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# Some Distance Measures of Single Valued Neutrosophic Hesitant Fuzzy Sets and Their Applications to Multiple Attribute Decision Making

## Abstract

Single-valued neutrosophic hesitant fuzzy set is a merged form of single-valued neutrosophic sets and hesitant fuzzy sets. This set is a useful tool to handle imprecise, incomplete and inconsistent information existing in multi-attribute decision making problems. In multi-attribute decision making, distance measures play an important role to take a decision regarding alternatives. In this paper we propose a variety of distance measures for single valued neutrosophic sets. Furthermore, we apply these measures to multi-attribute decision making problem with single-valued neutrosophic hesitant fuzzy set environment to find out the best alternative. We provide an illustrative example to validate and to show fruitfulness of the proposed approach. Finally, we compare the proposed approach with other existing methods for solving multi-attribute decision making problems.

## Keywords

Hesitant fuzzy sets, single-valued neutrosophic set, single-valued neutrosophic hesitant fuzzy set, distance measure, multi-attribute decision making problem.

## 1. Introduction

Distance and similarity measures are significant in a variety of scientific fields such as decision making, pattern recognition, and market prediction. Lots of studies have been done on fuzzy sets [1], intuitionistic fuzzy sets [2], and neutrosophic sets [3]. Among them the most widely used distance measure are Hamming distance and Euclidean distance. Generally when people make decision, they often hesitate to select for one thing or another to reach the final decision. Tora and Narukawa [4], Tora [5] introduced hesitant fuzzy set (HFS), an extension of fuzzy set, which allows the membership degree to assume a set of possible values. HFS can express the hesitant information compressively than other extensions of fuzzy sets. Xu and Xia [6] defined some distance measures on the basis of well-known Hamming distance and Euclidean distance and the Housdroff metric. They developed a class of hesitant distance measures and discussed some of their properties. Peng et al. [7] proposed the generalised hesitant fuzzy synergetic weighted distance measure and applied it to multi-attribute decision making (MADM) problem, where the best alternative. Having defined hesitancy degree, Li et al. [8] proposed some distance and similarity measures on HFSs and developed a TOPSIS method for MADM.

On the other hand, Zhu et al. [9] proposed a dual hesitant fuzzy set (DHFS) which consists of two parts – one is the membership hesitancy function and another is the non-membership hesitancy function. DHFS generalises fuzzy set (FS), intuitionistic fuzzy set (IFS), hesitant fuzzy set (HFS), and its membership degree and non-membership degree are presented by two set of possible values. Consequently, DHFS can represent imprecise and uncertain information existing in real decision making problem in more flexible way than FS, IFS, HFS. Singh [10] defined some distance and similarity measures of DHFSs on the basis of the geometric distance model, the set theoretic approach and the matching functions to study MADM with DHFSs.

However, HFSs and DHFSs cannot represent indeterminacy hesitant function for incomplete or inconsistent information. This type of function is an another issue to be considered in decision making and thus it should be included with membership hesitant and non-membership hesitant function to catch up imprecise, incomplete, inconsistent information found in decision making process. Ye [11] introduced single-valued neutrosophic hesitant fuzzy set (SVNHF) which consists of three parts – the truth membership hesitancy function, the indeterminacy membership hesitancy function, and falsity membership hesitancy function. This set can express imprecise, incomplete, inconsistent information with these three kinds of hesitancy functions in a more flexible way. In same discussions [11], Ye developed two aggregation operators for SVNHFS information and applied these operators to MADM problems. Sahin and Liu [12] defined correlation coefficient of SVNHFSs to solve MADM with SVNHFSs. Literature review suggests that the distance measures and similarity measures have not been studied, therefore we need to develop distance measures for SVNHFSs.

In this paper, we propose a class of distance measures for single-valued neutrosophic hesitant fuzzy sets and study their properties with variational parameters. We apply the weighted distance measures to calculate the distances between each alternative and ideal alternative in the MADM problems. With these distance values, we present the ranking order of alternatives for selecting the best one. We present an illustrative example to verify the proposed approach and to show its fruitfulness. Finally, we compare the proposed method with other existing methods for solving MADM under SVNHF environment.

The rest of the paper is organised as follows: Section 2 presents some basics of single-valued neutrosophic set and hesitant fuzzy sets and the existing distance measures for HFSs. Section 3 proposes Hamming distance measure, Euclidean distance measure, generalised distance measure, and Hausdroff distance. Section 4 devotes application of proposed distance measure to MADM with SVNHFS information. In Section 5, an illustrative example is given to validate and show effectiveness of the proposed approach. In Section 6, we present concluding remarks and future scope of research.

## 2. Preliminaries

In this section we review some basic definitions regarding single-valued neutrosophic sets and hesitant fuzzy sets to develop the present paper.

### 2.1. Single valued neutrosophic set

**Definition 1.** [13]

Let $X$ be a space of points (objects) with a generic element in $X$ denoted by $x$. A SVNS $A$ in $X$ is characterized by a truth membership function $T_A(\mathrm{x})$, an indeterminacy membership function $I_A(\mathrm{x})$, and a falsity membership function $F_A(\mathrm{x})$ and is denoted by

$$A = \{x, \langle T_A(\mathrm{x}), I_A(\mathrm{x}), F_A(\mathrm{x})\rangle \mid x \in X\}.$$

Here $T_A(\mathrm{x})$, $I_A(\mathrm{x})$ and $F_A(\mathrm{x})$ are real subsets of $[0,1]$ that is $T_A(\mathrm{x}): \mathrm{X} \to [0,1]$, $I_A(\mathrm{x}): \mathrm{X} \to [0,1]$ and $F_A(\mathrm{x}): \mathrm{X} \to [0,1]$. The sum of $T_A(\mathrm{x})$, $I_A(\mathrm{x})$ and $F_A(\mathrm{x})$ lies in $[0,3]$ that is $0 \le T_A(\mathrm{x}) + I_A(\mathrm{x}) + F_A(\mathrm{x}) \le 3$. For convenience, SVNS $A$ can be denoted by $\tilde{A} = \langle T_A(\mathrm{x}), I_A(\mathrm{x}), F_A(\mathrm{x})\rangle$ for all $x$ in $X$.

Now we mention some commonly used distance measures for two SNVS $A$ and $B$ on $X = \{x_1, x_2, ..., x_n\}$.

1. Normalized Hamming distance measure [14]:

$$D^N_{Ham}(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|\right) \tag{1}$$

2. Normalized Euclidean distance measure [14]:

$$D^N_{Euc}(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}\left((T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2\right)} \tag{2}$$

3. The Hausdroff metric [15]:

$$D^N_{Ham}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\} \tag{3}$$

### 2.2. Hesitant fuzzy sets

**Definition 2.** [4, 5, 16]

Let $X$ be a fixed set. A hesitant fuzzy set $A$ on $X$ is presented in terms of a function such that when applied to $X$ returns a subset of $[0,1]$, i.e.

$A = \{\langle x, h_A(x)\rangle \mid x \in X\}$, where $h_A(x)$ is a set of some different values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to $A$. For convenience, $h_A(x)$ is called a hesitant fuzzy element (HFE).

We have some well-known distance measures for two SNVS $A$ and $B$ on $X = \{x_1, x_2, ..., x_n\}$.

1. Generalized hesitant normalized distance:

$$D^N_G(A,B) = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_A^{\sigma(j)} - h_B^{\sigma(j)}\right|^{\lambda}\right)\right]^{1/\lambda}, \quad \lambda > 0 \tag{4}$$

2. Generalized hesitant normalized Hausdorff distance:

$$D^N_{Hau}(A,B) = \left[\frac{1}{n}\sum_{i=1}^{n}\max_j\left|h_A^{\sigma(j)} - h_B^{\sigma(j)}\right|^{\lambda}\right]^{1/\lambda}, \lambda > 0. \tag{5}$$

$l_{x_i} = \max\{l(h_A(x_i)), l(h_B(x_i))\}$ for each $x_i$ in $X$; $h_A^{\sigma(i)}(x_i)$ and $h_B^{\sigma(i)}(x_i)$ are the $j$th largest values in $h_A(x_i)$ and $h_B(x_i)$, respectively. $l(h_A(x_i))$ and $l(h_B(x_i))$ are the number of values in $h_A(x_i)$ and $h_B(x_i)$, respectively.

**Definition 3.** [16]

Let $A_1$, $A_2$ and $A_3$ be three HFSs on $X = \{\mathrm{x}_1, \mathrm{x}_2, ..., \mathrm{x}_n\}$, then the distance measure between $A_1$ and $A_2$ is defined as $d(A_1, A_2)$, which satisfies the following properties:

1. $0 \le d(A_1, A_2) \le 1$;
2. $d(A_1, A_2) = 0$ if and only if $A_1 = A_2$;
3. $d(A_1, A_2) = d(A_2, A_1)$;

The similarity measure between $A_1$ and $A_2$ is defined as $s(A_1, A_2)$, which satisfies the following properties:

1. $0 \le s(A_1, A_2) \le 1$;
2. $s(A_1, A_2) = 1$ if and only if $A_1 = A_2$;
3. $s(A_1, A_2) = s(A_2, A_1)$.

If $d(A_1, A_2)$ be the distance measure between two HFSs $A_1$ and $A_2$, then $s(A_1, A_2) = 1 - d(A_1, A_2)$ is the similarity measure between two HFSs $A_1$ and $A_2$. Similarly, if $s(A_1, A_2)$ be the similarity measure between two HFSs $A_1$ and $A_2$, then $d(A_1, A_2) = 1 - s(A_1, A_2)$ is the distance measure between two HFSs $A_1$ and $A_2$.

## 3. Distance measure of single valued neutrosophic sets

The neutrosophic set [3] theory pioneered by Smarandache has emerged as one of the research focus in many branches such as management sciences, engineering, applied mathematics. Neutrosophic set generalizes the concept of the crisp set, fuzzy set [1], interval valued fuzzy set [17], intuitionistic fuzzy set [2], and interval valued intuitionistic fuzzy set [18].

**Definition 4.** [11]

Let $X$ be a fixed set, then a single valued neutrosophic hesitant fuzzy set $N$ on $X$ is defined as follows:

$N = \{\langle x, t(x), i(x), f(x)\rangle \mid x \in X\}$, where, $t(x)$, $i(x)$, $f(x)$ are three sets of some values in $[0,1]$, denoting the respectively the possible truth, indeterminacy and falsity membership degrees of the element $x \in X$ to the set $N$. The membership degrees $t(x)$, $i(x)$ and $f(x)$ satisfy the following conditions:

$0 \le \delta, \gamma, \eta \le 1,\ 0 \le \delta + \gamma + \eta \le 3$

where, $\delta \in t(x),\ \gamma \in i(x),\ \eta \in f(x),\ \delta^+ \in t^+(x) = \bigcup_{\delta \in t(x)} \max t(x),\ \gamma^+ \in i^+(x) = \bigcup_{\gamma \in i(x)} \max i(x)$

and $\eta^+ \in f^+(x) = \bigcup_{\eta \in i(x)} \max f(x)$ for all $x \in X$.

For convenience of notation, the triple $n(x) = \langle t(x), i(x), f(x)\rangle$ is called a single valued neutrosophic hesitant fuzzy element (SVNHFE) and is denoted by $n = \langle t, i, f\rangle$. It is to be noted that the number of values for possible truth, indeterminacy and falsity membership degrees of the element in different SVNHFEs may be different.

**Definition 5.** [11]

Let $n_1 = \langle t_1, i_1, f_1\rangle$ and $n_2 = \langle t_2, i_2, f_2\rangle$ be two SVNHFEs, the following operational rules are defined as follows:

1. $n_1 \oplus n_2 = \left\langle \bigcup_{\delta_1 \in t_1, \gamma_1 \in i_1, \eta_1 \in f_1,\ \delta_2 \in t_2, \gamma_2 \in i_2, \eta_2 \in f_2} \{\{t_1 + t_2 - t_1 t_2\}, \{i_1 i_2\}, \{f_1, f_2\}\} \right\rangle$;

2. $n_1 \otimes n_2 = \left\langle \bigcup_{\delta_1 \in t_1, \gamma_1 \in i_1, \eta_1 \in f_1,\ \delta_2 \in t_2, \gamma_2 \in i_2, \eta_2 \in f_2} \{\{t_1 t_2\}, \{i_1 + i_2 - i_1 i_2\}, \{f_1 + f_2 - f_1 f_2\}\} \right\rangle;$

3. $\lambda n_1 = \left\langle \bigcup_{\delta_1 \in t_1, \gamma_1 \in i_1, \eta_1 \in f_1} \{\{1-(1-t_1)^{\lambda}\}, \{i_1^{\lambda}\}, \{f_1^{\lambda}\}\} \right\rangle, \lambda > 0$ ;

4. $n_1^{\lambda} = \left\langle \bigcup_{\delta_1 \in t_1, \gamma_1 \in i_1, \eta_1 \in f_1} \{\{t_1^{\lambda}\}, \{1-(1-i_1)^{\lambda}\}, \{1-(1-f_1)^{\lambda}\}\} \right\rangle, \lambda > 0.$

Motivating from the concept provided by Xu and Xia [16], we define a generalized single valued neutrosophic hesitant normalized distance:

$$D_G^N = \left[ \frac{1}{3n} \sum_{i=1}^{n} \left( \frac{1}{l_{x_i}} \left( \sum_{j=1}^{\#h_{x_i}} \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|^{\lambda} + \sum_{k=1}^{\#g_{x_i}} \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|^{\lambda} + \sum_{p=1}^{\#m_{x_i}} \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right|^{\lambda} \right) \right) \right]^{1/\lambda}, \lambda > 0 \quad (6)$$

where $l_{x_i} = \#h_{x_i} + \#g_{x_i} + \#m_{x_i}$ ; $\#h_{x_i}$ , $\#g_{x_i}$ and $\#m_{x_i}$ are the number of elements $t$, $i$, and $f$, respectively.

If $\lambda = 1$, Eq.(9) reduces to single valued neutrosophic hesitant normalized Hamming distance:

$$D_{GHam}^N = \left[ \frac{1}{3n} \sum_{i=1}^{n} \left( \frac{1}{l_{x_i}} \left( \sum_{j=1}^{\#h_{x_i}} \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right| + \sum_{k=1}^{\#g_{x_i}} \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right| + \sum_{p=1}^{\#m_{x_i}} \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right| \right) \right) \right] \quad (7)$$

If $\lambda = 2$, Eq.(9) reduces to single valued neutrosophic hesitant normalized Euclidean distance:

$$D_G^N = \left[ \frac{1}{3n} \sum_{i=1}^{n} \left( \frac{1}{l_{x_i}} \left( \sum_{j=1}^{\#h_{x_i}} \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|^{2} + \sum_{k=1}^{\#g_{x_i}} \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|^{2} + \sum_{p=1}^{\#m_{x_i}} \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right|^{2} \right) \right) \right]^{1/2} \quad (8)$$

However, if we consider, Hausdroff metric to the distance measure, then the generalized single valued neutrosophic hesitant normalized Hausdorff distance can be defined as follows:

$$D_{GHau}^N(A,B) = \left[ \frac{1}{n} \sum_{i=1}^{n} \max \left\{ \begin{array}{l} \max_j \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|^{\lambda}, \max_k \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|^{\lambda}, \\ \max_p \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right|^{\lambda} \end{array} \right\} \right]^{1/\lambda}, \lambda > 0. \quad (9)$$

For $\lambda = 1$, Eq.(12) reduces to single valued neutrosophic hesitant normalized Hamming Hausdorff distance:

$$D_{GHau}^N(A,B) = \left[ \frac{1}{3n} \sum_{i=1}^{n} \max \left\{ \begin{array}{l} \max_j \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|, \max_k \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|, \\ \max_p \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right| \end{array} \right\} \right]. \quad (10)$$

For $\lambda = 2$, Eq.(12) reduces to single valued neutrosophic hesitant normalized Euclidean Hausdorff distance:

$$D_{GHau}^N(A,B) = \left[ \frac{1}{3n} \sum_{i=1}^{n} \max \left\{ \begin{array}{l} \max_j \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|^{2}, \max_k \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|^{2}, \\ \max_p \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right|^{2} \end{array} \right\} \right]^{1/2}. \quad (11)$$

In some situations, we need weight of each element $x_i \in X$ , and then we present the following weighted distance measures for SVNHFs. Assume that the weight of the element $x_i \in X$ is $w_i (i = 1, 2, ..., n)$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, then we have a generalized SVNHF weighted distance:

$$D_G^N = \left[ \frac{1}{3} \sum_{i=1}^{n} \left( \frac{w_i}{l_{x_i}} \left( \sum_{j=1}^{\#h_{x_i}} \left| \delta_A^{\sigma(j)}(x_i) - \delta_B^{\sigma(j)}(x_i) \right|^{\lambda} + \sum_{k=1}^{\#g_{x_i}} \left| \gamma_A^{\sigma(k)}(x_i) - \gamma_B^{\sigma(k)}(x_i) \right|^{\lambda} + \sum_{p=1}^{\#m_{x_i}} \left| \eta_A^{\sigma(p)}(x_i) - \eta_B^{\sigma(p)}(x_i) \right|^{\lambda} \right) \right) \right]^{1/\lambda}, \lambda > 0 \quad (12)$$

and a generalized SVNH weighed Hausdroff distance:

$$D_{GHau}^{N}(A,B)=\left[\frac{1}{3}\sum_{i=1}^{n}\max\left\{\begin{array}{c}\max_j w_i\left|\delta_A^{\sigma(j)}(x_i)-\delta_B^{\sigma(j)}(x_i)\right|^{\lambda},\max_k w_i\left|\gamma_A^{\sigma(k)}(x_i)-\gamma_B^{\sigma(k)}(x_i)\right|^{\lambda},\\ \max_p w_i\left|\eta_A^{\sigma(p)}(x_i)-\eta_B^{\sigma(p)}(x_i)\right|^{\lambda}\end{array}\right\}\right]^{1/\lambda},\lambda>0. \quad (13)$$

## 4. Application of proposed distance measure in multi-attribute decision making

In this section we use the proposed distance measures to find out the best alternative in multi-attribute decision making with single valued neutrosophic hesitant fuzzy environment.

For a multi-attribute decision making problem, assume that $A=\{A_1,A_2,...,A_m\}$ be the set of m alternatives, $C=\{C_1,C_2,...,C_n\}$ be the set of n attributes, whose weight vector $w=(w_1,w_2,...,w_n)^T$, satisfies $w_j>0(j=1,2,...,n)$ and $\sum_{j=1}^{n}w_j=1$, where $w_j$ denotes the weight of the attribute $C_j$. The performance of the alternative $A_i$ with respect to the attribute $C_j$ is measured by an SVNHFE $n_{ij}=\{t_{ij},i_{ij},f_{ij}\}$, where $t_{ij}=\{\delta_{ij}\mid\delta_{ij}\in t_{ij},0\le\delta_{ij}\le 1\}$, $i_{ij}=\{\gamma_{ij}\mid\gamma_{ij}\in i_{ij},0\le\gamma_{ij}\le 1\}$, and $f_{ij}=\{\eta_{ij}\mid\eta_{ij}\in f_{ij},0\le\eta_{ij}\le 1\}$ are the possible truth, indeterminacy and falsity membership degree, respectively such that $0\le\delta_{ij}^{+}+\gamma_{ij}^{+}+\eta_{ij}^{+}\le 3$, where, $\delta_{ij}^{+}=\bigcup_{\delta_{ij}\in t_{ij}}\max\{\delta_{ij}\}$, $\gamma_{ij}^{+}=\bigcup_{\gamma_{ij}\in i_{ij}}\max\{\gamma_{ij}\}$, and $\eta_{ij}^{+}=\bigcup_{\eta_{ij}\in f_{ij}}\max\{\eta_{ij}\}$.

All $n_{ij}=\{t_{ij},i_{ij},f_{ij}\}(i=1,2,...,m;j=1,2,...,n)$ are contained in SVNHF decision matrix $N=(n_{ij})_{m\times n}$ (See Table 1.)

Table 1. SVNHF decision matrix

| | $C_1$ | $C_1$ | … | $C_n$ |
|---|---|---|---|---|
| $A_1$ | $n_{11}$ | $n_{11}$ | … | $n_{11}$ |
| $A_1$ | $n_{11}$ | $n_{11}$ | … | $n_{11}$ |
| ⋮ | ⋮ | ⋮ | ⋱ | ⋮ |
| $A_m$ | $n_{m1}$ | $n_{m1}$ | … | $n_{m1}$ |

Basically attributes are two types:
1. benefit type attributes,
2. cost type attributes.

In such cases, we propose the rating values of ideal alternatives $A_j^*$ as $n_j^*=\{t_j^*,i_j^*,f_j^*\}$ for $j=1,2,...,n$, where,

$n_j^*=\{\{1\},\{0\},\{0\}\}$ for benefit type attributes and $n_j^*=\{\{0\},\{1\},\{1\}\}$ for cost type attributes.

Then to determine the best alternatives, we propose the following steps:

**Step 1.** Determine the distance between an alternative $A_j(j=1,2,...,n)$ and the ideal alternative $A^*$ using proposed distance measure according to the nature of attributes.

**Step 2.** Rank the alternative on the basis of distance measure values.

**Step 3.** Obtain the best alternative according to the minimum value of distance measure.

## 5. Numerical example

In this section we consider the example adopted from Ye [11] to illustrate the application of the proposed GRA method for MADM proposed in Section 4. Consider an investment company that wants to invest a sum of money in the best option. There is a panel with four possible alternatives: (1) $A_1$ is the car company; (2) $A_2$ is the food company; (3) $A_3$ is the computer company; (4) $A_4$ is the arms company. To take a decision, the investment company consider three attributes: (1) $C_1$ is the risk analysis; (2) $C_2$ is the growth analysis; (3) $C_3$ is the environmental impact analysis.

The attribute weight vector is given as $W=(0.35,0.25.0.40)^T$ . The four possible alternatives $\{A_1,A_2,A_3,A_4\}$ are evaluated by using SVNHFEs under three attributes $C_j(j=1,2,3)$. We can arrange the rating values in a matrix form namely a SVNHF decision matrix $X=(x_{ij})_{4\times 3}$ that is shown in Table-1.

Table 1. *Single valued neutrosophic hesitant fuzzy decision matrix*

| $C_1$ | $C_2$ | $C_3$ |
|---|---|---|
| $\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$ | $\{\{0.5,0.6\},\{0.2,0.3\},\{0.3,0.4\}\}$ | $\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$ |
| $\{\{0.6,0.7\},\{0.1,0.2\},\{0.2,0.3\}\}$ | $\{\{0.6,0.7\},\{0.1\},\{0.3\}\}$ | $\{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\}$ |
| $\{\{0.5,0.6\},\{0.4\},\{0.2,0.3\}\}$ | $\{\{0.6\},\{0.3\},\{0.4\}\}$ | $\{\{0.5,0.6\},\{0.1\},\{0.3\}\}$ |
| $\{\{0.7,0.8\},\{0.1\},\{0.1,0.2\}\}$ | $\{\{0.6,0.7\},\{0.1\},\{0.2\}\}$ | $\{\{0.3,0.5\},\{0.2\},\{0.1,0.2,0.3\}\}$ |

Now we consider the following steps, described in Section-4, to find the best alternatives.

**Step 1.**Using Eq.(14), we calculate the SVNH weighted distance measure between alternatives $A_i(i=1,2,3,4)$ and ideal alternative $A^*$ for $\lambda=1,2,5,10$ which are shown in Table 2.:

Table 2. *Weighted distant measures for different values of* $\lambda$*'s*

| $\lambda$ | $A_1$ | $A_2$ | $A_3$ | $A_4$ | **Ranking** |
|---|---|---|---|---|---|
| $\lambda$ **=1** | 0.1361 | 0.0810 | 0.1089 | 0.0816 | $A_2 \succ A_4 \succ A_3 \succ A_1$ |
| $\lambda$ **=2** | 0.2687 | 0.1531 | 0.2065 | 0.1738 | $A_2 \succ A_4 \succ A_3 \succ A_1$ |
| $\lambda$ **=5** | 0.4487 | 0.2469 | 0.3192 | 0.3462 | $A_2 \succ A_3 \succ A_4 \succ A_1$ |
| $\lambda$ **=10** | 0.5677 | 0.3059 | 0.3841 | 0.4802 | $A_2 \succ A_3 \succ A_4 \succ A_1$ |

**Step 2.**Rank the alternative according to the value of SVNH weighted distance measure.

**Step 3.** Based on the minimum value of SVNH weighted distance measures for different values of $\lambda=1,2,5,10$, we conclude that $A_2$ as the best alternative, which is same as the results obtained in Ye [11] and Sahin and Liu [12].

From Table-2, we observe that ranking results change with different values of $\lambda$.Therefore taking the values of $\lambda$ according to decision maker's preference play a crucial role in ranking process. Ye [11] considered weighted cosine similarity measure of SVNHFs and Sahin and Liu [12] proposed weighted correlation co-efficient to determine the ranking order of alternatives. In both studies, we see that there is no option to consider different values of the attitudinal character $\lambda$ that can change the ranking result as we have seen in our study. Thus our method is more realistic and flexible over these two methods, furthermore our method is simple and effective.

## 6. Conclusion

In this paper, we develop a class of distance measures for single-valued neutrosophic hesitant fuzzy sets and discussed their properties with variational parameters. We apply the weighted distance measures to calculate the distances between each alternative and ideal alternative in the MADM problems. With these distance values, we obtain the ranking order of alternatives for selecting the best one. We provide an illustrative example to verify the proposed approach and to show its fruitfulness. Finally, we compared the proposed method with other existing methods for solving MADM under SVNHF environment. The proposed method is simple and effective to handle MADM under SVNHF. We hope that the proposed distance measures can be extended to interval neutrosophic hesitant fuzzy set, and can be applied in medical diagnosis, pattern recognition, and personal selection under neutrosophic hesitant fuzzy environment.

## References


1. L.A. Zadeh, Fuzzy sets, Information Control, 8(1965) 338–353.
2. K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986) 87–96.
3. F. Smarandache, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. American Research Press, Rehoboth, 1998.
4. V. Torra, Y. Narukawa, On hesitant fuzzy sets and decision in: The 18th IEEE International Conference on Fuzzy Systems, Jeju Island, Korea, 2009 1378-1382.
5. V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems 25(2010) 529-539.
6. M.M. Xia, Z.S. Xu, Hesitant fuzzy information aggregation in decision making, International Journal of Approximate Reasoning 52(2011) 395-407.
7. D.H. Peng, C.Y. Gao, Z. F. Gao, Generalized hesitant fuzzy synergetic weighted distance measures and their application to multiple criteria decision-making, Applied Mathematical Modelling 37(8)(2013) 5837-5850.
8. L. Wang, S. Xu, Q. Wang, M. Ni. Distance and similarity measures of dual hesitant fuzzy sets with their applications to multiple attribute decision making. In Progress in Informatics and Computing (PIC), 2014 International Conference on 2014 May 16 (88-92). IEEE.
9. B. Zhu, Z.S. Xu, M.M. Xia, Dual hesitant fuzzy sets, Journal of Applied Mathematics (2012) doi: 10.1155/2012/879629.
10. P. Singh, Distance and similarity measures for multiple-attribute decision making with dual hesitant fuzzy sets. Computational and Applied Mathematics (2013) doi: 10.1007/s40314-015-0219-2.
11. J. Ye, Multiple-attribute decision making under a single-valued neutrosophic hesitant fuzzy environment, Journal of Intelligent Systems (2014) doi: 10.1515/jisys-2014-0001.
12. R. Sahin, P Liu, Correlation coefficient of single-valued neutrosophic hesitant fuzzy sets and its applications in decision making, Neural Computing and Applications (2016) doi: 10.1007/s00521-015-2163-x.
13. H.Wang, F. Smarandache, R. Sunderraman, Y.Q. Zhang, Single-valued neutrosophic sets, Multi space and Multi structure. 4(2010) 410–413.
14. P. Majumdar, S.K. Samanta, On similarity and entropy of neutrosophic sets, Journal of Intelligent and fuzzy Systems, 26(3)(2014) 1245-1252.
15. S. Broumi,F. Smarandache, Several similarity measures of neutrosophic sets. Neutrosophic Sets and Systems, 1(1)(2013), 54-62.
16. Z. Xu,M. Xia, Distance and similarity measures for hesitant fuzzy sets. Information Sciences, 181(11)(2011), 2128-2138.
17. M.B. Gorzalczany, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems 21 (1987) 1–17.
18. K. Atanassov, G. Gargov, Interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 31(3) (1989), 343-349.