PHYSICAL PROPERTIES OF STARS AND STELLAR DYNAMICS

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ABSTRACT

The present study is an investigation of stellar physics based on observables such as mass, luminosity, radius, and photosphere temperature. We collected a dataset of these characteristics for 360 stars, and diagramed the relationships between their characteristics and their type (white dwarf, red dwarf, main sequence star, giant, supergiant, hypergiant, Wolf-Rayet, carbon star, etc.). For stars dominated by radiation pressure in the photosphere which follow the Eddington luminosity, we computed the opacity and cross section to photon flux per hydrogen nuclei in the photosphere. We considered the Sun as an example of star dominated by gas pressure in the photosphere, and estimated the density of the solar photosphere using limb darkening and assuming the adiabatic gradient of a monoatomic gas. We then estimated the cross section per hydrogen nuclei in the plasma of the solar photosphere, which we found to be about $2.66 \times 10^{-28} \, m^2$, whereas the cross section of neutral hydrogen as given by the Bohr model is $8.82 \times 10^{-21} \, m^2$. This result suggests that the electrons and protons in the plasma are virtually detached. Hence, a hydrogen plasma may be represented as a gas mixture of electrons and protons. If the stellar photosphere was made of large hydrogen atoms or ions such as the ones we find in gases, its surface would evaporate due to the high temperatures.

Keywords: photosphere; radiation pressure; stars

1. INTRODUCTION

The present study is an investigation of stellar physics based on characteristics such as mass, luminosity, radius, and photosphere temperature. We analysed a set of 360 stars for which we collected available data from the literature. The set included white dwarfs, red dwarfs, main sequence stars, giant stars, Wolf-Rayet stars, carbon stars, etc. Let us introduce the basics to get a sense of how stars regulate fusion reactions and the basic principles of stellar dynamics.

We can easily infer that stellar equilibrium is driven by hydrostatic pressure. The internal pressure of a star is determined by the radiation pressure and gas pressure, which counterbalance the hydrostatatic pressure from gravitation and prevent the star from collapsing. Radiation pressure and gas pressure are temperature dependent. When a star cools, it experiences a drop in internal pressure that causes the star to contract. This contraction will cause an increase in the hydrostatic pressure within the star. The gravitational force exerted by the inner mass of the star on a particle at a given radius is $F_g = \frac{GM_r m_p}{r^2}$, where r is the radius, M_r the interior mass of the star up to radius r, m_p the mass of the particle, and G the gravitational constant. Therefore, the more the star contracts, the higher the hydrostatic pressure. The increase in hydrostatic pressure increases the rate of fusion, which produces excess heat. In return, this excess heat increases the gas and radiation pressure in the star causing the star to expand. This process repeats until the star reaches a certain equilibrium.

Nuclear fusion, therefore, is driven by the hydrostatic pressure in stars. There are three possible mechanisms by which hydrostatic pressure could affect the fusion power of stars:

- Assuming that a minimum pressure or temperature is required to sustain fusion, the volume of the fusing core increases as hydrostatic pressure increases. According to the Arrhenius equation, reaction kinetics are highly dependent on temperature. Note that the Arrhenius equation assumes the Maxwell-Boltzmann distribution, and the relationship would be different for a Fermi gas.

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- The density in the core of the star increases as hydrostatic pressure increases. Hence, a larger quantity of matter would be subject to fusion in the core of the star.

- The kinetic rate of fusion (i.e. the reaction rate or speed) may increase as pressure increases.

These are the mechanisms we propose regulate a star. In some instances the volume and luminosity of the star oscillates. These are the so-called variable stars. A notable example of variable stars are the Cepheid variables. They are known for a method to measure distances based on the period of their oscillation. As there is a relationship between the period of the star's oscillations and its luminosity, one can infer the intrinsic luminosity and compute the distance. Several different theories explain the oscillations of variable stars. We enumerate some possible mechanisms below:

- The κ-mechanism or Eddington valve is the most popular theory explaining variable Cepheids (Smith 1984). According to this theory, doubly ionized helium is more opaque than single ionized helium. As helium in the star heats, it becomes more ionized and less transparent so that the heat is retained longer. As the star expands, it cools and its helium becomes less ionized and hence more transparent, allowing the heat to escape. Then the star contracts again and the process repeats.

- Another mechanism would be a change in the regime of the fusion reactions for certain thresholds in the hydrostatic pressure of the star. For example, fusion of heavier elements in the core of large stars could ignite at a certain temperature threshold and produce large temperature spikes causing the star to oscillate. This theory would be applicable to massive stars where fusion of heavy elements in the core occurs.

- The ageing model of the core could also explain variable stars. Let us consider a star fusing hydrogen into helium when the star has too low a mass to ignite helium fusion. As the star ages, the helium core grows, and the shell of fusing hydrogen around the core thins. Let us say the hydrogen shell heats the core, making it expand and push the hydrogen shell to the exterior; the temperature of the shell would fall below the ignition point, and switch off hydrogen fusion. Then the core would cool, returning the hydrogen shell to the ignition point and switching hydrogen fusion on again. This pattern would repeat in cycles. This theory would apply to stars with small cores and explain the type II Cepheids, which have about half the mass of the Sun and therefore are not massive enough to fuse helium.

- Temperature driven kinetics for fusion reactions may also induce stellar oscillations. If the kinetic rate of fusion increases as temperature increases, a small increase in temperature at the core would cause large temperature spikes. Then the star would expand over a long period of time before cooling and contracting again. Note that this process would cause stars to be unstable. The fact that the Sun is stable with very low oscillations of order 0.1 % of its luminosity would be a counter example of temperature driven fusion kinetics, unless the sensitivity of the fusion-kinetic rate with respect to temperature is very small.

When a star has exhausted the nuclear supply at its core, it will cool. This will eventually trigger a gravitational collapse. When the star contracts, the depleted nuclear fusion at its core would not be able to counterbalance the hydrostatic pressure. As the radius of the star diminishes, the gravitational force acting on the particles of the star increases proportionally to $\frac{1}{r^2}$. The gravitational collapse of the star can lead to the formation of a black hole on one extreme or a supernova at the other. The latter occurs if at a certain point during the collapse the pressure is so high that it triggers fusion reactions in series at a very fast rate, causing the star to explode and leading to the formation of up to the heaviest elements of the Mendeleev table such as uranium. A black hole would form if fusion does not halt the gravitational collapse. In some instances gravitational collapse stops before the formation of a black hole, producing a neutron star or white dwarf. These are intermediary stages before the formation of a black hole. White dwarfs are less dense than neutron stars, at an earlier stage of matter compression than neutron stars. Neutron stars are composed of neutronium, a compact pack of neutrons, and have densities around $4 \times 10^{17}\, kg/m^3$. White dwarfs have densities around 10^7 to $10^{10}\, kg/m^3$. Electron degeneracy pressure is the mechanism which supposedly prevents the further collapse of white dwarfs. Degeneracy of matter from gravitational collapse starts at the core of the star. Sometimes the core of the star collapses into a neutron star or a black hole while the outer shell of the star explodes into a supernova. Red giants of masses comparable to the Sun generally blow out their outer layer at the end of their life to form planetary nebulae, leaving a white dwarf in the core.

We find that stellar photosphere dynamics are crucial in the determination of the power of stars as measured by their luminosity. We cannot miss the notable work of Arthur Eddington on the dynamics of stars dominated by radiation

pressure in the photosphere, according to which, the luminosity of such stars is proportional to their mass. Using data from stars dominated by radiation pressure in their photosphere, we can estimate the opacity parameter. We also discuss models and factors which may affect opacity, as this is a preponderant parameter for radiative heat transfer, a key component of stellar models. For stellar models we also need boundary conditions such as the density of the photosphere. We show how to estimate the density of the solar photosphere using limb darkening. According to the standard solar model, there is a layer at the surface of the Sun where radiative heat transfer is not efficient enough and convection takes place. The photosphere can be viewed as a plasma surface; hence using a model of the surface we can compute the cross section per hydrogen nuclei in the photosphere. We computed the cross section per hydrogen nuclei from radiation pressure and gas pressure, and found that both values match closely. From the cross section per hydrogen nuclei we obtained, we can infer that in stellar plasma the electrons and nuclei are virually detached. Therefore, stellar plasma may be represented as a gas mixture of electrons and nuclei. We discuss the modelling implications of this representation of stellar plasma.

2. OVERVIEW OF STELLAR DATA

Stars form a very heterogeneous group having various luminosities, masses, temperatures, and densities. In the below diagrams we show the relationships between these characteristics for the stars in our catalog. In section 2.1 we introduce the classification of stars we used for the diagrams. Section 2.2 shows the stellar diagrams we obtained with a emphasise on their interpretation.

2.1. *Classification of stars*

Stars can be classified according to their spectra, color, and size. Stellar spectra provide precious information about their atmospheric composition by analyzing their spectral lines, and surface temperature from Planck's law of black-body spectrum. We divided the stars in our catalog according to the below groups:

- White dwarfs are degenerated stars which are very dense and composed mostly of electron-degenerate matter. They have masses comparable to that of the Sun, volumes comparable to that of Earth, and are very faint. Some white dwarfs are classified as helium stars as they have very strong helium lines and weak hydrogen lines (Liebert et al. 2004).

- Brown dwarfs have masses comprised in the range of 13 to 80 Jupiter masses. Their mass is below the threshold needed to fuse hydrogen, but enough to fuse deuterium.

- Red dwarfs have masses in the range of 0.075 to 0.6 solar masses, and surface temperatures below 4,000 K. A count the stars nearest to earth, it was estimated that red dwarfs comprise about 80% of the stars in the Milky Way.

- Yellow dwarfs are main-sequence stars of comparable mass to the Sun, with a surface temperature between 5,300 and 6,000 K. We created a broader group that we called yellow main sequence stars to include all stars with masses between 0.6 and 1.7 solar masses, and a temperature between 4,200 and 7,200 K.

- A-type stars are main-sequence stars of spectral type A of 1.4 to 2.1 solar masses, and a surface temperature between 7,600 and 11,500 K. Their spectra have strong hydrogen Balmer absorption lines.

- B-type stars are main-sequence stars of 2 to 16 solar masses, and a surface temperature between 10,000 and 30,000 K. Their spectra have non-ionized helium lines.

- Subgiants are stars at an intermediary stage of evolution before becoming giants. These stars are brighter than main-sequence stars but not as bright as giants.

- Red giants are evolved stars of 0.8 to 8 solar masses which have exhausted the hydrogen supply in their core and are fusing helium into carbon. They have high luminosities compared to their main-sequence peers, and inflated atmospheres making their radii large, resulting in low surface temperatures between 3,200 and 4,000 K. Orange giants are distinguished from red giants by their temperature, which ranges from 4,000 to 5,500 K.

- Carbon stars are red giants whose atmosphere contains more carbon than oxygen.

- S-type stars are giant stars with approximately equal quantities of carbon and oxygen. These are intermediaries between giants and carbon stars.

- Blue giants are hot giant stars with masses in the range of ten to hundreds of solar masses, and surface temperatures between 22,000 and 45,000 K.

- Supergiants are stars with luminosities between those of the giants and hypergiants on the Hertzsprung-Russell diagram. They are divided into red supergiants, orange supergiants, and blue supergiants according to their surface temperatures. The red ones have surface temperatures between 3,200 and 4,000 K, the orange ones between 4,000 and 7,000 K, and the blue ones between 7,000 and 50,000 K.

- Hypergiants are stars with tremendous luminosities on the high end of the Hertzsprung-Russell diagram. They are divided into red hypergiants, yellow hypergiants, and blue hypergiants according to their surface temperatures. The temperature ranges are the same as for supergiants with the yellow group replacing the orange stars of the supergiant category.

- Wolf-Rayet stars are evolved massive stars which are fusing helium or heavier elements in the core. They have spectra showing broad emission lines of highly ionized helium and nitrogen or carbon. Most Wolf-Rayet stars have lost their outer hydrogen and have an atmosphere predominantly made of helium. Their surface temperature ranges between 30,000 and 200,000 K. A subgroup of Wolf-Rayet stars referred to as WO stars have strong oxygen emission lines, indicating the star is on the oxygen sequence.

2.2. Stellar diagrams

In the current section we display several diagrams showing the relationship among the characteristics of stars along with their classification. Figure 1 shows the relationship between the luminosity and mass of stars, Figure 2 the relationship between the volume and the luminosity of stars, and Figure 3 the relationship between the average density of stars and temperature of the photosphere.

Figure 1 shows that red giants are much more luminous than their main-sequence star counterparts for the same mass. As red giants are evolved stars which fuse helium in the core, we can infer that the fusion of helium into carbon is much more exothermic than the fusion of hydrogen into helium. Red giants are also less dense than their main-sequence counterparts, meaning that helium fusion occurs in a domain at lower pressure than hydrogen fusion and produces more heat. In Figure 2, we see that main sequence stars expand when shifting on the helium burning sequence to form red giants, and contract when shifting from the main-sequence branch to Wolf Rayet stars. For Wolf-Rayet stars which fuse helium or heavier elements in the core, fusion occurs in a domain at higher pressure than their counterparts. This is especially pronounced for OW Wolf-Rayet stars on the oxygen sequence, where the fusion pressure domain is clearly higher than for helium fusion. There are also mass threadshods for fusion to occur. For example, red giants of mass less than 0.9 solar mass are never observed. This limit is commonly attributed to the age of the universe, because low mass main-sequence stars take longer to fuse the hydrogen in their core, and therefore it is hypothesized that stars below 0.9 solar masses did not have sufficient time to become red giants. However, this limit could also represent the minimum mass required to obtain the necessary conditions for helium fusion. Similarly, Wolf-Rayet stars have masses above the 8.0-9.0 solar mass limit. Thefore, low mass stars do have the necessary conditions to fuse elements heavier than helium in the core.

The red dwarfs in Figure 1, show a distribution in their luminosities. This might be due to ageing, as red dwarfs haven't sufficient mass to fuse the helium accumulating in their core. As a star exhausts its hydrogen supply and accumulates helium in its core, the core cools and contracts. As the core contracts, a new shell of fresh hydrogen fuel is formed at the periphery of the core. Fusion of this hydrogen shell maintains the temperature of the core, preventing it from contracting further. The fact that the atomic mass of helium is greater than that of hydrogen also plays a role. Helium nuclei are formed of four nucleons (two protons and two neutrons). Therefore, there is four times more mass in a helium gas than in a hydrogen gas at a given pressure, provided they obey the ideal gas law. As the star gets older, the core shrinks and grows ever denser by accumulating helium. Therefore, as red dwarfs age, they should become denser and less luminous. Common stellar age-dating methods, based on the main-sequence turnoff, are focused on main-sequence stars that become red giants. Such age calculation methods do not yield stellar ages older than about 15 billion years, perhaps because this is when a solar type main-sequence star becomes a red giant. No methods have been developed so far to estimate the age of red dwarfs, which could possibly be much older. Using stellar models would be an approach for age-datating of red dwarfs.
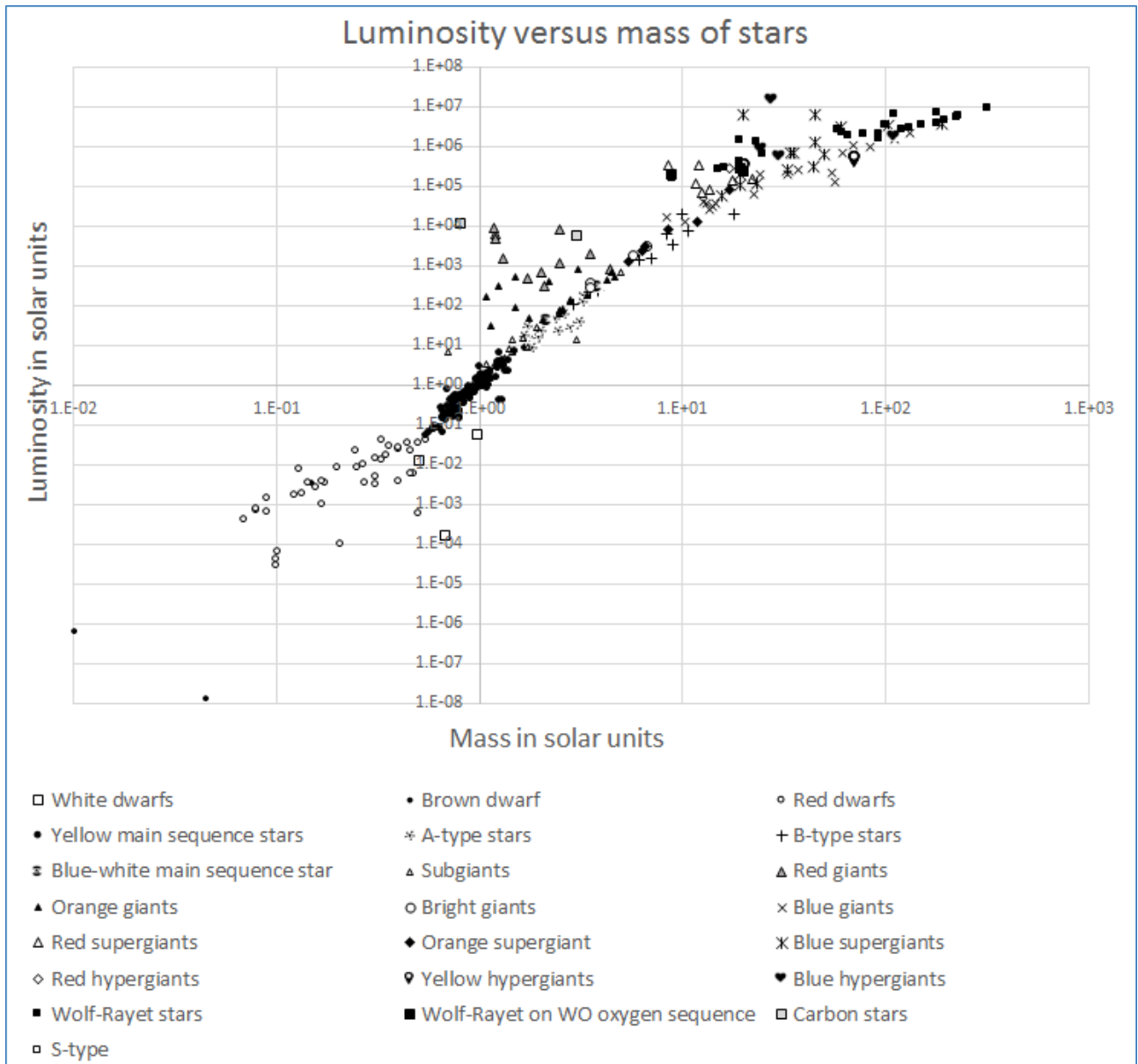
Luminosity versus mass of stars

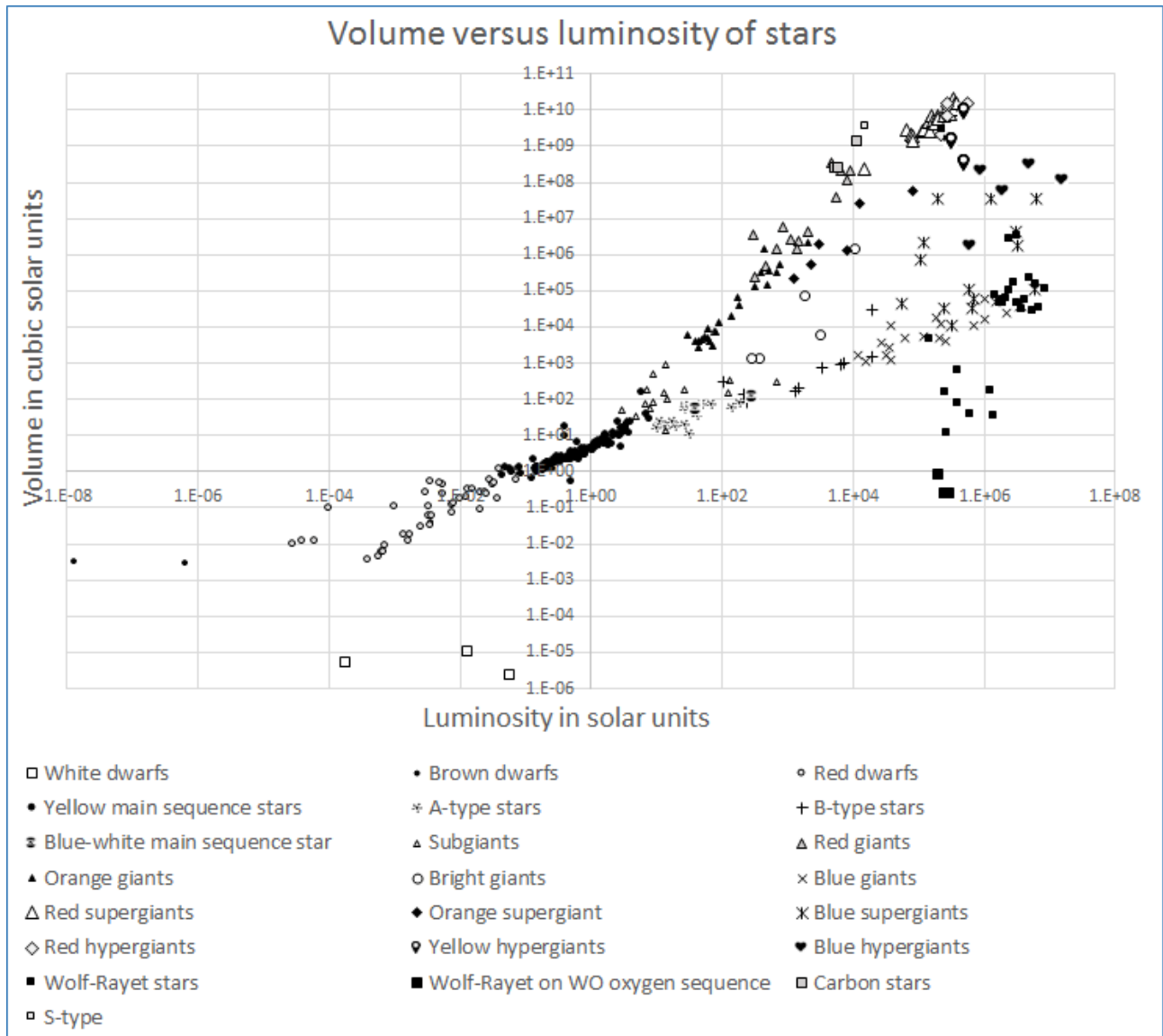**Figure 1**. Luminosity versus mass of stars. Mass and luminosity are in solar units.

Figure 2. Volume versus luminosity of stars. Volume and luminosity are in solar units.
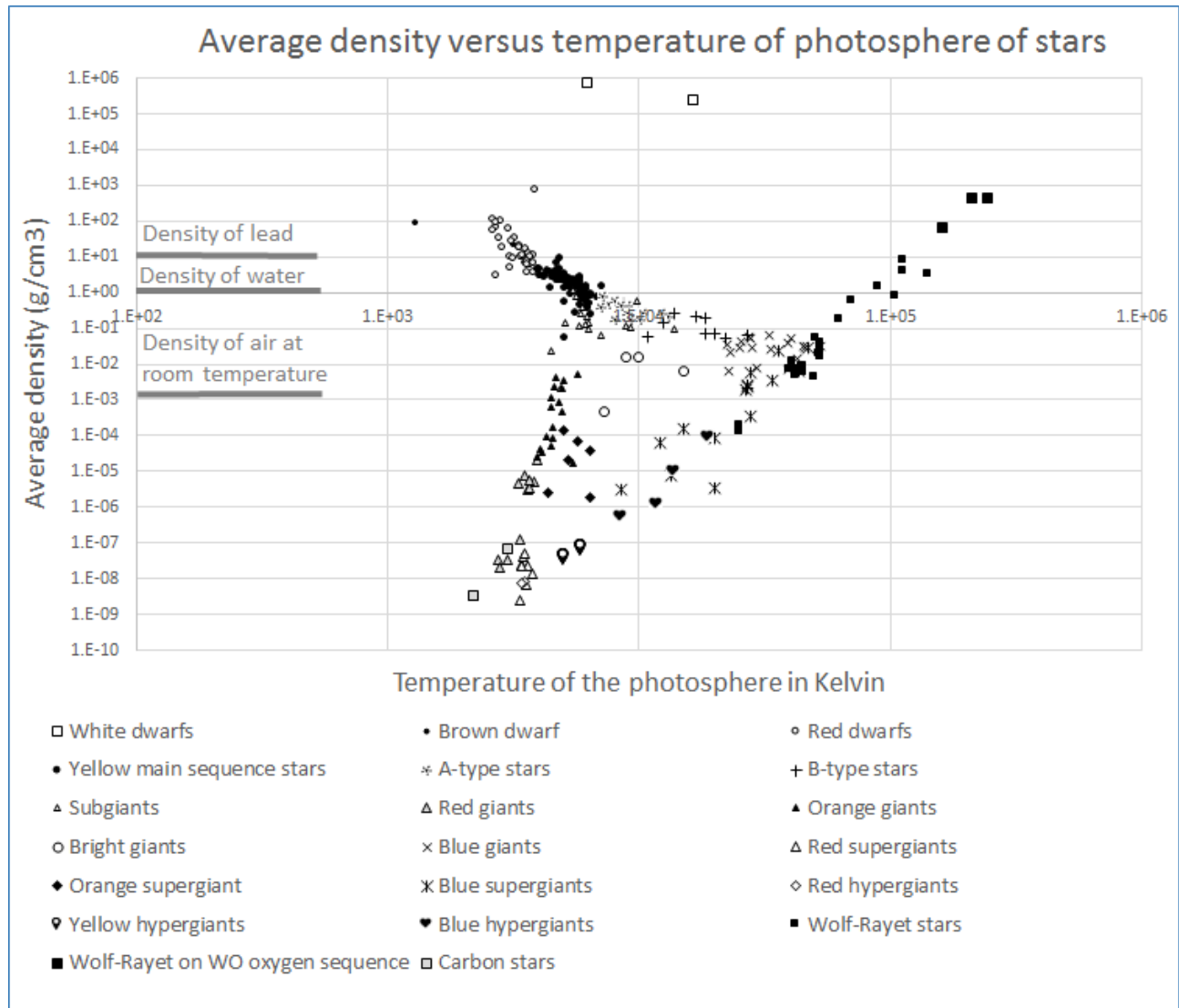
Average density versus temperature of photosphere of stars

Legend:
- □ White dwarfs
- • Brown dwarf
- ○ Red dwarfs
- • Yellow main sequence stars
- ✳ A-type stars
- + B-type stars
- ▵ Subgiants
- △ Red giants
- ▲ Orange giants
- ○ Bright giants
- × Blue giants
- △ Red supergiants
- ◆ Orange supergiant
- ✳ Blue supergiants
- ◇ Red hypergiants
- ⦿ Yellow hypergiants
- ♥ Blue hypergiants
- ■ Wolf-Rayet stars
- ■ Wolf-Rayet on WO oxygen sequence
- □ Carbon stars

Figure 3. Average density versus temperature of the photosphere of stars. Density is given in g/cm^3, and temperature in Kelvin.

3. STARS DOMINATED BY RADIATION PRESSURE IN THE PHOTOSPHERE

3.1. *Eddington luminosity*

Inside a star, the internal pressure acting against the hydrostatic pressure is the sum of the radiation pressure and gas pressure, hence:

$$P = \rho_n kT + \frac{1}{3}aT^4 \,, \tag{1}$$

where $\rho_n = \frac{N}{V}$, N is the number of molecules in the gas, V is the volume, $a = \frac{4\sigma}{c}$ is the radiation constant, k is the Boltzmann constant, σ is the Stefan-Boltzmann constant, T is the temperature, and c the speed of light.

When the radiation pressure is considerably higher than the gas pressure, the gas pressure term can be neglected, therefore we get:

$$\frac{\partial P_r}{\partial T} = \frac{4}{3}aT^3 \,, \tag{2}$$

The equation for radiative heat transfer is expressed as follows:

$$\frac{\partial T}{\partial r} = -\frac{3}{4}\frac{1}{ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2} \,, \tag{3}$$

where κ is the opacity, L is the luminosity, T is the temperature, r is the radius, ρ is the density, c is the speed of light, and a the radiation constant.

Rewriting (2), we get:

$$\frac{\partial P_r}{\partial r}\frac{\partial r}{\partial T} = \frac{4}{3}aT^3 \,, \tag{4}$$

Combining (3) and (4) we get:

$$\frac{\partial P_r}{\partial r} = -\frac{\kappa\rho}{c}\frac{L}{4\pi r^2} \,, \tag{5}$$

From hydrostatic equilibrium:

$$\frac{\partial P}{\partial r} = -\frac{GM_r\rho}{r^2} \,, \tag{6}$$

where G is the gravitational constant, M_r is the interior mass of the star at radius r, and ρ is the density.

By combining (5) and (6) we get:

$$L = \frac{4\pi cG}{\kappa}M \,, \tag{7}$$

which is the Eddington luminosity. Stars dominated by radiation pressure in their photosphere are fully determined by the photosphere, meaning that their luminosities will adjust to match the Eddington luminosity. For such stars luminosity is proportional to mass as shown by the Eddington luminosity equation. Should excess heat be generated, the star will lose matter through its photosphere, which may explain why many Wolf-Rayet stars have lost their outer hydrogen layer.

We can also express this equation in terms of temperature using Stefan-Boltzmann as:

$$Flux = \frac{L}{4\pi r^2} = \sigma T^4 \,, \tag{8}$$

Hence, combining (7) and (8), we get:

$$T = \left(\frac{cG}{\kappa\sigma}\right)^{1/4}\frac{M^{1/4}}{R^{1/2}} \,, \tag{9}$$

3.2. *Cross section of an hydrogen ion from photon flux*

There are two different methods to calculate the cross section of an ion exposed to photon flux in the photosphere; these are known respectively as the optical and the radiation pressure cross section approaches.

The optical cross section calculation considers the obscuration of a radiative flux travelling in an isotropic medium. Let us consider an isotropic gas with a radiative flux going through a surface A in the x-direction orthogonal to the surface. The flux at step $x + dx$ is equal to the flux at step x multiplied by one minus the proportion of the area that is obscured by the cross section of the atoms in the volume Adx. The number of atoms in the volume Adx is $\rho_n Adx$. We multiply the number of atoms in the volume by the cross section of the atom σ_p to give the total area obscured by the gas. Hence, we get:

$$F(x + dx) = F(x)\left(1 - \sigma_p \rho_n dx\right), \tag{10}$$

where $F(x)$ is the flux at step x, $F(x + dx)$ is the flux at step $x + dx$, ρ_n is the density in number of particles per volume, and σ_p is the cross section per particle.

As $dF = F(x + dx) - F(x)$, we get:

$$\frac{dF}{F} = -\sigma_p \rho_n dx, \tag{11}$$

We integrate (11) to obtain:

$$F(x) = F_0 \exp(-\sigma_p \rho_n x), \tag{12}$$

The opacity is defined from the attenuation of radiation intensity through a medium and is given by $I(x) = I_0 \exp(-\kappa \rho x)$, where I is the intensity, therefore:

$$\kappa = \frac{\sigma_p}{m_p}, \tag{13}$$

where κ is the opacity, σ_p is the cross section of a particle, and m_p is the mass of a particle.

The radiation pressure cross section considers an ion above the surface of a star. Let us assume that the ion is in equilibrium, meaning that the gravitational force exerted by the star on the atom is equal to the radiation pressure from the radiation flux coming from the surface of the star times the cross section of the ion. Therefore, we get:

$$\frac{GMm_p}{R^2} = \sigma_p \frac{1}{3} aT^4, \tag{14}$$

where G is the gravitational constant, M the mass of the star, R the radius of the star, m_p the mass of an ion, σ_p the cross section of an ion, T the temperature, and a the radiative constant.

Note that the radiation pressure just above the surface is the same as the radiation pressure below the surface. This can be proven but is outside scope of our discussion.

Combining (9) and (14) we get:

$$\kappa = \frac{4}{3} \frac{\sigma_p}{m_p}, \tag{15}$$

This equation differs slightly from (13) due to factors introduced in the derivation of the radiative heat transfer equation (3). The factor 3/4 in equation (3) comes from the fact that a collimated radiation flux was used to compute the radiation pressure dependency on the flux (Salaris & Cassisi 2005). The two cross section calculation approaches provide a consistency check across the different models. We see that the optical and radiation pressure cross sections mean the same thing; it is the cross section of an ion exposed to photon flux.

3.3. *Opacity and cross-section calculations*

Now let us confront the model for stars dominated by radiation pressure in the photosphere with actual data. The stars dominated by radiation pressure must be those with low average densities and high photosphere temperatures and include the most massive stars. We included in this group blue giants, carbon stars, all the supergiants and hyper giants (red to blue), and all the Wolf-Rayets. Then we did a linear regression of photosphere temperature against $M^{1/4}R^{-1/2}$, where temperature is in Kelvin, mass M in kilograms, and radius R in meters (see figure 4). We obtained

a linear equation with slope $\alpha = 35.87 \, [Kkg^{-1/4}m^{1/2}]$ and determination coefficient R^2 standing at 93%. Using the formalism of equation (14), we obtain the below cross section to particle mass ratio function of the slope α:

$$\frac{\sigma_p}{m_p} = \frac{3G}{a\alpha^4} , \tag{16}$$

The cross section σ_p expresses the surface of the ion exposed to photon flux.

By considering a hydrogen ion having a mass $m_p = 1.67 \times 10^{-27} \, kg$, we obtain a cross section $\sigma_p = 2.67 \times 10^{-28} \, m^2$. This cross section is equal to four times the Thomson cross section for the scattering of a free electron by radiation. The Thomson cross section of free electron scattering is expressed as follows:

$$\sigma_T = \frac{8\pi}{3} \left(\frac{q^2}{4\pi\epsilon_0 mc^2} \right)^2 = 6.65 \times 10^{-29} \, m^2 , \tag{17}$$

where q is the charge of the electron, ϵ_0 is the permittivity of free space , m is the mass of the electron, and c is the speed of light.

For comparison purpose, the radius of a proton is about $8.8 \times 10^{-16} \, m$, which works out to a cross section of $2.43 \times 10^{-30} \, m^2$, which is about hundred times less than the cross section we computed. The radius of a hydrogen atom from the Bohr model is about $5.3 \times 10^{-11} \, m$, or a cross section of $8.82 \times 10^{-21} \, m^2$, which is about 33 million times larger than the cross section we computed. In contrast, the cross section of hydrogen ion exposed to photon flux we computed is four times the Thomson cross section for the scattering of free electrons.
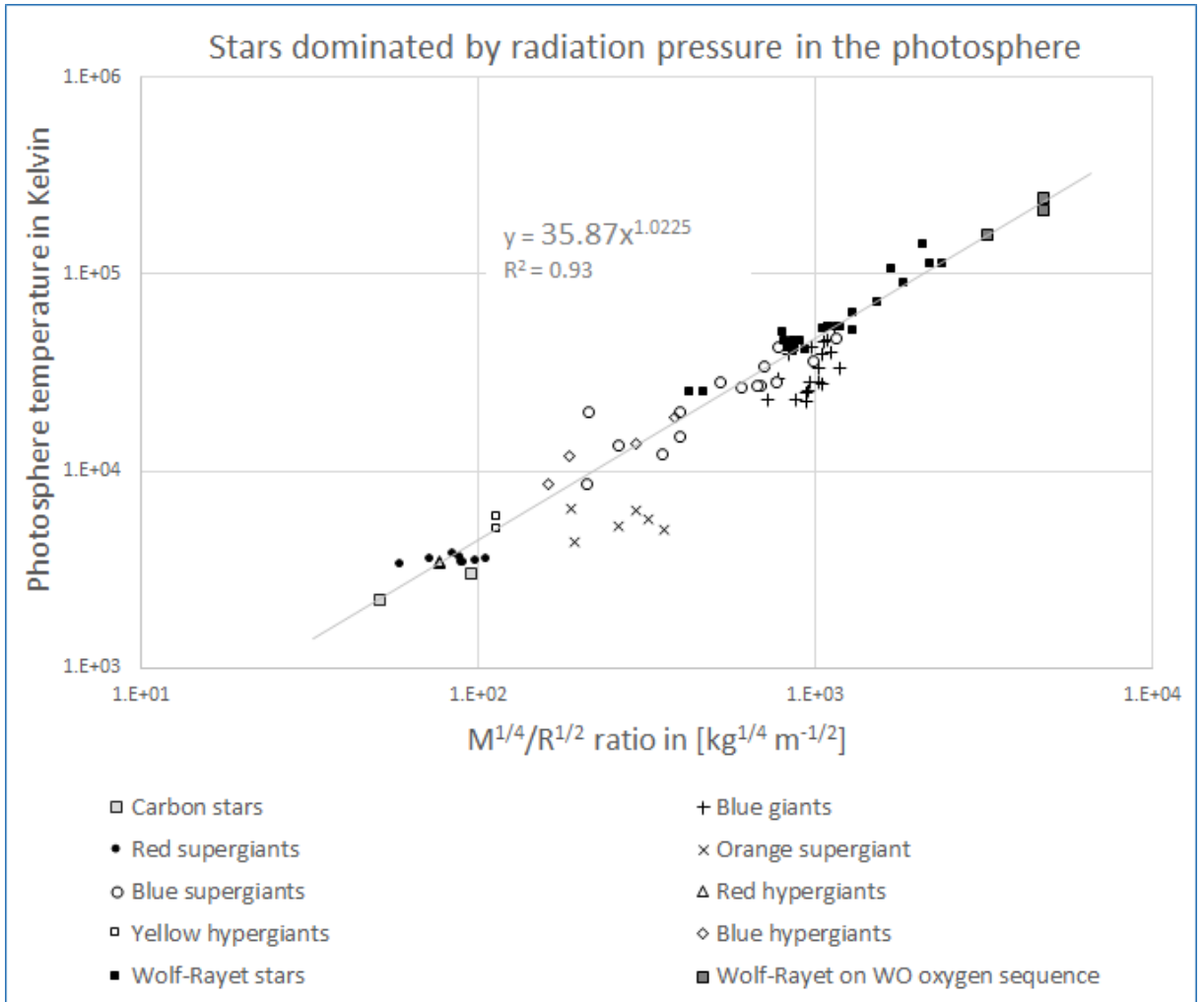

Figure 4. Photosphere's temperature versus $M^{1/4}R^{-1/2}$ ratio for stars dominated by radiation pressure in the photosphere.

The corresponding opacity κ is $0.160\,m^2 kg^{-1}$ given (13) or $0.213\,m^2 kg^{-1}$ given (15). Opacity remains fairly consistent across the range of photosphere temperatures (2,200 K to 245,000 K), and photosphere compositions (different hydrogen to helium ratio) for the stars in our sample. Wolf-Rayet stars generally exhibit strong helium lines in their atmosphere. For example, Wolf-Rayet star WR136 which is among our sample set was determined to have an atmospheric composition of 86.5% helium, 12% hydrogen and 1.5% nitrogen by mass based on analysis of its spectra (Hamann et al. 1994). The red hypergiant star WOH G64 has a broad number of emission lines in its spectrum including $\text{H}\alpha$, $H\beta$, [O I], [N I], [S II], [N II], and [O III] (Levesque et al. 2009). Despite the limited data available on helium to hydrogen ratio estimates for these stars, the variability of stellar spectra in our sample would suggest that opacity is not sensitive to the composition of the photosphere, unless all of these stars have lost their outer hydrogen layer. For example, if the ratio $\frac{\sigma_p}{m_p}$ is higher for hydrogen than for helium, according to (14), stars dominated by radiation pressure in the photosphere would preferentially lose hydrogen through their surface while retaining their helium.

Ionisation supposedly depends on temperature. However, the wide range of photosphere temperatures in the sample would suggest that the degree of ionisation is not relevent. This could be indicative of the process contributing to radiative opacity in the photosphere. For bound-free transitions which consist of the absorption of radiation by an electron bound to an ion, and free-free transitions which consist of the absorption of a photon by an unbound electron moving in the field of an ion, the Rosseland opacity is a function of the temperature and hydrogen fraction, and exhibits the dependency with temperature $\kappa \propto \rho T^{-7/2}$ as per Kramers' law. This is quite unexpected as the data do not show such a dependency; otherwise, the regression in Figure 4 would not be linear. Instead, temperature would be proportional to the square of $M^{1/4}R^{-1/2}$. As this is not the case, these opacity models do not seem to adequately describe stellar photosphere plasma.

4. STARS DOMINATED BY GAS PRESSURE IN THE PHOTOSPHERE

4.1. *Estimation of the density in the solar photosphere*

The density of the photosphere is an important parameter required to solve the heat transfer equation for stars. A way to probe the density of the photosphere of the Sun is by using limb darkening. Limb darkening is the observation of the center part of a star appearing brighter than the edge or limb of the luminous disk. This effect is due to the thermal gradient and transparency of the photosphere. The intensity of light at the center of the disk corresponds to the black-body spectrum at an optical depth of 2/3 because of the transparency of the photosphere. The intensity of light at the edge of the disk corresponds to the black-body spectrum at the surface of the photosphere, which is cooler than the temperature at an optical depth of 2/3. The intensity of light travelling through a semi-transparent medium is expressed as follows:

$$I(x) = I_0 \exp(-\kappa \rho x)\,, \tag{18}$$

where κ is the opacity, ρ the density, and x the depth of the medium.

Therefore, the distance from the surface at an optical depth of 2/3 corresponding to 1/3 of the intensity going through is expressed as follows:

$$l = -\frac{\ln(1/3)}{\kappa \rho}\,, \tag{19}$$

Let us say T_0 is the temperature at the limb which is the surface of the photosphere, and $T_{2/3}$ is the temperature at the center of the disk or an optical depth of 2/3. Hence, the temperature gradient is expressed as follows:

$$\frac{dT}{dr} = \frac{T_0 - T_{2/3}}{l}\,, \tag{20}$$

where l is given by (19).

Within a star heat transfer is dominated by the process having the lowest thermal gradient. We know that for the external layer of the Sun, the temperature is too low for radiative heat transfer to be efficient, and convective heat transfer dominates. The thermal gradient of convective heat transfer in a gas is the adiabatic gradient. From limb darkening we get T_0 and $T_{2/3}$. Therefore, using (19) and (20), we can estimate the density of the photosphere.

The ratio of the intensity at an angle θ to intensity at the center of the star from limb darkening is expressed as follows (Carroll & Ostlie 1996):

$$\frac{I(\theta)}{I(0)} = \frac{2}{5} + \frac{3}{5}\cos(\theta)\,, \tag{21}$$

The intensity at the limb is the intensity at an angle $\theta = \frac{\pi}{2}$. Therefore, the ratio of the intensity at the limb to the intensity at the center of the star is 0.4. From Stefan-Boltzmann law, we get the ratio of the temperature at the limb to the temperature at the center:

$$\frac{T_0}{T_{2/3}} = 0.4^{1/4}\,, \tag{22}$$

The average temperature of the solar photosphere is about 5,800 K. Let us say the temperature at the center of the disk is $T_{2/3} = 6,300$ K. Hence, the temperature at the limb is $T_0 = 5,010$ K.

The adiabatic gradient is the temperature gradient obtained for a gas parcel as it rises, assuming an ideal gas. For an ideal gas we have $P = (R/\mu)\rho T$, where R is the ideal gas constant and μ the molar weight. As we move a gas parcel upwards an infinitesimal distance, the variation in pressure is given by:

$$\frac{dP}{dr} = \frac{R}{\mu}\left(\rho\frac{dT}{dr} + T\frac{d\rho}{dr}\right) = \frac{P}{T}\frac{dT}{dr} + \frac{P}{\rho}\frac{d\rho}{dr}\,, \tag{23}$$

For an adiabatic gas, we also have $P = K\rho^\gamma$, hence:

$$\frac{dP}{dr} = K\gamma\rho^{\gamma-1}\frac{d\rho}{dr} = \gamma\frac{P}{\rho}\frac{d\rho}{dr}\,. \tag{24}$$

Combining (23) and (24) we get:

$$\frac{dT}{dr} = (\gamma-1)\frac{T}{P}\frac{P}{\rho}\frac{d\rho}{dr} = \left(\frac{\gamma-1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}\,. \tag{25}$$

From hydrostatic pressure, we have:

$$\frac{dP}{dr} = -\frac{GM}{R^2}\rho\,. \tag{26}$$

Combining (25) and (26) with $P = \frac{\rho}{m_p}kT$ we get:

$$\frac{dT}{dr} = -\left(\frac{\gamma-1}{\gamma}\right)\frac{GM}{R^2}\frac{m_p}{k}\,. \tag{27}$$

which is the adiabatic gradient at the stellar surface, where k is the Boltzmann constant, G the gravitational constant, M the mass of the star, R the radius of the star, m_p the mass of a gas molecule.

For a monoatomic gas $\gamma = \frac{5}{3}$. Hence, the adiabatic gradient at the surface of the sun is $0.013\,K/m$. In contrast, the standard solar model uses an adiabatic gradient of $0.010\,K/m$.

Hence, the density of the photosphere of the Sun from (19) and (20) is:

$$\rho = -\frac{1}{\kappa}\frac{\ln(1/3)}{\left(T_{2/3} - T_0\right)}\frac{dT}{dr}\,, \tag{28}$$

which yields a density of $6.92\times10^{-5}\ kg/m^3$, whereas the standard solar model uses a photosphere density of about $10^{-6}\ kg/m^3$ (Eddy 1979). For the calculation, we used the opacity obtained in section 3.3.

4.2. Calculation of the cross section per hydrogen nuclei from gas pressure

Let us consider an ion above the stellar surface. A condition to have a stable surface is that the gravitational force exerted by the star on the ion is offset by the repulsive force due to gas pressure. Assuming an ideal gas, we get:

$$\frac{GMm_p}{R^2} = \sigma_{ef}\frac{\rho\,k\,T}{m_p}\,, \tag{29}$$

where σ_{ef} is the effective cross section, m_p is the mass per hydrogen nuclei, M is the mass of the star, G is the gravitational constant, R is the radius of the star, ρ is the mass density in the photosphere, k is the Boltzmann comstant, and T is the temperature.
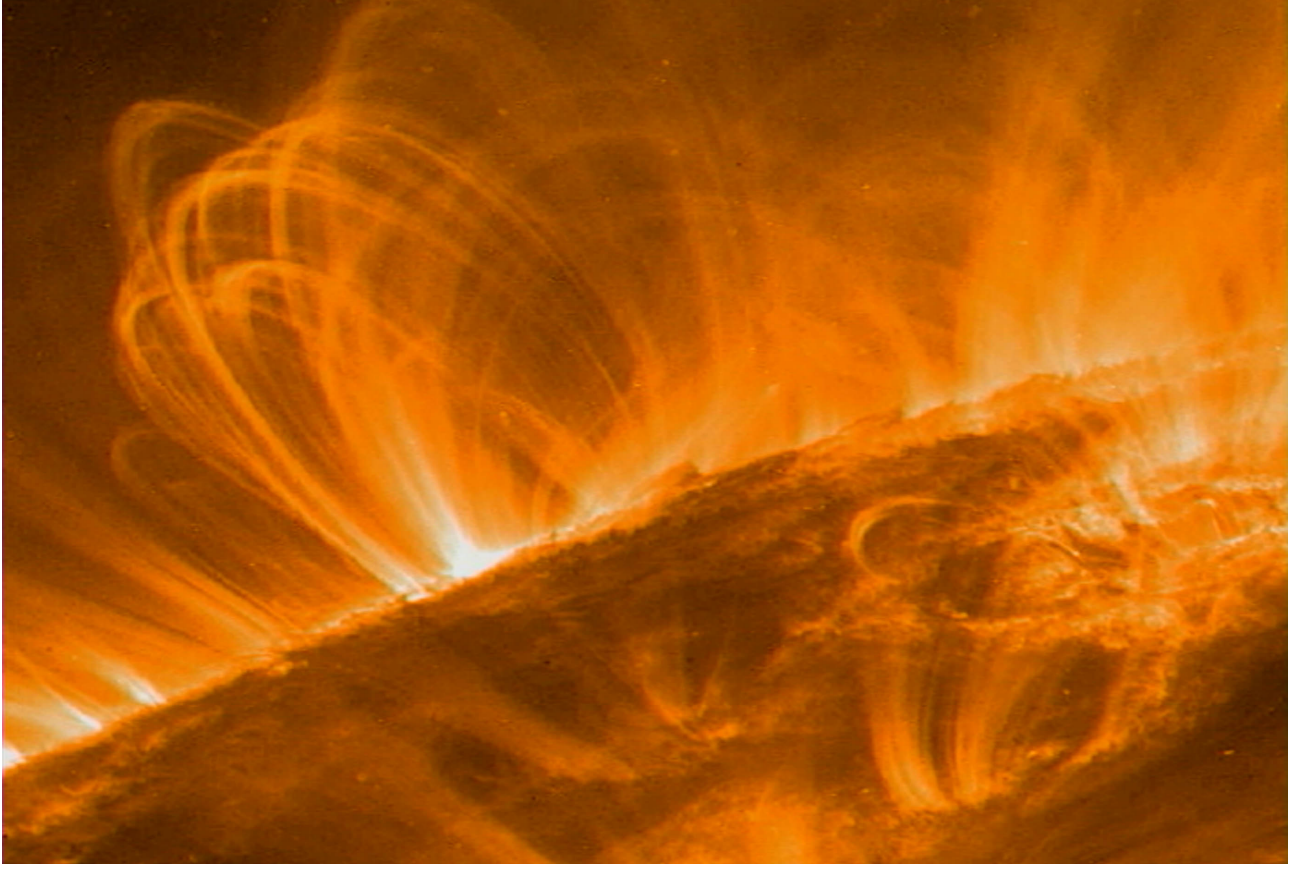
Figure 5. Image of the solar surface. Credit: NASA/GSFC (December 2000)

Although the photosphere is about 500 km thick, modelling the photosphere as a surface makes sense. As shown in figure 5, we can see a clear surface of dense plasma at the photosphere of the Sun. Note that in equation (29) we did not consider the electromagnetic forces. Because free electrons are lighter than the protons, they should tend to escape the surface much easier. However, the plasma may have mechanisms in place to keep its neutrality. For example, a positively charged surface would retain the electrons while pushing out the protons. Equation (29) provides a net cross section from gravity alone and does not model such an effect.

The gas pressure due to molecular collisions is somehow different than radiation pressure. When a photon collides with a surface, the momentum vector is applied in the direction of the trajectory of the photon. For molecular gas collisions, it is like playing pool. Considering molecules of spherical shape, the momentum vector is normal to the sphere, meaning it is applied along the axis between the point of impact of the collision and the center of the sphere. Therefore, we need to introduce a shape coefficient to relate the effective cross section to the geometrical cross section of the molecule.

Let us consider a force f exerted on a sphere of radius r. The surface element is $dS = r^2 \sin(\theta)\, d\theta\, d\varphi$. The projection of the force f on the z-axis is $f_z = f \cos(\theta)$, where θ is the angle between the z-direction and and the vector f. The effective force is the average of f_z over the half sphere. Hence, the effective force is computed as follows:

$$f_{ef} = \frac{1}{2\pi r^2} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} f \cos(\theta) r^2 \sin(\theta)\, d\theta\, d\varphi\,. \tag{30}$$

Because $\sin(\theta)\cos(\theta) = \frac{\sin(2\theta)}{2}$, we get:

$$f_{ef} = \frac{f}{4\pi} \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \sin(2\theta)\, d\theta\, d\varphi\,. \tag{31}$$

We get:

$$f_{ef} = \frac{f}{2} \, . \tag{32}$$

Therefore, the geometric cross section is twice the effective cross section from gas pressure: $\sigma_g = 2\sigma_{ef}$, where σ_g is the geometric cross section and σ_{ef} the effective cross section.

From (29) and the density we obtained in section 4.1, we get an effective cross section of $1.33 \times 10^{-28} \, m^2$ or a geometric cross section of $2.66 \times 10^{-28} \, m^2$. In section 3.3, we obtained a cross section to photon flux of 2.67×10^{-28} m^2. Hence, in the plasma the cross section per hydrogen nuclei from gas pressure is virtually the same as the cross section from radiation pressure.

Neutral hydrogen atoms in the Bohr model are represented with the nucleus at the center and an electron in orbit around the nucleus. The Bohr model yields a radius of $5.3 \times 10^{-11} \, m$ for the hydrogen atom with a corresponding cross section of $8.82 \times 10^{-21} \, m^2$. Hydrogen cross sections have been obtained from electron collisions yielding cross sections on the order of $10^{-21} \, m^2$ for ionized hydrogen (Yoon et al. 2008). A precise value was measured by Frolov (2015), who obtained a cross section of $3.86 \times 10^{-21} \, m^2$ using photodetachment of negatively charged hydrogen ions H^-, that is in close agreement with the Bohr model. The fact that we obtained a much smaller cross section per hydrogen nuclei suggests that in a plasma, the electrons are virtually detached from the nuclei. Therefore, a hydrogen plasma may be represented as a gas mixture of electrons and protons. Hence, the total pressure would be equal to the sum of the partial pressure of the electrons and protons.

Assuming that the electrons and protons are at the same temperature, the adiabatic gradient we computed with eq. (27) should be divided by two, and the density in the photosphere would be half the estimate we obtained, leaving the cross section unchanged. For the proton and electron temperatures to equilibrate, the Coulomb collision rates would need to dominate to allow energy transfer between the electrons and protons. Most plasmas are considered weakly collisional, which means that the Coulomb collision rates are negligible compared to other processes that control the velocity distributions. Therefore, if we assume that the temperature of the electrons is much lower than the temperature of the protons, we can neglect the electron pressure; and if it is the reverse, then we can neglect the proton pressure, provided that both particles are on the ideal gas domain.

Electrons and protons are fermions, meaning they are modelled as a Fermi gas. Fermions are particles described by the Fermi-Dirac distribution thus obeying the Pauli exclusion principle. Whenever the average interparticular separation is much larger than the average de Broglie wavelength of the particules, the Fermi-Dirac distribution can be approximated by the Maxwell-Boltzmann distribution, and the Fermi gas behaves similarly to an ideal gas (Reif 2010):

$$\bar{R} \gg \frac{h}{\sqrt{3mkT}} \, , \tag{33}$$

where $\bar{R}$ is the average interparticle separation, h the Planck's constant, m the mass of the particle, k the Boltzmann constant, and T the temperature.

This condition is satisfied in the solar photosphere for both the electrons and protons, hence we can use the ideal gas equation as an approximation in the photosphere.

Note that if the particles in the plasma of the solar photosphere were made of large ions or atoms such as the ones we find in gases, according to (29), the surface of the Sun would evaporate due to the high temperatures.

5. CONCLUSION

In the present study we collected stellar data (mass, radius, luminosity and surface temperature) for a set of 360 stars. From stars dominated by radiation pressure in the photosphere, we estimated the opacity, a key parameter for radiative heat transfer. As radiative heat transfer is no longer efficient in the solar convective zone where heat transfer occurs by convection, we assumed the adiabatic gradient of a monoatomic gas for the solar photosphere. We then estimated the density in the photosphere of the Sun using limb darkening. Photosphere density is a boundary parameter required for the solar model. We also considered that the stellar photosphere can be modelled as a surface. Hence, for an hydrogen ion in equilibrium in the photosphere, the force exerted by the gravitation of the star on the ion should be offset by the radiation and gas pressure. Therefore, we computed the cross section per hydrogen nuclei from radiation pressure for stars dominated by radiation pressure in the photosphere, and from gas pressure for stars dominated by gas pressure in the photosphere. We found that the cross section per hydrogen nuclei in stellar plasma is about $2.66 \times 10^{-28} \, m^2$ from both radiation and gas pressure. The cross section of neutral hydrogen as given by the Bohr model for an electron in orbit around the nucleus is $8.82 \times 10^{-21} \, m^2$, which suggests that the electrons and

protons in the plasma are virtually detached. Hence, a hydrogen plasma may be represented as a gas mixture of electrons and protons. If the stellar photosphere was made of large hydrogen atoms or ions such as the ones we find in gases, the surface of the photosphere would evaporate due to the high temperatures. This result could impact stellar models as we would have to add together the partial pressures of the electrons and the protons in the plasma.

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