Two conjectures involving Harshad numbers, primes and powers of 2

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Abstract. In this paper I make the following two conjectures: (I) For any prime p, p > 5, there exist n positive integer such that the sum of the digits of the number p*2^n is divisible by p; (II) For any prime p, p > 5, there exist an infinity of positive integers m such that the sum of the digits of the number p*2^m is prime.

Conjecture I:

For any prime p, p > 5, there exist n positive integer such that the sum of the digits of the number p*2^n is divisible by p.

The least n for which s(p*2^n) is divisible by p, for few primes p:

: n = 14 for p = 7, because s(7*2^14) = s(114688) = 28, divisible by 7;
: n = 6 for p = 11, because s(11*2^6) = s(704) = 11, divisible by 11;
: n = 6 for p = 13, because s(13*2^6) = s(832) = 13, divisible by 13;
: n = 6 for p = 17, because s(17*2^6) = s(1088) = 17, divisible by 17;
: n = 19 for p = 19, because s(19*2^19) = s(9961472) = 38, divisible by 19;
: n = 12 for p = 23, because s(23*2^12) = s(94208) = 23, divisible by 23;
: n = 12 for p = 29, because s(29*2^12) = s(118784) = 29, divisible by 29;
: n = 12 for p = 31, because s(31*2^12) = s(126976) = 31, divisible by 31;
: n = 149 for p = 37, because s(37*2^149) = s(26404082315060257799578290434815660023080812544) = 185, divisible by 37;
: n = 30 for p = 41, because s(41*2^30) = s(44023414784) = 41, divisible by 41;
: n = 24 for p = 47, because s(47*2^24) = s(788529152) = 47, divisible by 47.

Note the interesting thing that:

: s(11*2^5) = 10, while s(11*2^6) = 11;
: s(29*2^11) = 28, while s(29*2^12) = 29;
: s(47*2^23) = 46, while s(47*2^24) = 47.

Conjecture II:

For any prime p, p > 5, there exist an infinity of positive integers m such that the sum of the digits of the number p*2^m is prime.

The sequence of the primes s(p*2^m) for p = 7:

: 5(= s(7*2^1) = s(14)), 11(= s(7*2^3) = s(56)), 23(= s(7*2^7) = s(896)), 19(= s(7*2^8) = s(1792)), 17(= s(7*2^11) = s(14336)) (...)

The sequence of the primes s(p*2^m) for p = 11:

: 11(= s(11*2^6) = s(704)), 13(= s(11*2^7) = s(1408)), 17(= s(11*2^8) = s(2816)), 19(= s(11*2^11) = s(22528)), 13(= s(11*2^13) = s(90112)) (...)