The special nature of the magnetic field

around a moving line charge

Yannan YANG[*]

Shanghai Jinjuan Information Science & Technology Co., Ltd.，Shanghai, China

Semeatech (Shanghai) Co., Ltd.，Shanghai, China

Email: ynyang98@163.com; ynyang@semeatech.com

Abstract

Two types of electric currents and related magnetic fields were analyzed deeply in this paper. One is the current in a neutral wire, which is very familiar to us. The other current is the result of directional motion of one type (positive or negative) charges suspended in the space. The analysis results show that the two types of currents and related magnetic fields have same properties to observers in stationary frame. But, to the observers in a moving frame, the two types of currents and related magnetic field appear very different behaviors. In the second situation, it looks not sufficient to determine a magnetic field to a given point in space only by its magnitude and direction. In order to sufficiently determine the magnetic, the source of the magnetic field is also required to be considered into.

Keywords: The current in a moving line charge; the magnetic field around a moving line charge.

---

[*] Corresponding author: Yannan Yang (E-mail: ynyang@semeatech.com).

1. Introduction

We all know that an electric current will produce a magnetic field around it as shown in Fig.1. An electric current can be created either by moving relative to one another of the positive and negative charges in a neutral wire or by directional moving of one type (positive or negative) of charges in the space.[1,2] Are there any differences between the two currents produced by the two methods? Are the magnetic fields of the two currents are identical? Followings, we will perform deep analysis in the view of observers who are in different motion state.
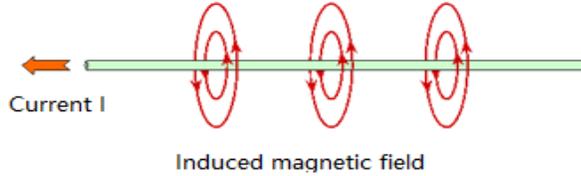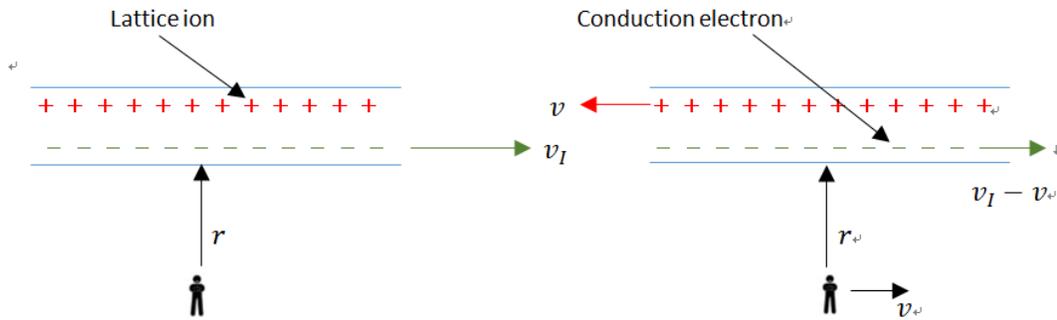

Current I

Induced magnetic field

Fig.1. Magnetic field around an electric current

2. Analysis and discussion

2.1 The current in a neutral wire and its magnetic field

The current in a neutral wire is the result of moving relatively to one another of its positive and negative charges. Such as for a metal wire, the current comes from the conduction electrons' moving relative to the lattice positive ions. For an observer nearby the wire, no matter he is in stationary or moving along the wire, he will always see the same magnitude of the current. The current value depends only on the relative motion rate of the electrons with respect to the lattice ions. This is explained in Fig.2.

Left: Lab frame (S), where the observer is still. Right: Moving frame (S'), where the observer is moving to the right.

Fig.2. Two observers, one in the lab frame and the other in the moving frame to see the current in a neutral wire.

For the observer in S frame, the magnitude of the current is

$$I = \rho_e v_I A$$

Here, ρ_e is conduction electron density; v_I is the drifting rate of the conduction electrons in the lab frame (S). A is the cross section area of the wire.

For an observer in S' frame, the magnitude of the current is

$$I' = \rho_e \,(v_I - v)\, A \;+\; \rho_{ion}\, v\, A$$

Because the wire is neutral, so $\rho_e = \rho_{ion}$, thus

$$I' = \rho_e v_I A = I$$

The magnetic field strength in the place with the distance of r to the wire is

$$B = \frac{1}{4\pi\,\epsilon_o\, c^2}\,\frac{2I}{r}$$

Since the current keeps constant with respect to the moving speed of the observer, the magnetic field is invariable with respect to the moving speed of the observer. So, for an electric current in a long neutral wire, the current to an observer is invariable with respect to his motion state. Along the wire direction, no matter you are at rest or moving, you will always feel the same current, so does the magnetic field. The current you feel is only determined by relative motion rate between the positive and negative charges in the wire, and has nothing to do with observer's motion state.

For a charged particle nearby, the magnetic force applied is

$$F = qvB$$

To a given charged particle, the force and the velocity of the particle show linear relationship.

2.2. The electric current produced by directional motion of one type (positive or negative) charges in the space and its related magnetic field

As shown in Fig.3, the current is the result of directional motion of positive charges suspended in the space.
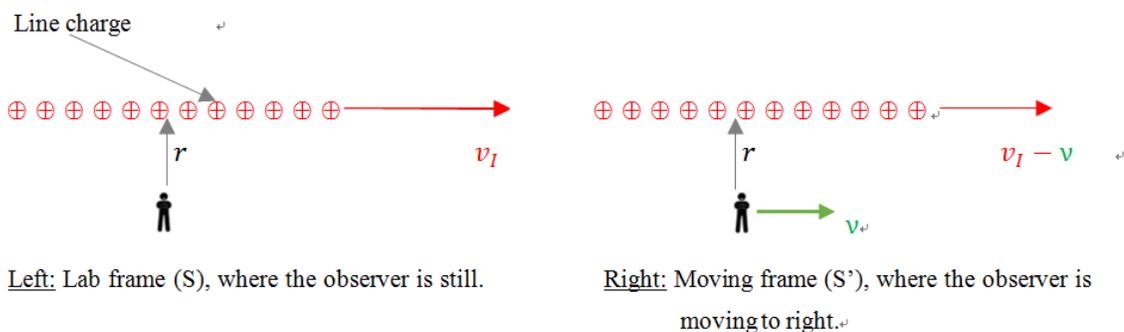

Left: Lab frame (S), where the observer is still.   Right: Moving frame (S'), where the observer is moving to right.

Fig.3. The current of moving positive line charge

For an observer in S frame, the current is

$$I = \rho_+ v_I A$$

Here, ρ_+ is charge density; A is cross section area of the line charge. The magnetic field strength is

$$B = \frac{1}{4\pi \in_o c^2} \frac{2I}{r} = \frac{1}{4\pi \in_o c^2} \frac{2\rho_+ v_I A}{r}$$

For an observer in S' frame, the current is

$$I' = \rho_+ (v_I - v) A$$

The magnetic field strength is

$$B' = \frac{1}{4\pi \in_o c^2} \frac{2I}{r} = \frac{1}{4\pi \in_o c^2} \frac{2\rho_+ (v_I - v) A}{r}$$

From the above, the current observed in moving frame S' is smaller than that observed in stationary frame S. The magnetic field observed in S' frame is also smaller than that observed in S frame. Here, the current and magnetic field is no longer constant with respect to the motion state of the observer. They vary with the moving speed of the observer. For a place with the distance of r to the line charge, the magnetic field strength can be written as

$$B = \alpha (v_I - v) \qquad (\alpha = \frac{1}{4\pi \in_o c^2} \frac{2\rho A}{r}. \text{ It is a constant.})$$

The magnetic field strength linearly changes versus the speed of the observer, as shown in Fig.4. In the stationary frame ($v = 0$), the magnetic field is

$$B = \frac{1}{4\pi \in_o c^2} \frac{2\rho_+ v_I A}{r}$$

When the observer starts to move in the same direction of the line charge, the magnetic field increase with the velocity of the observer and reaches to zero as the $v = v_I$. As $v > v_I$, the magnetic field reverses direction and then keeps increasing its magnitude as the v rises.

From the analysis above, in a stationary frame, a magnetic field with the same value may be produced by a line charge with different charge density and moving speed. So, if there are two magnetic fields with the same direction and magnitude in a stationary frame, they may show different direction and magnitude in moving frame.
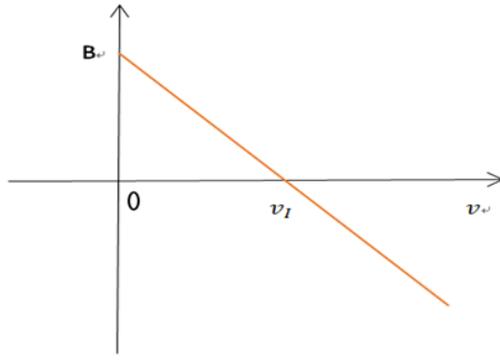
Fig.4. Magnetic field strength (**B**) changes versus the speed of the observer (*v*).

So, for a current produced by directional motion of one type charges in the space, the current and the magnetic field are variable with respect to the motion state of the observer. The observer will feel different current and magnetic field when they are in different motion state.

Now, let's analyze the force applied to a positively charged particle nearby this current. In Fig.5, there is a particle with electricity q nearby the current at the distance r to the line charge. The line charge is moving to right in velocity v_I to form a current $I = \rho v_I A$. The charged particle is moving to right in velocity v. To eliminate the Coulomb interaction between the line charge and the charged particle, the line charge is placed within a long conductor tube. So now, there will be only magnetic interaction between them.
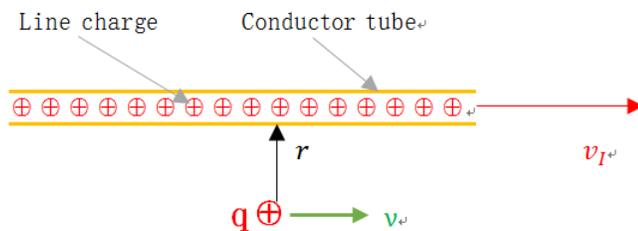

Fig.5. a positively charged particle is nearby a positive line charge. They both move to right.

The magnetic field strength that the particle feels is

$$B = \frac{1}{4\pi \, \epsilon_o \, c^2} \frac{2I}{r} = \frac{1}{4\pi \, \epsilon_o \, c^2} \frac{2\rho \, (v_I - v) \, A}{r}$$

For the place with constant distance to the current,

$$B = \alpha \, (v_I - v) \qquad (\alpha = \frac{1}{4\pi \epsilon_o c^2} \frac{2\rho A}{r}. \text{ It is a constant.})$$

According to the Lorentz law, a magnetic force towards to the current will be applied to the particle. The magnitude of the force is

$$F = qvB = \alpha qv(v_I - v)$$

Instead of a linear relationship in previous case, the force is a quadratic function of the particle velocity. The change rule of the magnetic force versus the particle velocity is shown in Fig.6. When the particle is still, there is no magnetic force on the particle. As the particle moving in the same direction of the line charge, the force increase in a parabolic curve rule and reaches to a peak point when the particle's velocity equals to half of the velocity of the line charge. And then the force begins to decrease with the particle's velocity, until it reaches to zero when the velocity of the particle equals to the velocity of the line charge. When the velocity of the particle surpasses the velocity of the line charge, the force reverses direction from attraction to repulsion and keeps increasing the magnitude as increasing of the particle's velocity.
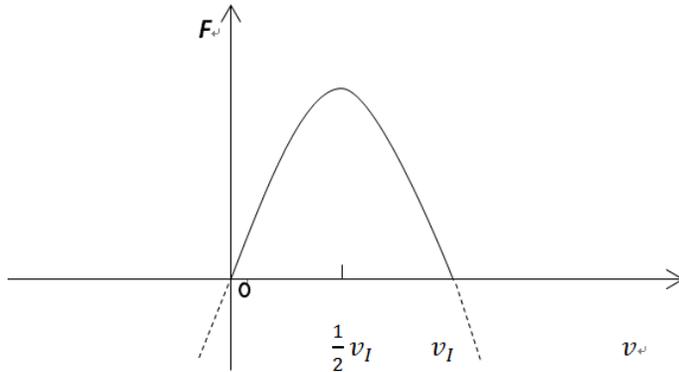

Fig. 6 Magnetic force versus the speed of the charged particle

2.3. The magnetic field of a current loop

For the current in neutral wire, when it forms a loop a magnetic field through the loop is symmetrically distributed in any directions. No matter you see it in a stationary frame or in a moving frame, as in Fig.7.

But for the loop of a current produced by directional moving of one type charges in the space, the magnetic field will show asymmetrical in different directions to an observer who is in moving frame, because the current becomes different in different directions. As in Fig.8, for an observer in a frame moving to right, the up current is larger than the bottom current. This causes the magnetic field stronger in up area and weaker in bottom area.
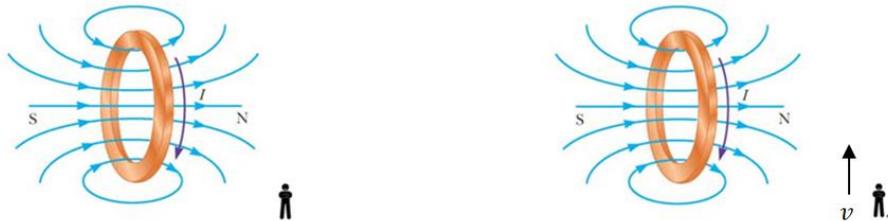
Left: Lab frame (S), where the observer is still.   Right: Moving frame (S'), where the observer is moving to the top.

Fig.7. Two observers, one in the lab frame and the other in the moving frame to see the magnetic field of a current loop of a neutral wire.
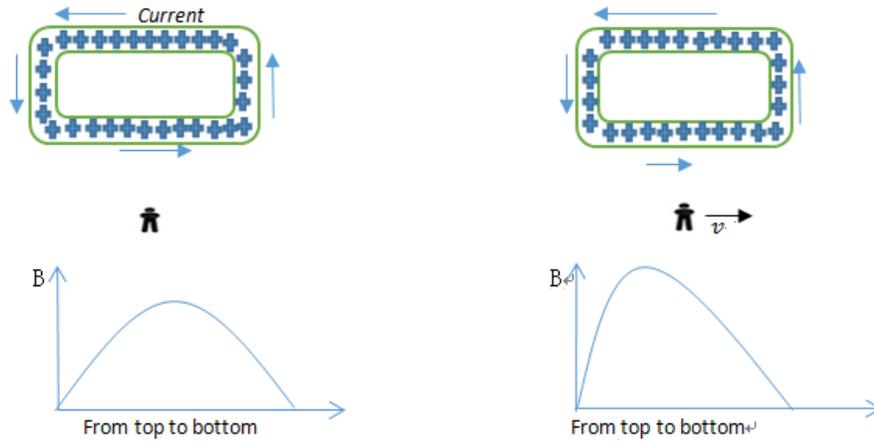
Left: Lab frame (S), where the observer is still.   Right: Moving frame(S'), where the observer is moving to right.

Fig.8. Two observers, one in the lab frame and the other in the moving frame to see the magnetic field of a current loop. The current is formed by a moving line charge.

3. Conclusion

Electric currents can be created either by moving relative to one another of the positive and negative charges in a neutral wire or by directional moving of one type (positive or negative) of charges in the space. The currents created by the two methods have different properties with respect to the observer's motion state and so do the magnetic fields created by the two ways. The current in a neutral wire and the magnetic field created thereby are invariable with respect to the observer's motion state. However, the current created by directional motion of one type charges is variable with respect to the observer's motion state. So do the magnetic field produced by this way. The magnetic field produced by the first way appears the same behavior as the magnetic field of permanent magnets. The magnetic field produced by the second way does not. Because the magnetic fields from the two methods have different behaviors with respect to observer's motion state, it is not sufficient to determine a magnetic field to a given point in space only by its magnitude and direction. For sufficiently to determine a magnetic field in a given point in space, the source of the magnetic field is also required to be considered into.

References

1. Feynman, R. et., al. "Feynman lectures on physics volume 2, Mainly Electromagnetism and Matter, Chapter 13", Addison-Wesley，2011.

2. Purcell, Edward M. and Morin, David J. "Electricity and Magnetism", chapter 5, (Berkeley Physics Course, Vol. 2) 3 edition, Cambridge University Press; 3 edition (January 21, 2013)