Conjecture involving Harshad numbers and sexy primes

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Abstract. In this paper I conjecture that for any pair of sexy primes (p, p + 6) there exist a prime q = p + 6*n, where n > 1, such that the number p*(p + 6)*(p + 6*n) is a Harshad number.

Conjecture:

For any pair of sexy primes (p, p + 6) there exist a prime q = p + 6*n, where n > 1, such that the number p*(p + 6)*(p + 6*n) is a Harshad number.

Note: see the sequence A005349 in OEIS for Harshad numbers and the sequence A023201 for sexy primes.

The least such prime q for the first nine pairs of sexy primes:

: q = 17 for (p, p + 6) = (5, 11), because 5*11*17 = 935 is a Harshad number, divisible by 17;
: q = 19 for (p, p + 6) = (7, 13), because 7*13*19 = 1729 is a Harshad number, divisible by 19;
: q = 107 for (p, p + 6) = (11, 17), because 11*17*107 = 20009 is a Harshad number, divisible by 11;
: q = 61 for (p, p + 6) = (13, 19), because 13*19*61 = 15067 is a Harshad number, divisible by 19;
: q = 29 for (p, p + 6) = (17, 23), because 17*23*29 = 11339 is a Harshad number, divisible by 17;
: q = 41 for (p, p + 6) = (23, 29), because 23*29*41 = 27347 is a Harshad number, divisible by 23;
: q = 61 for (p, p + 6) = (31, 37), because 31*37*61 = 69967 is a Harshad number, divisible by 37;
: q = 157 for (p, p + 6) = (37, 43), because 37*43*157 = 249787 is a Harshad number, divisible by 37;
: q = 311 for (p, p + 6) = (41, 47), because 41*47*311 = 599297 is a Harshad number, divisible by 41.

The sequence of Harshad numbers of the form p*(p + 6)*(p + 6*n), where p, p + 6 and p + 6*n are primes and n > 1:

: 935 (= 5*11*17), 1729 (= 7*13*19), 2821 (= 7*13*31), 10505 (= 5*11*191), 11339 (= 17*23*29), 15067 (= 13*19*61), 18031 (= 13*19*73), 19201 (= 7*13*211), 20009 (= 11*17*107) (...)