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Distance and Similarity Measures for Multiple Attribute Decision Making with Single-Valued Neutrosophic Hesitant Fuzzy Information

Abstract

With respect to a combination of hesitant sets, and single-valued neutrosophic sets which are a special case of neutrosophic sets, the single valued neutrosophic hesitant sets (SVNHFS) have been proposed as a new theory set that allows the truth-membership degree, indeterminacy membership degree and falsity-membership degree including a collection of crisp values between zero and one, respectively. There is no consensus on the best way to determine the order of a sequence of single-valued neutrosophic hesitant fuzzy elements. In this paper, we first develop an axiomatic system of distance and similarity measures between single-valued neutrosophic hesitant fuzzy sets and also propose a class of distance and similarity measures based on three basic forms such that the geometric distance model, the set-theoretic approach, and the matching functions. Then we utilize the distance measure between each alternative and ideal alternative to establish a multiple attribute decision making method under single-valued neutrosophic hesitant fuzzy environment. Finally, a numerical example of investment alternatives is provided to show the effectiveness and usefulness of the proposed approach. The advantages of the proposed distance measure over existing measures have been discussed.

Keywords

Single-valued neutrosophic set, hesitant fuzzy set, single-valued neutrosophic hesitant fuzzy set, distance measure, similarity measure, multiple attribute decision making.

1. Introduction

Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. However, there are many complicated problems in economics, engineering, environment, social science, medical science, etc., that involve data which are not always all crisp. Classical methods cannot successfully handle uncertainty, because the uncertainties appearing in these domains may be of various types. Zadeh (1965) introduced fuzzy sets (FS) and applied them in many fields including uncertainty. As a generalization of the fuzzy sets, Atanassov (1986) introduced the concept of intuitionistic fuzzy set (IFS). Then Atanassov and Gargov (1989) extended the concept of IFS to interval-valued intuitionistic fuzzy set (IVIFS). Current literature has very large number of distance and similarity measures for FSs and IFSs (Atanassov 1986; Xuecheng 1992; Chen et al. 1995; Liu et al. 2015; Farhadinia 2014; Wang 1997; Szmidt and Kacprzyk 2000; Khaleie and Fasanghari 2012; Grzegorzewski 2004; Wang and Xin 2005; Xu 2007; Hung and Yang 2007; Li 2007; Şahin 2015; Tan 2011).

Torra and Narukawa (2009) and Torra (2010) proposed the concept of hesitant fuzzy set (HFS), discussed the relationship between hesitant fuzzy set and intuitionistic fuzzy set and showed that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. The membership degree of an element in hesitant fuzzy set includes a set of possible values between zero and one. Since its appearance, the hesitant fuzzy information has been used to solve multiple attribute decision making problems. Xia and Xu (2011) defined some techniques for aggregating hesitant fuzzy information and utilized their performances in decision making. Based on the relationship between HFS and IFS, they proposed the set-theoretic laws of HFSs. Xu and Xia (2011) defined a collection of distance measures for HFSs and generated the similarity measures associated with the proposed distance measures.

Furthermore, Zhu et al. (2012) introduced dual hesitant fuzzy set (DHFS) as a generalization of FSs, IFSs, HFSs, and fuzzy multisets (FMSs) and presented some basic operations of DHFSs. A DHFS are characterized by two class of possible values, the membership degrees and nonmembership degrees. Therefore, DHFSs include FSs, IFSs, HFSs, and FMSs under certain conditions, and so they have the desirable performances and advantages of its own and appear to be a more favorable method than aforementioned sets because of considering much more information given by decision makers. Singh (2013) introduced a comprehensive family of distance measures and related similarity measures for DHFSs.

As a new branch of philosophy that combines the knowledge of logics, philosophy, set theory, and probability, Smarandache (1999, 2005) proposed the concept of neutrosophic sets (NSs) as a further generalization of uncertainty modeling tools. Unlike the aforementioned sets, a neutrosophic set consists of three membership functions such that the truth-membership function, the indeterminacy-membership function and the falsity membership function. Additionally, the uncertainty presented here, i.e. the indeterminacy factor, is independent on the truth and falsity values, whereas the incorporated uncertainty is dependent on the degrees of belongingness and non-belongingness of existing sets. The structure of NSs is not appropriate to apply to real-life situations. Therefore, Wang et al. (2005, 2010) developed single-valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs), which are an extension of NSs. Şahin (2014) proposed a neutrosophic hierarchical clustering algorithm based on relationship between SVNSs. Şahin and Küçük (2014) defined a subsethood measure for SVNSs and applied it in a decision making problem. The correlation coefficients of SVNSs as well as a decision-making method using SVNSs were proposed by Ye (2013). In addition, Ye (2014b) investigated the concept of simplified neutrosophic sets (SNSs), which can be expressed by three real numbers in the real unit interval $[0,1]$, provided the set-theoretic operators of SNSs, and developed a multi criteria decision making

(MCDM) method based on the aggregation operators of SNSs. But, Peng et al. (2015) showed that some operations of Ye (2014b) may also be unrealistic in special cases, and defined the novel operations and aggregation operators and applied them to MCDM problems. Also, Ye (2014a) proposed the single valued neutrosophic cross-entropy for solving multicriteria decision making (MCDM) problems with single valued neutrosophic information. Broumi and Smarandache (2013) extended the correlation coefficient to INSs. Zhang et al. (2014) developed a MCDM method based on aggregation operators within an interval neutrosophic environment. Furthermore, Majumdar and Samanta (2014) proposed the distance and similarity measures between SVNSs. Ye (2014d) extended these measures to INSs as based on the relationship between similarity measures and distances. Liu and Wang (2014) discussed a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator based on Bonferroni mean, the weighted Bonferroni mean (WBM), and the normalized WBM. Peng et al. (2015) introduced the multi-valued neutrosophic sets (MVNSs) and developed the operations of multi-valued neutrosophic numbers (MVNNs) based on Einstein operations.

Recently, Ye (2015c) proposed the concept of single valued neutrosophic hesitant fuzzy set (SVNHFS) as a generalization of FSs, IFSs, HFSs, FMSs, and also SVNSs and discussed the basic operations and properties of SVNHFSs. SVNHFSs consist of three parts, first is the truth-membership hesitancy function, second is the indeterminacy-membership hesitancy function, and third is the falsity-membership hesitancy function. The current sets, including FSs, IFSs, HFSs, FMSs, and SVNSs can be regarded as special cases of SVNHFSs. In a SVNHFS, the truth-membership hesitancy degrees, indeterminacy-membership hesitancy degrees and falsity-membership hesitancy degrees are represented by three sets of possible values between zero and one, respectively. Therefore, it is not only more general than aforementioned set but only more suitable for solving MADM problems due to considering much more information provided by decision makers.

From above analysis, we cannot utilize the current measures for dealing with distance and similarity measure between SVNHFSs. Therefore, we need to develop new distance and similarity measures for SVNHFSs, because a SVNHFS consists of three basic membership function such that the truth-membership hesitancy function and indeterminacy-membership hesitancy function and falsity-membership hesitancy function. In this paper, we first define a compressive class of distance measures between SVNHFSs and then proposed the similarity measures based on the geometric distance model, the set-theoretic approach and the matching functions. Also, we show that the proposed measures satisfies the axiom definition of distance and similarity measures developed for SVNHFSs. Finally, we utilize the proposed distance measure to solve a MADM problem with single valued neutrosophic hesitant fuzzy information. The rest of this paper is organized as follows. In section 2, we introduce some basic concepts related to HFS, SVNS and SVNHFS, and some operational and theoretical laws. In Section 3, we propose a variety class of distance measures of SVNHFSs as a further generalization of the existing distance measure for HFSs, DHFSs, IFSs, and SVNSs. Based on the geometric distance model, the set-theoretic approach and the matching functions, we present some similarity measures between SVNHFSs. Section 4 develops a MADM method with single valued neutrosophic hesitant fuzzy information based on the proposed distance measure for SVNHFSs. In Section 6, an illustrative example is provided to demonstrate the application and effectiveness of the developed method. Section 7 gives related comparative analysis. Finally, conclusions and future work are given in Section 8.

2. Preliminaries

In this subsection, we give some concepts related to NSs and SVNSs.

2.1 Neutrosophic set

Definition 1. (Smarandache 2005) Let X be a universe of discourse, then a neutrosophic set is defined as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}, \tag{1}$$

which is characterized by a truth-membership function $T_A: X \to]0^-, 1^+[$, an indeterminacy-membership function $I_A: X \to]0^-, 1^+[$and a falsity-membership function $F_A: X \to]0^-, 1^+[$.

There is not restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

In the following, we adopt the representations $t_A(x)$, $\imath_A(x)$ and $f_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively.

Wang et al. (2010) defined the single valued neutrosophic set which is an instance of neutrosophic set.

2.2.Single valued neutrosophic sets

Definition 2. Wang et al. (2010) Let X be a universe of discourse, then a single valued neutrosophic set is defined as:

$$A = \{\langle x, t_A(x), \imath_A(x), f_A(x)\rangle : x \in X\}, \tag{2}$$

where $t_A: X \to [0,1]$, $\imath_A: X \to [0,1]$ and $f_A: X \to [0,1]$ with $0 \leq t_A(x) + \imath_A(x) + f_A(x) \leq 3$ for all $x \in X$. The values $t_A(x), \imath(x)$ and $f_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

2.3.Hesitant fuzzy sets

Definition 3. (Torra 2010) A hesitant fuzzy set M on X is defined in terms of a function h_M when applied to X, which returns a finite subset of [0,1], i.e.,

$$M = \{\langle x, h_M(x)\rangle : x \in X\}, \tag{3}$$

where $h_M(x)$ is a set of some different values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to M.

2.4.Single-valued neutrosophic hesitant sets

Definition 4. (Ye 2014c) Let X be a fixed set, then a single-valued neutrosophic hesitant fuzzy set A on X is defined as,

$$A = \left\{\langle x, \left(\tilde{t}_A(x), \tilde{\imath}_A(x), \tilde{f}_A(x)\right)\rangle : x \in X\right\} \tag{4}$$

in which $\tilde{t}_A(x), \tilde{\imath}_A(x)$, and $\tilde{f}_A(x)$ are three sets of some different values in [0,1], denoting the truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $x \in X$ to A, respectively, with the conditions $0 \leq \gamma, \delta, \eta \leq 1$ and $0 \leq \gamma^+ + \delta^+ + \eta^+ \leq 3$, where $\gamma \in \tilde{t}_A(x)$, $\delta \in \tilde{\imath}(x)$, $\eta \in \tilde{f}_A(x)$, $\gamma^+ \in \tilde{t}_A^+(x) =$

$\cup_{\gamma \in \tilde{t}_A(x)} \max\{\gamma\}$, $\delta^+ \in \tilde{\imath}_A^+(x) = \cup_{\delta \in \tilde{\imath}_A(x)} \max\{\delta\}$, and $\eta^+ \in \tilde{f}_A^+(x) = \cup_{\eta \in \tilde{f}_A(x)} \max\{\eta\}$ for $x \in X$.

For convenience, the three tuple $A = \{\left(\tilde{t}_A(x), \tilde{\imath}_A(x), \tilde{f}_A(x)\right)\}$ is called a single-valued neutrosophic hesitant fuzzy element (SVNHFE) or a triple hesitant fuzzy element, which is denoted by the simplified symbol $A = \{\left(\tilde{t}_A, \tilde{\imath}_A, \tilde{f}_A\right)\}$.

Now, we give the following definitions to propose the distance and similarity measures between SVNHFSs.

Definition 5 Let A, B and C be three SVNHSs on $X = \{x_1, x_2, \dots, x_n\}$, then the distance measure between A and B is defined as $\tilde{d}(A, B)$, which satisfies the following properties:

(1) $0 \le \tilde{d}(A, B) \le 1$;
(2) $\tilde{d}(A, B) = 0$ if and only if $A = B$;
(3) $\tilde{d}(A, B) = \tilde{d}(B, A)$.
(4) $\tilde{d}(A, B) \le \tilde{d}(A, C)$ and $\tilde{d}(B, C) \le \tilde{d}(A, C)$, if $A \subseteq B \subseteq C$.

Definition 6. Let A, B and C be three SVNHSs on $X = \{x_1, x_2, \dots, x_n\}$, then the similarity measure between A and B is defined as $\tilde{s}(A, B)$, which satisfies the following properties:

(1) $0 \le \tilde{s}(A, B) \le 1$;
(2) $\tilde{s}(A, B) = 1$ if and only if $A = B$;
(3) $\tilde{s}(A, B) = \tilde{s}(B, A)$.
(4) $\tilde{s}(A, B) \ge \tilde{s}(A, C)$ and $\tilde{s}(B, C) \ge \tilde{s}(A, C)$, if $A \subseteq B \subseteq C$.

From Definitions 5 and 6, it is noted that $\tilde{s}(A, B) = 1 - \tilde{d}(A, B)$.
Similar to HFS, in most of the cases, the number of values in different SVNHFEs might be different, i.e., $l_{\tilde{t}_A}(x_i) \neq l_{\tilde{t}_B}(x_i)$, $l_{\tilde{\imath}_A}(x_i) \neq l_{\tilde{\imath}_B}(x_i)$ and $l_{\tilde{f}_A}(x_i) \neq l_{\tilde{f}_B}(x_i)$. Let $l_{\tilde{t}}(x_i) = \max\{l_{\tilde{t}_A}(x_i), l_{\tilde{t}_B}(x_i)\}$, $l_{\tilde{\imath}}(x_i) = \max\{l_{\tilde{\imath}_A}(x_i), l_{\tilde{\imath}_B}(x_i)\}$ and $l_{\tilde{f}}(x_i) = \max\{l_{\tilde{f}_A}(x_i), l_{\tilde{f}_B}(x_i)\}$ for each $x_i \in X$. We can make them have the same number of elements through adding some elements to the SVNHFE which has less number of elements. The selection of this operation mainly depends on the decision makers' risk preferences. Pessimists expect unfavorable outcomes and may add the minimum of the truth-membership degree and maximum value of indeterminacy-membership degree and falsity-membership degree. Optimists anticipate desirable outcomes and may add the maximum of the truth-membership degree and minimum value of indeterminacy-membership degree and falsity-membership degree That is, according to the pessimistic principle, if $l_{\tilde{t}_A}(x_i) < l_{\tilde{t}_B}(x_i)$, then the least value of $\tilde{t}_A(x_i)$ or $\tilde{t}_B(x_i)$ will be added to $\tilde{t}_A(x_i)$. Moreover, if $l_{\tilde{\imath}_A}(x_i) < l_{\tilde{\imath}_B}(x_i)$, then the largest value of $l_{\tilde{t}_A}(x_i)$ or $l_{\tilde{t}_B}(x_i)$ will be inserted in $\tilde{\imath}_A(x_i)$ for $x_i \in X$. Similarity, if $l_{\tilde{f}_A}(x_i) < l_{\tilde{f}_B}(x_i)$, then the largest value of $l_{\tilde{f}_A}(x_i)$ or $l_{\tilde{f}_B}(x_i)$ will be inserted in $\tilde{f}_A(x_i)$ for $x_i \in X$.

3. Some distance measures for SVNHFSs

In this section, we give some distance measures between two SVNHFSs.

Based on the geometric distance model for SVNHFSs, we define the following distance measures.

(1) Generalized single valued neutrosophic hesitant normalized distance (GN), for $\lambda > 0$;

$$\tilde{d}_{GN}=\left(\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda}+\frac{1}{l_{\tilde{\iota}}(x_i)}\sum_{j=1}^{l_{\tilde{\iota}}(x_i)}\left|\tilde{\iota}_A^{\sigma(j)}(x_i)-\tilde{\iota}_B^{\sigma(j)}(x_i)\right|^{\lambda}+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}, \quad (5)$$

where $\tilde{t}_A^{\sigma(j)}(x_i), \tilde{t}_B^{\sigma(j)}(x_i)$; $\tilde{\iota}_A^{\sigma(j)}(x_i), \tilde{\iota}_B^{\sigma(j)}(x_i)$ and $\tilde{f}_A^{\sigma(j)}(x_i), \tilde{f}_B^{\sigma(j)}(x_i)$ are the jth largest values of truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of A and B, respectively.

i. If λ = 1, Eq. (5) reduces a single valued neutrosophic hesitant normalized Hamming distance (NH):

$$\tilde{d}_{NH}=\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\iota}}(x_i)}\sum_{j=1}^{l_{\tilde{\iota}}(x_i)}\left|\tilde{\iota}_A^{\sigma(j)}(x_i)-\tilde{\iota}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\right). \quad (6)$$

ii. If λ = 2, Eq. (5) reduces a single valued neutrosophic hesitant normalized Euclidean distance (NE)

$$\tilde{d}_{NE}=\left(\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{2}+\frac{1}{l_{\tilde{\iota}}(x_i)}\sum_{j=1}^{l_{\tilde{\iota}}(x_i)}\left|\tilde{\iota}_A^{\sigma(j)}(x_i)-\tilde{\iota}_B^{\sigma(j)}(x_i)\right|^{2}+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{2}\right)\right)^{\frac{1}{2}}. \quad (7)$$

Equation (5) can be viewed as a most generalized case of distance measures. We can see that if there is no indeterminacy in SVNHFS, then the indeterminacy-membership value of SVNHFS will disappear, hence, Eqs. (5), (6), and (7) are reduced to a generalized dual hesitant normalized distance, a dual hesitant normalized Hamming distance and a dual hesitant normalized Euclidean distance, respectively (i.e., the distance measures proposed by Singh 2013). In addition, if there is no both indeterminacy and nonmembership in SVNHFS, then both indeterminacy-membership value and falsity-membership value of SVNHFS will disappear, hence, Eqs. (5), (6), and (7) are reduced to a generalized hesitant normalized distance, a hesitant normalized Hamming distance and a hesitant normalized Euclidean distance, respectively (i.e., the distance measure proposed by Xu and Xia 2011).

If we apply the Hausdorff metric to the distance measure, we obtain that

(2)Generalized single valued neutrosophic hesitant normalized Hausdorff distance (GNH):

$$\tilde{d}_{GNH} = \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda}, \left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda}, \left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}. \quad (8)$$

i. If $\lambda = 1$, Eq. (6) reduces a single valued neutrosophic hesitant normalized Hamming–Hausdorff distance (NHH):

$$\tilde{d}_{NHH} = \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|, \left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|, \left|\tilde{f}_A^{\sigma(j)}(x_i) - f_B^{\sigma(j)}(x_i)\right|\right)\right). \quad (9)$$

ii. If $\lambda = 2$, Eq. (6) reduces a single valued neutrosophic hesitant normalized Euclidean–Hausdorff distance (NEH):

$$\tilde{d}_{NEH} = \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^{2}, \left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{2}, \left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|^{2}\right)\right)^{\frac{1}{2}}. \quad (10)$$

In many practical situations, the weight of each element $x_i \in X$ should be taken into account. For instance, in MADM problems, the considered attribute usually has different importance, thus needs to be assigned with different weights. Since in SVNHFSs, we have three types of degree, one is truth-membership degree, other is indeterminacy-membership and final is falsity-membership degree. Since three degrees may have different importance, according to decision maker, different weights can be assigned to each element in each degree. Assume that the weights $\omega = (\omega_1, \omega_2, \ldots \omega_n)^T$ with $\omega_j \in [0,1]$, $\sum_{i=1}^{n}\omega_i = 1$; $\psi = (\psi_1, \psi_2, \ldots \psi_n)^T$ with $\psi_i \in [0,1]$, $\sum_{i=1}^{n}\psi_i = 1$ and $\phi = (\phi_1, \phi_2, \ldots \phi_n)^T$ with $\phi_i \in [0,1]$, $\sum_{i=1}^{n}\phi_i = 1$ denote the weights assigned to truth-membership degree, indeterminacy-membership degree and falsity-membership degree, respectively, of SVNHFS.

Now, we present the following weighted distance measures for SVNHFSs.

(3) Generalized single valued neutrosophic hesitant weighted distance (GW):

$$\tilde{d}_{GW} = \left(\frac{1}{3}\sum_{i=1}^{n}\left(\omega_i\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right) + \psi_i\left(\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right) + \phi_i\left(\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)\right)^{\frac{1}{\lambda}}. \quad (11)$$

i. If $\lambda = 1$, then we get a single valued neutrosophic hesitant weighted Hamming distance (WH):

$$\tilde{d}_{WH} = \frac{1}{3}\sum_{i=1}^{n}\left(\omega_j\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|\right) + \psi_j\left(\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|\right) + \phi_j\left(\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\right). \quad (12)$$

ii. If $\lambda = 2$, then we get a single valued neutrosophic hesitant weighted Euclidean distance (WE):

$$\tilde{d}_{WE} = \left(\frac{1}{3}\sum_{i=1}^{n}\left(\omega_j\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^2\right) + \psi_j\left(\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|^2\right) + \phi_j\left(\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|^2\right)\right)\right)^{\frac{1}{2}}. \quad (13)$$

(4) Generalized single valued neutrosophic hesitant weighted Hausdorff distance (GWH), for $\lambda > 0$;

$$\tilde{d}_{GWH} = \left(\frac{1}{3}\sum_{i=1}^{n}\max_j\left(\omega_i\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^\lambda, \psi_j\left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|^\lambda, \phi_j\left|\tilde{f}_A^{\sigma(j)}(x_i) - f_B^{\sigma(j)}(x_i)\right|^\lambda\right)\right)^{\frac{1}{\lambda}}.$$

(14)

i. $\lambda = 1$, then we get a single valued neutrosophic hesitant weighted Hamming–Hausdorff distance (WHH):

$$\tilde{d}_{WHH} = \left(\frac{1}{3}\sum_{i=1}^{n}\max_j\left(\omega_i\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|, \psi_i\left|\tilde{\imath}_A^{\sigma(j)}(x_i) - \tilde{\imath}_B^{\sigma(j)}(x_i)\right|, \phi_i\left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\right).$$

(15)

ii. $\lambda = 2$, then we get a single valued neutrosophic hesitant weighted Euclidean–Hausdorff distance (WEH):

$$\tilde{d}_{WEH}=\left(\frac{1}{3}\sum_{i=1}^{n}\max_{j}\left(\omega_i\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^2\right),\psi_i\left(\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^2\right),\phi_i\left(\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^2\right)\right)\right)^{\frac{1}{2}}. \quad (16)$$

Next, we shall show that the proposed distance measures satisfy axiom definition of distance measure.

Theorem 7. Let A, B and C be any SVNHFSs, then $\tilde{d}_{NH}(A,B)$ is a distance measure.

Proof. We should prove that $\tilde{d}_{NH}(A,B)$ satisfies axioms (D1)-(D4).

(D1) Suppose that A and B are two SVNHFSs with n attributes, then

$\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|\geq 0$, $\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\geq 0$ and $\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\geq 0$

and so

$$\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|\geq 0,$$

$$\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\geq 0 \text{ and } \frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\geq 0.$$

Thus, we have that

$$\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\geq 0$$

and $\tilde{d}_{NH}(A,B)\geq 0$.

On the other hand, since

$\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|\leq 1$, $\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\leq 1$ and $\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\leq 1$,

we get

$$\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\leq 1$$

and so $\tilde{d}_{NH}(A,B)\leq 1$.

Then it implies that $0\leq\tilde{d}_{NH}(A,B)\leq 1$.

D2)

$$\begin{aligned}\tilde{d}_{NH}(A,B) &= \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\right.\\ &\qquad\left.+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\\ &= \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_B^{\sigma(j)}(x_i)-\tilde{t}_A^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_B^{\sigma(j)}(x_i)-\tilde{\imath}_A^{\sigma(j)}(x_i)\right|\right.\\ &\qquad\left.+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_B^{\sigma(j)}(x_i)-\tilde{f}_A^{\sigma(j)}(x_i)\right|\right)\\ &= \tilde{d}_{NH}(B,A).\end{aligned}$$

(D3) Consider $A = B$, then

$$\begin{aligned}A = B &\Leftrightarrow \frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\tilde{t}_A^{\sigma(j)}(x_i)+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\tilde{\imath}_A^{\sigma(j)}(x_i)+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\tilde{f}_A^{\sigma(j)}(x_i)\\ &= \frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\tilde{t}_B^{\sigma(j)}(x_i)+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\tilde{\imath}_B^{\sigma(j)}(x_i)+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\tilde{f}_B^{\sigma(j)}(x_i)\\ &\Leftrightarrow \frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\\ &\qquad+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right| = 0\\ &\Leftrightarrow \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_B^{\sigma(j)}(x_i)-\tilde{t}_A^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_B^{\sigma(j)}(x_i)-\tilde{\imath}_A^{\sigma(j)}(x_i)\right|\right.\\ &\qquad\left.+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_B^{\sigma(j)}(x_i)-\tilde{f}_A^{\sigma(j)}(x_i)\right|\right) = 0\\ &\Leftrightarrow \tilde{d}_{NH}(A,B) = 0.\end{aligned}$$

(D4) Since $A \subseteq B \subseteq C$, we have

$$\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\tilde{t}_A^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\tilde{t}_B^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\tilde{t}_C^{\sigma(j)}(x_i),$$

$$\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\tilde{\imath}_C^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\tilde{\imath}_B^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\tilde{\imath}_A^{\sigma(j)}(x_i),$$

$$\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\tilde{f}_C^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\tilde{f}_B^{\sigma(j)}(x_i) \le \frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\tilde{f}_A^{\sigma(j)}(x_i).$$

Then it follows that

$$\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|\le\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_C^{\sigma(j)}(x_i)\right|,$$
$$\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\le\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_C^{\sigma(j)}(x_i)\right|,$$
$$\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\le\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_C^{\sigma(j)}(x_i)\right|$$

and so

$$\begin{aligned}\tilde{d}_{NH}(A,B)&=\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\right.\\&\qquad\left.+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\right)\\&\le\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_C^{\sigma(j)}(x_i)\right|+\frac{1}{l_{\tilde{\imath}}(x_i)}\sum_{j=1}^{l_{\tilde{\imath}}(x_i)}\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_C^{\sigma(j)}(x_i)\right|\right.\\&\qquad\left.+\frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_C^{\sigma(j)}(x_i)\right|\right)\\&=\tilde{d}_{NH}(A,C).\end{aligned}$$

Similarly, we can prove $\tilde{d}_{NH}(B,C)\le\tilde{d}_{NH}(A,C)$.

Theorem 8. Let A and B be two SVNHSs, then $\tilde{d}_{GH}(A,B)$ and $\tilde{d}_{NE}(A,B)$ are two distance measures.

Proof. By the similar proof manner of Theorem 7, we can also give the proof of Theorem 8 (omitted).

Theorem 9. Let A, B and C be any SVNHFSs, then $\tilde{d}_{GNH}(A,B)$ is the distance measure.

Proof. We should prove that $\tilde{d}_{GNH}(A,B)$ satisfies axioms (D1)-(D4).

(D1) Suppose that A and B are two SVNHFSs with n attributes, then

$\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|\ge 0$, $\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|\ge 0$ and $\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|\ge 0$

and so

$$\left(\frac{1}{3n}\sum_{i=1}^{n}\max_j\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-f_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\ge 0.$$

Thus, we have $\tilde{d}_{GNH}(A,B)\ge 0$.

(D2)

$$\begin{aligned}\tilde{d}_{GNH}(A,B) &= \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\ &= \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_B^{\sigma(j)}(x_i)-\tilde{t}_A^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_B^{\sigma(j)}(x_i)-\tilde{\imath}_A^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_B^{\sigma(j)}(x_i)-\tilde{f}_A^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\ &= \tilde{d}_{GNH}(B,A)\end{aligned}$$

(D3) Let $\tilde{d}_{GNH}(A,B)=0$, then

$$\Leftrightarrow \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}=0$$
$$\Leftrightarrow \tilde{t}_A^{\sigma(j)}(x_i)=\tilde{t}_B^{\sigma(j)}(x_i),\qquad \tilde{\imath}_A^{\sigma(j)}(x_i)=\tilde{\imath}_B^{\sigma(j)}(x_i)\quad \text{and}\quad \tilde{f}_A^{\sigma(j)}(x_i)=\tilde{f}_B^{\sigma(j)}(x_i)$$
$$\Leftrightarrow A=B.$$

(D4) Since $A\subseteq B\subseteq C$, we have

$\tilde{t}_A^{\sigma(j)}(x_i)\le \tilde{t}_B^{\sigma(j)}(x_i)\le \tilde{t}_C^{\sigma(j)}(x_i), \tilde{\imath}_C^{\sigma(j)}(x_i)\le \tilde{\imath}_B^{\sigma(j)}(x_i)\le \tilde{\imath}_A^{\sigma(j)}(x_i)$ and $\tilde{f}_C^{\sigma(j)}(x_i)\le \tilde{f}_B^{\sigma(j)}(x_i)\le \tilde{f}_A^{\sigma(j)}(x_i)$.

Then it follows that

$$\begin{aligned}&\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\\ &\le \max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_C^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_C^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_C^{\sigma(j)}(x_i)\right|^{\lambda}\right)\end{aligned}$$

and so

$$\begin{aligned}\tilde{d}_{GNH}(A,B) &= \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_B^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\ &\le \left(\frac{1}{3n}\sum_{i=1}^{n}\max_{j}\left(\left|\tilde{t}_A^{\sigma(j)}(x_i)-\tilde{t}_C^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{\imath}_A^{\sigma(j)}(x_i)-\tilde{\imath}_C^{\sigma(j)}(x_i)\right|^{\lambda},\left|\tilde{f}_A^{\sigma(j)}(x_i)-\tilde{f}_C^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}}\\ &= \tilde{d}_{GNH}(A,C).\end{aligned}$$

Similarly, we can prove $\tilde{d}_{GNH}(B,C)\le \tilde{d}_{GNH}(A,C)$.

Theorem 10. Let A and B be two SVNHSs, then $\tilde{d}_{NHH}(A,B)$ and $\tilde{d}_{NEH}(A,B)$ are two distance measures.

Proof. By the similar proof manner of Theorem 9, we can also give the proof of Theorem 10 (omitted).

4. Some similarity measures for SVNHFSs

In this section, we present some similarity measures based on the proposed distance measures between SVNHFSs.

4.1. The similarity measures based on geometric distance model for SVNHFSs

With respect to Eq. (5), the similarity measure can be defined as follows:

(1) Similarity measure based on generalized single valued neutrosophic hesitant normalized distance:

$$\tilde{s}_{GN}(A,B) = 1 - \tilde{d}_{GN}(A,B) = 1 -$$

$$\tilde{d}_{GN} = \left(\frac{1}{3n}\sum_{i=1}^{n}\left(\frac{1}{l_{\tilde{t}}(x_i)}\sum_{j=1}^{l_{\tilde{t}}(x_i)}\left|\tilde{t}_A^{\sigma(j)}(x_i) - \tilde{t}_B^{\sigma(j)}(x_i)\right|^{\lambda} + \frac{1}{l_{\tilde{\iota}}(x_i)}\sum_{j=1}^{l_{\tilde{\iota}}(x_i)}\left|\tilde{\iota}_A^{\sigma(j)}(x_i) - \tilde{\iota}_B^{\sigma(j)}(x_i)\right|^{\lambda} + \frac{1}{l_{\tilde{f}}(x_i)}\sum_{j=1}^{l_{\tilde{f}}(x_i)}\left|\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right|^{\lambda}\right)\right)^{\frac{1}{\lambda}} \quad (17)$$

Similarly, we give another similarity measures based on distance measure as follows:

(i) Similarity measure based on single valued neutrosophic hesitant normalized Hamming distance:

$$\tilde{s}_{NH}(A,B) = 1 - \tilde{d}_{NH}(A,B) \quad (18)$$

(ii) Similarity measure based on single valued neutrosophic hesitant normalized Euclidian distance:

$$\tilde{s}_{NE}(A,B) = 1 - \tilde{d}_{NE}(A,B) \quad (19)$$

(2) Similarity measure based on generalized single valued neutrosophic hesitant normalized Hausdorff distance:

$$\tilde{s}_{GNH}(A,B) = 1 - \tilde{d}_{GNH}(A,B) \quad (20)$$

(i) Similarity measure based on single valued neutrosophic hesitant normalized Hamming–Hausdorff distance:

$$\tilde{s}_{NHH}(A,B) = 1 - \tilde{d}_{NHH}(A,B) \quad (21)$$

(ii) Similarity measure based on single valued neutrosophic hesitant normalized Euclidian–Hausdorff distance:

$$\tilde{s}_{NEH}(A,B) = 1 - \tilde{d}_{NEH}(A,B) \quad (22)$$

(3) Similarity measure based on generalized single valued neutrosophic hesitant weighted Hausdorff distance:

$$\tilde{s}_{GW}(A,B) = 1 - \tilde{d}_{GW}(A,B) \quad (23)$$

(i) Similarity measure based on single valued neutrosophic hesitant weighted Hamming distance:

$$\tilde{s}_{WH}(A,B) = 1 - \tilde{d}_{WH}(A,B) \quad (24)$$

(ii) Similarity measure based on single valued neutrosophic hesitant weighted Euclidian distance:

$$\tilde{s}_{WE}(A,B) = 1 - \tilde{d}_{WE}(A,B) \quad (25)$$

4.2. Similarity measure based on the set-theoretic approach

Let A and B be two SVNHFSs, then we define a similarity measure from the point of set-theoretic view as follows:

$$\tilde{s}_{ST}(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{l_{\tilde{t}}(x_i)}(\min\Delta\tilde{t}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{\iota}}(x_i)}(\min\Delta\tilde{\iota}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{f}}(x_i)}\left(\min\Delta\tilde{f}_{AB}(x_i)\right)}{\sum_{j=1}^{l_{\tilde{t}}(x_i)}(\max\Delta\tilde{t}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{\iota}}(x_i)}(\max\Delta\tilde{\iota}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{f}}(x_i)}\left(\max\Delta\tilde{f}_{AB}(x_i)\right)}, \quad (26)$$

where $\Delta\tilde{t}_{AB}(x_i) = \left(\tilde{t}_A^{\sigma(j)}(x_i), \tilde{t}_B^{\sigma(j)}(x_i)\right)$, $\Delta\tilde{\iota}_{AB}(x_i) = \left(\tilde{\iota}_A^{\sigma(j)}(x_i) - \tilde{\iota}_B^{\sigma(j)}(x_i)\right)$, $\Delta\tilde{f}_{AB}(x_i) = \left(\tilde{f}_A^{\sigma(j)}(x_i) - \tilde{f}_B^{\sigma(j)}(x_i)\right)$

By taking into account the weight of each element $x_i \in X$ for truth-membership function, indeterminacy-membership function and falsity membership function, we define a similarity measure as:

$$\tilde{s}_{WST}(A,B) = \frac{1}{3n}\sum_{i=1}^{n} \frac{\sum_{j=1}^{l_{\tilde{t}}(x_i)} \omega_j(\min \Delta\tilde{t}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{\imath}}(x_i)} \psi_j(\min \Delta\tilde{\imath}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{f}}(x_i)} \phi_j(\min \Delta\tilde{f}_{AB}(x_i))}{\sum_{j=1}^{l_{\tilde{t}}(x_i)} \omega_j(\max \Delta\tilde{t}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{\imath}}(x_i)} \psi_j(\max \Delta\tilde{\imath}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{f}}(x_i)} \phi_j(\max \Delta\tilde{f}_{AB}(x_i))} \tag{27}$$

4.3. Similarity measure based on matching function

The concept of similarity between FSs based on a matching function was defined by Chen et al. (1995). Then Xu (2007) extended the matching function to deal with the similarity measures for IFSs. In the following, we propose the similarity measure for SVNHFSs based on the matching function.

Suppose that A and B are two SVNHFSs, then we define a similarity measure based on the matching function as follows:

$$\tilde{s}_{MF}(A,B) = \frac{1}{3n}\sum_{i=1}^{n} \frac{\sum_{j=1}^{l_{\tilde{t}}(x_i)} \nabla\tilde{t}_{AB}(x_i) + \sum_{j=1}^{l_{\tilde{\imath}}(x_i)} \nabla\tilde{\imath}_{AB}(x_i) + \sum_{j=1}^{l_{\tilde{f}}(x_i)} \nabla\tilde{f}_{AB}(x_i)}{\max\left(\sum_{j=1}^{l_{\tilde{t}}(x_i)} \varDelta\tilde{t}_{AB}(x_i), \sum_{j=1}^{l_{\tilde{t}}(x_i)} \varDelta\tilde{\imath}_{AB}(x_i), \sum_{j=1}^{l_{\tilde{t}}(x_i)} \varDelta\tilde{f}_{AB}(x_i)\right)} \tag{28}$$

where $\nabla\tilde{t}_{AB}(x_i) = \left(\tilde{t}_A^{\sigma(j)}(x_i)\times\tilde{t}_B^{\sigma(j)}(x_i)\right)$, $\nabla\tilde{\imath}_{AB}(x_i) = \left(\tilde{\imath}_A^{\sigma(j)}(x_i)\times\tilde{\imath}_B^{\sigma(j)}(x_i)\right)$ and $\nabla\tilde{f}_{AB}(x_i) = \left(\tilde{f}_A^{\sigma(j)}(x_i)\times \tilde{f}_B^{\sigma(j)}(x_i)\right)$, and $\varDelta\tilde{t}_{AB}(x_i) = \left(\tilde{t}_A^{\sigma(j)}(x_i)\right)^2 + \left(\tilde{t}_B^{\sigma(j)}(x_i)\right)^2$, $\varDelta\tilde{\imath}_{AB}(x_i) = \left(\tilde{\imath}_A^{\sigma(j)}(x_i)\right)^2 + \left(\tilde{\imath}_B^{\sigma(j)}(x_i)\right)^2$ and $\varDelta\tilde{f}_{AB}(x_i) = \left(\tilde{f}_A^{\sigma(j)}(x_i)\right)^2 + \left(\tilde{f}_B^{\sigma(j)}(x_i)\right)^2$.

If we consider weight of each $x \in X$, then we get

$$\tilde{s}_{WMF}(A,B) = \frac{1}{3n}\sum_{i=1}^{n} \frac{\sum_{j=1}^{l_{\tilde{t}}(x_i)} \omega_j(\nabla\tilde{t}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{\imath}}(x_i)} \psi_j(\nabla\tilde{\imath}_{AB}(x_i)) + \sum_{j=1}^{l_{\tilde{f}}(x_i)} \phi_j(\nabla\tilde{f}_{AB}(x_i))}{\max\left(\sum_{j=1}^{l_{\tilde{t}}(x_i)} \omega_j(\varDelta\tilde{t}_{AB}(x_i)), \sum_{j=1}^{l_{\tilde{t}}(x_i)} \psi_j(\varDelta\tilde{\imath}_{AB}(x_i)), \sum_{j=1}^{l_{\tilde{t}}(x_i)} \phi_j(\varDelta\tilde{f}_{AB}(x_i))\right)} \tag{29}$$

It is clear that $\tilde{s}_{WMF}(A,B)$ satisfies all the properties described in Definition 6.

5. Decision-making method based on the single-valued neutrosophic hesitant fuzzy information

In this section, we apply the developed distance and similarity measures to a MADM problem with single-valued neutrosophic hesitant fuzzy information.

For the MADM problem, let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be a set of attributes. Suppose that $\omega = (\omega_1, \omega_2, \dots \omega_n)^T$, $\psi = (\psi_1, \psi_2, \dots \psi_n)^T$ and $\phi = (\phi_1, \phi_2, \dots \phi_n)^T$ are the potential weighting vector assigned to the truth-membership, the indeterminacy-membership and the falsity-membership, respectively, in each alternative, where $\omega_j \geq 0, \psi_j \geq 0$ and $\phi_j \geq 0$, $j = 1,2,\dots,n$, $\sum_{j=1}^{n} \omega_j = 1$, $\sum_{j=1}^{n} \psi_j = 1$ and $\sum_{j=1}^{n} \phi_j = 1$. If the decision makers provide several values for the alternative $A_i (i = 1,2,\dots,m)$ under the attribute C_j $(j = 1,2,\dots,n)$, these values can be characterized as a SVNHFN $e_{ij} = \{t_{ij}, \imath_{ij}, f_{ij}\}$ $(j = 1,2,\dots,n; i = 1,2,\dots,m)$. Assume that $E = [e_{ij}]_{m\times n}$ is the decision matrix, where e_{ij} is expressed by a single-valued neutrosophic hesitant fuzzy element.

In multiple attribute decision-making environments, we can utilize the concept of ideal point to determine the best alternative in the decision set. Although the ideal alternative does not exist in real world, it does provide a useful theoretical construct against which to evaluate alternatives. Therefore, we propose each ideal SVNHFN in the ideal alternative $A^* = \{\langle C_j, e_j^*\rangle : C_j \in C\}$ as $e_j^* = \{\tilde{t}_j^*, \tilde{\imath}_j^*, \tilde{f}_j^*\} = \{\{1\},\{0\},\{0\}\}$ $(j = 1,2,\dots,n)$.

Thus, we can develop a procedure for the decision maker to select the best choice with single valued neutrosophic hesitant fuzzy information, which can be given as follows:

Step1. *Compute the distance (similarity) measure between an alternative* A_i $(i = 1,2,\ldots,m)$ *and the ideal alternative* A^* *by using proposed distance (similarity) measure.*

Step 2. *Rank all of the alternative with respect to the values of distance (similarity) measure.*

Step 3. *Choose the best alternative with respect to the minimum value of distance (maximum value of similarity).*

Step4. *End.*

6. Practical example

Here, an example for the multicriteria decision-making problem of alternatives is used as the demonstration of the application of the proposed decision-making method, as well as the effectiveness of the proposed method.

We take the example adopted from Ye (2014c) to illustrate the utility of the proposed distance and similarity measures. Also, we show that the results obtained using the proposed distance measure are more reasonable than the results obtained using Ye's (2014c) cosine similarity measure.

Example 11. Suppose that an investment company that wants to invest a sum of money in the best option. There is a panel with four possible alternatives in which to invest the money: (1) A_1 is a car company, (2) A_2 is a food company, (3) A_3 is a computer company, and (4) A_4 is an arms company. The investment company must make a decision according to the three attributes: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis, and (3) C_3 is the environmental impact analysis. Suppose that $\omega = (0.35, 0.25, 0.40)$, $\psi = (0.35, 0.40, 0.25)$, and $\phi = (0.30, 0.40, 0.30)$ are the attribute weight vector for truth-membership degree, the indeterminacy-membership degree and the falsity membership degree, respectively. The four possible alternatives are to be evaluated under these three attributes and are presented in the form of single valued neutrosophic hesitant fuzzy information by decision maker according to three attributes C_j $(j = 1,2,3)$, as expressed in the following single valued neutrosophic hesitant fuzzy decision matrix E:

Table 1: *Decision matrix E*

$$E = \begin{pmatrix} \{\{0.3,0.4,0.5\},\{0.1\},\{0.3,0.4\}\} & \{\{0.5,0.6\},\{0.2,0.3\},\{0.3,0.4\}\} & \{\{0.2,0.3\},\{0.1,0.2\},\{0.5,0.6\}\} \\ \{\{0.6,0.7\},\{0.1,0.2\},\{0.2,0.3\}\} & \{\{0.6,0.7\},\{0.1\},\{0.3\}\} & \{\{0.6,0.7\},\{0.1,0.2\},\{0.1,0.2\}\} \\ \{\{0.5,0.6\},\{0.4\},\{0.2,0.3\}\} & \{\{0.6\},\{0.3\},\{0.4\}\} & \{\{0.5,0.6\},\{0.1\},\{0.3\}\} \\ \{\{0.7,0.8\},\{0.1\},\{0.1,0.2\}\} & \{\{0.6,0.7\},\{0.1\},\{0.2\}\} & \{\{0.3,0.5\},\{0.2\},\{0.1,0.2,0.3\}\} \end{pmatrix}$$

To get the best alternative(s), the following steps are involved:

Step 1. Using Eq. (12), we can compute the single valued neutrosophic hesitant weighted Hamming distance between the alternatives and the ideal alternative as:

$$\tilde{d}_{NH}(A_1, A^*) = 0.4370, \tilde{d}_{NH}(A_2, A^*) = 0.2383, \tilde{d}_{NH}(A_3, A^*) = 0.3679, \tilde{d}_{NH}(A_4, A^*) = 0.2654.$$

Step 2. With respect to the values of weighted Hamming distance, we can rank the alternatives as $A_2 \succ A_4 \succ A_3 \succ A_1$.

Step 3. The alternative A_2 is the optimal choice according to the minimum value among weighted Hamming distances, which is not in agreement with the one obtained in Ye (2014c).

Above example clearly shows that the developed method is effective and applicable under single-valued neutrosophic hesitant fuzzy environment.

7. Related comparative analysis

Case 1. Ye (2014c) proposed a method based on single-valued neutrosophic hesitant fuzzy aggregation operators and cosine measure function to find the best alternative. This method lacks the decision makers' risk factor, which causes the distortion of similarity between an alternative and the ideal alternative and makes the proposed method more realistic. Therefore, the results of the proposed method don't coincide with the existing method Ye (2014c). In proposed method, we not only consider the decision makers' risk case but also the individual weighting vectors of truth-membership, indeterminacy-membership, and falsity-membership degrees of each element in decision space, separately. From Table 2, we can see that the rankings are changed according to different parameters λ, consequently, the proposed distance measure can provide a more flexible decision and more choice for decision makers because of the decision maker' risk factor and the individual weighting vector of membership degrees. Combining the analyses above, our method is more precise and reliable than the result produced in Ye (2014c).

Table 2: *Results obtained by Eq. (5) corresponding different λ values*

λ	A_1	A_2	A_3	A_4	Ranking
$\lambda = 1$	0.4370	0.2383	0.3679	0.2654	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 2$	0.6611	0.5256	0.5942	0.5619	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 5$	0.8683	0.8102	0.8320	0.8455	$A_2 \succ A_3 \succ A_4 \succ A_1$
$\lambda = 10$	0.9395	0.9140	0.9214	0.9320	$A_2 \succ A_3 \succ A_4 \succ A_1$

Case 2. In order to validate the feasibility of the proposed decision making method, we give another comparative study between our method and existing methods and use the concept of weighted Euclidian distance. Xu and Xia's method (2011) is used to rank the HFSs which are only characterized by a set of the membership degrees, whereas Singh's method (2013) is applied to DHFSs which are taken into account both the membership hesitant degree and the non-membership hesitant degree in decision making process.

Table 3: *Relationships between existing methods and proposed method*

Methods	Rankings	The best alternative(s)	The worst alternative(s)
Xu and Xia's method (2011)	$A_2 \succ A_3 \succ A_4 \succ A_1$	A_2	A_1
Singh's method (2015)	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1
Ye's method (2005)	$A_4 \succ A_2 \succ A_3 \succ A_1$	A_4	A_1
Our method	$A_2 \succ A_4 \succ A_3 \succ A_1$	A_2	A_1

On the other hand, we also utilize the weighted Euclidian distance to determine the final ranking order of all the alternatives associated with SVNHFS, which are expressed by the truth-membership hesitant degree, indeterminacy-membership hesitant degree, and falsity-membership hesitant degree, to calculate the distance measures and to rank all of the alternatives according to these values. Using the MCDM problem in Example 11, the results with different methods are shown in Table 3.

According to the results presented in Table 3, if the distance methods in Xu and Xia (2011) and the Singh (2013) are used, then the best alternatives are A_2 and the worst one is A_1, respectively. Ye's method (2014c) say that the best ones are A_4 and the worst one is A_1. With respect to proposed method in this paper, the best one is A_2 and A_1 is the worst one. But, there are some small differences in the ranking of the alternatives due to definition of set theories. Additionally, from results of Table 3, we can say that the concept of distance measure is more remarkable and more useable than cosine measure to determine the order of the alternatives.

As mentioned above, the single valued neutrosophic hesitant fuzzy set is a generalization of FSs, IFSs, HFSs, FMSs, DHFSs and also SVNSs. Therefore a SVNHFS (truth-membership hesitant degree, indeterminacy-membership hesitant degree, and falsity-membership hesitant degree) contains more information than the HFS (membership hesitant degree), the IFS (both membership degree and nonmembership degree), the DHFS (membership hesitant degree and nonmembership hesitant degree), and also SVNS (truth-membership degree, indeterminacy-membership degree, and falsity-membership degree). Then, the proposed distance and similarity measures of SVNHFSs is a further generalization of the distance and similarity measures of FSs, IFSs, HFSs, FMSs, DHFSs and also SVNSs. In other words, the distance and similarity measures of FSs, IFSs, HFSs, FMSs, DHFSs and also SVNSs are special cases of the distance and similarity measures of SVNHFSs proposed in this paper. Therefore, the discrimination measures for SVNHFSs can be used to solve not only distance and similarity measures with SVNHFSs but also the problems of fuzzy environment, hesitant fuzzy environment, intuitionistic fuzzy environment, dual hesitant fuzzy environment and single valued neutrosophic environment, whereas the methods in Xu and Xia (2011), Xu (2007), Singh (2013) and Majumdar and Samanta (2014) are only sustainable for problems with HFSs, IFSs, DHFSs, and SVNSs, respectively. Moreover, since SVNHFSs include the aforementioned fuzzy sets, the decision-making method using the proposed distance and similarity measures is more general and more feasibility than existing decision-making methods in fuzzy setting, intuitionistic fuzzy setting, hesitant fuzzy setting, dual hesitant fuzzy setting, and single-valued neutrosophic setting.

8. Conclusions

Based on the combination of both HFSs and SVNSs as a further generalization of fuzzy concepts, the SVNHFS contains more information because it takes into account the information of its truth-membership hesitant degree, indeterminacy-membership hesitant degree, and falsity-membership hesitant degree, whereas the HFS only contains the information of its membership hesitant degrees and DHFS contains the information of its membership hesitant degree and nonmembership hesitant degree. Therefore, it has the desirable characteristics and advantages of its own, appears to be a more flexible method than the existing methods and include much more information given by decision makers. Based on the geometric distance model, the set-theoretic approach, and the matching functions, this paper proposed some distance and similarity measures between SVHNSs as a new extension of discrimination measures between fuzzy sets, hesitant fuzzy sets, dual hesitant fuzzy sets and the single-valued neutrosophic sets. In a multiple attribute decision making process with single-valued neutrosophic hesitant fuzzy information, the proposed distance measure between each alternative and the ideal alternative was used to rank the alternatives and determine the best one(s) according to the measure values. Finally, a numeric example was given to verify the proposed approach and to show its practicality and favorable. The developed method has useable and effective calculation, and presents a new model for handling decision-making problems under the single-valued neutrosophic hesitant fuzzy environment. In the future, we shall further develop more discrimination measures such as correlation coefficient, entropy and cross-entropy for SVNHFSs and apply them to solve practical applications in these areas, such as group decision making, expert system, clustering, information fusion system, fault diagnoses, and medical diagnoses.

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