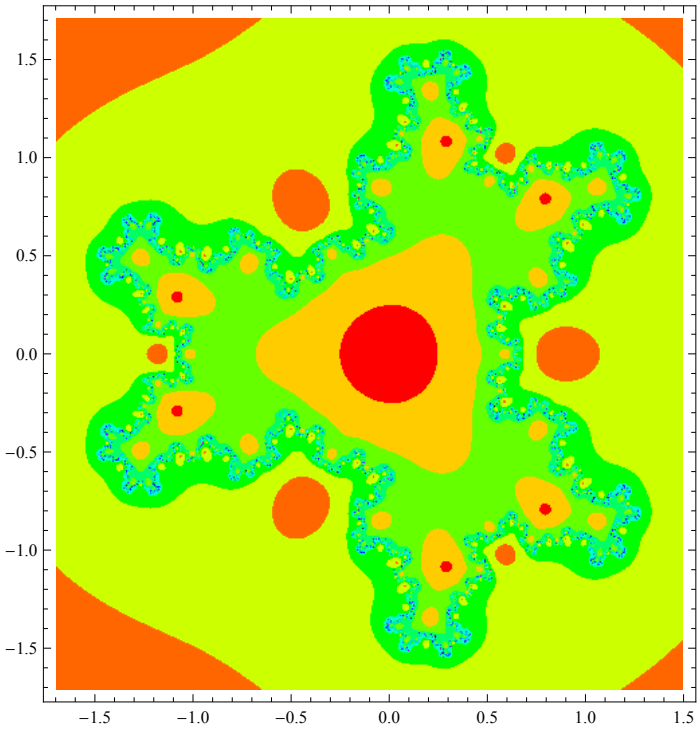
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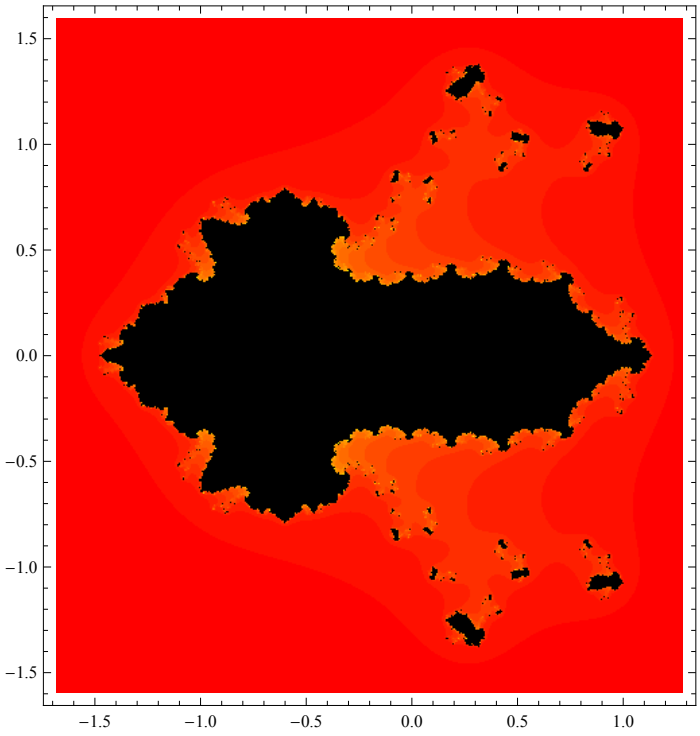
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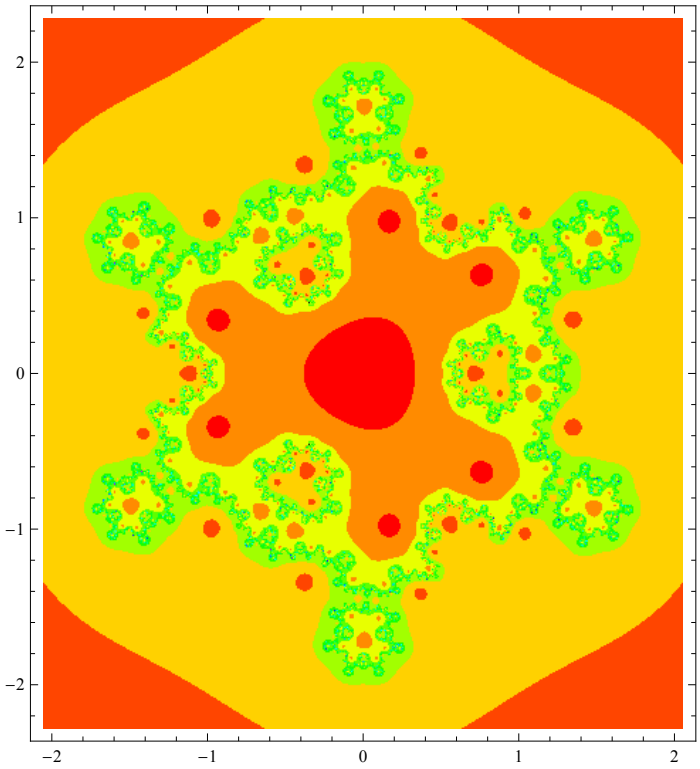
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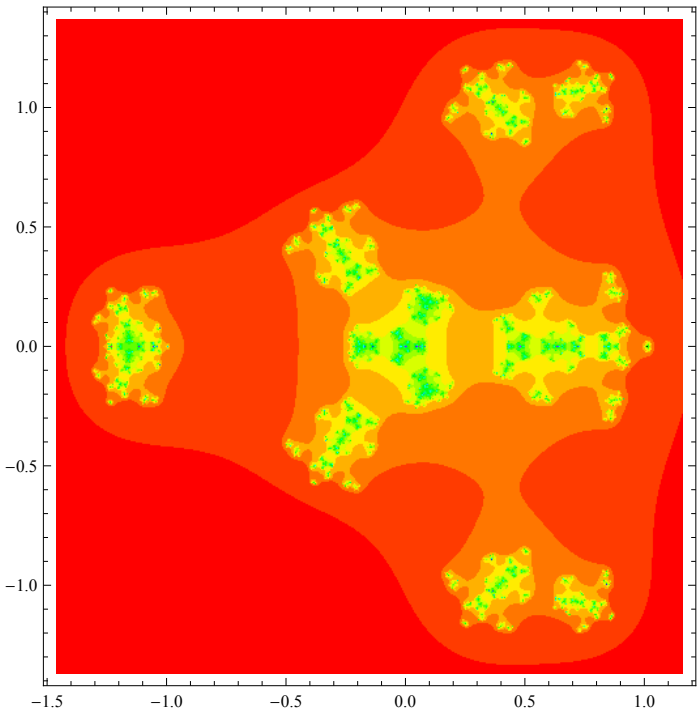
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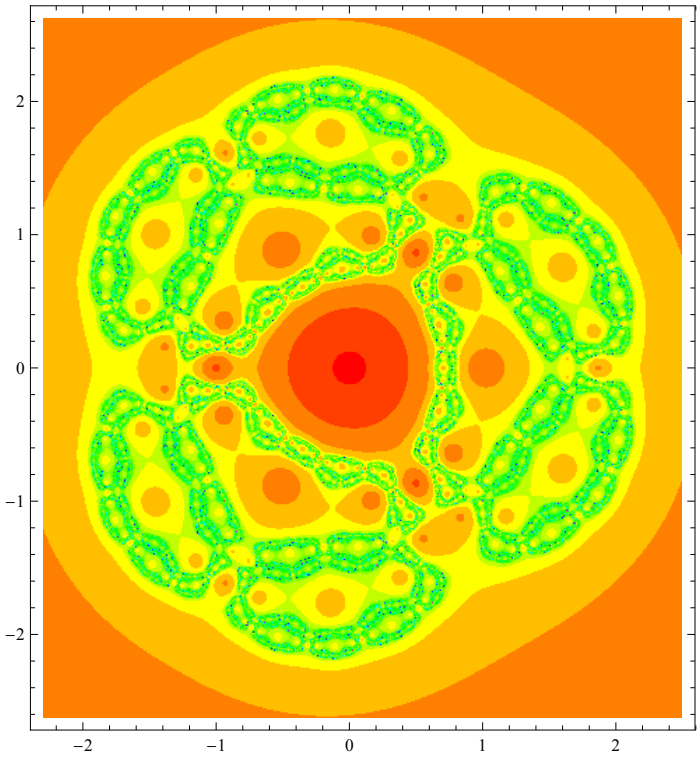

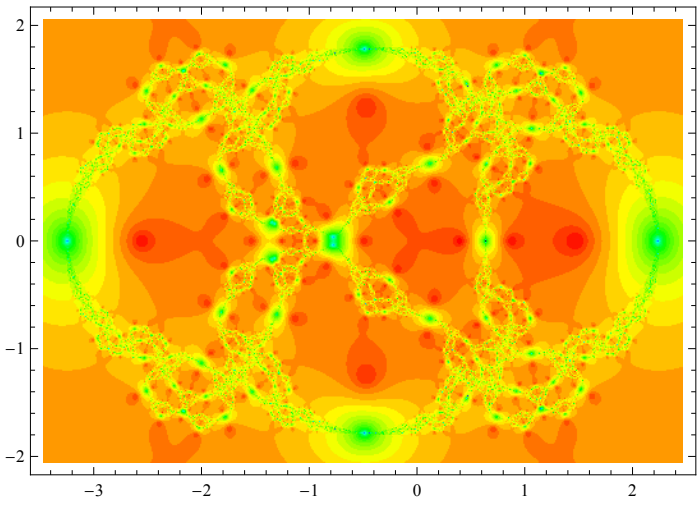

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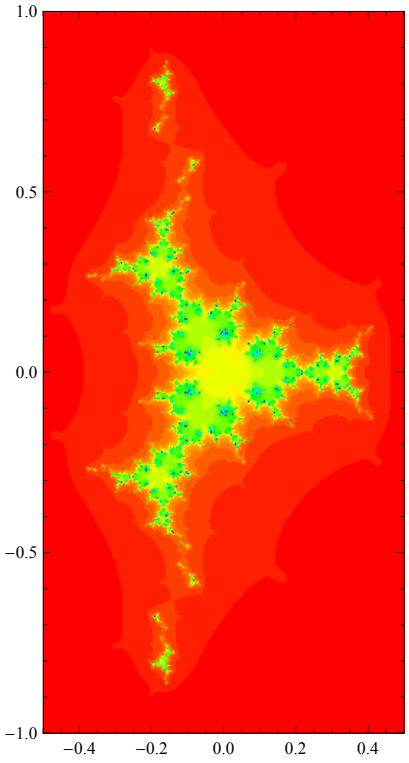
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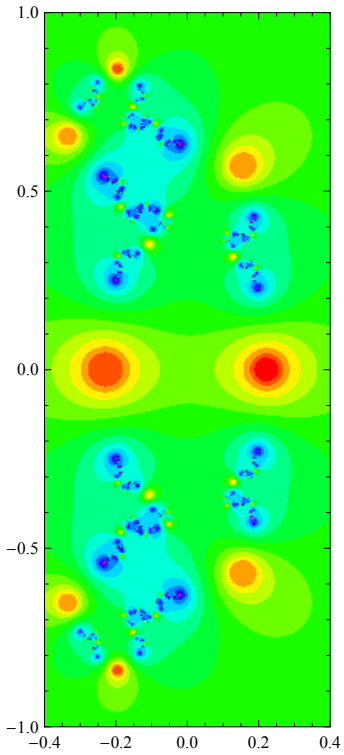
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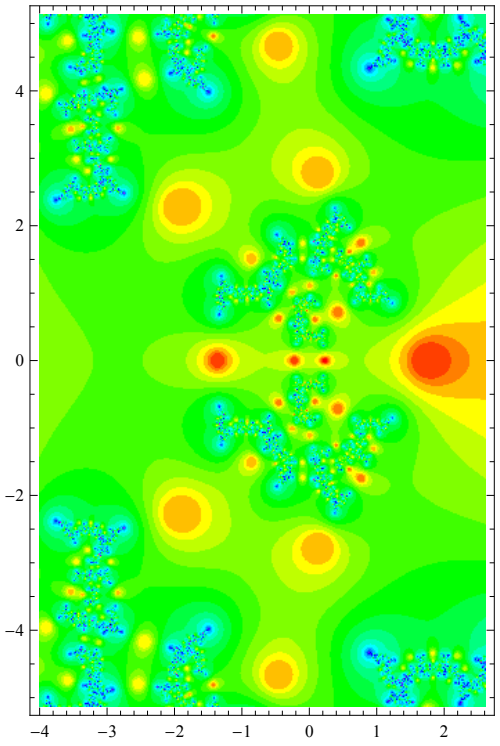
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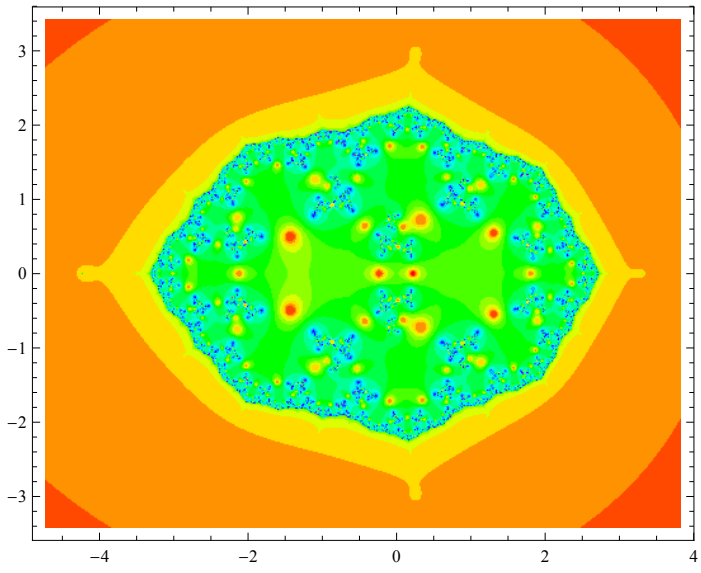
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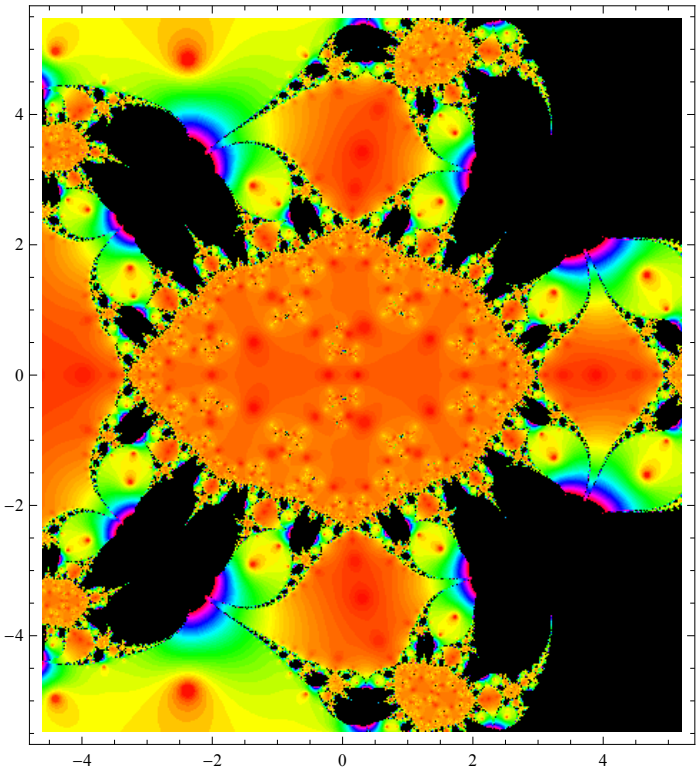

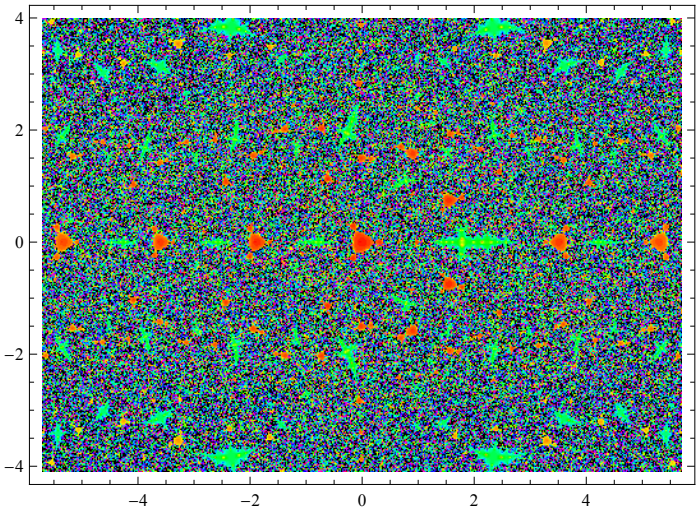

Referencia

A. Valdebenito , E. : Ramanujan's Radicals , unpublished note , 2016.