# Comment on "*New formulation of the two body problem using a continued fractional potential* "

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**Abstract:** In a relatively recent article by F.A. Abd El-Salam et al. [Astrophys. Space Sci. **350**, 507 (2014)], the authors claimed a new formulation of the two-body problem *via* the introduction of the continued fractional potential. Even if the idea of applying the continued fraction procedure to the gravitational physics is by itself a novelty, the study presented in their work suffers both from mathematical and physical issues. These issues are discussed in this comment.

**Keywords:** continued fraction, two-body problem, continued fractional potential

## 1. Introduction

In their original article entitled "*New formulation of two body problem using a continued fractional potential*", F.A. Abd El-Salam *et al.*(2014) claimed the reformulation of two-body problem in the context of Newtonian gravity by using a continued fraction procedure; and step by step, they derived many equations (1-43), which are supposed by the authors as a new formulation of the two-body problem. The main reason that eventually led the authors to the erroneous equations has been identified – it is the accidental confusion between mathematics and physics in addition to the negligence of the dimensional analysis.

Before tackling the paper under discussion, it is judged important to begin by recalling briefly the profound difference between mathematics and physics. First, Mathematics is not Physics, and Physics is not Mathematics. The 'inhabitants' of the mathematical world are purely abstract objects characterized by an absolute freedom. However, the 'inhabitants' of the physical world are purely concrete objects – in the theoretical sense and/or in the experimental/observational sense – and are characterized by very relative and restricted freedom. When applied outside its original context, mathematics should play the role of an accurate language and useful tool, and gradually should lose its abstraction.

From all that, we arrive at the following assertion: there is an explicit distinction between a physical equation (an equation written in a purely physical context) and a mathematical equation (an equation written in a purely mathematical context). For example, the physical equations are permanently subject to dimensional analysis (DA). The principal role of DA is to check and verify the correctness and the coherence of the physical equations in their proper context. Unfortunately, many physics students and professional physicists ignore or neglect the veritable goal and usefulness of DA. For instance, the ignorance or negligence of DA is well reflected by the fact that we can find in many specialized textbooks and research articles some fundamental physical equations written without $c^2$ and $G$, which are the speed of light squared and the Newton's gravitational constant respectively. They are, by common ill-convention, supposed to be $c=1$ and $G=1$, which, perhaps, is an acceptable

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trick to facilitate mathematical calculations. But physically, it represents a loss of information and can lead to confusion, and such equations cannot be checked by DA.

## 2. Proofs of fatal errors

Now, we arrive at our main subject namely the scrutiny of the paper under consideration "*New formulation of two body problem using a continued fractional potential"*. Recall that our first major objection is that the authors failed to derive the correct equations supposed to be a new formulation of two-body problem *via* the introduction of the continued fractional potential. We focus our attention only on the principal equations derived by the authors , which are clearly the cornerstone of their supposed 'new formulation'.

In order to make our scrutiny more comprehensible, we are obliged to rewrite the authors' central claims, word by word. In their introduction (pages 508-509), the authors wrote: " *... In order to keep the problem simple we will further assume that the potential of each body is that of a mass $m_1$ and $m_2$ respectively with a perturbating continued fractional potential of the form (1). In it, r is the mutual distance between the bodies and $\mu$ is the product of the gravitational constant G times the sum of the bodies' masses $m_1 + m_2$.*

*Retaining the first two terms of the series we get*

$$U = \frac{\mu}{r + \frac{c_1\mu}{r}} = \frac{\mu r}{r^2 + c_1\mu} = \frac{\mu r}{r^2 + \lambda^2}, \qquad c_1\mu = \lambda^2. \tag{2}$$

*The units of $c_1$ is* $\mathrm{s^2\,kg^{-1}m^{-1}}$."

In order to prove that the so-called continued fractional potential (2) is not a potential at all, let us recall the expression of Newtonian (gravitational) potential

$$V \equiv V(r) = -\frac{GM}{r}, \tag{i}$$

In SI units, the 2014 CODATA-recommended value of the gravitational constant (with standard uncertainty in parentheses) is:

$$G = 6.67408(31)\times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}. \tag{ii}$$

Thus, by applying the dimensional analysis (DA) to (i), we get

$$[V] = \frac{\mathrm{L^3M^{-1}T^{-2}M}}{\mathrm{L}} = \mathrm{L^2\,T^{-2}}. \tag{iii}$$

Relation (iii) means that the Newtonian (gravitational) potential (i) has the physical dimensions of velocity squared.

Now, we have according to the authors $[c_1] = \mathrm{T^2\,M^{-1}\,L^{-1}}$ and $[\mu] = [G(m_1 + m_2)] = \mathrm{L^3\,T^{-2}}$ consequently the product gives

$$[c_1][\mu] = \mathrm{T}^2\,\mathrm{M}^{-1}\,\mathrm{L}^{-1}\,\mathrm{L}^3\,\mathrm{T}^{-2} = \mathrm{L}^2\,\mathrm{M}^{-1}. \tag{iv}$$

Therefore, contrary to the authors' supposition, that is to say, the quantity $c_1\mu = \lambda^2$ in (2) has not the geometrical dimensions of length squared ($\mathrm{L}^2$) and the denominator $\left(r^2 + \lambda^2\right)$ in (2) has no sense geometrically and physically. For that reason, the expression (2) is not a potential in any way, and dimensionally we have as a result

$$[U] \neq [V]. \tag{v}$$

Consequently, all the equations containing the meaningless expressions $\left(r^2 - \lambda^2\right)$ and $\left(r^2 + \lambda^2\right)$ are also meaningless mathematically and physically.

In the second section entitled 'Angular momentum integral' (page 509) the authors accidentally confounded the potential with potential energy because they used the same notation for both concepts. They wrote: " *The kinetic and potential energies of the system are given by*

$$T = \frac{1}{2}m_1(\dot{\vec{r}}_1 \cdot \dot{\vec{r}}_1) + \frac{1}{2}m_2(\dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2), \tag{3}$$

$$U = \frac{G m_1 m_2}{\left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right| + \dfrac{c_1 G m_1 m_2}{\left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|}} = \frac{G m_1 m_2 \left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|}{\left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|^2 + c_1 G m_1 m_2}, \tag{4}$$

$$U = \frac{\mu\left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|}{\left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|^2 + \lambda^2} = \frac{\mu r}{r^2 + \lambda^2}, \tag{5}$$

$$\mu = G m_1 m_2,\; r = \left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|,\; c_1\mu = \lambda^2 .\text{"}$$

If we set apart the typographical errors, *e.g.*, we should have $r = \left|\vec{r}_1 - \vec{r}_2\right|$ instead of $r = \left|\dot{\vec{r}}_1 - \dot{\vec{r}}_2\right|$. It is worthwhile to note that the expression on right hand side of (2) is identical to the expression on right hand side of (5). Consequently, the authors created a confusion between the potential and potential energy because they adopted the same notation for both concepts. Recall, the gravitational potential and the gravitational potential energy have, respectively, the following dimensions $\mathrm{L}^2\,\mathrm{T}^{-2}$ and $\mathrm{M}\,\mathrm{L}^2\,\mathrm{T}^{-2}$. In passing, for the case $\mu = G m_1 m_2$, the quantity $c_1\mu = \lambda^2$ has the correct geometrical dimensions of length squared ($\mathrm{L}^2$). In the same section, the authors wrote: "*Thus the Lagrangian of the two body problem with continued fractions*

$$\mathcal{L} = T - U = \frac{1}{2}m_1(\dot{\vec{r}}_1 \cdot \dot{\vec{r}}_1) + \frac{1}{2}m_2(\dot{\vec{r}}_2 \cdot \dot{\vec{r}}_2) - \frac{\mu r}{r^2 + \lambda^2} . \tag{6}$$

*Substitution of these equations into Euler-Lagrange equation yields directly the two body equations of motion*

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\vec{r}}} - \frac{\partial \mathcal{L}}{\partial \vec{r}} = 0 \;\Rightarrow\; \ddot{\vec{r}} + \nabla U = 0. \tag{7}$$

"

Contrary to the authors' claim, Eqs.(7) are not the two-body problem equations of motion purely and simply because they are physico-mathematically incorrect. To show this incorrectness, let us rewrite Eqs.(7): $\ddot{\vec{r}} + \nabla U = 0$, where $\ddot{\vec{r}}$ should be the total acceleration of system $\{m_1, m_2\}$, but $\nabla U$ should be a force since, according to (5), $U = \mu r\left(r^2 + \lambda^2\right)^{-1}$ is the potential energy, with $\mu = G m_1 m_2$. However, if we suppose $\ddot{\vec{r}}$ to be the total average acceleration of system under consideration, in this case, the correct expression should be $\ddot{\vec{r}} + m_1^{-1}\nabla U = 0$ if $m_2$ is the central body and $\ddot{\vec{r}} + m_2^{-1}\nabla U = 0$ if $m_1$ is the central body.

The authors accidentally propagated the errors by replacing the dimensionally correct potential energy (5) with the incorrect potential (2) and used *it* as a cornerstone for their formalism. That's why the totality of their equations containing the wrong expressions $\left(r^2 - \lambda^2\right)$ and $\left(r^2 + \lambda^2\right)$ both derived from (2). Hence, the angular momentum integral, the integral of the center of mass, the integral of total mechanical energy and equation of orbit are incorrect since, according to the authors, they are derived from *the equation of motion of two body problem* (12):

$$\frac{d^2\vec{r}}{dt^2} - \frac{\mu(r^2 - \lambda^2)}{r(r^2 + \lambda^2)^2}\vec{r} = 0 \ , \tag{12}$$

which is manifestly incorrect for the case when $\mu = G(m_1 + m_2)$ and $c_1\mu = \lambda^2$ because the units of $c_1$ are $\mathrm{s}^2\,\mathrm{kg}^{-1}\mathrm{m}^{-1}$ or dimensionally $[c_1] = \mathrm{T}^2\,\mathrm{M}^{-1}\,\mathrm{L}^{-1}$, and for the other case when $\mu = G m_1 m_2$.

## 3. Conclusion

In this comment, we have scrutinized the paper "*New formulation of two body problem using a continued fractional potential"* and proved that this paper is physico-mathematically meaningless because *it* contains fatal errors. Consequently, the authors' formalism is exceedingly questionable mathematically and physically.

## Reference

[1] Abd El-Salam, F.A., Abd El-Bar, S.E., Rasem, M., Alamri, S.Z.: Astrophys. Space Sci. **350**, 507 (2014)