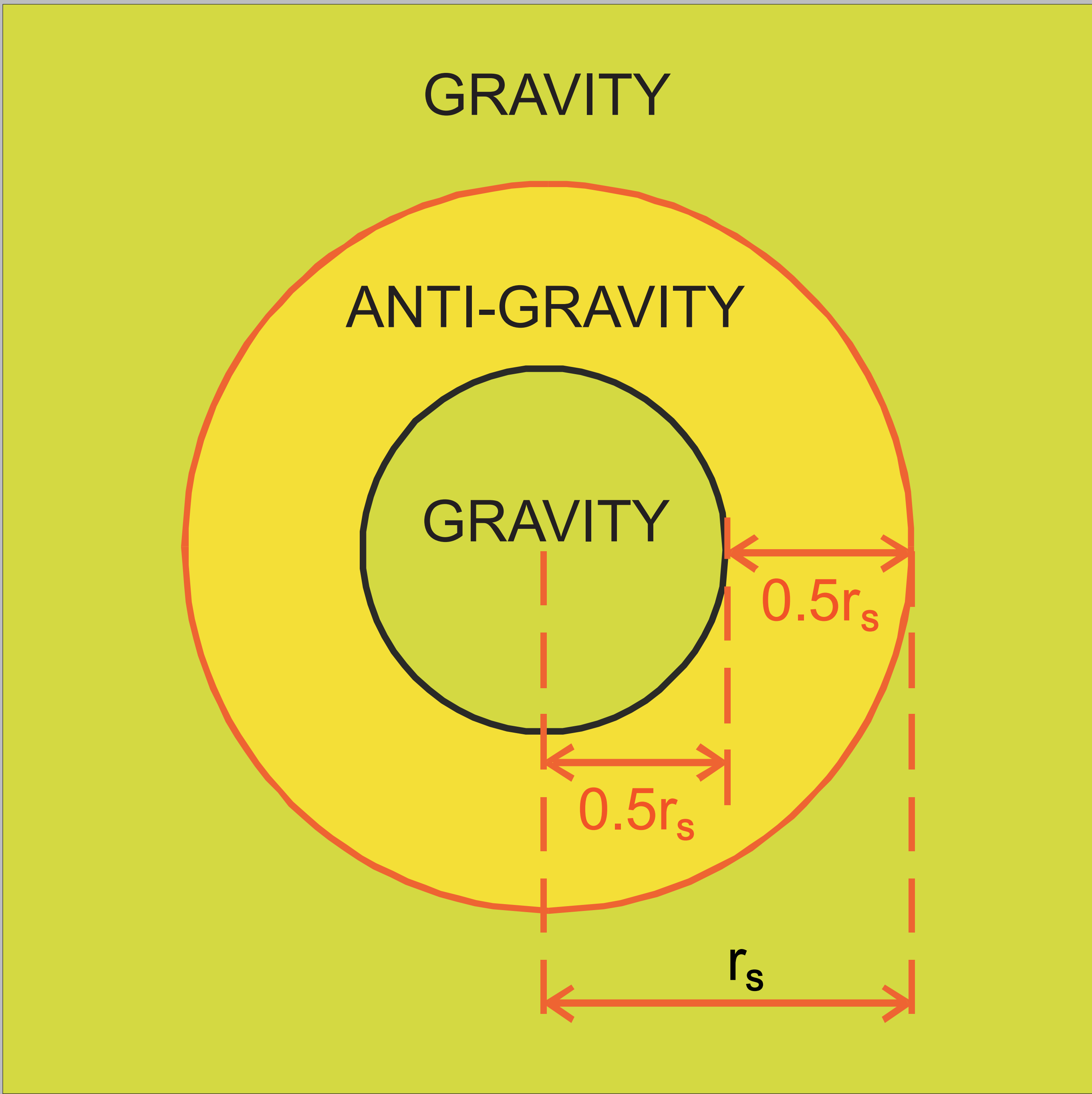

Zbigniew Osiak

ANTI-GRAVITY

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Zbigniew Osiak

ANTI-GRAVITY

ABOUT HOW GENERAL RELATIVITY
HIDES ANTI-GRAVITY – HEURISTIC APPROACH

Mathematics should be a servant, not a queen.

To Margaret,
my daughter,
I dedicate

Anti-gravity

by Zbigniew Osiak

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Portraits (drawings) of Newton, Gauss, Einstein and Schwarzschild – Małgorzata Osiak

Portrait of the author on back cover – Rafał Pudło

Translated by Rafał Rodziewicz

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ANTI-GRAVITY

1 INTRODUCTION

- **Entry**

Couple of reports on the topic of anti-gravity, that I presented in the circle of fellow theoretical physicsts, met with extremely sharp critic and ignorance. Speeches for non professionals, by contrast, didn't awake any interest in listeners. I think that many „hunters" of anti-gravity wrongly assume that, this phenomenon is an exact opposite phenomenon to gravity. They justify such view by postulating an analogy to electrostatic interactions, which in all space can be both attractive and repulsive.

Despite that, I decided to present my views about anti-gravity, because they seem to be coherent and logically correct. Main assumptions of general theory of relativity show that, external Schwarzschild solution is true for $r \geq \frac{1}{2} r_S$. Especially, for $\frac{1}{2} r_S \leq r < r_S$ it describes anti-gravity, and for $r > r_S$ – gravity. In other words, behind event horizon of a black hole, there is area where anti-gravity occurs. Gravity and anti-gravity have layer-like nature, so they differ significantly from attractive and repulsive electrostatic interactions.

- **Limitations for components of a metric tensor**

Examples served in tome „General Theory of Relativity" [1] based on an assumption, that all components of a metric tensor are non-negative and, in relation with

$$\mathbf{e}_\mu \cdot \mathbf{e}_\nu = g_{\mu\nu} \geq 0, \quad (\mu, \nu = 1,2,3,4),$$

it guaranteed real values of local base vectors.

- **Cause-and-effect link between two events**

Two events (x^1, x^2, x^3, x^4) and $(x^1 + dx^1, x^2 + dx^2, x^3 + dx^3, x^4 + dx^4)$ stay in cause-and-effect link, if space distance of those events is not greater than their time distance. In case of metric with spatial-temporal zero-components, we can write this conditions as:

$$\left| g_{\alpha\beta} dx^\alpha dx^\beta \right| \leq \left| g_{44} dx^4 dx^4 \right|, \quad x^4 = ict, \quad (\alpha, \beta = 1,2,3).$$

- **Physical spacetime**

In spacetime there are areas, inside which

$$g_{\mu\nu} \geq 0, \quad (\mu, \nu = 1,2,3,4),$$

and also areas, in which

$$g_{\mu\nu} \leq 0, \quad (\mu, \nu = 1,2,3,4).$$

If both events stay in cause-and-effect link, then in every of those areas we have as follows:

$$g_{\mu\nu} \geq 0\,, \quad (ds)^2 \leq 0\,, \quad (\mu, \nu = 1,2,3,4)$$

and

$$g_{\mu\nu} \leq 0\,, \quad (ds)^2 \geq 0\,, \quad (\mu, \nu = 1,2,3,4).$$

Areas that fulfill above conditions, we will call **physical spacetime**.

In areas, inside which

$$g_{\mu\nu} \geq 0\,, \quad (ds)^2 > 0\,, \quad (\mu, \nu = 1,2,3,4)$$

or

$$g_{\mu\nu} \leq 0\,, \quad (ds)^2 < 0\,, \quad (\mu, \nu = 1,2,3,4),$$

there is no single pair of events that stays in cause-and-effect link.

- **Relations between components of a metric tensor and local basis vectors**

If we want to get local basis vectors with real values in physical spacetime, then we need to adopt new relation between those vectors and components of a metric tensor.

$$\mathbf{e}_\mu \cdot \mathbf{e}_\nu = -\left(\operatorname{sgn} ds^2\right) g_{\mu\nu} \geq 0\,, \quad (ds)^2 \neq 0\,, \quad (\mu, \nu = 1,2,3,4)$$

It causes a necessity to change a definition of dot product and associated terms. Those changes, forced by physics, will cause only small complication of some formulas.

In cases when

$$(ds)^2 < 0 \quad \text{and} \quad g_{\mu\nu} \geq 0\,,$$

above modifications doesn't lead to any changes, in previously quoted by Me tome titled „General Theory of Relativity” [1].

$$\mathbf{e}_\mu \cdot \mathbf{e}_\nu = -\left(\operatorname{sgn} ds^2\right) g_{\mu\nu} \geq 0\,, \quad (ds)^2 \neq 0\,, \quad (\mu, \nu = 1,2,3,4)$$

$$\Big\downarrow \quad (ds)^2 < 0$$

$$\mathbf{e}_\mu \cdot \mathbf{e}_\nu = g_{\mu\nu} \geq 0$$

- **Dot product**

Dot product of vectors $\mathbf{A} = A^\mu \mathbf{e}_\mu$ and $\mathbf{B} = A^\nu \mathbf{e}_\nu$ we will call phrase

$$\mathbf{A} \cdot \mathbf{B} = A^\mu B^\nu \mathbf{e}_\mu \cdot \mathbf{e}_\nu$$

$$\Big\downarrow \quad \mathbf{e}_\mu \cdot \mathbf{e}_\nu = -\left(\operatorname{sgn} ds^2\right) g_{\mu\nu} \geq 0\,, \quad (ds)^2 \neq 0\,, \quad (\mu, \nu = 1,2,3,4)$$

$$\mathbf{A} \cdot \mathbf{B} = -\left(\operatorname{sgn} ds^2\right) g_{\mu\nu} A^\mu B^\nu$$

- **Vector value**

$$\mathbf{A}\cdot\mathbf{A} = A^{\mu}A^{\nu}\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = -\left(\operatorname{sgn} ds^2\right)g_{\mu\nu}A^{\mu}A^{\nu}, \quad (ds)^2 \neq 0, \quad (\mu,\nu = 1,2,3,4)$$

$$A = |\mathbf{A}| = \sqrt{\mathbf{A}\cdot\mathbf{A}} = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\nu}A^{\mu}A^{\nu}}$$

- **Physical (true) vector components**

$$\mathbf{A} = A^{\mu}\mathbf{e}_{\mu}$$

$$\mathbf{A} = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}\, A^{\mu}\, \frac{\mathbf{e}_{\mu}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}}, \quad (ds)^2 \neq 0, \quad (\mu,\nu = 1,2,3,4)$$

$$\hat{A}^{\mu} = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}\, A^{\mu} \quad \text{Physical vector components}$$

$$\hat{\mathbf{e}}_{\mu} = \frac{\mathbf{e}_{\mu}}{e_{\mu}} = \frac{\mathbf{e}_{\mu}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}}, \quad \left|\hat{\mathbf{e}}_{\mu}\right| = 1$$

$$\mathbf{A} = \hat{A}^{\mu}\,\hat{\mathbf{e}}_{\mu}$$

- **Vector value expressed by physical vector components**

$$\mathbf{A}\cdot\mathbf{A} = A^{\mu}A^{\nu}\,\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu}$$

$$A^{\mu}A^{\nu} = \frac{\hat{A}^{\mu}\hat{A}^{\nu}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\nu\nu}}}$$

$$\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = -\left(\operatorname{sgn} ds^2\right)g_{\mu\nu} \geq 0, \quad (ds)^2 \neq 0, \quad (\mu,\nu = 1,2,3,4)$$

$$\frac{-\left(\operatorname{sgn} ds^2\right)g_{\mu\nu}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}}\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\nu\nu}}} = \frac{g_{\mu\nu}}{\sqrt{g_{\mu\mu}}\sqrt{g_{\nu\nu}}}$$

$$|\mathbf{A}| = A = \sqrt{\mathbf{A}\cdot\mathbf{A}} = \sqrt{\frac{\hat{A}^{\mu}\hat{A}^{\nu}\,g_{\mu\nu}}{\sqrt{g_{\mu\mu}}\sqrt{g_{\nu\nu}}}}$$

- **Cosine of an angle between local basis vectors**

$$\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = \left|\mathbf{e}_{\mu}\right|\left|\mathbf{e}_{\nu}\right|\cos\left(\mathbf{e}_{\mu},\mathbf{e}_{\nu}\right)$$

$$\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = -\left(\operatorname{sgn} ds^2\right)g_{\mu\nu} \geq 0, \quad (ds)^2 \neq 0, \quad (\mu,\nu = 1,2,3,4)$$

$$\left|\mathbf{e}_{\mu}\right| = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\mu\mu}} \geq 0$$

$$\left|\mathbf{e}_{\nu}\right| = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\nu\nu}} \geq 0$$

$$\cos\left(\mathbf{e}_\mu, \mathbf{e}_\nu\right) = \frac{\mathbf{e}_\mu \cdot \mathbf{e}_\nu}{\left|\mathbf{e}_\mu\right|\left|\mathbf{e}_\nu\right|} = \frac{g_{\mu\nu}}{\sqrt{g_{\mu\mu}}\sqrt{g_{\nu\nu}}}$$

- **Stationary metric with spatial-temporal zero-components**

For the sake of simplicity, let's limit detailed contemplations to a stationary metric with spatial-temporal zero-components, that is:

$$(ds)^2 = g_{\alpha\beta} dx^\alpha dx^\beta + g_{44} dx^4 dx^4, \quad \frac{\partial g_{\alpha\beta}}{\partial x^4} = 0, \quad \frac{\partial g_{44}}{\partial x^4} = 0, \quad (\alpha, \beta = 1,2,3).$$

Example of such metric is external Schwarzschild metric, which depending on output coordinate system, can be written in equivalent forms:

$$(ds)^2 = \left\{\delta_{\alpha\beta} + \frac{x^\alpha x^\beta}{r^2}\left[\left(1 - \frac{r_S}{r}\right)^{-1} - 1\right]\right\} dx^\alpha dx^\beta + \left(\delta_{44} - \frac{r_S}{r}\right) dx^4 dx^4, \quad (\alpha, \beta = 1,2,3),$$

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = ict, \quad r_S = \frac{2GM}{c^2}$$

or

$$(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2 + \left(1 - \frac{r_S}{r}\right) dx^4 dx^4,$$

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = ict, \quad r_S = \frac{2GM}{c^2}.$$

Quoted works

[1] Z. Osiak: *Ogólna Teoria Względności (General Theory of Relativity).*
Self Publishing (2012), ISBN: 978-83-272-3515-2, http://vixra.org/abs/1804.0178

2 EQUATIONS OF MOTION

- **Quadratic Differential Form of spacetime with stationary metric with spatial-temporal zero-components**

$$(ds)^2 = g_{\alpha\beta}dx^\alpha dx^\beta + g_{44}dx^4dx^4, \quad \frac{\partial g_{\alpha\beta}}{\partial x^4} = 0, \quad \frac{\partial g_{44}}{\partial x^4} = 0, \quad (\alpha,\beta = 1,2,3).$$

- **4-velocity**

$$\tilde{\mathbf{v}} = \tilde{v}^\lambda \, \mathbf{e}_\lambda, \quad (\lambda = 1,2,3,4)$$

$$\tilde{v}^\lambda \overset{df}{=} \sqrt{\operatorname{sgn} ds^2}\; c\,\frac{dx^\lambda}{ds}, \quad (ds)^2 \neq 0, \quad (\lambda = 1,2,3,4)$$

$$ds = c\sqrt{-g_{44}}\sqrt{1 - \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}\frac{1}{c^2}}\; dt = c\sqrt{-g_{44}}\,\gamma_G^{-1}\; dt, \quad (\alpha,\beta = 1,2,3)$$

$$\frac{\sqrt{\operatorname{sgn} ds^2}}{\sqrt{-g_{44}}} = \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}, \quad \frac{1}{\gamma_G} \overset{df}{=} \sqrt{1 - \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}\frac{1}{c^2}}$$

$$\tilde{v}^\lambda = \frac{\gamma_G}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}\frac{dx^\lambda}{dt}, \quad (\lambda = 1,2,3,4), \quad \tilde{v}^4 = \frac{\gamma_G}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}\, ic$$

- **3-velocity**

$$\mathbf{v} = v^\alpha \, \mathbf{e}_\alpha, \quad (\alpha = 1,2,3)$$

$$v^\alpha \overset{df}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}\frac{dx^\alpha}{dt}, \quad (ds)^2 \neq 0$$

- **3-velocity value**

$$v^2 = \mathbf{v}\cdot\mathbf{v}$$

$$\mathbf{v} = v^\alpha \, \mathbf{e}_\alpha, \quad (\alpha = 1,2,3)$$

$$v^2 = \mathbf{v}\cdot\mathbf{v} = v^\alpha \, \mathbf{e}_\alpha \cdot v^\beta \, \mathbf{e}_\beta = v^\alpha v^\beta \, \mathbf{e}_\alpha \cdot \mathbf{e}_\beta, \quad (\alpha,\beta = 1,2,3)$$

$$v^\alpha = \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}\frac{dx^\alpha}{dt}, \quad v^\beta = \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\, g_{44}}}\frac{dx^\beta}{dt}, \quad (\alpha,\beta = 1,2,3)$$

$$\mathbf{e}_\alpha \cdot \mathbf{e}_\beta = -\left(\operatorname{sgn} ds^2\right) g_{\alpha\beta} \geq 0, \quad (ds)^2 \neq 0$$

$$v^2 = \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}, \quad v = \sqrt{\frac{g_{\alpha\beta}}{g_{44}}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt}}$$

- **Physical (true) componets of 3-vector**

$$\mathbf{v} = v^{\alpha}\,\mathbf{e}_{\alpha}, \qquad (\alpha = 1,2,3)$$

$$\mathbf{v} = \sqrt{-(\operatorname{sgn} ds^2)g_{\alpha\alpha}}\; v^{\alpha} \frac{\mathbf{e}_{\alpha}}{\sqrt{-(\operatorname{sgn} ds^2)g_{\alpha\alpha}}}, \qquad (ds)^2 \neq 0$$

$$\hat{v}^{\alpha} = \sqrt{-(\operatorname{sgn} ds^2)g_{\alpha\alpha}}\; v^{\alpha} = \frac{\sqrt{g_{\alpha\alpha}}}{\sqrt{g_{44}}}\frac{dx^{\alpha}}{dt}$$ Physical components of 3-vector

$$\hat{\mathbf{e}}_{\alpha} = \frac{\mathbf{e}_{\alpha}}{e_{\alpha}} = \frac{\mathbf{e}_{\alpha}}{\sqrt{-(\operatorname{sgn} ds^2)g_{\alpha\alpha}}}, \qquad |\hat{\mathbf{e}}_{\alpha}| = 1$$

$$\mathbf{v} = \hat{v}^{\alpha}\,\hat{\mathbf{e}}_{\alpha} = \frac{\sqrt{g_{\alpha\alpha}}}{\sqrt{g_{44}}}\frac{dx^{\alpha}}{dt}\hat{\mathbf{e}}_{\alpha}$$

- **Lorentz factor**

$$\frac{1}{\gamma_G} = \sqrt{1 - \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt}\frac{1}{c^2}}$$

$$v^2 = \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt}, \qquad (\alpha,\beta = 1,2,3)$$

$$\gamma_G = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- **Components of 4-vector expressed by components of 3-vector**

$$\tilde{v}^{\lambda} = \frac{\gamma_G}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{dx^{\lambda}}{dt}, \qquad (\lambda = 1,2,3,4)$$

$$v^{\alpha} \overset{df}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{dx^{\alpha}}{dt}, \qquad (\alpha = 1,2,3)$$

$$x^4 = ict, \qquad v^4 \overset{df}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{dx^4}{dt} = \frac{ic}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}$$

$$\tilde{v}^{\lambda} = \gamma_G v^{\lambda}, \qquad (\lambda = 1,2,3,4)$$

- **Physical (true) components of 4-vector**

$$\tilde{\mathbf{v}} = \tilde{v}^{\lambda}\,\mathbf{e}_{\lambda}, \quad (\lambda = 1,2,3,4)$$

$$\tilde{\mathbf{v}} = \sqrt{-(\operatorname{sgn} ds^2)g_{\lambda\lambda}}\;\tilde{v}^{\lambda}\,\frac{\mathbf{e}_{\lambda}}{\sqrt{-(\operatorname{sgn} ds^2)g_{\lambda\lambda}}}, \quad (ds)^2 \neq 0$$

$$\hat{\tilde{v}}^{\lambda} = \sqrt{-(\operatorname{sgn} ds^2)g_{\lambda\lambda}}\;\tilde{v}^{\lambda} = \gamma_G \frac{\sqrt{g_{\lambda\lambda}}}{\sqrt{g_{44}}}\frac{dx^{\lambda}}{dt}$$ Physical components of 4-vector

$$\hat{\mathbf{e}}_{\lambda} = \frac{\mathbf{e}_{\lambda}}{e_{\lambda}} = \frac{\mathbf{e}_{\lambda}}{\sqrt{-(\operatorname{sgn} ds^2)g_{\lambda}}}, \quad |\hat{\mathbf{e}}_{\lambda}| = 1$$

$$\tilde{\mathbf{v}} = \hat{\tilde{v}}^{\lambda}\,\hat{\mathbf{e}}_{\lambda} = \gamma_G \frac{\sqrt{g_{\lambda\lambda}}}{\sqrt{g_{44}}}\frac{dx^{\lambda}}{dt}\hat{\mathbf{e}}_{\lambda}$$

- **Total acceleration 4-vector**

$$\tilde{\mathbf{a}}_{total} = \tilde{\mathbf{a}} = \tilde{a}^{\lambda}_{total}\,\mathbf{e}_{\lambda} = \tilde{a}^{\lambda}\,\mathbf{e}_{\lambda}, \quad (\lambda = 1,2,3,4)$$

$$\tilde{a}^{\lambda}_{total} = \tilde{a}^{\lambda} \overset{df}{=} (\operatorname{sgn} ds^2)c^2 \frac{d^2x^{\lambda}}{ds^2}, \quad (ds)^2 \neq 0, \quad (\lambda = 1,2,3,4)$$

$$\tilde{v}^{\lambda} = \sqrt{\operatorname{sgn} ds^2}\; c\,\frac{dx^{\lambda}}{ds}, \quad (\lambda = 1,2,3,4)$$

$$\tilde{a}^{\lambda} = \sqrt{\operatorname{sgn} ds^2}\; c\,\frac{d\tilde{v}^{\lambda}}{ds}$$

$$ds = c\sqrt{-g_{44}}\sqrt{1 - \frac{g_{\alpha\beta}}{g_{44}}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt}\frac{1}{c^2}}\;dt = c\sqrt{-g_{44}}\;\gamma_G^{-1}\;dt$$

$$\tilde{a}^{\lambda} = \frac{\gamma_G}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{d\tilde{v}^{\lambda}}{dt} = \frac{\gamma_G^2}{-(\operatorname{sgn} ds^2)\,g_{44}}\left[\frac{d^2x^{\lambda}}{dt^2} - \frac{1}{2\,g_{44}}\frac{dx^{\lambda}}{dt}\frac{dg_{44}}{dt} + \frac{1}{\gamma_G}\frac{d\gamma_G}{dt}\frac{dx^{\lambda}}{dt}\right]$$

- **Total acceleration 3-vector**

$$\mathbf{a}_{total} = \mathbf{a} = a^{\alpha}\,\mathbf{e}_{\alpha}$$

$$a^{\alpha} \overset{df}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)g_{44}}}\frac{dv^{\alpha}}{dt}, \quad v^{\alpha} = \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{dx^{\alpha}}{dt}, \quad (ds)^2 \neq 0, \quad (\alpha = 1,2,3)$$

$$\mathbf{a}_{total} = \mathbf{a} = \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)g_{44}}}\frac{dv^{\alpha}}{dt}\mathbf{e}_{\alpha} = \frac{1}{-(\operatorname{sgn} ds^2)\,g_{44}}\left[\frac{d^2x^{\lambda}}{dt^2} - \frac{1}{2\,g_{44}}\frac{dx^{\lambda}}{dt}\frac{dg_{44}}{dt}\right]\mathbf{e}_{\alpha}$$

- **Value of total acceleration 3-vector**

$$a^2 = \mathbf{a}\cdot\mathbf{a}$$

$$\mathbf{a} = a^{\alpha}\,\mathbf{e}_{\alpha}\,,\quad \mathbf{a} = a^{\beta}\,\mathbf{e}_{\beta}\,,\quad (\alpha = 1,2,3)$$

$$a^2 = \mathbf{a}\cdot\mathbf{a} = a^{\alpha}\,\mathbf{e}_{\alpha}\cdot a^{\beta}\,\mathbf{e}_{\beta} = a^{\alpha}a^{\beta}\,\mathbf{e}_{\alpha}\cdot\mathbf{e}_{\beta}\,,\quad (\alpha,\beta = 1,2,3)$$

$$a^{\alpha} \overset{\mathrm{df}}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)g_{44}}}\frac{dv^{\alpha}}{dt}\,,\quad a^{\beta} \overset{\mathrm{df}}{=} \frac{1}{\sqrt{-(\operatorname{sgn} ds^2)g_{44}}}\frac{dv^{\beta}}{dt}\,,\quad (\alpha,\beta = 1,2,3)$$

$$\mathbf{e}_{\alpha}\cdot\mathbf{e}_{\beta} = -\left(\operatorname{sgn} ds^2\right)g_{\alpha\beta} \geq 0\,,\quad (ds)^2 \neq 0$$

$$a^2 = \frac{g_{\alpha\beta}}{g_{44}}\frac{dv^{\alpha}}{dt}\frac{dv^{\beta}}{dt}\,,\qquad a = \sqrt{\frac{g_{\alpha\beta}}{g_{44}}\frac{dv^{\alpha}}{dt}\frac{dv^{\beta}}{dt}}$$

- **Physical (true) components of total acceleration 3-vector**

$$\mathbf{a} = a^{\alpha}\,\mathbf{e}_{\alpha}\,,\quad (\alpha = 1,2,3)$$

$$\mathbf{a} = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}\; a^{\alpha}\,\frac{\mathbf{e}_{\alpha}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}}\,,\quad (ds)^2 \neq 0$$

$$\hat{a}^{\alpha} = \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}\; a^{\alpha} = \frac{\sqrt{g_{\alpha\alpha}}}{\sqrt{g_{44}}}\frac{dv^{\alpha}}{dt}$$ Physical components of total acceleration 3-vector

$$\hat{\mathbf{e}}_{\alpha} = \frac{\mathbf{e}_{\alpha}}{e_{\alpha}} = \frac{\mathbf{e}_{\alpha}}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}}\,,\quad \left|\hat{\mathbf{e}}_{\alpha}\right| = 1$$

$$\mathbf{a} = \hat{a}^{\alpha}\,\hat{\mathbf{e}}_{\alpha} = \frac{\sqrt{g_{\alpha\alpha}}}{\sqrt{g_{44}}}\frac{dv^{\alpha}}{dt}\hat{\mathbf{e}}_{\alpha}$$

- **Components of total acceleration 4-vector expressed by components of total acceleration 3-vector**

$$\tilde{a}^{\alpha} = \gamma_G^2 a^{\alpha} + \frac{\gamma_G}{-\left(\operatorname{sgn} ds^2\right)g_{44}}\frac{dx^{\lambda}}{dt}\frac{d\gamma_G}{dt}\,,\quad (\alpha = 1,2,3)$$

$$\tilde{a}^{4} = \frac{\gamma_G^2}{\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{44}}}\frac{dv^4}{dt} + \frac{\gamma_G}{-\left(\operatorname{sgn} ds^2\right)g_{44}}\frac{dx^4}{dt}\frac{d\gamma_G}{dt}$$

- **Physical (true) components of total acceleration 4-vector**

$$\tilde{\mathbf{a}} = \tilde{a}^{\lambda}\,\mathbf{e}_{\lambda}, \quad (\lambda = 1,2,3,4)$$

$$\tilde{\mathbf{a}} = \sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}\;\tilde{a}^{\lambda}\,\frac{\mathbf{e}_{\lambda}}{\sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}}, \quad (ds)^2 \neq 0$$

$$\hat{\tilde{a}}^{\lambda} = \sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}\;\tilde{a}^{\lambda}$$

$\hat{\tilde{a}}^{\lambda}$ = Physical components of total acceleration 4-vector

$$\hat{\mathbf{e}}_{\lambda} = \frac{\mathbf{e}_{\lambda}}{e_{\lambda}} = \frac{\mathbf{e}_{\lambda}}{\sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}}, \quad |\hat{\mathbf{e}}_{\lambda}| = 1$$

$$\tilde{a}^{\lambda} = \tilde{a}^{\lambda}_{\mathrm{total}} \overset{\mathrm{df}}{=} (\mathrm{sgn}\,ds^2)c^2\,\frac{d^2x^{\lambda}}{ds^2}$$

$$\tilde{a}^{\lambda} = \frac{\gamma_G}{\sqrt{-(\mathrm{sgn}\,ds^2)\,g_{44}}}\,\frac{d\tilde{v}^{\lambda}}{dt}$$

$$\tilde{v}^{\lambda} = \gamma_G v^{\lambda}$$

$$\hat{\tilde{a}}^{\lambda} = \frac{\gamma_G^2\sqrt{-(\mathrm{sgn}\,ds^2)\,g_{\lambda\lambda}}}{-(\mathrm{sgn}\,ds^2)\,g_{44}}\left[\frac{d^2x^{\lambda}}{dt^2} - \frac{1}{2g_{44}}\frac{dx^{\lambda}}{dt}\frac{dg_{44}}{dt} + \frac{1}{\gamma_G}\frac{d\gamma_G}{dt}\frac{dx^{\lambda}}{dt}\right]$$

$$\tilde{\mathbf{a}} = \hat{\tilde{a}}^{\lambda}\,\hat{\mathbf{e}}_{\lambda} \qquad \tilde{\mathbf{a}} = \sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}\;\tilde{a}^{\lambda}\,\hat{\mathbf{e}}_{\lambda}$$

$$\tilde{\mathbf{a}} = \sqrt{-(\mathrm{sgn}\,ds^2)g_{\lambda\lambda}}\;(\mathrm{sgn}\,ds^2)c^2\,\frac{d^2x^{\lambda}}{ds^2}\,\hat{\mathbf{e}}_{\lambda}$$

$$\tilde{\mathbf{a}} = \frac{\gamma_G\sqrt{g_{\lambda\lambda}}}{\sqrt{g_{44}}}\frac{d\tilde{v}^{\lambda}}{dt}\,\hat{\mathbf{e}}_{\lambda} \qquad \tilde{\mathbf{a}} = \left(\gamma_G^2\frac{\sqrt{g_{\lambda\lambda}}}{\sqrt{g_{44}}}\frac{dv^{\lambda}}{dt} + \frac{\gamma_G\sqrt{g_{\lambda\lambda}}}{\sqrt{g_{44}}}\,v^{\lambda}\,\frac{d\gamma_G}{dt}\right)\hat{\mathbf{e}}_{\lambda}$$

$$\tilde{\mathbf{a}} = \frac{\gamma_G^2\sqrt{-(\mathrm{sgn}\,ds^2)\,g_{\lambda\lambda}}}{-(\mathrm{sgn}\,ds^2)\,g_{44}}\left[\frac{d^2x^{\lambda}}{dt^2} - \frac{1}{2g_{44}}\frac{dx^{\lambda}}{dt}\frac{dg_{44}}{dt} + \frac{1}{\gamma_G}\frac{d\gamma_G}{dt}\frac{dx^{\lambda}}{dt}\right]\hat{\mathbf{e}}_{\lambda}$$

- **4-dimensional equations of motion of a test particle**

Components of acceleration 4-vector of a test particle with mass (m) in given point of
1. curved Riemann spacetime or
2. flat Minkowski spacetime in regard to non-inertial reference frame

are described by equations:

$$\frac{\tilde{F}^\alpha}{m} = \tilde{a}^\alpha_{force} \stackrel{df}{=} \left(\operatorname{sgn} ds^2\right)c^2\left(\frac{d^2x^\alpha}{ds^2} + \tilde{k}\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right), \quad ds^2 = g_{\mu\nu}dx^\mu dx^\nu \neq 0, \quad \left(\operatorname{sgn} ds^2\right)g_{\mu\nu} \leq 0$$

$\tilde{F}^\alpha$ = components of net force 4-vector, with ommision of „gravitational" and „inertial" forces

$g_{\mu\nu}$ = metric tensor of curved spacetime (components of this tensor are solutions of field equations) or metric tensor of non-inertial reference frame in flat Minkowski spacetime

$$\tilde{k} = \begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}$$

Postulated equations of motion of a test particle have interpretation as follows:

$$\tilde{a}^\alpha_{total} = \frac{\tilde{F}^\alpha}{m} + \tilde{a}^\alpha_{grav\&iner}$$

$$\tilde{a}^\alpha_{total} = \tilde{a}^\alpha = \left(\operatorname{sgn} ds^2\right)c^2\frac{d^2x^\alpha}{ds^2}$$

Component (corresponding with index α) of total acceleration of a test particle

$$\tilde{a}^\alpha_{grav\&iner} = -\left(\operatorname{sgn} ds^2\right)c^2\tilde{k}\,\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}$$

Sum of components (corresponding with index α) of gravitational and inertial acceleration of a test particle

In case of stationary metric with spatial-temporal zero-components, which is metric of type:

$$(ds)^2 = g_{\kappa\lambda}dx^\kappa dx^\lambda + g_{44}dx^4dx^4, \quad \frac{\partial g_{\kappa\lambda}}{\partial x^4} = 0, \quad \frac{\partial g_{44}}{\partial x^4} = 0, \quad (\kappa,\lambda = 1,2,3),$$

equations of motion, inter alia, can be also written in such forms:

$$\frac{\tilde{\mathbf{F}}}{m} = \left[\frac{\gamma_G\sqrt{g_{\alpha\alpha}}}{\sqrt{g_{44}}}\frac{d\tilde{v}^\alpha}{dt} + \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}\;\tilde{k}\,\Gamma^\alpha_{\mu\nu}\tilde{v}^\mu\tilde{v}^\nu\right]\hat{\mathbf{e}}_\alpha, \quad (\alpha,\mu,\nu = 1,2,3,4)$$

$$\frac{\tilde{\mathbf{F}}}{m} = \frac{\gamma_G^2\sqrt{-(\operatorname{sgn} ds^2)\,g_{\alpha\alpha}}}{-(\operatorname{sgn} ds^2)\,g_{44}}\left[\frac{d^2x^\lambda}{dt^2} - \frac{1}{2g_{44}}\frac{dx^\lambda}{dt}\frac{dg_{44}}{dt} + \frac{1}{\gamma_G}\frac{d\gamma_G}{dt}\frac{dx^\alpha}{dt} + \tilde{k}\,\Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right]\hat{\mathbf{e}}_\alpha$$

Let's remind:

$$\tilde{v}^\alpha \stackrel{df}{=} \frac{\gamma_G}{\sqrt{-(\operatorname{sgn} ds^2)\,g_{44}}}\frac{dx^\alpha}{dt}, \quad \frac{1}{\gamma_G} = \sqrt{1 - \frac{g_{\kappa\lambda}}{g_{44}}\frac{dx^\kappa}{dt}\frac{dx^\lambda}{dt}\frac{1}{c^2}}, \quad \hat{\mathbf{e}}_\alpha = \mathbf{e}_\alpha\Big/\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{\alpha\alpha}}, \quad \left|\hat{\mathbf{e}}_\alpha\right| = 1$$

3 FIELD EQUATIONS

- **Introduction**

From classical physics we know, that absolute value of gravitational field strength in center of homogeneous ball (that has constant density) is equal to zero. Together with growth of distance from the center – strength grows linearly, reaching its maximal value on the surface of a ball. With further growth of distance – it decreases inversely squared.

If we want to get the same result, within the frames of Einstein's general theory of relativity, then we have to notice that stationary gravitational field is a 2-potential field.

$$\frac{\partial \mathbf{E}}{\partial t}=0, \quad \mathrm{rot}\mathbf{E}=0, \quad \mathrm{rot\,grad}\,\varphi=0 \quad \Rightarrow \quad \begin{array}{lll} \mathbf{E}^{in}=\mathrm{grad}\,\varphi^{in}=-\tilde{k}\,\mathrm{grad}\,\varphi^{in}, & 0\le r<R, & \lim\limits_{r\to 0}\varphi^{in}=0 \\ \mathbf{E}^{ex}=-\mathrm{grad}\,\varphi^{ex}=-\tilde{k}\,\mathrm{grad}\,\varphi^{ex}, & r\ge R, & \lim\limits_{r\to\infty}\varphi^{ex}=0 \end{array}$$

$$E_r^{in}=-\frac{4}{3}\pi G\rho r, \quad \varphi^{in}=-\frac{2}{3}\pi G\rho r^2, \quad E_r^{ex}=-\frac{GM}{r^2}, \quad \varphi^{ex}=-\frac{GM}{r}.$$

On the surface of a ball, we have

$$\varphi^{in}-\varphi^{ex}=\frac{GM}{2R}, \quad \mathbf{E}^{in}-\mathbf{E}^{ex}=0.$$

$\mathbf{E}^{in}$, $\mathbf{E}^{ex}$ = gravitational field strenght respectively inside and outside of a ball
φ^{in}, φ^{ex} = gravitational field potential respectively inside and outside of a ball
M = mass of a ball, R = radius of a ball, ρ = density

$$\tilde{k}=\begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}$$

- **Field equations**

First accurate exterior and interior solutions of Einstein's field equations [1] was served by Schwarzschild [2, 3]. Other known interior solutions differ over form of energy momentum tensor, such as in Tolman's work [4].

Equations of gravitational field we will write as:

$$R_{\mu\nu}=-\kappa\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right) \quad \text{or} \quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\kappa T_{\mu\nu},$$

where

$$R_{\mu\nu}=\frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}}-\frac{\partial\Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}}+\Gamma^{\beta}_{\mu\alpha}\Gamma^{\alpha}_{\beta\nu}-\Gamma^{\beta}_{\mu\nu}\Gamma^{\alpha}_{\beta\alpha}, \quad \Gamma^{\alpha}_{\mu\nu}=\frac{1}{2}g^{\alpha\sigma}\left(\frac{\partial g_{\mu\sigma}}{\partial x^{\nu}}+\frac{\partial g_{\nu\sigma}}{\partial x^{\mu}}-\frac{\partial g_{\mu\nu}}{\partial x^{\sigma}}\right),$$

$$\kappa=\frac{8\pi G}{c^4}=2.073\cdot 10^{-43}\frac{s^2}{kg\cdot m}, \quad T\overset{df}{=}g^{\alpha\beta}T_{\alpha\beta}, \quad R\overset{df}{=}g^{\alpha\beta}R_{\alpha\beta}, \quad R=\kappa T,$$

$$x^1=r, \quad x^2=\theta, \quad x^3=\varphi, \quad x^4=ict.$$

In case where uniformly distributed mass in a ball, is a source of a gravitational field, we postulate existence of the solution in the following form:

$$(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + g_{44}(dx^4)^2,$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \quad g^{11} = \frac{1}{g_{11}}, \quad g^{22} = \frac{1}{g_{22}}, \quad g^{33} = \frac{1}{g_{33}}, \quad g^{44} = \frac{1}{g_{44}},$$

$$T_{\alpha\beta} = -\frac{1}{2}\rho c^2 g_{\alpha\beta}, \quad T \overset{df}{=} g^{\alpha\beta}T_{\alpha\beta} = -2\rho c^2, \quad \rho = \mathrm{const}.$$

The divergence of the tensor ($T_{\alpha\beta}$) should be equal to zero, which actually happens:

$$T_{\alpha\beta;\beta} = \left(-\frac{1}{2}\rho c^2 g_{\alpha\beta}\right)_{;\beta} = -\frac{1}{2}\rho c^2 (g_{\alpha\beta})_{;\beta} = 0.$$

Adopted assumptions let us reduce number of field equations to two.

$$\frac{r}{2}\frac{\partial^2 g_{44}}{\partial r^2} + \frac{\partial g_{44}}{\partial r} = -\frac{1}{2}\kappa\rho c^2 r$$

$$r\frac{\partial g_{44}}{\partial r} + g_{44} - 1 = -\frac{1}{2}\kappa\rho c^2 r^2$$

Equations are fulfilled, when

$$0 \le r < R, \quad \rho = \mathrm{const} > 0, \quad g_{44} = 1 - \frac{4\pi G\rho}{3c^2}r^2 = 1 - \frac{GM}{c^2R^3}r^2 = 1 - \frac{r_S}{2R^3}r^2,$$

$$r \ge R, \quad \rho = 0, \quad g_{44} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_S}{r}, \quad r \ne r_S.$$

$$M = \frac{4}{3}\pi\rho R^3$$

R = radius of a ball, inside which a mass source is located

$r_S = \dfrac{2GM}{c^2}$ = **Schwarzschild radius**

Presented solutions of field equations fulfill below boundary values.

$$0 \le r < R, \quad \lim_{r\to 0} g_{44} = 1$$

$$r \ge R, \quad \lim_{r\to\infty} g_{44} = 1$$

Quoted works

[1] A. Einstein: *Die Feldgleichungen der Gravitation.* Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften **2**, 48 (1915) 844-847.

[2] K. Schwarzschild: *Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie.* Sitzungsberichte der Königlich Preuβischen Akademie der Wissenschaften **1**, 7 (1916) 189-196.

[3] C. Schwarzschild: *Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssigkeit nach der Einsteinschen Theorie.* Sitzungsberichte der Königlich Preuβischen Akademie der Wissenschaften **1**, 18 (1916) 424-434.

[4] Richard C. Tolman: *Static Solutions of Einstein's Field Equations for Spheres of Fluid.* Physical Review **55**, 4 (February 15, 1939) 364-373.

4 GRAVITATIONAL FIELD OUTSIDE MASS SOURCE

- **Exterior Schwarzschild metric**

Space metric outside mass source ($r \geq R$, $\rho = 0$) is being described by exteror Schwarzschild metric [1]:

$$(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + \left(1 - \frac{r_S}{r}\right)(dx^4)^2, \quad x^4 = ict, \quad r \neq r_S = \frac{2GM}{c^2}$$

- **Speed of light propagation and exterior Schwarzschild metric**

Exterior Schwarzschild metric, for

$$\theta = \text{const}, \quad d\theta = 0, \quad \varphi = \text{const}, \quad d\varphi = 0,$$

reduces itself to

$$(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1}(dr)^2 - \left(1 - \frac{r_S}{r}\right)c^2(dt)^2.$$

We designate speed (v) of light propagation from condition:

$$(ds)^2 = 0$$

or equivalent

$$v^2 = \left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{r_S}{r}\right)^2 = c^2\left(1 - \frac{2GM}{rc^2}\right)^2.$$

Note that

$$\left[0 < \left(\frac{dr}{dt}\right)^2 \leq c^2\right] \Leftrightarrow \left[r \geq \frac{1}{2}r_S, \quad r \neq r_S\right].$$

It means that exterior Schwarzschild metric is correct if and only if

$$r \geq \frac{1}{2}r_S, \quad r \neq r_S.$$

- **Gravitational acceleration of free fall outside mass source**

We will designate radial component of gravtational acceleration of free falling test particle by equation of motion

$$\tilde{a}^r = \tilde{a}^1 = -\tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right), \quad r \neq r_S, \quad (ds)^2 \neq 0.$$

Taking into account, that

$$g_{44} = 1 - \frac{r_S}{r}, \quad r_S = \frac{2GM}{c^2}, \quad \tilde{k} = +1,$$

$$\Gamma^1_{11} = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r} = -\left(1 - \frac{r_S}{r}\right)^{-1} \cdot \frac{GM}{c^2 r^2}, \quad \Gamma^1_{44} = -\frac{1}{2} g_{44}\frac{\partial g_{44}}{\partial r} = -\left(1 - \frac{r_S}{r}\right) \cdot \frac{GM}{c^2 r^2},$$

$$1 = \left(1 - \frac{r_S}{r}\right)^{-1}\left(\frac{dr}{ds}\right)^2 + \left(1 - \frac{r_S}{r}\right)\left(\frac{dx^4}{ds}\right)^2,$$

we get

$$\boxed{\tilde{a}^r = \tilde{a}^1 = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r} = \left(\operatorname{sgn} ds^2\right)\frac{GM}{r^2}}.$$

Physical (true) component of gravitational acceleration of free fall

$$\hat{a}^r \stackrel{df}{=} \sqrt{-\left(\operatorname{sgn} ds^2\right) g_{rr}}\ \tilde{a}^r,$$

where

$$g_{rr} = g_{11} = \left(1 - \frac{r_S}{r}\right)^{-1},$$

finally can be written in the form:

$$\boxed{\hat{a}^r = \sqrt{-\left(\operatorname{sgn} ds^2\right) g_{rr}}\ \left(\operatorname{sgn} ds^2\right)\ \frac{GM}{r^2}}.$$

- **Gravity and anti-gravity**
  Above equation has interesting physical interpretation. For $r > r_S$ it describes gravity and for $\frac{1}{2} r_S \le r < r_S$ – anti-gravity.

Gravity

$$r > r_S = \frac{2GM}{c^2}, \quad g_{rr} = \left(1 - \frac{r_S}{r}\right)^{-1} > 0, \quad (ds)^2 < 0, \quad \boxed{\hat{a}^r = -\frac{GM}{r^2} \cdot \frac{1}{\sqrt{1 - \frac{r_S}{r}}}}$$

Anti-gravity

$$\frac{1}{2} r_S \le r < r_S = \frac{2GM}{c^2}, \quad g_{rr} = \left(1 - \frac{r_S}{r}\right)^{-1} < 0, \quad (ds)^2 > 0, \quad \boxed{\hat{a}^r = +\frac{GM}{r^2} \cdot \frac{1}{\sqrt{\frac{r_S}{r} - 1}}}$$

- **Main hypothesis**

......Anti-gravity works in such a way that free test particle located in external gravitational field (of a non rotating mass source) in certain area gets acceleration directed from the center of that mass source.

In areas, where $g_{\mu\nu} \geq 0$, $(ds)^2 < 0$, $(\mu, \nu = 1,2,3,4)$, graviy occurs.	In areas, where $g_{\mu\nu} \leq 0$, $(ds)^2 > 0$, $(\mu, \nu = 1,2,3,4)$, anti-graviy occurs.

Quoted works

[1] K. Schwarzschild: *Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie.* Sitzungsberichte der Königlich Preuβischen Akademie der Wissenschaften **1**, 7 (1916) 189-196.

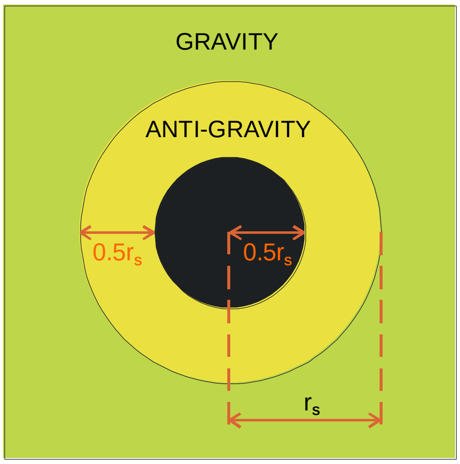
5 BLACK HOLE WITH MAXIMAL ANTI-GRAVITY HALO

- **Black hole with maximal anti-gravity halo**

Black hole with maximal anti-gravity halo we will call homogeneous ball with mass (M) and radius (R), for which

$$\frac{M}{R}=\frac{c^2}{G}\approx 1.3466\times 10^{27}\,\frac{kg}{m}.$$

Spatial radius of a black hole with maximal anti-gravity halo is equal to half of Schwarzschild radius. Area of outside black hole with maximal anti-gravity halo consist of two layers, in first $(0.5\,r_S \le r < r_S)$ anti-gravity occurs and in second $(r > r_S)$ – gravity. Thickness of this anti-gravity shell is equal to spatial radius of black hole with maximal anti-gravity halo. Inside of anti-gravity layer acceleration is directed from the center of mass source, and inside of gravity layer – towards the center



Inside of anti-gravity layer acceleration is directed from the center of a mass source and it grows to the border with gravity layer. Then acceleration changes direction, and its absolute value decreases together with a growth of distance from the center.
This model can be called „layer model”: anti-gravity – gravity.

First time model of black hole with maximal anti-gravity halo was proposed by me in 2005 on the V Forum of Unconventional Inventions, Constructions and Ideas in Wrocław on the occasion of the 100th anniversary of Einstein’s formulation of special theory of relativity. This Forum was organized by Janusz Zagórski.

6 GRAVITATIONAL FIELD INSIDE MASS SOURCE

- **Spacetime metric inside mass source**
  Spacetime metric inside mass source ($0 \le r < R$, $\rho = const > 0$) is given by:

$$(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta\,(d\varphi)^2 + g_{44}(dx^4)^2,$$

where

$$x^4 = ict, \quad r_S = \frac{2GM}{c^2}, \quad g_{11} = \frac{1}{g_{44}}, \quad g_{44} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2 .$$

- **Speed of light propagation in virtual vacuum tunnel that is inside mas source**
  Spacetime metric inside mass source for

$$\theta = const, \quad d\theta = 0, \quad \varphi = const, \quad d\varphi = 0,$$

reduces itself to a form:

$$(ds)^2 = g_{11}(dr)^2 - g_{44}c^2(dt)^2, \quad g_{11} = \frac{1}{g_{44}}, \quad g_{44} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2 .$$

We will designate speed (v) of light propagation in virtual vacuum tunnel from condition

$$(ds)^2 = 0$$

or equivalent

$$\boxed{v^2 = \left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{r_S}{2R^3} r^2\right)^2 = c^2\left(1 - \frac{GM}{c^2R^3} r^2\right)^2}.$$

Notice that

$$\left[0 < \left(\frac{dr}{dt}\right)^2 \le c^2, \quad R \ge \frac{1}{2} r_S\right] \Leftrightarrow [r < R].$$

It means that interior metric is correct if and only if

$$\boxed{r < R, \quad R \ge \frac{1}{2} r_S}.$$

- **Gravitational acceleration of free fall inside mass source**

Radial component of gravitational acceleration of freely falling test particle inside virtual vacuum tunnel, which is located inside mass source, we will get from equation of motion

$$\tilde{a}^r = \tilde{a}^1 = -\tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right), \quad 0 \le r < R, \quad (ds)^2 \ne 0.$$

Taking into account that

$$\tilde{k} = -1, \quad \operatorname{sgn} ds^2 = -1, \quad R \ge \frac{1}{2}r_S, \quad r_S = \frac{2GM}{c^2},$$

$$\Gamma^1_{11} = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r},$$

$$\Gamma^1_{44} = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r},$$

$$g_{44} = 1 - \frac{4\pi G\rho}{3c^2}r^2 = 1 - \frac{GM}{c^2R^3}r^2 = 1 - \frac{r_S}{2R^3}r^2,$$

$$1 = g_{44}^{-1}\left(\frac{dr}{ds}\right)^2 + g_{44}\left(\frac{dx^4}{ds}\right)^2,$$

we get

$$\tilde{a}^r = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r} = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{GM}{R^3}r = -\frac{GM}{R^3}r.$$

Physical (true) component of gravitational acceleration of free fall

$$\hat{a}^r \overset{df}{=} \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{rr}}\;\tilde{a}^r,$$

where

$$g_{rr} = g_{11} = \left(1 - \frac{r_S}{2R^3}r^2\right)^{-1},$$

in the end, can be written in a form:

$$\hat{a}^r = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{rr}}\;\frac{GM}{R^3}r = -\frac{GM}{R^3}r\cdot\frac{1}{\sqrt{1 - \frac{r_S}{2R^3}r^2}}.$$

7 GRAVITATIONAL FIELD INSIDE BLACK HOLE WITH MAXIMAL ANTI-GRAVITY HALO

- **Spacetime metric inside black hole with maximal anti-gravity halo**
  Spacetime metric inside black hole with maximal anti-gravity halo is given by:

$$(ds)^2 = \left(1 - \frac{r^2}{R^2}\right)^{-1} (dr)^2 + r^2 (d\theta)^2 + r^2 \sin^2\theta \,(d\varphi)^2 + \left(1 - \frac{r^2}{R^2}\right) (dx^4)^2 ,$$

where

$$x^4 = ict , \quad 0 \le r < R = \frac{1}{2} r_S , \quad r_S = \frac{2GM}{c^2} , \quad g_{11} = \frac{1}{g_{44}} , \quad g_{44} = 1 - \frac{r^2}{R^2} , \quad \rho = const > 0 .$$

- **Speed of light propagation in virtual vacuum tunnel that is inside black hole with maximal anti-gravity halo**
  Spacetime metric inside black hole with maximal anti-gravity halo for

$$\theta = const , \quad d\theta = 0 , \quad \varphi = const , \quad d\varphi = 0 ,$$

reduces to a form

$$(ds)^2 = \left(1 - \frac{r^2}{R^2}\right)^{-1} (dr)^2 - \left(1 - \frac{r^2}{R^2}\right) c^2 (dt)^2 .$$

We will designate speed (v) of light propagation in virtual vacuum tunnel from condition

$$(ds)^2 = 0$$

or equivalent

$$\boxed{v^2 = \left(\frac{dr}{dt}\right)^2 = c^2 \left(1 - \frac{r^2}{R^2}\right)^2} .$$

Notice that

$$\left[0 < \left(\frac{dr}{dt}\right)^2 \le c^2\right] \Leftrightarrow [r < R] .$$

- **Gravitational acceleration of free fall inside black hole with maximal anti-gravity halo**

Radial component of gravitational acceleration of freely falling test particle inside virtual vacuum tunnel, which is located inside black hole with maximal anti-gravity halo, we will get from equation of motion

$$\tilde{a}^r = \tilde{a}^1 = -\tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right), \quad 0 \le r < R, \quad (ds)^2 \ne 0.$$

Taking into account that

$$\tilde{k} = -1, \quad \operatorname{sgn} ds^2 = -1, \quad R = \frac{1}{2}r_S, \quad r_S = \frac{2GM}{c^2}, \quad \frac{M}{R} = \frac{c^2}{G},$$

$$\Gamma^1_{11} = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r},$$

$$\Gamma^1_{44} = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r},$$

$$g_{44} = 1 - \frac{r^2}{R^2},$$

$$1 = g^{-1}_{44}\left(\frac{dr}{ds}\right)^2 + g_{44}\left(\frac{dx^4}{ds}\right)^2,$$

we get

$$\tilde{a}^r = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r} = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{R^2}r = -\frac{c^2}{R^2}r.$$

Physical (true) component of gravitational acceleration of free fall

$$\hat{a}^r \stackrel{df}{=} \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{rr}}\;\tilde{a}^r,$$

where

$$g_{rr} = g_{11} = \left(1 - \frac{r^2}{R^2}\right)^{-1},$$

in the end, can be written in a form:

$$\hat{a}^r = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\sqrt{-\left(\operatorname{sgn} ds^2\right)g_{rr}}\;\frac{c^2}{R^2}r = -\frac{c^2}{R^2}r\cdot\frac{1}{\sqrt{1 - \frac{r^2}{R^2}}}.$$

8 GRAPHIC ANALYSIS OF THE FULL SOLUTION

- **Charts that show how time-time component of metric tensor and physical (true) component of gravitational acceleration of free fall depends on the distance from the center of black hole with maximal anti-gravity halo**

We will make those charts for case, where spatial radius of black hole is half of Schwarzschild radius

$$R = \frac{1}{2} r_S = \frac{GM}{c^2},$$

i.e. for black hole with maximal anti-gravity halo.

Time-time component of metric tensor and physical (true) component of gravitational acceleration of free fall, in three distance intervals from the center of black hole, are given by below relations.

GRAVITY

$$0 \le r < \frac{1}{2} r_S, \quad g_{44} = \left(1 - \frac{r^2}{R^2}\right) > 0, \quad \hat{a}^r = -\frac{c^2}{R^2} r \cdot \frac{1}{\sqrt{1 - \frac{r^2}{R^2}}}$$

ANTI-GRAVITY

$$\frac{1}{2} r_S \le r < r_S, \quad g_{44} = \left(1 - \frac{r_S}{r}\right) < 0, \quad \hat{a}^r = +\frac{GM}{r^2} \cdot \frac{1}{\sqrt{\frac{r_S}{r} - 1}}$$

GRAVITY

$$r > r_S, \quad g_{44} = \left(1 - \frac{r_S}{r}\right) > 0, \quad \hat{a}^r = -\frac{GM}{r^2} \cdot \frac{1}{\sqrt{1 - \frac{r_S}{r}}}$$

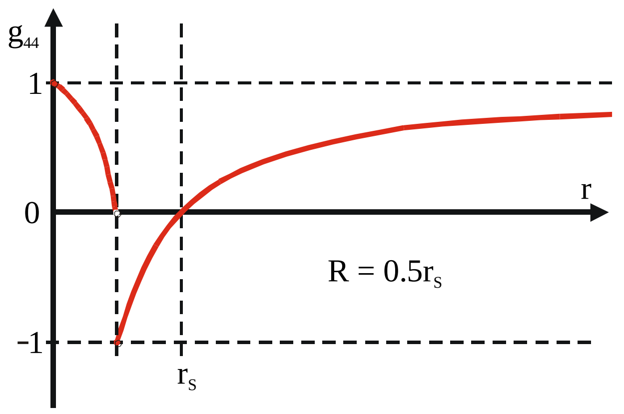


Chart that shows how time-time component (g_{44}) of metric tensor depends on the distance (r) from the center of black hole with maximal anti-gravity halo.

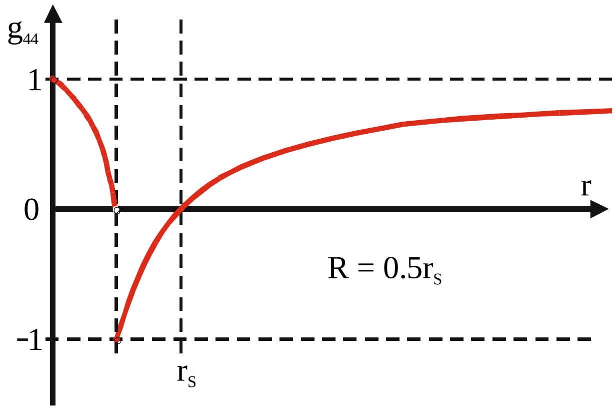


Chart that shows how time-time component (g_{44}) of metric tensor depends on the distance (r) from the center of bkack hole with maximal anti-gravity halo.

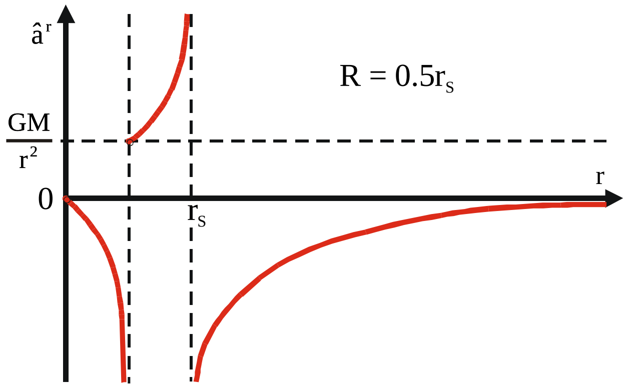


Chart of dependence of radial component ($\hat{a}^r$) of physical (true) gravitational acceleration of free fall on the distance (r) from the center of black hole with maximal anti-gravity halo.

ATTENTION
First graph is doublet for better clearness on the same page.

From the charts we can see that:

$0 \le r < 0.5 \cdot r_S \Rightarrow$ gravity

$r = 0.5 \cdot r_S \Rightarrow$ transition from gravity to anti-gravity

$0.5 \cdot r_S < r < r_S \Rightarrow$ anti-gravity

$r = r_S \Rightarrow$ transition from anti-gravity to gravity

$r > r_S \Rightarrow$ gravity

Gravity and anti-gravity has layer-like nature.

9 FIELD EQUATIONS AND EQUATIONS OF MOTION

- **Equations of motion are contained in field equations**

How should one formulate equations of motion, so that unscaled radial components of 4-acceleration could be served with below forms?

$$\tilde{a}^r_{in} \overset{df}{=} (\operatorname{sgn} ds^2) c^2 \frac{d^2 r}{ds^2} = -\frac{GM}{R^3} r = -\tilde{k} (\operatorname{sgn} ds^2) \frac{GM}{R^3} r, \quad 0 \le r < R, \quad \tilde{k} = -1, \quad (\operatorname{sgn} ds^2) < 0$$

$$\tilde{a}^r_{ex} \overset{df}{=} (\operatorname{sgn} ds^2) c^2 \frac{d^2 r}{ds^2} = -\frac{GM}{r^2} = \tilde{k} (\operatorname{sgn} ds^2) \frac{GM}{r^2}, \quad r > R, \quad \tilde{k} = +1, \quad (\operatorname{sgn} ds^2) < 0$$

While answering to this question, we will use second of two field equations.

$$\frac{r}{2} \frac{\partial^2 g^{in}_{44}}{\partial r^2} + \frac{\partial g^{in}_{44}}{\partial r} = -\frac{1}{2} \kappa \rho c^2 r$$

$$r \frac{\partial g^{in}_{44}}{\partial r} + g^{in}_{44} - 1 = -\frac{1}{2} \kappa \rho c^2 r^2$$

$$g^{in}_{44} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2 R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2, \quad \kappa = \frac{8\pi G}{c^4}, \quad r_S = \frac{2GM}{c^2}$$

$$\frac{c^2}{2} \frac{\partial g^{in}_{44}}{\partial r} = -\frac{4}{3} \pi G \rho r = -\frac{GM}{R^3} r$$

$$\tilde{a}^r_{in} \overset{df}{=} (\operatorname{sgn} ds^2) c^2 \frac{d^2 r}{ds^2} = -\tilde{k} (\operatorname{sgn} ds^2) \frac{GM}{R^3} r = -\frac{GM}{R^3} r$$

$$\tilde{a}^r_{in} = \tilde{k} (\operatorname{sgn} ds^2) \frac{c^2}{2} \frac{\partial g^{in}_{44}}{\partial r}$$

$$\Gamma^1_{11} \frac{dr}{ds} \cdot \frac{dr}{ds} + \Gamma^1_{44} \frac{dx^4}{ds} \cdot \frac{dx^4}{ds} = -\frac{1}{2} \frac{\partial g^{in}_{44}}{\partial r}$$

$$\tilde{a}^r_{in} = -\tilde{k} (\operatorname{sgn} ds^2) c^2 \left(\Gamma^1_{11} \frac{dr}{ds} \cdot \frac{dr}{ds} + \Gamma^1_{44} \frac{dx^4}{ds} \cdot \frac{dx^4}{ds} \right)$$

Analogical considerations for component ($\tilde{a}^r_{ex}$) lead to a conclusion, that equations of motion should have the form served below:

$$\tilde{a}^\alpha \overset{df}{=} (\operatorname{sgn} ds^2) c^2 \frac{d^2 x^\alpha}{ds^2} = -\tilde{k} (\operatorname{sgn} ds^2) c^2 \Gamma^\alpha_{\mu\nu} \frac{dx^\mu}{ds} \cdot \frac{dx^\nu}{ds}$$

$$\tilde{k} = \begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}$$

- **Equations of motion and field equations in General Relativity, and 2-potential nature of Newton's stationary gravitational field**

Postulated by us, within the frames of General Relativity, equations of motion of free test particle lead to a conclusion that Newton's stationary gravitational field is a 2-potential field. We will show it on an example of gravitational field, which source is a mass distributed homogeneously in a volume of a ball with radius (R).

$$\frac{\tilde{F}^{\alpha}}{m} = \tilde{a}^{\alpha} + \tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\,\Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}, \quad ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \neq 0, \quad \left(\operatorname{sgn} ds^2\right)g_{\mu\nu} \leq 0$$

$$\tilde{F}^{\alpha} = 0$$

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = ict$$

$$(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + g_{44}\left(dx^4\right)^2$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta$$

$$\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds} = -\frac{1}{2}\frac{\partial g_{44}}{\partial r}$$

$$\tilde{a}^r = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r}$$

a^r_{in}, a^r_{ex} = radial acceleration respectively inside and outside of a ball

g^{in}_{44}, g^{ex}_{44} = metric tensor components respectively inside and outside of a ball

$$g^{in}_{44} = 1 - \frac{4\pi G\rho}{3c^2}r^2 = 1 - \frac{GM}{c^2R^3}r^2 = 1 - \frac{r_S}{2R^3}r^2, \quad 0 \leq r < R, \quad \rho = \text{const} > 0$$

$$g^{ex}_{44} = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_S}{r}, \quad r \geq R, \quad r \geq R, \quad \rho = 0$$

$$\tilde{a}^r_{in} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r} = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{GM}{R^3}r, \quad 0 \leq r < R$$

$$\tilde{a}^r_{ex} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g^{ex}_{44}}{\partial r} = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{GM}{r^2}, \quad r \geq R$$

$$r >> r_S, \quad v^2 << c^2, \quad r_S = \frac{2GM}{c^2}$$

$$a^r = \frac{d^2 r}{dt^2}, \quad \operatorname{sgn} ds^2 = -1, \quad \tilde{k} = \begin{cases} +1 & \text{outside of homogeneous ball} \\ -1 & \text{inside of homogeneous ball} \end{cases}$$

$$a^r_{in} = -\tilde{k}\,\frac{c^2}{2}\frac{\partial g^{in}_{44}}{\partial r} = -\frac{GM}{R^3}r, \quad 0 \leq r < R$$

$$a^r_{ex} = -\tilde{k}\,\frac{c^2}{2}\frac{\partial g^{ex}_{44}}{\partial r} = -\frac{GM}{r^2}, \quad r \geq R$$

By substitution in last two equations

$$g_{44}^{in}=1+\frac{2\varphi^{in}}{c^2}\,,\qquad g_{44}^{ex}=1+\frac{2\varphi^{ex}}{c^2}\,,$$

where (φ^{in}) and (φ^{ex}) are potentials of gravitational field respectively inside and outside of a ball, we will get

$$a_{in}^{r}=-\tilde{k}\,\frac{\partial\varphi^{in}}{\partial r}=\frac{\partial\varphi^{in}}{\partial r}=-\frac{GM}{R^3}r\,,\quad 0\le r<R\,,\quad \varphi^{in}=-\frac{GM}{2R^3}r^2,\quad \lim_{r\to 0}\varphi^{in}=0$$

$$a_{ex}^{r}=-\tilde{k}\,\frac{\partial\varphi^{ex}}{\partial r}=-\frac{\partial\varphi^{ex}}{\partial r}=-\frac{GM}{r^2}\,,\quad r\ge R\,,\quad \varphi^{ex}=-\frac{GM}{r}\,,\quad \lim_{r\to\infty}\varphi^{ex}=0$$

Now lets serve definition of potentials (φ^{in}) and (φ^{ex}) that corresponds with standard definition of gravitational potential.

$$\varphi^{in}\overset{df}{=}\int_0^{r<R}a_{in}^{r}\,dr=-\frac{GM}{2R^3}r^2$$

$$\varphi^{ex}\overset{df}{=}-\int_{r\ge R}^{\infty}a_{ex}^{r}\,dr=-\frac{GM}{r}$$

In the field equations inside the mass source, we will replace the time-time component (g_{44}^{in}) of the metric tensor with the potential (φ^{in}).

$$\frac{r}{2}\frac{\partial^2 g_{44}^{in}}{\partial r^2}+\frac{\partial g_{44}^{in}}{\partial r}=-\frac{1}{2}\kappa\rho c^2 r$$

$$r\frac{\partial g_{44}^{in}}{\partial r}+g_{44}^{in}-1=-\frac{1}{2}\kappa\rho c^2 r^2$$

$$\Big\downarrow\quad g_{44}^{in}=1+\frac{2\varphi^{in}}{c^2}$$

$$r^2\cdot\frac{\partial^2\varphi^{in}}{\partial r^2}+2r\cdot\frac{\partial\varphi^{in}}{\partial r}=-4\pi G\rho r^2$$

$$2r\frac{\partial\varphi^{in}}{\partial r}+2\varphi^{in}=-4\pi G\rho r^2$$

Poisson’s equation for the potential (φ^{in})

First of those equations is Poisson’s equation for the potential (φ^{in}) in spherical coordinate system. From classical Poisson’s equation it differs only in sign of right side. Then, from both equations it appears that

$$r^2\cdot\frac{\partial^2\varphi^{in}}{\partial r^2}=2\varphi^{in}$$

Now we will analize second field equation for potential (φ^{in}).

$$r\cdot\frac{\partial\varphi^{in}}{\partial r}=-2\pi G\rho r^2-\varphi^{in}$$

$$\varphi^{in}=-\frac{2}{3}\pi G\rho r^2=-\frac{GM}{2R^3}r^2$$

$$\frac{\partial\varphi^{in}}{\partial r}=-\frac{4}{3}\pi G\rho r=-\frac{GM}{R^3}r$$

$$a^r_{in}=-\frac{4}{3}\pi G\rho r=-\frac{GM}{R^3}r$$

$$\frac{\partial\varphi^{in}}{\partial r}=a^r_{in}$$

Equation of motion for radial component of free fall acceleration inside of mass source

By analyzing Poisson's equation for the potential (φ^{in}) we showed that the equations of motion are contained in field equations inside the source mass.

In the field equations outside mass source we will substitute the time-time component (g^{ex}_{44}) of the metric tensor with the potential (φ^{ex}).

$$\frac{r}{2}\frac{\partial^2 g^{ex}_{44}}{\partial r^2}+\frac{\partial g^{ex}_{44}}{\partial r}=-\frac{1}{2}\kappa\rho c^2 r$$

$$r\frac{\partial g^{ex}_{44}}{\partial r}+g^{ex}_{44}-1=-\frac{1}{2}\kappa\rho c^2 r^2$$

$$g^{ex}_{44}=1+\frac{2\varphi^{ex}}{c^2},\qquad \rho=0$$

$$r^2\cdot\frac{\partial^2\varphi^{ex}}{\partial r^2}+2r\cdot\frac{\partial\varphi^{ex}}{\partial r}=0$$

$$2r\cdot\frac{\partial\varphi^{ex}}{\partial r}+2\varphi^{ex}=0$$

Poisson's equation for the potential (φ^{ex})

First of those equations is Poisson's equation for the potential (φ^{ex}) in spherical coordinate system. Then, from both equations it appears that

$$r^2\cdot\frac{\partial^2\varphi^{ex}}{\partial r^2}=2\varphi^{ex}$$

Now let's analize second field equation for potential (φ^{ex}).

$$r\frac{\partial \varphi^{ex}}{\partial r} = -\varphi^{ex}$$

$$\varphi^{ex} = -\frac{GM}{r}$$

$$\frac{\partial \varphi^{ex}}{\partial r} = \frac{GM}{r^2}$$

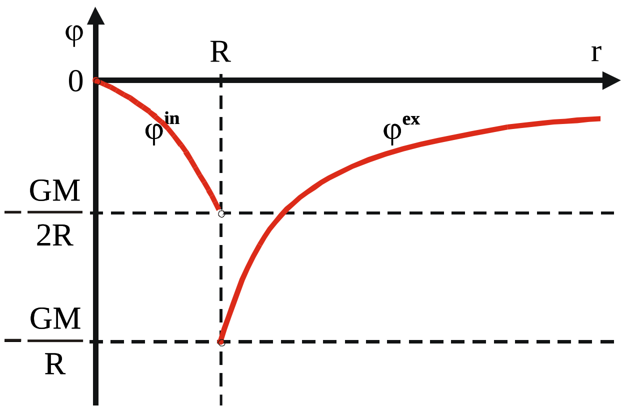
$$a^{r}_{ex} = -\frac{GM}{r^2}$$

$$\frac{\partial \varphi^{ex}}{\partial r} = -a^{r}_{ex}$$

Equation of motion for radial component of free fall acceleration outside of mass source

Analyzing Poisson's equations for the potentials (φ^{in}) and (φ^{ex}) we obtained analogous results. The equations of motion are included in field equations.

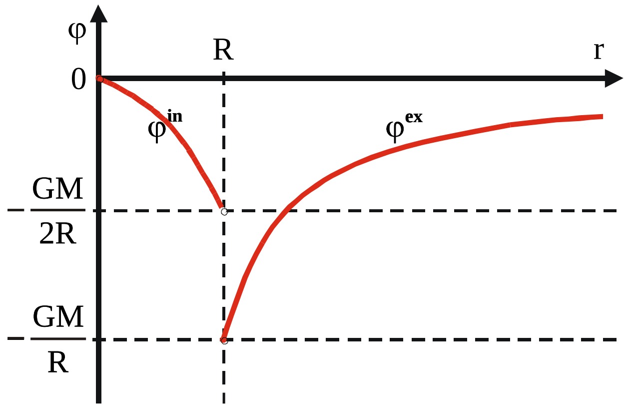
Introducing two potentials into Newton's theory of gravitation allowed to find solution of field and motion equations (within the frames of General Relativity), which in boundary case ($r >> r_S$, $v^2 << c^2$) lead to compabillity of both theories, both inside as well as outside of mass source.



$$\varphi^{in} = -\frac{GM}{2R^3}r^2, \quad 0 \le r < R, \quad \lim_{r\to 0}\varphi^{in} = 0$$

$$\varphi^{ex} = -\frac{GM}{r}, \quad r \ge R, \quad \lim_{r\to\infty}\varphi^{ex} = 0$$

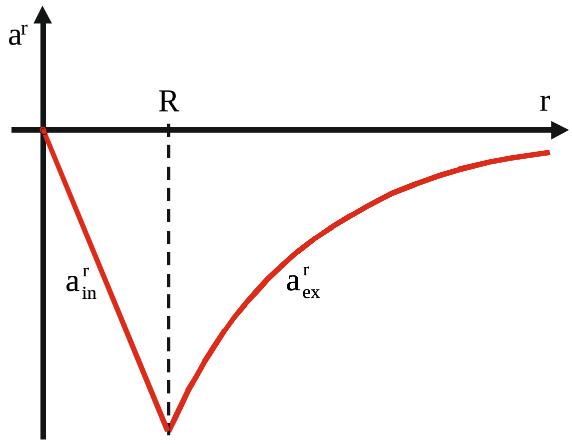
Diagram that shows how potentials (φ^{in}) and (φ^{ex}) depend on the distance (r) from the center of mass source in case of ($r >> r_S$) and ($v^2 << c^2$).



$$\varphi^{in} = -\frac{GM}{2R^3} r^2, \quad 0 \le r < R, \quad \lim_{r \to 0} \varphi^{in} = 0$$

$$\varphi^{ex} = -\frac{GM}{r}, \quad r \ge R, \quad \lim_{r \to \infty} \varphi^{ex} = 0$$

Diagram that shows how potentials (φ^{in}) and (φ^{ex}) depend on the distance (r) from the center of mass source in case of ($r >> r_S$) and ($v^2 << c^2$).



$$a^r_{in} = -\frac{GM}{R^3} r, \quad 0 \le r < R$$

$$a^r_{ex} - \frac{GM}{r^2}, \quad r \ge R$$

Diagram that shows how radial components of free fall acceleration (a^r_{in}) and (a^r_{ex}) depend on the distance (r) from the center of mass source in case of ($r >> r_S$) and ($v^2 << c^2$).

ATTENTION
For clearness, we have also featured diagrams from previous page.

10 NEW TEST OF GENERAL RELATIVITY

- **The proposition of the experiment**

How to show, in Earth conditions, 2-potential nature of gravitational field and also prove the existence of black holes with anti-gravity halo? In this purpose we have to measure ratio of distance passed by light to the time of flight, in a vertically positioned vacuum cylinder, right under and right above the surface of Earth (of the sea level). If the difference of squares of those measurements will be equal to the square of escape speed, then it will prove existence of black holes with anti-gravity halo. This experiment would be a new test of General Theory of Relativity.

Below we will justify purpose of this experiment:

$$\left(\frac{dr}{dt}\right)_{in}^2 - \left(\frac{dr}{dt}\right)_{ex}^2 = ?$$

$\left(\frac{dr}{dt}\right)_{in}$ i $\left(\frac{dr}{dt}\right)_{ex}$ – ratio of path travelled by the light to the time of travel measured respectively right under and right over Earth surface

$$\left(\frac{dr}{dt}\right)_{in}^2 = c^2\left(1-\frac{r_S}{2R^3}r^2\right)^2$$

$$\left(\frac{dr}{dt}\right)_{ex}^2 = c^2\left(1-\frac{r_S}{r}\right)^2$$

$$r \approx R$$

$$r_S = \frac{2GM}{c^2}$$

$$\frac{r_S}{R} << 1$$

$$\left(\frac{dr}{dt}\right)_{in} + \left(\frac{dr}{dt}\right)_{ex} \approx 2c$$

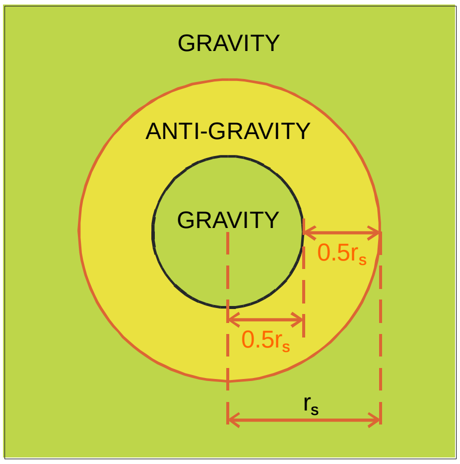
$$\frac{GM}{R} = \left(7.91\frac{km}{s}\right)^2$$

$$c = 2.99792458\cdot 10^8\,\frac{m}{s} \approx 3\cdot 10^8\,\frac{m}{s}$$

$$\left(\frac{dr}{dt}\right)_{in}^2 - \left(\frac{dr}{dt}\right)_{ex}^2 \cong \frac{2GM}{R} \qquad \left(\frac{dr}{dt}\right)_{in} - \left(\frac{dr}{dt}\right)_{ex} \cong \frac{GM}{cR} \cong 0.208\,\frac{m}{s}$$

11 BLACK HOLE MODEL OF OUR UNIVERSE

- **Our Universe as a black hole with maximal anti-gravity halo**



In the center of black hole, gravitational acceleration of free particle is equal to zero which corresponds with Gauss law. Later, absolute value of acceleration grows together with growth of distance from the center. In anti-gravity layer, gravitational acceleration is directed from the center of mass source and it grows to the border with gravitational layer. Then, acceleration turns direction, and its absolute value decreases with growth of distance from the center.
This model can be called „layer model":
gravity – anti-gravity – gravity.

Our Universe can be treated as a gigantic, homogeneous Black Hole. It is isolated from the rest of universe with a space area where anti-gravity occurs. Our galaxy together with Solar system and Earth (which in cosmological scales can be treated merely as a point) should be located near the center of Black Hole. Once again let's remind that in the center of black hole, gravitational acceleration of free particle, leaving out local gravitational fields, is equal to zero. In other words: Black Hole is not generating gravitational field in its center.

- **Radius of Our Universe**

Below, let's estimate radius of Our Universe, assuming that, its average density is the same as density (n_p) of protons in every cubic meter.

$$R = \frac{1}{2} r_S = \frac{GM}{c^2}$$

$$M = \frac{4}{3}\pi\rho R^3$$

$$R = \sqrt{\frac{3c^2}{4\pi G}} \cdot \sqrt{\frac{1}{\rho}}$$

$$c = 2.99792458 \cdot 10^8 \frac{m}{s} \approx 3 \cdot 10^8 \frac{m}{s}, \quad G = 6.6742 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \approx 6.7 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$$

$$\rho \approx n_p \cdot 1.67 \cdot 10^{-27} \frac{kg}{m^3}$$

n_p = average amount of protons in every cubic meter

$m_p = 1.67262171 \cdot 10^{-27} kg$ = proton mass

light year $\approx 0.95 \cdot 10^{16} m$

$$R \approx \sqrt{\frac{19}{n_p}} \cdot 10^{26} m \approx 10.5 \cdot \sqrt{\frac{19}{n_p}} \cdot 10^9 \text{ light years}$$

12 PHOTON PARADOX

- **Introduction**

Theory of relativity, both special and general, is strictly connected with wave theory of light (electomagnetic waves). Attempt to explaining gravitational redshift on a ground of photon light theory in spacetimes other than conformally flat leads to a paradox.

- **Gravitational redshift**

Gravitational redshift is the phenomenon that the spectrum of light reaching the Earth from the Sun or other stars is shifted towards the longer wavelengths relative to analogical spectrum of light that comes from emitter which is located on the Earth.

- **Conformally flat spacetime**

Conformally flat spacetime is a spacetime with metric type like:

$$(ds)^2 = \left[K\left(x^1, x^2, x^3, x^4\right)\right]^2 \quad \left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 + \left(dx^4\right)^2\right].$$

$$x^1 = x, \quad x^2 = y, \quad x^3 = z, \quad x^4 = ict$$

$$K = K\left(x^1, x^2, x^3, x^4\right) = \text{ conformal factor}$$

- **Schwarzschild spacetime**

Gravitational field outside of mass source (M) is being described by Schwarzschil metric [3].

$$(ds)^2 = \left\{\delta_{\alpha\beta} + \frac{x^\alpha x^\beta}{r^2}\left[\left(1 - \frac{r_S}{r}\right)^{-1} - 1\right]\right\} dx^\alpha dx^\beta + \left(\delta_{44} - \frac{r_S}{r}\right) dx^4 dx^4, \quad (\alpha, \beta = 1,2,3)$$

$$\delta_{\alpha\beta} = \begin{cases} 1 \Leftrightarrow \alpha = \beta \\ 0 \Leftrightarrow \alpha \neq \beta \end{cases}$$

$$r^2 = \left(x^1\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2$$

$$r_S = \frac{2GM}{c^2}$$

- **Friedman-Lemaître-Robertson-Walker spacetime**

FLRW metric [4, 5, 6, 8, 9] describes spatially isotropic and homegeneous universe.

$$ds^2 = B^2 L^2 \left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right] + \left(dx^4\right)^2$$

$$x^4 = ict$$

$$L = L(t) = \text{ dimensionless time scale factor}$$

$$B = \frac{1}{1+\frac{1}{4}\frac{L^2 r^2}{L^2 a^2}} = \frac{1}{1+\frac{1}{4}\frac{r^2}{a^2}} = \frac{1}{1+\frac{1}{4}kr^2}$$
$$r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$
$$k = \frac{1}{a^2}, \quad \operatorname{sgn} k = \operatorname{sgn}\frac{1}{a^2} = -1,\ 0,\ +1$$
a^2 = square of unscaled constant radius of space curavutre

- **Photon paradox**

Idea of photon in the context of Schwarzschild and Friedman-Lemaître-Robertson-Walker metrics leads to a paradox. Calculating the influence of those metrics on photon energy from the equation $E = \frac{h}{T}$ or equivalent $E = \frac{hc}{\lambda}$, we get different results, depending on used equation. For simplification, let's assume that $dx^2 = dx^3 = 0$. Then we get:

$$\hat{E} = \frac{h}{\hat{T}}, \quad \hat{E} = \frac{hc}{\hat{\lambda}}$$
$\hat{T} = KT, \quad \hat{\lambda} = K\lambda$ for conformally flat metric

$\hat{T} = \sqrt{g_{44}}\,T, \quad \hat{\lambda} = \frac{\lambda}{\sqrt{g_{44}}}$ for Schwarzschild metric

$\hat{T} = T, \quad \hat{\lambda} = BL\lambda$ for FLRW metric

This paradox was called by me „photon paradox”.

- **Does photons have memory?**

Photon paradox doesn't occur in conformally flat spacetimes. In other spacetimes, one of solutions for photon paradox is an assumption that photon energy depends on point of space-time, inside which its emission happened and it remains constant during photon movement. It means that photons have memory, or more scholarly – photon energy is invariant. Wherein, in stronger gravitational field given source should send photons with less energy, than the same source located in weaker field.

- **Energy of photon in Newton's gravitational field**

Let's remind a deduction of equation for total energy ($\hat{E}$) of body source with mass (M) and the photon emitted by given atom, which in emission moment is at distance (R) from the center of mass souce.

$$\hat{E} = h\nu - \frac{GMm}{R} = \text{const}$$
$$\downarrow \quad m = \frac{h\nu}{c^2}$$
$$\hat{E} = h\nu - \frac{GMh\nu}{Rc^2} = h\nu\left(1 - \frac{GM}{Rc^2}\right) = \text{const}$$
$$\downarrow \quad 1 - \frac{GM}{Rc^2} \approx \sqrt{1 - \frac{2GM}{Rc^2}}$$
$$\downarrow$$

$$\hat{E} = h\nu\sqrt{1-\frac{2GM}{Rc^2}} = \text{const}$$

$$\nu = \frac{1}{T}, \quad \nu = \frac{c}{\lambda}$$

$$h\nu = \frac{h}{T} = \frac{hc}{\lambda} = \frac{\hat{E}}{\sqrt{1-\frac{2GM}{Rc^2}}} = \frac{\text{const}}{\sqrt{1-\frac{2GM}{Rc^2}}}$$

$$h = \text{const}$$
$$c = \text{const}$$
$$\hat{E} = \text{const}$$

$$R\uparrow, \quad \nu\downarrow, \quad T\uparrow, \quad \lambda\uparrow$$

Interpretation of gravitational redshift (or blueshift) based on above equations, relies on an assumption that **photon energy in the moment of emission doesn't depend on gravitational field, inside which given atom was placed**. It should be ephasized that those relations aren't generatig photon paradox.

Analogical reasoning to above, has been served for the first time by Einstein [1] and I think it has only heuristic meaning.

$$\hat{E} = h\nu - \frac{GMm}{R} = \text{const}$$

$$m = \frac{2h\nu}{c^2}$$

$$\hat{E} = h\nu - \frac{2GMh\nu}{Rc^2} = h\nu\left(1-\frac{2GM}{Rc^2}\right) = \text{const}$$

$$\nu = \frac{1}{T}, \quad \nu = \frac{c}{\lambda}$$

$$h\nu = \frac{h}{T} = \frac{hc}{\lambda} = \frac{\hat{E}}{1-\frac{2GM}{Rc^2}} = \frac{\text{const}}{1-\frac{2GM}{Rc^2}}$$

$$h = \text{const}$$
$$c = \text{const}$$
$$\hat{E} = \text{const}$$

$$R\uparrow, \quad \nu\downarrow, \quad T\uparrow, \quad \lambda\uparrow$$

In a frame on the left, previous calculations were repeated, based on new form of expression for rest energy [13, 14].

$$E = \frac{1}{2}mc^2$$

- **Hydrogen atom in Schwarzschild gravitational field – heuristic approach**

We will estimate energy of photon emitted by hydrogen atom inside gravitational field described by exterior Schwarzschild mertic.

$$\hat{E} \sim -\frac{1}{\hat{r}}$$

$$\hat{r} = r\sqrt{g_{11}}\,, \quad g_{11} = \left(1 - \frac{2GM}{c^2 R}\right)^{-1} = \left(1 - \frac{R_S}{R}\right)^{-1}, \quad g_{44} = \frac{1}{g_{11}}, \quad R_S = \frac{2GM}{c^2}, \quad E \sim -\frac{1}{r}$$

$$\hat{E} \sim -\frac{1}{r\sqrt{g_{11}}}\,, \quad \hat{E} = \frac{E}{\sqrt{g_{11}}} = \sqrt{g_{44}}\, E$$

$$\boxed{\Delta\hat{E} = \frac{\Delta E}{\sqrt{g_{11}}} = \sqrt{g_{44}}\,\Delta E}, \quad \left(\frac{2GM}{c^2 R}\right)_{\text{Earth}} \approx 1.4\cdot 10^{-9}, \quad \left(\frac{2GM}{c^2 R}\right)_{\text{Sun}} \approx 4.3\cdot 10^{-6}$$

E = energy on allowed (stationary) orbit under the absence of gravitational field
r = radius of allowed (stationary) orbit under the absence of gravitational field
$\hat{E}$ = energy on allowed (stationary) orbit in exterior gravitational field
$\hat{r}$ = radius of allowed (stationary) orbit in exterior gravitational field
g_{11}, g_{44} = meric tensor components of exterior gravitational field
R = distance of photon emission place from the center of body source
M = mass of body source
G = gravitational constant
c = speed of light in vacuum
R_S = Schwarzschild radius of body source

Einstein couldn't serve such reasoning in 1911 [1], because Bohr atom model was formulated by Bohr in 1913 [2] and Schwarzschild metric showed on in 1916 [3].

- **How to define redshift?**

$$\boxed{z^* \overset{\text{df}}{=} \frac{E_{\text{lab}} - E_{\text{out}}}{E_{\text{out}}} = \frac{E_{\text{lab}}}{E_{\text{out}}} - 1}$$

$$E_{\text{lab}} = \frac{E_{\max}}{\sqrt{g_{11}^{\text{lab}}}}$$

$$E_{\text{out}} = \frac{E_{\max}}{\sqrt{g_{11}^{\text{out}}}}$$

$$\boxed{z^* = \frac{\sqrt{g_{11}^{\text{out}}}}{\sqrt{g_{11}^{\text{lab}}}} - 1}$$

E_{lab} = photon energy emitted from a source that is in laboratory
E_{out} = photon energy emitted from a source that is outside laboratory
E_{max} = photon energy emitted from a source in the absence of gravitational field
g_{11}^{lab} = component of metric tensor in laboratory in a place of photon detection
g_{11}^{out} = component of metric tensor outside of laboratory in a place of photon emission

- **Redshift of light which comes to Earth from the Sun**

$$z^* = \frac{\sqrt{g_{11}^{out}}}{\sqrt{g_{11}^{lab}}} - 1$$

$$g_{11}^{out} = \left[1 - \left(\frac{2GM_S}{c^2R_S}\right)\right]^{-1}, \quad g_{11}^{lab} = \left[1 - \left(\frac{2GM_E}{c^2R_E}\right)\right]^{-1}$$

$$\left(\frac{2GM_S}{c^2R_S}\right) \approx 4.3 \cdot 10^{-6}, \quad \left(\frac{2GM_E}{c^2R_E}\right) \approx 1.4 \cdot 10^{-9}$$

$$z^* \approx \frac{1}{2}\left(\frac{2GM_S}{c^2R_S}\right) - \frac{1}{2}\left(\frac{2GM_E}{c^2R_E}\right) - \frac{1}{4}\left(\frac{2GM_S}{c^2R_S}\right)\left(\frac{2GM_E}{c^2R_E}\right) \approx 2.15 \cdot 10^{-6}$$

Sun-light redshift is a local effect, thats why for components of metric tensor we used equations right for exterior Schwarzschild metric, which depends on local mass sources and their sizes. Also we assumed that given light source is located either on Sun or Eartth surface.

- **Redshift of light which comes to Earth from a distant galaxy**

$$z^* = \frac{\sqrt{g_{11}^{out}}}{\sqrt{g_{11}^{lab}}} - 1$$

$$g_{11}^{out} = \left(1 - \frac{r^2}{R^2}\right)^{-1}$$

$$g_{11}^{lab} = \left[1 - \left(\frac{2GM_{Earth}}{c^2R_{Earth}}\right)\right]^{-1} \approx \frac{1}{1 - 1.4 \cdot 10^{-9}}$$

$$z^* = \frac{\sqrt{1 - \left(\frac{2GM_{Earth}}{c^2R_{Earth}}\right)}}{\sqrt{1 - \frac{r^2}{R^2}}} - 1 \approx \frac{\sqrt{1 - 1.4 \cdot 10^{-9}}}{\sqrt{1 - \frac{r^2}{R^2}}} - 1$$

On the next page there is a graph of redshift (z*) dependence on the distance (r) of the source from the center of Our Universe, (R) is the radius of Our Universe.

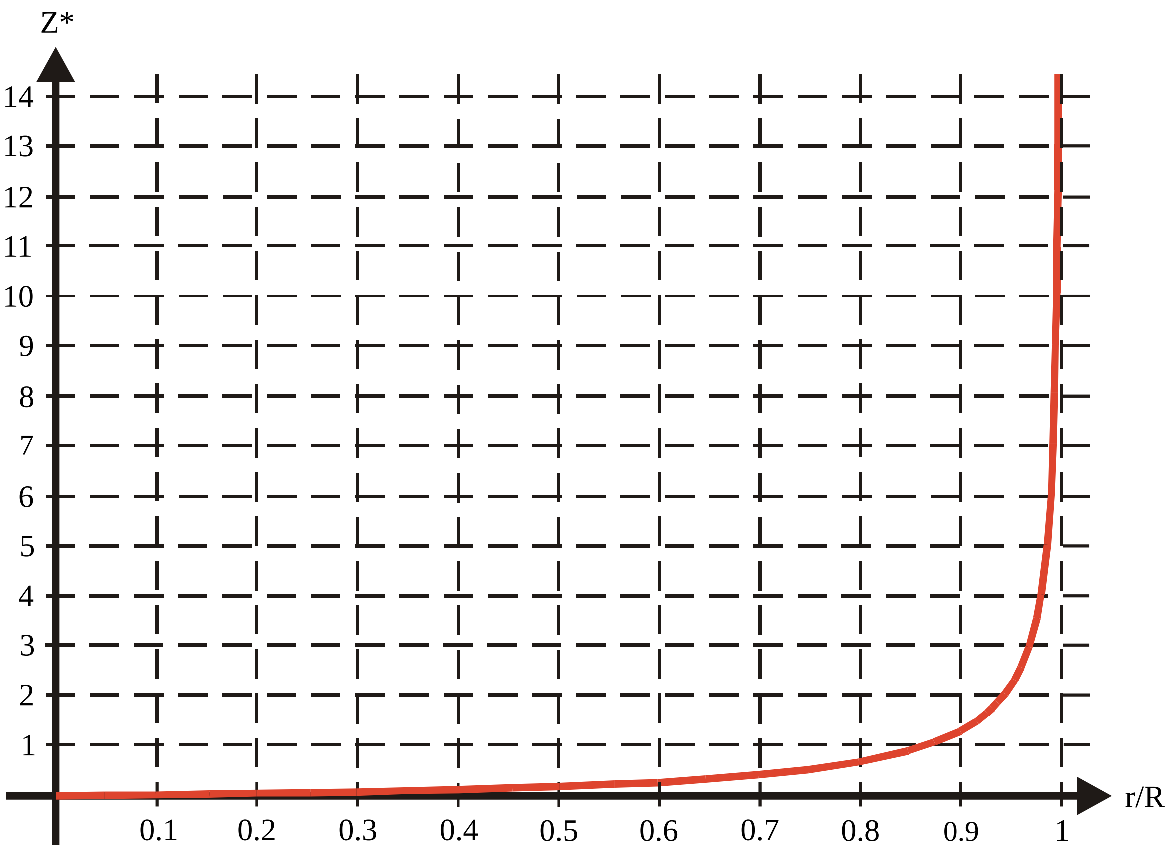


Diagram that shows dependence of redshift (z*) versus the distance (r) of the source from the center of Our Universe. [Attension: (z*) has negative values for ratio (r/R) approximate less than $3.74\cdot 10^{-5}$.]

Below let's estimate redshift (z*) for case, when ($r^2 << R^2$).

$$z^* \approx \frac{\sqrt{1-1.4\cdot 10^{-9}}}{\sqrt{1-\frac{r^2}{R^2}}} - 1 \approx \frac{1}{\sqrt{1-\frac{r^2}{R^2}}} - 1 = \frac{1}{\sqrt{1-\frac{r}{R}}\cdot\sqrt{1+\frac{r}{R}}} - 1$$

$$\left(1-\frac{r}{R}\right)^{-\frac{1}{2}} \approx 1+\frac{1}{2}\cdot\frac{r}{R}$$

$$z^* = \frac{1+\frac{1}{2}\cdot\frac{r}{R}}{\sqrt{1+\frac{r}{R}}} - 1$$

$$\sqrt{1+\frac{r}{R}} \approx 1$$

$$z^* \approx \frac{1}{2}\cdot\frac{r}{R}$$

- **Hubble's law**

First let's remind a definition of redshift (z) which bases on an assumption that photon energy doesn't depend on place of emission and changes during the movement of photon.

$$z \overset{df}{=} \frac{E_{emitted} - E_{observed}}{E_{observed}} = \frac{E_{emitted}}{E_{observed}} - 1$$

$$E = h\nu = \frac{h}{T} = \frac{hc}{\lambda}$$

$$z = \frac{\nu_{emitted}}{\nu_{observed}} - 1 = \frac{T_{observed}}{T_{emitted}} - 1 = \frac{\lambda_{observed}}{\lambda_{emitted}} - 1$$

Hubble's law [7] is a result of connection between Hubble's observations and non-relativistic Doppler law for light.

$$z = \frac{\lambda_{observed}}{\lambda_{emitted}} - 1$$

Hubble's observations: $z = k_H r$, $k_H \approx 0.81 \cdot 10^{-26} m^{-1}$, k_H = Hubble's coefficient

non-relativistic Doppler law for light: $z = \frac{v}{c}$

$$v = ck_H r = Hr, \quad H \overset{df}{=} ck_H \approx 75\,km \cdot s^{-1} \cdot Mpc^{-1} \approx 2.43 \cdot 10^{-18} s^{-1}$$ Hubble's law

In literature [12] given values of Hubble's constant (H) are contained in wide range: $H = \left(60\,km \cdot s^{-1} \cdot Mpc^{-1} \approx 1.944 \cdot 10^{-18} s^{-1}\right) \div \left(75\,km \cdot s^{-1} \cdot Mpc^{-1} \approx 2.43 \cdot 10^{-18} s^{-1}\right)$

Let's emphasize that proposed by us definition of redshift (z*) was basing on assumption that photon energy depends on place of emission and doesn't change during movement of photon. Redshift values (z) and (z*) are the same.

- **Radius of Our Universe according to Hubble's observation**

Knowing (z*) and (r), we can designate radius of Our Universe (R).

$$R = \frac{r}{\sqrt{1 - \left(\frac{1}{z^* + 1}\right)^2}}$$

$$r^2 << R^2$$

$$R \approx \frac{r}{2z^*}$$

Hubble's observations: $z = k_H r$, $k_H \approx 0.81 \cdot 10^{-26} m^{-1}$ for $H = 75\,km \cdot s^{-1} \cdot Mpc^{-1}$

$z = z^*$

light year $\approx 0.95 \cdot 10^{16} m$

$$R \approx \frac{1}{2k_H} \approx 0.6 \cdot 10^{26} m \approx 6.31 \text{ billion light years}$$

- **Average density of Our Universe according to Hubble's observation**

Now let's calculate average density of Our Universe, using estimated above radius value of Our Universe and equation from page 40.

$$R = \sqrt{\frac{3c^2}{4\pi G}} \cdot \sqrt{\frac{1}{\rho}}$$

$$\rho = \frac{3c^2}{4\pi G} \cdot \frac{1}{R^2} \approx \frac{3c^2 k_H^2}{\pi G} = \frac{3H^2}{\pi G}$$

This density is 8 times bigger than critical density of Friedman's Universe.

$R = R_H \approx 0.6 \cdot 10^{26} m$ = radius of Our Universe for $H = 75\, km \cdot s^{-1} \cdot Mpc^{-1}$

$\rho = \rho_H$ = density of Our Universe for $H = 75\, km \cdot s^{-1} \cdot Mpc^{-1}$

$\rho_F \approx 4.97 \cdot 10^{-27}\, kg/m^3$ that is almost 3 protons per every cubic meter

ρ_F = density of Friedman's Universe according to [11] for $H = 75\, km \cdot s^{-1} \cdot Mpc^{-1}$ and $\Omega = 0.47$

$$\rho_H \approx 84.59 \cdot 10^{-27} kg \cdot m^{-3}$$

that is almost 51 protons/m^3

$$\frac{\rho_H}{\rho_F} \approx 17.02$$

For Hubble's constant $H = 75\, km \cdot s^{-1} \cdot Mpc^{-1}$ and density parameter $\Omega = 0.47$ density of Black Hole Universe is over 17 times bigger than density of Friedman's Universe.
Our model doesn't require us to assume the existence of dark energy.

- **Constancy of photon energy and Pound-Rebka experiment**

The purpose of Pound-Rebka experiment [10] was to prove a hypothesis, that photon energy doesn't depend on place of emission and changes during movement in Newton's gravitational field. Therefore result from this hyphothesis is also: energy absorbed by atom doesn't depend on place of absorption.

$$h\nu \sim \frac{1}{1 - \frac{GM}{Rc^2}}$$

$$z \stackrel{df}{=} \frac{\nu_{emitted}}{\nu_{absorbed}} - 1$$

$$\nu_e \sim \frac{1}{1 - \frac{GM}{Rc^2}} \approx 1 + \frac{GM}{Rc^2}$$

$$\nu_a \sim \frac{1}{1-\frac{GM}{(R \pm L)c^2}}$$

R = distance from Earth's center
L = distance between photon's absorption and emission points
± plus, when emission point was closer to the center than absorption point
± minus, when emission point was further from the center than absorption point

$$z \approx \frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R \pm L}\right)$$

$$g = \frac{GM}{R^2}$$ = gravitational acceleration in distance (R) from Earth's center

$$z \approx \pm \frac{gL}{c^2} \frac{1}{1 \pm \frac{L}{R}}$$

$$L << R$$

$$z \approx \pm \frac{gL}{c^2}$$

Last relation was confirmed experimentally by Pound and Rebka in 1960, using Mössbauer and Doppler phenomenons. Gamma ray emiter ($h\nu = 14.4\,\mathrm{keV}$) was ^{57}Co, and ^{57}Fe was an absorber. Emitter and absorber was located alternately, in turns. First, emitter on botton and absorber on top, then vice-versa. Gravitational redshift was being neutralized by Doppler shift. Energy differences of appropriate energetical atom levels of emitter and absorber (which was in the same distance from the center of Eatrth) was identical.

Now let's make analogical deduction, granted this time, that energy of photon depends on place of emission and is constant during its migration in Schwarzschild gravitational field. Let's notice that energy absorbed by atom also depends on place of absorbtion.

$$h\nu \sim \sqrt{1-\frac{2GM}{Rc^2}}$$

$$z^* \overset{df}{=} \frac{E_{absorbed} - E_{emitted}}{E_{emitted}} = \frac{E_{absorbed}}{E_{emitted}} - 1 = \frac{\nu_{absorbed}}{\nu_{emitted}} - 1$$

$$\nu_{absorbed} \sim \sqrt{1-\frac{2GM}{(R \pm L)c^2}} \approx 1 - \frac{GM}{(R \pm L)c^2}$$

$$\frac{1}{\nu_{emitted}} \sim \frac{1}{\sqrt{1-\frac{2GM}{Rc^2}}} \approx 1 + \frac{GM}{Rc^2}$$

R = distance from Earth's center
L = distance between photon's absorption and emission points
± plus, when emission point was closer to the center than absorption point
± minus, when emission point was further from the center than absorption point

$$z^* \approx \frac{GM}{c^2}\left(\frac{1}{R} - \frac{1}{R \pm L}\right)$$

$$g = \frac{GM}{R^2}$$ = gravitational acceleration in distance (R) from Earth center

$$z^* \approx \pm \frac{gL}{c^2} \frac{1}{1 \pm \frac{L}{R}}$$

$$L << R$$

$$z^* \approx \pm \frac{gL}{c^2}$$

Pound-Rebka experiment does not settle which hypothesis is correct.

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13 EARTH GRAVITATIONAL FIELD AND UNIVERSE GRAVITATIONAL FIELD

- **Influence of gravitational field on space and time distances**

Gravitational fields studied by us can be clearly characterized by time-time component of metric tensor.

In big distances from the center of Our Universe, spacetime metric

$$(g_{44})_{Universe} = \left(1 - \frac{r^2}{R^2}\right)_{Universe}$$

has different form than locally near Earth

$$(g_{44})_{Earth} = \left(1 - \frac{r_S}{r}\right)_{Earth} \quad .$$

From above equations results, that:

1. In cosmological scale: the further from Earth, the stronger gravitational field is. Locally near Earth we observe opposite situation.
2. Space distance between two closely placed events is the greater the stronger gravitational field is.

$$r \downarrow \Rightarrow (g_{11})_{Earth}(dr)^2 \uparrow \qquad r \uparrow \Rightarrow (g_{11})_{Universe}(dr)^2 \uparrow \qquad (g_{11}) = (g_{44})^{-1}$$

This phenomenon can be called „gravitational dilatation of space distance".

3. Time distance between two closely placed events is the lower the stronger gravitational field is.

$$r \downarrow \Rightarrow (g_{44})_{Earth}(cdt)^2 \downarrow \qquad r \uparrow \Rightarrow (g_{44})_{Universe}(cdt)^2 \downarrow$$

This phenomenon can be called „gravitational dilatation of time distance".

- **Local properties of redshift**

We will determine the distance (r_0) from the center of Our Universe, in which there should be a given light source emitting photons with the same energy as an identical source located on the Earth surface. For this purpose, we compare the time-time components of metric tensors that characterize the gravitational fields of the Universe and Earth, respectively.

$$1 - \frac{r_0^2}{R_U^2} = 1 - \frac{r_S^E}{R_E}$$

r_S^E = Schwarzschild radius for Earth

R_E = radius of Earth

R_U = radius of Our Universe

$$r_0 = \sqrt{\frac{r_S^E}{R_E}} \cdot R_U$$

$$\sqrt{\frac{r_S^E}{R_E}} \approx 3.74 \cdot 10^{-5}, \quad R_U \approx 0.6 \cdot 10^{26} \text{m}, \quad \text{light year} \approx 0.95 \cdot 10^{16} \text{m}$$

$$r_0 \approx 2.245 \cdot 10^{21} \text{m} \approx 2.363 \cdot 10^5 \text{ light years} = 236300 \text{ light years}$$

In distance (r_0) from the center of Our Universe, time-time component of metric tensor is equal to analogical component on Earth surface.

- **Conclusions**

1. In a distance from the center of Earth, approximately equal to (r_0) redshift (z^*) measured in regard to Our Planet changes sign from negative to positive.
2. Light that comes to Earth from Our Galaxy, which radius is around 50000 ly, and its thickness around 12000 ly, should be shifted towards violet in regard to the light emitted on the surface of Earth. Wherein negative value of reshift (z^*) should depend on the direction of observation.

14 OLBERS' PARADOX

- **Olbers' paradox**

Olbers' paradox, also called photometric paradox, was formulated by Olbers in 1826 [1]: „If universe is static, homogeneous and infinite in time and space, then why sky at night is dark?"

Olbers tried to explain this paradox, assuming that interstellar matter is absorbing light that is heading towards Earth.

......According to commonly accepted idea, formulated within the frames of Friedman's expanding universe theory [2, 3] and based on it hypothesis about Bing Bang, sky at night is dark, because age of universe is finite and light from distant stars didn't manage to reach us yet, in addition its spectrum is redshifted.

In my view, above statement is wrong in a part reffering to the age of universe. According to this statement, sky at night should get brighter over time. In turn, redshift can be explained within the scope of other hypotheses.

- **Probability of photon hitting Earth**

Portion of photons that come to Earth at night is only a negligable amount of all photons, especially photons emitted in very far distances from Earth in comparision with Earth radius.

Dividing by 4π Earth's solid angle of view from the emission point of a photon, denoted by (α), we obtain the probability that the photon „hit" the Earth.

$$R_E << r \Rightarrow \alpha \approx \frac{2\pi R_E^2}{r^2}$$

$$\boxed{R_E << r \Rightarrow \frac{\alpha}{4\pi} \approx \frac{1}{2}\frac{R_E^2}{r^2}}$$

$$R_E \approx 6.371 \cdot 10^6 \text{ m}$$
$$r \approx 0.6 \cdot 10^{26} \text{ m}$$

$$\frac{\alpha}{4\pi} \approx \frac{1}{2}\frac{R_E^2}{r^2} = \frac{1}{2}\left(\frac{6.371 \cdot 10^6 \text{ m}}{0.6 \cdot 10^{26} \text{ m}}\right)^2 \approx 1.388 \cdot 10^{-40}$$

Probability that photon emitted on the edge of Our Universe will „hit" Earth is only $\boxed{1.388 \cdot 10^{-40}}$.

- **Bohr hydrogen atom in Our Universe**

Below we will serve a formula for hydrogen atom energy in Our Universe, when electron is on n-th allowed (stationary) orbit.

$$\hat{E}_n = \sqrt{g_{44}^{in}}\, E_n$$

$$E_n = \frac{1}{n^2} E_1$$

$$\hat{E}_n = \frac{1}{n^2}\sqrt{g_{44}^{in}}\, E_1$$

$$E_1 = -13.6\,\mathrm{eV}$$

$$g_{44}^{in} = 1 - \frac{4\pi G\rho}{3c^2} r^2 = 1 - \frac{GM}{c^2 R^3} r^2 = 1 - \frac{r_S}{2R^3} r^2 = 1 - \frac{r^2}{R^2}$$

$$R = R_H \approx 0.6 \cdot 10^{26}\mathrm{m} = \text{ Our Universe radius for } H = 75\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}$$

$$\hat{E}_n = -\frac{1}{n^2}\sqrt{g_{44}^{in}}\,13.6\,\mathrm{eV} = -\frac{1}{n^2}\sqrt{1 - \frac{r^2}{R^2}}\,13.6\,\mathrm{eV} = -\frac{1}{n^2}\sqrt{1 - \frac{r^2}{0.36 \cdot 10^{52}\mathrm{m}^2}}\,13.6\,\mathrm{eV}$$

E_n = energy on n-th stationary orbit in the absence of gravitational field
$\hat{E}_n$ = energy on n-th stationary orbit of hydrogen atom which is in distance (r) from the center of Our Universe
g_{44}^{in} = component of metric tensor of gravitational field in distance (r) from the center of Our Universe
r = distance of photon emission place from the center of Our Universe
M = mass of Our Universe
R = radius of Our Universe
ρ = density of Our Universe
G = gravitational constant
c = speed of light in vacuum
r_S = Schwarzschild radius of Our Universe

EXAMPLE
In what distance from Earth should hydrogen atom be, so that energy of emitted photon which correspond in Earth conditions to shortwave edge of Lyman series (13.6 eV) was equal to the upper limit of energy which correspond in Earth conditions to infrared radiation (1.59 eV)?

$$1.59\,\mathrm{eV} = \sqrt{1 - \frac{r^2}{R^2}}\,13.6\,\mathrm{eV}$$

$$\frac{r}{R} \approx 0.9$$

Light emitted by hydrogen atoms on the edges of Our Universe ($r \approx 0.9 \cdot R$) lies completly in infrared.

REMINDER
Infrared radiation (Infrared) are photons with energies from $1.24 \cdot 10^{-3}\,\mathrm{eV}$ to $1.59\,\mathrm{eV}$

- **Illuminance of Earth surface at night**

  Luminous intensity of Earth's horizontal surface by starry sky at night is $3 \cdot 10^{-4}\,\mathrm{lx}$ [4, page 204].

To appoint Earth's horizontal surface luminous intensity caused by the starry sky at night with light emitted by hydrogen atoms, let's hypothetically assume that there are 51 hydrogen atoms in every cubic meter of Our Universe. Also let's take into account energy corresponding to given spectral line, which would be emitted by hydrogen atom in Earth conditions, fraction of hydrogen atoms that in a time unit emitt photons which correspond to given spectral line, probability for a photon hitting the Earth and redshift.

$$dI_i = \frac{1}{4\pi R_E^2} \cdot \frac{\rho_H}{m_H} \cdot 4\pi r^2 dr \cdot E_i \cdot \eta_i \cdot \frac{R_E^2}{2r^2} \cdot \sqrt{1 - \frac{r^2}{R^2}}$$

$$I_i = \frac{\pi}{2} \frac{\rho_H}{m_H R} E_i \eta_i \int_{R_Z}^{R} \sqrt{R^2 - r^2} dr$$

$$I_i \approx \frac{\pi}{8} \frac{R\rho_H}{m_H} E_i \eta_i$$

In the end, for the Earth's horizontal surface luminous intensity by the starry sky at night, we will get below formula:

$$I \approx \sum_i I_i = \frac{\pi}{8} \frac{R\rho_H}{m_H} \sum_i E_i \eta_i$$

I_i = Earth's horizontal surface luminous intensity at night, with light emitted by hydrogen atoms corresponding to given spectral line
I = Earth's horizontal surface luminous intensity by the starry sky at night
$4\pi R_E^2$ = Earth surface area
$\frac{\rho_H}{m_H}$ = amount of hydrogen atoms in one cubic meter
$4\pi r^2 dr$ = spherical shell volume with thickness dr
E_i = energy corresponding to given spectral line which would be emitted by hydrogen atom in Earth conditions
η_i = fraction of hydrogen atoms which emit (in time unit) photons corresponding to given spectral line
$[\eta] = s^{-1}$
$\frac{2\pi R_E^2}{r^2}$ = probability that photon will hit Earth
$\sqrt{1 - \frac{r^2}{R^2}}$ = time-time component of Our Universe metric tensor
ρ_H = Our Universe density
m_H = hydrogen atom mass
R_E = Earth radius
R = Our Universe radius
r = distance from the center of Our Universe

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15 MICROVAWE BACKGROUND RADIATION

- **Background radiation**

Background radiation is a microwave radiation that corresponds to 2.7 Kelvin degree temperature, which comes to Earth almost equally from all directions. It's being called relict radiation or residual radiation.

Background radiation was discovered by Penzias and Wilson in 1965 [2]. Back then, they were working for Bell Laboratories, probing a radio connection with satelittes. In that purpose they were using 6 meter directional antenna. Noise popping out from that antenna was proven to be a microwave isotropic background radiation.

Detailed studies of cosmic microwave background radiation has been done with instruments which was placed on a COBE satelitte. Cosmic Background Explorer has been launched on 18 November 1989. Initial results of measurements were known two months later [3]. It turned out that spectrum of cosmic microwave background radiation is almost identical with spectrum of black body radiation with 2.735 Kelvin degree temperature with 0.06 K error margin. According to other data from COBE [4, 5] there is a quadrupole effect in Our Galaxy, and there are only negligible fluctuactions in spatial distribution of background radiation temperature.

- **Background radiation in Big-Bang theory**

According to commonly accepted opinion, discovery of cosmic microwave background radiation confirms hypothesis about existence of relict radiation as remant of Big Bang. Such hypothesis was first formulated by Gamov in 1948 [1].

- **Background radiation in Black Hole Universe**

We postulate that cosmic microwave background radiation in black hole with maximal anti-gravity halo universe model, are photons emitted by atoms and molecules which are on the edges of Our Universe.

EXAMPLE

Here we will use expression (given in the chapter 12 of this lecture) for energy of hydrogen atom in Our Universe, when electron is on a n-th allowed (stationary) orbit.

$$\hat{E}_n = -\frac{1}{n^2}\sqrt{1-\frac{r^2}{R^2}}\,13.6\,\text{eV}$$

$\hat{E}_n$ = energy on n-th stationary orbit of hydrogen atom which is in distance (r) from the center of Our Universe
r = photon emission distance from the center of Our Universe
R = radius of Our Universe

What distance from Earth is needed for hydrogen atom, so that energy of emitted photon which in Earth conditions corresponds to shortwave Lyman series limit (13.6 eV) was equal to energy $6.6\cdot10^{-4}\,\text{eV}$, which in Earth conditions corresponds to maximum radiation intensity of black body that is in 2,7 Kelvin degree temperature?

$$6.6\cdot10^{-4}\,\text{eV} = \sqrt{1-\frac{r^2}{R^2}}\,13.6\,\text{eV}\,, \qquad r \approx 0.999999999\cdot R$$

REMINDING

Microwaves correspond to photons with energies from $4.14\cdot10^{-6}\text{eV}$ to $1.24\cdot10^{-3}\text{eV}$.

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16 AVERAGE UNIVERSE DENSITY IN FRIEDMAN'S THEORY

- **Critical density**

Critical density of universe in Friedman' theory [2] is a density, where universe becomes spatially flat.

$$\rho_c = \frac{3H^2}{\kappa c^4} = \frac{3H^2}{8\pi G}$$

$$H = 75\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1} \approx 2.43\cdot 10^{-18}\mathrm{s}^{-1}$$

$$\kappa = \frac{8\pi G}{c^4} = 2.073\cdot 10^{-43}\frac{\mathrm{s}^2}{\mathrm{kg}\cdot\mathrm{m}}, \quad c = 2.99792458 \cdot 10^8\frac{\mathrm{m}}{\mathrm{s}} \approx 3\cdot 10^8\frac{\mathrm{m}}{\mathrm{s}}$$

$$\rho_c \approx 1.058\cdot 10^{-26}\frac{\mathrm{kg}}{\mathrm{m}^3}$$

- **Density parameter and current average density of Friedman's universe**

The density parameter is the ratio of current average density (ρ_F) of Friedman's Universe to its critical density (ρ_c).

$$\Omega \overset{df}{=} \frac{\rho_F}{\rho_c}$$

Knowing the value of density parameter and critical density, we can state current average density of Friedman's Universe.

$$\rho_F = \Omega\rho_c$$

$\Omega = 0.47$ according to [1]

$$\rho_c \approx 1.058\cdot 10^{-26}\frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$\rho_F \approx 4.97\cdot 10^{-27}\frac{\mathrm{kg}}{\mathrm{m}^3}$$

that is almost 3 protons per cubic meter

- **Dark energy**

Interpretation of observattional data within Friedman's Model of Universe forced cosmologists to hypothesise about existence of dark energy.

ATTENTION

For $H = 75\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{Mpc}^{-1}$ density of Universe in Our Model is over 17 times bigger than in Friedman's Model of Universe. Our Model doesn't require us to assume existence of dark energy.

Quoted works

[1] J. H. Oort: *The Density of the Universe.* Astronomy & Astrophysics **7** (09/1970) 405-407.

[2] Z. Osiak: *Ogólna Teoria Względności (General Theory of Relativity).*
Self Publishing (2012), ISBN: 978-83-272-3515-2, http://vixra.org/abs/1804.0178

17 ANTI-GRAVITY IN OTHER MODELS OF UNIVERSE

- **Gravitational acceleration of free particle corresponding with F-L-R-W metric** [1]

$$\tilde{a}^{\alpha}_{grav\&iner} = -\left(\operatorname{sgn} ds^2\right) c^2 \tilde{k}\, \Gamma^{\alpha}_{\mu\nu} \frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}$$

$$\operatorname{sgn} ds^2 \le 0$$

$$\frac{\tilde{a}^{\alpha}_{grav\&iner}}{c^2\tilde{k}} = \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}$$

$$\frac{\tilde{a}^{1}_{grav\&iner}}{c^2\tilde{k}} = \Gamma^{1}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} =$$

$$= \Gamma^1_{11}\frac{dx^1}{ds}\frac{dx^1}{ds} + \Gamma^1_{12}\frac{dx^1}{ds}\frac{dx^2}{ds} + \Gamma^1_{13}\frac{dx^1}{ds}\frac{dx^3}{ds} + \Gamma^1_{14}\frac{dx^1}{ds}\frac{dx^4}{ds} +$$

$$+ \Gamma^1_{21}\frac{dx^2}{ds}\frac{dx^1}{ds} + \Gamma^1_{22}\frac{dx^2}{ds}\frac{dx^2}{ds} + \Gamma^1_{23}\frac{dx^2}{ds}\frac{dx^3}{ds} + \Gamma^1_{24}\frac{dx^2}{ds}\frac{dx^4}{ds} +$$

$$+ \Gamma^1_{31}\frac{dx^3}{ds}\frac{dx^1}{ds} + \Gamma^1_{32}\frac{dx^3}{ds}\frac{dx^2}{ds} + \Gamma^1_{33}\frac{dx^3}{ds}\frac{dx^3}{ds} + \Gamma^1_{34}\frac{dx^3}{ds}\frac{dx^4}{ds} +$$

$$+ \Gamma^1_{41}\frac{dx^4}{ds}\frac{dx^1}{ds} + \Gamma^1_{42}\frac{dx^4}{ds}\frac{dx^2}{ds} + \Gamma^1_{43}\frac{dx^4}{ds}\frac{dx^3}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\frac{dx^4}{ds}$$

Expressions for $\Gamma^{\alpha}_{\mu\nu}$ come from [1].

$$ds^2 = B^2L^2\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right] + \left(dx^4\right)^2$$

$x^4 = ict$, $L = L(t)$ = dimensionless time scale factor

$$B = \frac{1}{1+\frac{1}{4}\frac{L^2r^2}{L^2a^2}} = \frac{1}{1+\frac{1}{4}\frac{r^2}{a^2}} = \frac{1}{1+\frac{1}{4}kr^2}, \quad r^2 = \left(x^1\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2, \quad k = \frac{1}{a^2}$$

a^2 = square of unscaled constant radius of space curvature

$$\operatorname{sgn} k = \operatorname{sgn}\frac{1}{a^2} = -1,\ 0,\ +1$$

$$\gamma_F = \left\{1 - \frac{B^2L^2}{c^2}\left[\left(\frac{dx^1}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)^2 + \left(\frac{dx^3}{dt}\right)^2\right]\right\}^{-\frac{1}{2}}$$

$$ds = ic\gamma_F^{-1}dt$$

$$\tilde{a}^1_{grav\&iner} = -2\tilde{k}\gamma_F^2\frac{1}{L}\frac{\partial L}{\partial t}\frac{dx^1}{dt} +$$

$$-kB^2\tilde{k}\gamma_F^2\left[\frac{1}{2}x^1\left(\frac{dx^1}{dt}\frac{dx^1}{dt} + \frac{dx^2}{dt}\frac{dx^2}{dt} + \frac{dx^3}{dt}\frac{dx^3}{dt}\right) - \frac{dx^1}{dt}\left(x^1\frac{dx^1}{dt} + x^2\frac{dx^2}{dt} + x^3\frac{dx^3}{dt}\right)\right]$$

Now let's determine physical component of gravitational acceleration of free particle in spacetime with F-L-R-W metric.

$$\hat{a}^{\alpha}_{grav\&iner} = \sqrt{g_{11}}\,\tilde{a}^{\alpha}_{grav\&iner}$$

$$\Big\downarrow \quad g_{11} = B^2L^2$$

$$\hat{a}^{\alpha}_{grav\&iner} = BL\,\tilde{a}^{\alpha}_{grav\&iner}$$

$$\hat{a}^{1}_{grav\&iner} = -2\tilde{k}\gamma_F^2\,B\frac{\partial L}{\partial t}\frac{dx^1}{dt} + \\ -kB^3L\tilde{k}\gamma_F^2\left[\frac{1}{2}x^1\left(\frac{dx^1}{dt}\frac{dx^1}{dt}+\frac{dx^2}{dt}\frac{dx^2}{dt}+\frac{dx^3}{dt}\frac{dx^3}{dt}\right)-\frac{dx^1}{dt}\left(x^1\frac{dx^1}{dt}+x^2\frac{dx^2}{dt}+x^3\frac{dx^3}{dt}\right)\right]$$

In case of spatially flat spacetime (k = 0):

$$\hat{a}^{1}_{grav\&iner} = -2\tilde{k}\gamma_F^2\frac{\partial L}{\partial t}\frac{dx^1}{dt}, \quad \gamma_F^2\frac{\partial L}{\partial t} > 0, \quad \tilde{k} = +1$$

Let's notice, that

$$\frac{dx^1}{dt} > 0 \Rightarrow \hat{a}^{1}_{grav\&iner} < 0 \qquad \frac{dx^1}{dt} < 0 \Rightarrow \hat{a}^{1}_{grav\&iner} > 0$$

From the relation between signs of velocity components and gravitational acceleration of free particle, emerges that occurrence of gravity or anti-gravity depends on the velocity direction of test particle in spatially flat Friedman's Universe.

- **Gravitational acceleration of free particle corresponding with metric of simple model of expanding spacetime** [1]

$$\tilde{a}^{\alpha}_{grav\&iner} = -\left(\operatorname{sgn} ds^2\right)c^2\tilde{k}\,\Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}$$

$$\Big\downarrow \quad \operatorname{sgn} ds^2 \le 0$$

$$\frac{\tilde{a}^{\alpha}_{grav\&iner}}{c^2\tilde{k}} = \Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}$$

$$\frac{\tilde{a}^{1}_{grav\&iner}}{c^2\tilde{k}} = \Gamma^{1}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds} =$$

$$= \Gamma^{1}_{11}\frac{dx^1}{ds}\frac{dx^1}{ds} + \Gamma^{1}_{12}\frac{dx^1}{ds}\frac{dx^2}{ds} + \Gamma^{1}_{13}\frac{dx^1}{ds}\frac{dx^3}{ds} + \Gamma^{1}_{14}\frac{dx^1}{ds}\frac{dx^4}{ds} +$$

$$+ \Gamma^{1}_{21}\frac{dx^2}{ds}\frac{dx^1}{ds} + \Gamma^{1}_{22}\frac{dx^2}{ds}\frac{dx^2}{ds} + \Gamma^{1}_{23}\frac{dx^2}{ds}\frac{dx^3}{ds} + \Gamma^{1}_{24}\frac{dx^2}{ds}\frac{dx^4}{ds} +$$

$$+\Gamma^1_{31}\frac{dx^3}{ds}\frac{dx^1}{ds}+\Gamma^1_{32}\frac{dx^3}{ds}\frac{dx^2}{ds}+\Gamma^1_{33}\frac{dx^3}{ds}\frac{dx^3}{ds}+\Gamma^1_{34}\frac{dx^3}{ds}\frac{dx^4}{ds}+$$

$$+\Gamma^1_{41}\frac{dx^4}{ds}\frac{dx^1}{ds}+\Gamma^1_{42}\frac{dx^4}{ds}\frac{dx^2}{ds}+\Gamma^1_{43}\frac{dx^4}{ds}\frac{dx^3}{ds}+\Gamma^1_{44}\frac{dx^4}{ds}\frac{dx^4}{ds}$$

Expressions for $\Gamma^{\alpha}_{\mu\nu}$ come from [1].

$$ds^2 = L^2\left[\left(dx^1\right)^2+\left(dx^2\right)^2+\left(dx^3\right)^2+\left(dx^4\right)^2\right]$$

$x^4 = ict$, $L = L(t)$ = dimensionless time scale factor

$$\gamma = \left\{1-\frac{1}{c^2}\left[\left(\frac{dx^1}{dt}\right)^2+\left(\frac{dx^2}{dt}\right)^2+\left(\frac{dx^3}{dt}\right)^2\right]\right\}^{-\frac{1}{2}}, \quad ds = icL\gamma^{-1}dt$$

$$\tilde{a}^1_{grav\&iner} = -2\tilde{k}\gamma^2\frac{1}{L^3}\frac{\partial L}{\partial t}\frac{dx^1}{dt}$$

Let's now determine physical component of gravitational acceleration of free particle in spacetime with metric that represents a metric of a simple expanding spacetime model.

$$\hat{\tilde{a}}^{\alpha}_{grav\&iner} = \sqrt{g_{11}}\,\tilde{a}^{\alpha}_{grav\&iner}$$

$$g_{11} = L^2$$

$$\hat{\tilde{a}}^{\alpha}_{grav\&iner} = L\,\tilde{a}^{\alpha}_{grav\&iner}$$

$$\hat{\tilde{a}}^1_{grav\&iner} = -2\tilde{k}\gamma^2\frac{1}{L^2}\frac{\partial L}{\partial t}\frac{dx^1}{dt}, \quad \gamma^2\frac{1}{L^2}\frac{\partial L}{\partial t} > 0, \quad \tilde{k} = +1$$

Let's notice, that

$$\frac{dx^1}{dt} > 0 \Rightarrow \hat{\tilde{a}}^1_{grav\&iner} < 0 \qquad \frac{dx^1}{dt} < 0 \Rightarrow \hat{\tilde{a}}^1_{grav\&iner} > 0$$

From the relation between signs of velocity components and gravitational acceleration of free particle, emerges that occurrence of gravity or anti-gravity depends on the velocity direction of test particle in spacetime with metric that represents a metric of a simple model of expanding spacetime.

Quoted works

[1] Z. Osiak: *Ogólna Teoria Względności (General Theory of Relativity)*.
Self Publishing (2012), ISBN: 978-83-272-3515-2, http://vixra.org/abs/1804.0178

18 OUR UNIVERSE AS AN EINSTEIN'S SPACE

- **Other form of field equations – Our Universe as an Einstein's space**

$$R_{\alpha\beta} = -\kappa\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T\right), \qquad \kappa = \frac{8\pi G}{c^4} = 2.073\cdot 10^{-43}\frac{s^2}{kg\cdot m}$$

$$g_{12} = g_{21} = g_{13} = g_{31} = g_{14} = g_{41} = g_{23} = g_{32} = g_{24} = g_{42} = g_{34} = g_{43} = 0$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \quad g^{\alpha\alpha} = \frac{1}{g_{\alpha\alpha}}$$

$$T_{\alpha\alpha} = -\frac{1}{2}g_{\alpha\alpha}\rho c^2, \quad T \overset{df}{=} \sum_{\alpha=1}^{4} g^{\alpha\alpha}T_{\alpha\alpha} = -2\rho c^2, \quad T_{\alpha\alpha} - \frac{1}{2}g_{\alpha\alpha}T = \frac{1}{2}g_{\alpha\alpha}\rho c^2$$

$$R_{12} = R_{21} = R_{13} = R_{31} = R_{14} = R_{41} = R_{23} = R_{32} = R_{24} = R_{42} = R_{34} = R_{43} = 0$$

$$R_{11} = \frac{1}{g_{44}}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r} + \frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)$$

$$R_{22} = -1 + g_{44} + r\frac{\partial g_{44}}{\partial r}$$

$$R_{33} = \sin^2\theta\left(-1 + g_{44} + r\frac{\partial g_{44}}{\partial r}\right)$$

$$R_{44} = g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r} + \frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)$$

$$R_{\alpha\alpha} = -\tfrac{1}{2}\rho c^2\kappa g_{\alpha\alpha}, \quad R_{\mu\nu} = 0, \quad (\alpha,\mu,\nu = 1,2,3,4; \quad \mu \neq \nu)$$

Spacetime described by above equations, in which every component of Ricci tensor is proportional to adequate component of metric tensor is an Einstein's space [1]. So spacetime of Our Universe is an Einstein's space.

ATTENTION
All mixed components of Ricci tensor are identically equal to zero. Set of remaining equations can be reduced to only two independent ones.

$$\begin{matrix} R_{11} = -\tfrac{1}{2}\rho c^2\kappa g_{11} \\ R_{22} = -\tfrac{1}{2}\rho c^2\kappa g_{22} \end{matrix} \quad \Rightarrow \quad \begin{matrix} \dfrac{1}{r}\dfrac{\partial g_{44}}{\partial r} + \dfrac{1}{2}\dfrac{\partial^2 g_{44}}{\partial r^2} = -\dfrac{1}{2}\kappa\rho c^2 \\ -1 + g_{44} + r\dfrac{\partial g_{44}}{\partial r} = -\dfrac{1}{2}\kappa\rho c^2 r^2 \end{matrix} \qquad \begin{matrix} \dfrac{\partial g_{44}}{\partial r} + \dfrac{r}{2}\dfrac{\partial^2 g_{44}}{\partial r^2} = -\dfrac{1}{2}\kappa\rho c^2 r \\ -1 + g_{44} + r\dfrac{\partial g_{44}}{\partial r} = -\dfrac{1}{2}\kappa\rho c^2 r^2 \end{matrix}$$

Quoted works
[1] А. З. Петров: *Пространства Эйнштейна*. Физматгиз, Москва 1961.
There is an English translation of the above book:
A. Z. Petrov: *Einstein Spaces*. Pergamon Press, Oxford 1969.

19 ASSUMPTIONS

- **Main postulates of General Relativity**

General Theory of Relativity (General Relativity) studies conclusions that emerge from assumptions:
1. Maximum speed of signals propagation is the same in every frame of reference.
2. Definitions of physical quantities and physics equations can be formulated in a way so that their overall forms are the same in all frames of reference.
3. Spacetime metric depends on density distribution of source masses.
4. Inertial mass is equal to gravitational mass.

- **2-potential nature of stationary gravitational field**

Stationary gravitational field is a 2-potential field:

$$\frac{\partial \mathbf{E}}{\partial t} = 0, \quad \text{rot}\mathbf{E} = 0$$
$$\text{rot grad}\varphi = 0$$

$$\Rightarrow$$

$$\mathbf{E}^{in} = \text{grad}\varphi^{in} = -\tilde{k}\text{grad}\varphi^{in}, \quad 0 \le r < R, \quad \lim_{r\to 0}\varphi^{in} = 0$$
$$\mathbf{E}^{ex} = -\text{grad}\varphi^{ex} = -\tilde{k}\text{grad}\varphi^{ex}, \quad r \ge R, \quad \lim_{r\to\infty}\varphi^{ex} = 0$$

$$\tilde{k} = \begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}$$

- **Poisson's equation and 2-potential nature of stationary gravitational field**

$$\frac{\partial \mathbf{E}}{\partial t} = 0$$
$$\mathbf{E}^{in} = -\tilde{k}\text{grad}\varphi^{in}$$

Gauss law:

$$\text{div}\mathbf{E}^{in} = -4\pi G\rho$$
$$\text{div}\alpha\mathbf{A} = \alpha\text{div}\mathbf{A} + \mathbf{A}\text{grad}\alpha$$
$$\text{div grad}\varphi = \nabla^2\varphi = \Delta\varphi$$
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Δ = Laplace operator
laplacian

Poisson's equation

$$\Delta\varphi^{in} = 4\pi\tilde{k}G\rho = -4\pi G\rho$$

In empty space, outside of a mass source, right-hand side of Poisson's equation is equal to zero.

Laplace's equation

$$\Delta\varphi^{ex} = 0$$

- **Signes of right-hand sides of Poisson's equations and boundary conditions**

In Newton's theory of gravity, boundary conditions for the gravitational potentials

$$0 \le r < R, \quad \lim_{r\to 0}\varphi^{in} = 0$$
$$r \ge R, \quad \lim_{r\to\infty}\varphi^{ex} = 0$$

in spherical coordinates correspond to the following forms of Poisson's equation

$$0 \le r < R, \quad r^2 \cdot \frac{\partial^2 \varphi^{in}}{\partial r^2} + 2r \cdot \frac{\partial \varphi^{in}}{\partial r} + \frac{1}{\sin^2\theta} \cdot \frac{\partial^2 \varphi^{in}}{\partial \varphi^2} + \frac{\partial^2 \varphi^{in}}{\partial \theta^2} + \mathrm{ctg}\theta \cdot \frac{\partial \varphi^{in}}{\partial \theta} = -4\pi G \rho r^2$$

$$r \ge R, \quad r^2 \cdot \frac{\partial^2 \varphi^{ex}}{\partial r^2} + 2r \cdot \frac{\partial \varphi^{ex}}{\partial r} + \frac{1}{\sin^2\theta} \cdot \frac{\partial^2 \varphi^{ex}}{\partial \varphi^2} + \frac{\partial^2 \varphi^{ex}}{\partial \theta^2} + \mathrm{ctg}\theta \cdot \frac{\partial \varphi^{ex}}{\partial \theta} = 0$$

Equivalents of those relations in General Relativity are:

$$0 \le r < R, \quad \lim_{r \to 0} g_{44} = 1$$

$$r \ge R, \quad \lim_{r \to \infty} g_{44} = 1$$

$$0 \le r < R, \quad R_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)$$

$$r \ge R, \quad R_{\mu\nu} = 0$$

Signes of right-hand sides of Poisson's equations depend on assumed boundary conditions, which are connected with 2-potential nature of stationary gravitational field.

- **Hypothesis about memory of photons**

We assume that energy of photon depends on point in spacetime, where that photon was emitted and it stays constant during the movement. It means that photons have „memory", or more scholarly – energy of photon is invariant. Wherein, in stronger gravitational field, given source should send photons with lower energy than the same source in weaker gravitational field.

Photon energy, emitted in certain point of spacetime, is given by equation:

$$E = \frac{E_{max}}{\sqrt{|g_{11}|}}$$

E_{max} = photon energy emitted in nondeformed spacetime

g_{11} = component of metric tensor in photon's emission point

20 STABILITY OF THE MODEL

- **Stability of the model**

In order for proposed by me model was real, we should additionally assume that, each element of Universe moves in such way, so that it guarantees its stability. It’s possible when gravitational acceleration $\tilde{a}^{r}_{grav}$ acts as a centripetal acceleration $\tilde{a}^{r}_{cent}$.

$$\tilde{a}^{r}_{grav} = \tilde{a}^{r}_{cent}$$

$$\tilde{a}^{r}_{grav} = -\frac{c^2}{R^2} r \quad \text{(page 30)}$$

$$R = \sqrt{\frac{3c^2}{4\pi G}} \cdot \sqrt{\frac{1}{\rho}} \quad \text{(page 40)}$$

$$\tilde{a}^{r}_{grav} = -\frac{4}{3}\pi G \rho r$$

$$\tilde{a}^{r}_{cent} = -\omega^2 r$$

$$\omega^2 = \frac{c^2}{R^2} = \frac{4}{3}\pi G \rho$$

$$\omega = \frac{2\pi}{T}$$

$$T^2 = \frac{4\pi^2 R}{c^2} = \frac{3\pi}{G\rho}$$

R = radius of Our Universe
r = distance from the center of Our Universe
G = gravitational constant
c = maximal speed of signals propagation
ω = angular velocity of circulation of given element around the center of Our Universe
ρ = density of Our Universe
T = period of circulation of given element around the center of Our Universe

It follows from the foregoing that period of circulation of given element around the center of Our Universe does not depend on the distance from the center.

- **Gravitational Gauss law and galaxies**

Performing analogical calculations for given galaxy, within the limits of Newton’s gravitation theory, we will get that: period of circulation of given star around the galactic center does not depend on distance (r) of this element from the center. In below calculations we assumed that galaxy is a ball with constant density (ρ), and star with mass (m) interacts gravitationally with part of galaxy with radius (r) (according to Gauss law, gravitational interactions with the remaining part of galaxy is eliminated).

The force of gravity F_{grav} act as centripetal force F_{cent}.

$$F_{grav} = F_{cent}$$

$$F_{grav} = \frac{GMm}{r^2} \quad \text{(Newton's gravity force)}$$

$$M = \frac{4}{3}\pi r^3 \rho$$

$$F_{grav} = \frac{4}{3}\pi G m r$$

$$F_{cent} = m\omega^2 r \quad \text{(centripetal force)}$$

$$\boxed{\omega^2 = \frac{4}{3}\pi G \rho}$$

$$\omega = \frac{2\pi}{T}$$

$$\boxed{T^2 = \frac{3\pi}{G\rho}}$$

R = radius of galaxy
r = distance from the galaxy center
M = part of galaxy mass with radius r
m = mass of given star
ρ = galaxy density
G = gravitational constant
ω = angular velocity of circulation of given star around the galactic center
T = period of circulation of given star around the galactic center

21 MAIN RESULTS

- **Hypothesis about existence of anti-gravity**

Quadratic differential form of spacetime can be interpreted as square of spacetime distances of two points, called „events", that are located very close to each other in spacetime. Spacetime distance between two events is not a magnitude that can be directly measured, because we don't have „4 dimensional rulers". Also, depending on the sign of the quadratic differential form of spacetime, spacetime distances can be imaginary or real.

> Sign change (from negative to positive) of quadratic differential form of spacetime means transition from gravity to anti-gravity.
>
> $$ds^2 < 0 \Leftrightarrow \text{GRAVITY}$$
> $$ds^2 > 0 \Leftrightarrow \text{ANTI-GRAVITY}$$

- **Black hole with maximal anti-gravity halo**

It has been shown that exterior (vacuum) solution of gravitational field equations, which has been served by Schwarzschild in 1916, hides anti-gravity. Spatial radius of black hole with maximal anti-gravity halo is equal to half of Schwarzschild radius. Space outside of the black hole with maximal anti-gravity halo consists of two layers, in first ($0.5\, r_S \le r < r_S$) anti-gravity occurs, in second ($r > r_S$) – gravity. Thickness of anti-gravity layer is equal to the spatial radius of black hole with maximal anti-gravity layer. Inside this anti-gavity layer, acceleration is pointed away from the center of mass source, in gravitational part – to the center.

- **Hypothesis about 2-potential nature of stationary gravitational field**

Acceptance of hypothesis about 2-potential nature of stationary gravitational field made it possible to find interior solution for field equations that correspond with gravitational Gauss law and also made it possible to accomplish a modification of field equations.

- **Field equations contain equations of motion**

Has been justified that equations of motion of free test particle are contained in gravitational field equations (that is in spacetime metric equations).

- **Black Hole model of Our Universe**

Our Universe can be treated as a gigantic homogeneous black hole with spatial radius equal to half of Schwarzschild radius. It is isolated from the rest of universe with an area of spacetime where anti-gravity occurs.

Our Galaxy, together with Solar system and Earth, which in cosmological scales can be viewed only as a small point, should be located near the center of Black Hole Universe. Let's emphasize again, that in the center of black hole, gravitational acceleration of free test particle, ignoring local gravitational fields, is equal to zero. In other words: black hole is not creating gravitational field in its center.

The gravitational field is stronger, the space is more stretched. For Earth observer, the local space with increasing distance from Earth is becoming less stretched. In cosmic scale we have different situation: the further away from the Earth, the space is more stretched.

- **Size of Our Universe**

For $H = 75\,km \cdot s^{-1} \cdot Mpc^{-1}$ radius of Our Universe has value:

$$R \approx 0.6 \cdot 10^{26} m \approx 6.31 \cdot 10^{9} \text{ light years}$$

In a distance from the center of Earth approximate equal to:

$$r_0 \approx 2.245 \cdot 10^{21} m \approx 2.363 \cdot 10^{5} \text{ light years} = 236300 \text{ light years}$$

redshift measured in regard to our planet, changes its sign from negative to positive.

Light that comes to Earth from Our Galaxy (which radius is around 50000 light years and thickness is around 12000 light years) should be shifted towards violet in regard to light that is emitted on Earth surface. Wherein negative value of redshift should depend on direction of observation.

- **Density of Our Universe**

For $H = 75\,km \cdot s^{-1} \cdot Mpc^{-1}$ density of Universe in Our Model is over 17 times bigger than in Friedman's Universe Model and is: $\rho = 84.59 \cdot 10^{-27} kg \cdot m^{-3}$, that almost 51 protons per cubic meter.

ATTENTION

Our Model doesn't require assumption about existence of dark energy.

- **Photon paradox**

Commonly accepted assumption, that photon energy doesn't depend on place of emission, leads to a paradox. In spacetimes other than conformally flat, length and oscillation period of electromagnetic wave modelled by photon depends in different way on respective components of metric tensor. However photon energy depends on length or period in analogical way (that is inversely proportional).

- **Hypothesis about memory of photons**

Within the model of Our Universe as a black hole with maximum anti-gravity halo, hypothesis about memory of photons explains:

1. photon paradox,
2. Olbers' photometric paradox,
3. redshift dependence on the distance from Earth:
   A. nonlinear growth of redshift for big distances from Earth,
   B. existence of distance from Earth, where redshift changes its sign from negative to positive,
4. origin of cosmic microwave background radiation,
5. why hypothesis about existence of dark energy is unnecessary.

ATTENTION

It should be highlighted that Pound-Rebka experiment doesn't exlude hypothesis about memory of photons.

- **Our Universe is an Einstein's space**

It has been shown that field equations which are modelling Our Universe and black hole with maximum anti-gravity halo can be brought to a form describing the so-called Einstein's space.

- **Anti-gravity in other cosmological models**

From the relation between signs of components of velocity and gravitational acceleration of free test particle emerges that occurrence of gravity or anti-gravity depends on the direction of velocity of test particle in spatially flat Friedman's Universe as well as in spacetime with metric of a simple expanding spacetime model [1].

- **New tests of General Relativity**

To show in Earth conditions, existence of a black holes with anti-gravity halo and also 2-potential nature of gravitational field, we need to measure ratio of distance passed by light to the time of flight, in a vertically positioned vacuum cylinder, right under and right above the surface of Earth (of the sea level). If the difference of squares of those measurements will be equal to the square of escape speed, then it will prove existence of black holes with anti-gravity halo. This experiment would be a new test of General Theory of Relativity.

If Our Universe is a black hole with anti-gravity halo, then in the distance from the center of Earth approximately equal to 236000 light years redshift measured in regard to Our Planet changes its sign from negative to positive. Light that comes to Earth from Our Galaxy (radius of Milky Way is around 50000 light years and its thickness is around 12000 light years) should be shifted towards violet in regard to light that is emitted on Earth. Wherein negative value of redshift should depend on direction of observation

Quoted works

[1] Z. Osiak: *Ogólna Teoria Względności (General Theory of Relativity).*
Self Publishing (2012), ISBN: 978-83-272-3515-2, http://vixra.org/abs/1804.0178

22 ADDITION – MATHEMATICS

- **Explicit form of Christoffel symbols of first and second kind**

$x^1 = r, \quad x^2 = \theta, \quad x^3 = \varphi, \quad x^4 = ict,$

$ds^2 = g_{11}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\varphi^2 + g_{44}dx^4dx^4,$

$$\boxed{g_{11} = \frac{1}{g_{44}}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \quad g^{11} = \frac{1}{g^{44}}, \quad g^{22} = \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2\sin^2\theta},$$

$$\boxed{g_{11} = g_{11}(x^1)}$$

$$\left[{1\,1 \atop 1}\right] = -\frac{1}{2g_{44}g_{44}}\frac{\partial g_{44}}{\partial x^1} = -\frac{1}{2g_{44}g_{44}}\frac{\partial g_{44}}{\partial r}$$

$$\left[{2\,2 \atop 1}\right] = -\frac{1}{2}\frac{\partial g_{22}}{\partial x^1} = -r$$

$$\left[{3\,3 \atop 1}\right] = -\frac{1}{2}\frac{\partial g_{33}}{\partial x^1} = -r\sin^2\theta$$

$$\left[{3\,3 \atop 2}\right] = -\frac{1}{2}\frac{\partial g_{33}}{\partial x^2} = -r^2\sin\theta\cos\theta$$

$$\left[{4\,4 \atop 1}\right] = -\frac{1}{2}\frac{\partial g_{44}}{\partial x^1} = -\frac{1}{2}\frac{\partial g_{44}}{\partial r}$$

$$\left[{1\,2 \atop 2}\right] = \left[{2\,1 \atop 2}\right] = \frac{1}{2}\frac{\partial g_{22}}{\partial x^1} = r$$

$$\left[{1\,3 \atop 3}\right] = \left[{3\,1 \atop 3}\right] = \frac{1}{2}\frac{\partial g_{33}}{\partial x^1} = r\sin^2\theta$$

$$\left[{1\,4 \atop 4}\right] = \left[{4\,1 \atop 4}\right] = \frac{1}{2}\frac{\partial g_{44}}{\partial x^1} = \frac{1}{2}\frac{\partial g_{44}}{\partial r}$$

$$\left[{2\,3 \atop 3}\right] = \left[{3\,2 \atop 3}\right] = \frac{1}{2}\frac{\partial g_{33}}{\partial x^2} = r^2\sin\theta\cos\theta$$

$$\Gamma^1_{11} = \frac{1}{g_{11}}\left[{1\,1 \atop 1}\right] = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r}$$

$$\Gamma^1_{22} = \frac{1}{g_{11}}\left[{2\,2 \atop 1}\right] = -g_{44}r$$

$$\Gamma^1_{33} = \frac{1}{g_{11}}\left[{3\,3 \atop 1}\right] = -g_{44}r\sin^2\theta$$

$$\Gamma^1_{44} = \frac{1}{g_{11}}\left[{4\,4 \atop 1}\right] = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{g_{22}}\left[{1\,2 \atop 2}\right] = \frac{1}{g_{22}}r = \frac{1}{r}$$

$$\Gamma^2_{33} = \frac{1}{g_{22}}\left[{3\,3 \atop 2}\right] = -\frac{1}{g_{22}}r^2\sin\theta\cos\theta = -\sin\theta\cos\theta$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{g_{33}}\left[{1\,3 \atop 3}\right] = \frac{1}{g_{33}}r\sin^2\theta = \frac{1}{r}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \frac{1}{g_{33}}\left[{2\,3 \atop 3}\right] = \frac{1}{g_{33}}r^2\sin\theta\cos\theta = \mathrm{ctg}\,\theta$$

$$\Gamma^4_{14} = \Gamma^4_{41} = \frac{1}{g_{44}}\left[{1\,4 \atop 4}\right] = \frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r}$$

- **Explicit form of covariant Ricci curvature tensor**

$$R_{11} = \frac{\partial \Gamma^2_{12}}{\partial x^1} + \frac{\partial \Gamma^3_{13}}{\partial x^1} + \frac{\partial \Gamma^4_{14}}{\partial x^1} + \Gamma^2_{12}\Gamma^2_{12} + \Gamma^3_{13}\Gamma^3_{13} + \Gamma^4_{14}\Gamma^4_{14} - \Gamma^1_{11}\Gamma^2_{12} - \Gamma^1_{11}\Gamma^3_{13} - \Gamma^1_{11}\Gamma^4_{14}$$

$$\boxed{R_{11} = \frac{1}{g_{44}}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r} + \frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)}$$

$$R_{22} = \frac{\partial \Gamma^3_{23}}{\partial x^2} - \frac{\partial \Gamma^1_{22}}{\partial x^1} + \Gamma^2_{12}\Gamma^1_{22} + \Gamma^3_{23}\Gamma^3_{23} - \Gamma^1_{22}\Gamma^1_{11} - \Gamma^1_{22}\Gamma^3_{13} - \Gamma^1_{22}\Gamma^4_{14}$$

$$\boxed{R_{22} = -1 + g_{44} + r\frac{\partial g_{44}}{\partial r}}$$

$$R_{33}=-\frac{\partial\Gamma^1_{33}}{\partial x^1}-\frac{\partial\Gamma^2_{33}}{\partial x^2}+\Gamma^3_{13}\Gamma^1_{33}+\Gamma^3_{23}\Gamma^2_{33}-\Gamma^1_{33}\Gamma^1_{11}-\Gamma^1_{33}\Gamma^2_{12}-\Gamma^1_{33}\Gamma^4_{14}$$

$$\boxed{R_{33}=\sin^2\theta\left(-1+g_{44}+r\frac{\partial g_{44}}{\partial r}\right)}$$

$$R_{44}=-\frac{\partial\Gamma^1_{44}}{\partial x^1}+\Gamma^4_{14}\Gamma^1_{44}-\Gamma^1_{44}\Gamma^1_{11}-\Gamma^1_{44}\Gamma^2_{12}-\Gamma^1_{44}\Gamma^3_{13}$$

$$\boxed{R_{44}=g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)}$$

$$\boxed{\begin{aligned}R_{12}&=R_{21}=0\\R_{13}&=R_{31}=0\\R_{14}&=R_{41}=0\\R_{23}&=R_{32}=0\\R_{24}&=R_{42}=0\\R_{34}&=R_{43}=0\end{aligned}}$$

- **Field equations**

$$\boxed{R_{\alpha\alpha}=-\kappa\left(T_{\alpha\alpha}-\frac{1}{2}g_{\alpha\alpha}T\right)},\qquad \kappa=\frac{8\pi G}{c^4}=2.073\cdot10^{-43}\frac{s^2}{kg\cdot m}$$

$$\boxed{T_{\alpha\alpha}=-\frac{1}{2}g_{\alpha\alpha}\rho c^2},\quad \boxed{T\overset{df}{=}\sum_{\alpha=1}^{4}g^{\alpha\alpha}T_{\alpha\alpha}=-2\rho c^2},\quad \boxed{T_{\alpha\alpha}-\frac{1}{2}g_{\alpha\alpha}T=\frac{1}{2}g_{\alpha\alpha}\rho c^2}$$

$$R_{11}=\frac{1}{g_{44}}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right),\qquad R_{22}=-1+g_{44}+r\frac{\partial g_{44}}{\partial r}$$

$$R_{33}=\sin^2\theta\left(-1+g_{44}+r\frac{\partial g_{44}}{\partial r}\right),\qquad R_{44}=g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)$$

$$\frac{1}{g_{44}}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)=-\frac{1}{2}\kappa\frac{1}{g_{44}}\rho c^2$$

$$-1+g_{44}+r\frac{\partial g_{44}}{\partial r}=-\frac{1}{2}\kappa r^2\rho c^2$$

$$\sin^2\theta\left(-1+g_{44}+r\frac{\partial g_{44}}{\partial r}\right)=-\frac{1}{2}\kappa r^2\sin^2\theta\rho c^2$$

$$g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)=-\frac{1}{2}\kappa g_{44}\rho c^2$$

Above four equations reduce to two equations.

$$\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}=-\frac{1}{2}\kappa\rho c^2$$

$$-1+g_{44}+r\frac{\partial g_{44}}{\partial r}=-\frac{1}{2}\kappa\rho c^2 r^2$$

$$\frac{\partial g_{44}}{\partial r}+\frac{r}{2}\frac{\partial^2 g_{44}}{\partial r^2}=-\frac{1}{2}\kappa\rho c^2 r$$

$$-1+g_{44}+r\frac{\partial g_{44}}{\partial r}=-\frac{1}{2}\kappa\rho c^2 r^2$$

- **Curvature scalar**

$$R\overset{df}{=}g^{\alpha\beta}R_{\alpha\beta}$$

$$R=g^{11}R_{11}+g^{22}R_{22}+g^{33}R_{33}+g^{44}R_{44}$$

$$g^{11}=\frac{1}{g^{44}},\quad g^{22}=\frac{1}{r^2},\quad g^{33}=\frac{1}{r^2\sin^2\theta}$$

$$g_{11}g_{44}=1$$

$$R_{11}=g_{11}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right),\quad R_{22}=-1+g_{44}+r\frac{\partial g_{44}}{\partial r}$$

$$R_{33}=R_{22}\sin^2\theta\ ,\quad R_{44}=R_{11}g_{44}g_{44}$$

$$R=\frac{\partial^2 g_{44}}{\partial r^2}+\frac{4}{r}\frac{\partial g_{44}}{\partial r}+\frac{2g_{44}}{r^2}-\frac{2}{r^2}$$

- **Field equations expressed by curvature scalar**

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\kappa T_{\mu\nu}$$

$$R_{11}=g_{11}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right),\quad R_{22}=-1+g_{44}+r\frac{\partial g_{44}}{\partial r}$$

$$R_{33}=R_{22}\sin^2\theta\ ,\quad R_{44}=R_{11}g_{44}g_{44}$$

$$R=\frac{\partial^2 g_{44}}{\partial r^2}+\frac{4}{r}\frac{\partial g_{44}}{\partial r}+\frac{2g_{44}}{r^2}-\frac{2}{r^2}$$

$$\kappa=\frac{8\pi G}{c^4}=2.073\cdot 10^{-43}\frac{s^2}{kg\cdot m}$$

$$T_{\alpha\beta}=-\frac{1}{2}g_{\alpha\beta}\rho c^2$$

$$g_{11}g_{44}=1,\quad g_{22}=r^2,\quad g_{33}=r^2\sin^2\theta$$

$$R_{11}-\frac{1}{2}g_{11}R=-\kappa T_{11}$$

$$-\frac{g_{11}}{r}\frac{\partial g_{44}}{\partial r}-\frac{1}{r^2}+\frac{g_{11}}{r^2}=\frac{1}{2}\rho c^2\kappa g_{11}$$

After multiplying both sides of above equations by $(-g_{44}r^2)$, we get

$$r\frac{\partial g_{44}}{\partial r}+g_{44}-1=-\frac{1}{2}\rho c^2\kappa r^2$$

$$R_{22}=-\kappa\left(T_{22}-\frac{1}{2}g_{22}T\right)$$

After multiplying both sides of penultimate equation by ($\sin^2\theta$), we get

$$\sin^2\theta\left(r\frac{\partial g_{44}}{\partial r}+g_{44}-1\right)=-\frac{1}{2}\rho c^2\kappa r^2\sin^2\theta$$

$$R_{33}=-\kappa\left(T_{33}-\frac{1}{2}g_{33}T\right)$$

Now let's examine next field equation expressed by scalar curvature.

$$R_{22}-\frac{1}{2}g_{22}R=-\kappa T_{22}$$

$$-r\frac{\partial g_{44}}{\partial r}-\frac{1}{2}r^2\frac{\partial^2 g_{44}}{\partial r^2}=\frac{1}{2}\rho c^2\kappa r^2$$

After multiplying both sides of above equations by ($-\frac{g_{11}}{r^2}$), we get

$$g_{11}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)=-\frac{1}{2}\rho c^2\kappa g_{11}$$

$$R_{11}=-\kappa\left(T_{11}-\frac{1}{2}g_{11}T\right)$$

$$g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right)=-\frac{1}{2}\rho c^2\kappa g_{44}$$

$$R_{44}=-\kappa\left(T_{44}-\frac{1}{2}g_{44}T\right)$$

- **Laplace operator (Laplacian) in Cartesian and spherical coordinate systems**

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$x = r\sin\theta\cos\varphi, \quad y = r\sin\theta\sin\varphi, \quad z = r\cos\theta$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial}{\partial r} + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2}{\partial \varphi^2} + \frac{1}{r^2}\cdot\frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2}\mathrm{ctg}\theta\cdot\frac{\partial}{\partial \theta}$$

- **Used relations**

$$ds^2 = g_{11}drdr + g_{44}dx^4dx^4$$

$$g_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + g_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds} = 1$$

$$g_{11}g_{44} = 1$$

$$\frac{\partial g_{11}}{\partial r} = -\frac{1}{g_{44}g_{44}}\frac{\partial g_{44}}{\partial r}$$

$$g^{11}g_{11} = 1$$

$$g^{44}g_{44} = 1$$

$$\Gamma^1_{11} = -\frac{1}{2}g_{11}\frac{\partial g_{44}}{\partial r}$$

$$\Gamma^1_{44} = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r}$$

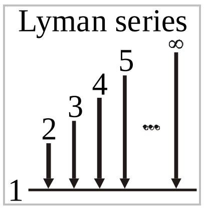
$$\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds} = -\frac{1}{2}\frac{\partial g_{44}}{\partial r}$$

23 ADDITION – PHYSICS

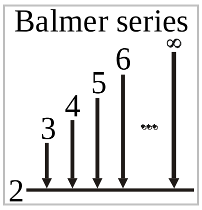
- **Hydrogen spectral series in Earth conditions**

Energies of photons emitted by hydrogen atom in Earth conditions are contained in the range from 1.89 eV to 13.6 eV.

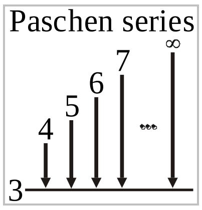
Energies of photons in Lyman series are contained in range from 10.20 eV to 13.60 eV.



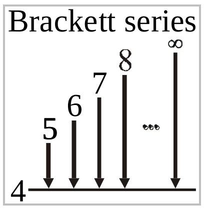
Energies of photons in Balmer series are contained in range from 1.89 eV to 3.40 eV.



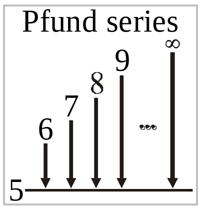
Energies of photons in Paschen series are contained in range from 0.66 eV to 1.51 eV.



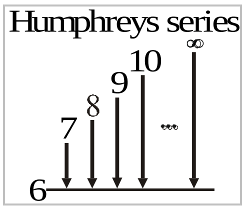
Energies of photons in Brackett series are contained in the range from 0.31 eV to 0.85 eV.



Energies of photons in Pfund series are contained in range from 0.17 eV to 0.54 eV.



Energies of photons in Humphreys series are contained in range from 0.10 eV to 0.38 eV.



- **Hydrogen spectral series in Our Universe**

$$\hat{E}_n = \frac{1}{n^2}\sqrt{g_{44}^{in}}\, E_1 = -\frac{1}{n^2}\sqrt{1-\frac{r^2}{R^2}}\, 13.6\,\text{eV}$$

$$\hat{E}_m = \frac{1}{m^2}\sqrt{g_{44}^{in}}\, E_1 = -\frac{1}{m^2}\sqrt{1-\frac{r^2}{R^2}}\, 13.6\,\text{eV}$$

$$\hat{E}_{n\to m} = \hat{E}_n - \hat{E}_m = -\sqrt{g_{44}^{in}}\, E_1\left(\frac{1}{m^2}-\frac{1}{n^2}\right) = \sqrt{1-\frac{r^2}{R^2}}\left(\frac{1}{m^2}-\frac{1}{n^2}\right)13.6\,\text{eV}, \quad n > m$$

- **Electromagnetic spectrum**

Radio waves
correspond to photons with energies from $1.24\cdot 10^{-11}\text{eV}$ to $1.24\cdot 10^{-3}\text{eV}$.

Microwaves
correspond to photons with energies from $4.14\cdot 10^{-6}\text{eV}$ to $1.24\cdot 10^{-3}\text{eV}$.

Light
correspond to photons of energies in the range $1.24\cdot 10^{-3}\ \text{eV} \div 124\,\text{eV}$.

Infrared radiation
correspond to photons with energies from $1.24\cdot 10^{-3}\text{eV}$ to $1.59\,\text{eV}$.

Visible light
correspond to photons with energies from $1.59\,\text{eV}$ to $3.26\,\text{eV}$.

Ultraviolet radiation
correspond to photons with energies from $3.26\,\text{eV}$ to $124\,\text{eV}$.

X-rays
correspond to photons with energies from 124 eV to 12.4 keV.

Gamma radiation
correspond to photons of energies greater than 12.4 keV.

- **Choosen concepts, constants and units**

Speed of light in vacuum

$$c = 2.99792458 \cdot 10^8 \frac{m}{s} \approx 3 \cdot 10^8 \frac{m}{s}$$

Planck's constant

$$h = 6.6260693 \cdot 10^{-34} J \cdot s = 4.13566743 \cdot 10^{-15} eV \cdot s$$

Gravitational constant

$$G = 6.6742 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \approx 6.7 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$$

Einstein's constant

$$\kappa = \frac{8\pi G}{c^4} = 2.073 \cdot 10^{-43} \frac{s^2}{kg \cdot m}$$

Light year (ly), distance passed by light in vacuum during average sun year

$$1\,ly = 9.460536 \cdot 10^{15} m \approx 9.5 \cdot 10^{15} m$$

$$1\,pc = 3.086 \cdot 10^{16} m$$

$$1\,year = 3.156 \cdot 10^7 s$$

Mass of proton

$$m_p = 1.67262171 \cdot 10^{-27} kg$$

Mass of the Earth

$$M_E = M_\oplus = 5.9736 \cdot 10^{24} kg \approx 6 \cdot 10^{24} kg$$

Mass of the Sun

$$M_S = 1.9891 \cdot 10^{30} kg \approx 2 \cdot 10^{30} kg$$

Hubble's constant unit

$$\frac{km}{s \cdot Mpc} = 0.324 \cdot 10^{-19} \frac{1}{s}$$

Critical density of Universe in Friedman's models expresssed by Hubble's constant (H) and gravitational constant (G)

$$\rho_c = \frac{3H^2}{\kappa c^4} = \frac{3H^2}{8\pi G}$$

Average radius of the Sun

$$R_S = 0.696 \cdot 10^9 m$$

Average radius of the Earth

$R_E = 6.371 \cdot 10^6 \text{m}$

Radius of Bohr hydrogen atom

$R_B = 0.5291772083 \cdot 10^{-10} \text{m}$

$1\,\text{eV} = 1.60217653 \cdot 10^{-19} \text{J}$

$\frac{3c^2}{4\pi G} \approx 0.32 \cdot 10^{27} \text{kg} \cdot \text{m}^{-1}$, $\frac{4\pi G}{3c^2} \approx 3.125 \cdot 10^{-27} \text{kg}^{-1} \cdot \text{m}$

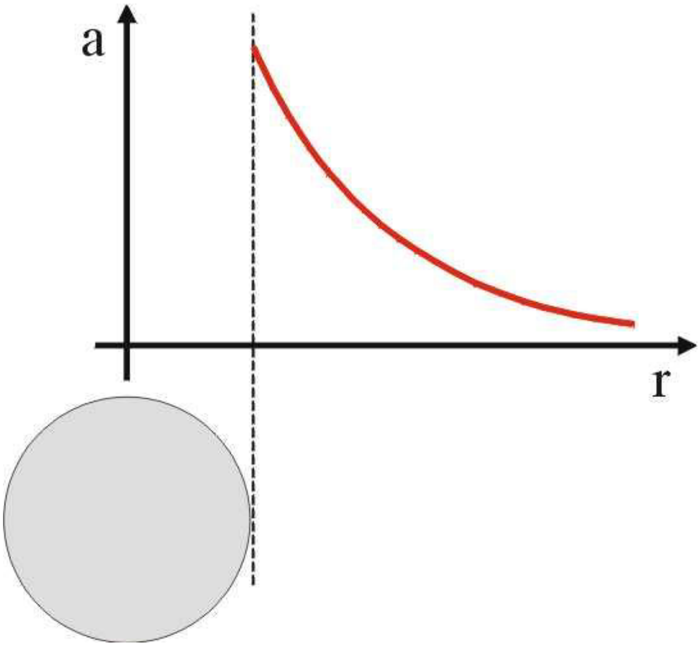
24 ADDITION – HISTORY

- **Newton's law of gravitation**

First theory of gravitation comes from Newton. It describes in a simple way, gravitational pull effects which occur between two material points, between two homogeneous balls, between homogeneous ball and test particle.

According to Newton's law, gravitational acceleration value of free particle (outside of a mass source which homogeneous ball is) decreases inversely to the square of distance from the center of the ball.

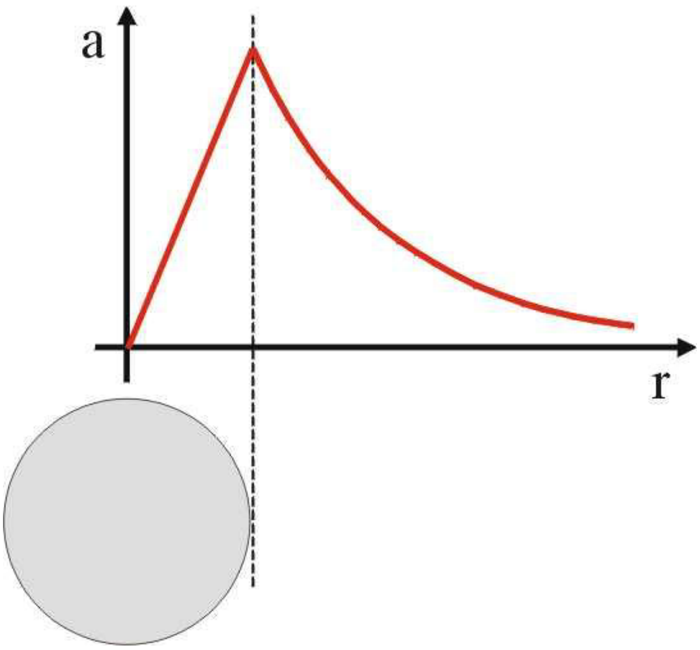
Isaac Newton
(1642-1727)



- **Gauss law of gravitation**

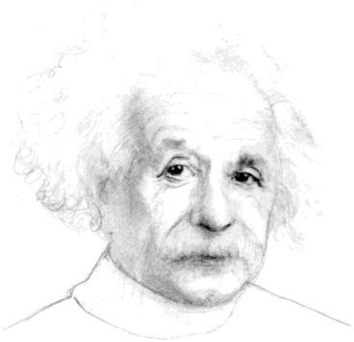
From gravitational Gauss law we know that gravitational acceleration value in center of homogeneous ball with constant density is equal to zero. It grows linearly with growth of distance from the center, reaching its maximal value on a surface of a ball. With further growth of distance – it decreases inversely squared.

Carl F. Gauss
(1777-1855)



- **Einstein's General Theory of Relativity**

General Theory of Relativity (General Relativity) explains gravity as a result of deformation of spacetime, which depends on density distribution of source masses. The free test particles move in space along the tracks, which correspond to geodetic lines in spacetime. General Relativity forces us to revise, inter alia, concepts such as gravitational forces, the forces of inertia and inertial frames.



Albert Einstein
(1879-1955)

General Relativity was finally formulated by Einstein on 25 November 1915. This theory bases on four postulates:

1. Maximum speed of signals propagation is the same in every frame of reference.

2. Definitions of physical quantities and physics equations can be formulated in a way so that their overall forms are the same in all frames of reference.

3. Spacetime metric depends on density distribution of source masses.

4. Inertial mass is equal to gravitational mass.

- **External (vacuum) Schwarzschild solution**

Carl Schwarzschild
(1873-1916)

Precise external (vacuum) solution for gravitational field equations was served by Schwarzschild in 1916:

$$g_{44} = \frac{1}{g_{11}} = 1 - \frac{r_S}{r}, \quad r \geq R$$

M = mass of a homogeneous ball with constant density
R = radius of a ball
$r_S = \frac{2GM}{c^2}$ = Schwarzschild radius

Anti-gravity is hidden in this solution.

It is easy to see that, for $r > r_S$ sign of quadratic differential form of spacetime

$$(ds)^2 = g_{11}(dr)^2 + g_{44}(dx^4)^2$$

is negative, and for $r < r_S$ this sign is positive.

25 ENDING

- **Author's reflections**

The spherical coordinate system used in this paper, for (r = 0) and (θ = 0º, 180º), generates apparent singularities in expressions such as

$$g^{22} = \frac{1}{r^2}, \quad g^{33} = \frac{1}{r^2 \sin^2\theta}$$

and in the original Poisson's equation. We applied this coordinate system despite the mentioned pathologies, because it is convenient in calculations.

I used general scheme of searching for solutions of field equations describing anti-gravity to known external (vacuum) Schwarzschild metric reduced by me to only two elements. Such heuristic approach should be treated as an initial stage in exploring anti-gravity phenomenon.

As an author I'm fully aware of all imperfections and defects of a model of black hole with anti-gravity halo and Our Universe model.

Experiments proposed by me – measurement of speed of light, over and under Earth surface in vertical vacuum cylinder, and also observations of light spectrum that comes to Earth from Our Galaxy – will confirm or disprove presented here hypotheses.

Zbigniew Osiak

I belong to a generation of physicists, whose idols were Albert Einstein, Lev Davidovich Landau and Richard P. Feynman. Einstein enslaved me with the power of his intuition. I admire Landau for reliability, precision, simplicity of his arguments and instinctual feel for the essence of the problem. Feynman magnetized me with the lightness of narration and subtle sense of humor.