VECTOR LORENTZ TRANSFORMATIONS

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This article presents the vector Lorentz transformations of time, space, velocity and acceleration.

Introduction

If we consider two inertial reference frames (S and S') whose origins coincide at time zero (in both frames) then the time (t'), the position $(\mathbf{r}')$, the velocity $(\mathbf{v}')$ and the acceleration $(\mathbf{a}')$ of a (massive or non-massive) particle relative to the inertial reference frame S' are given by:

$$t' = \gamma \left(t - \frac{\mathbf{r} \cdot \mathbf{V}}{c^2} \right)$$

$$\mathbf{r}' = \left[\mathbf{r} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{r} \cdot \mathbf{V})\, \mathbf{V}}{c^2} - \gamma\, \mathbf{V}\, t \right]$$

$$\mathbf{v}' = \left[\mathbf{v} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{v} \cdot \mathbf{V})\, \mathbf{V}}{c^2} - \gamma\, \mathbf{V} \right] \frac{1}{\gamma\, (1 - \frac{\mathbf{v} \cdot \mathbf{V}}{c^2})}$$

$$\mathbf{a}' = \left[\mathbf{a} - \frac{\gamma}{\gamma + 1} \frac{(\mathbf{a} \cdot \mathbf{V})\, \mathbf{V}}{c^2} + \frac{(\mathbf{a} \times \mathbf{v}) \times \mathbf{V}}{c^2} \right] \frac{1}{\gamma^2\, (1 - \frac{\mathbf{v} \cdot \mathbf{V}}{c^2})^3}$$

where $(t,\ \mathbf{r},\ \mathbf{v},\ \mathbf{a})$ are the time, the position, the velocity and the acceleration of the particle relative to the inertial reference frame S, $(\mathbf{V})$ is the velocity of the inertial reference frame S' relative to the inertial reference frame S and (c) is the speed of light in vacuum. $(\mathbf{V})$ is a constant and $\gamma = (1 - \mathbf{V} \cdot \mathbf{V} / c^2)^{-1/2}$

- $\mathbf{v}' = \dfrac{d\mathbf{r}'}{dt'} = \dfrac{d\mathbf{r}'}{dt'}\dfrac{dt}{dt} = \dfrac{d\mathbf{r}'}{dt}\dfrac{dt}{dt'} = \left(\dfrac{d\mathbf{r}'}{dt}\right)\dfrac{1}{\left(\dfrac{dt'}{dt}\right)}$

- $\mathbf{a}' = \dfrac{d\mathbf{v}'}{dt'} = \dfrac{d\mathbf{v}'}{dt'}\dfrac{dt}{dt} = \dfrac{d\mathbf{v}'}{dt}\dfrac{dt}{dt'} = \left(\dfrac{d\mathbf{v}'}{dt}\right)\dfrac{1}{\left(\dfrac{dt'}{dt}\right)}$

- $dt' = \gamma\left(dt - \frac{d\mathbf{r}\cdot\mathbf{V}}{c^2}\right)$

- $\left(\dfrac{dt'}{dt}\right) = \gamma\left(1 - \frac{\mathbf{v}\cdot\mathbf{V}}{c^2}\right)$

- $d\mathbf{r}' = \left[\, d\mathbf{r} + \frac{\gamma^2}{\gamma+1}\frac{(d\mathbf{r}\cdot\mathbf{V})\,\mathbf{V}}{c^2} - \gamma\,\mathbf{V}\,dt \,\right]$

- $\left(\dfrac{d\mathbf{r}'}{dt}\right) = \left[\, \mathbf{v} + \frac{\gamma^2}{\gamma+1}\frac{(\mathbf{v}\cdot\mathbf{V})\,\mathbf{V}}{c^2} - \gamma\,\mathbf{V} \,\right]$

- $d\mathbf{v}' = \left[\, d\mathbf{m}\cdot n - \mathbf{m}\cdot dn \,\right]\dfrac{1}{n^2}$

- $\mathbf{m} = \left[\, \mathbf{v} + \frac{\gamma^2}{\gamma+1}\frac{(\mathbf{v}\cdot\mathbf{V})\,\mathbf{V}}{c^2} - \gamma\,\mathbf{V} \,\right]$

- $d\mathbf{m} = \left[\, d\mathbf{v} + \frac{\gamma^2}{\gamma+1}\frac{(d\mathbf{v}\cdot\mathbf{V})\,\mathbf{V}}{c^2} \,\right]$

- $n = \left[\, \gamma\,(1 - \frac{\mathbf{v}\cdot\mathbf{V}}{c^2}) \,\right]$

- $dn = \left[\, -\gamma\,\frac{d\mathbf{v}\cdot\mathbf{V}}{c^2} \,\right]$

- $\left(\dfrac{d\mathbf{v}'}{dt}\right) = \left[\, \mathbf{a} - \frac{\gamma}{\gamma+1}\frac{(\mathbf{a}\cdot\mathbf{V})\,\mathbf{V}}{c^2} + \frac{(\mathbf{a}\times\mathbf{v})\times\mathbf{V}}{c^2} \,\right]\frac{1}{\gamma\,(1-\frac{\mathbf{v}\cdot\mathbf{V}}{c^2})^2}$

Bibliography

https://it.wikipedia.org/wiki/Trasformazione_di_Lorentz

https://en.wikipedia.org/wiki/Lorentz_transformation

https://arxiv.org/abs/physics/0507099

https://arxiv.org/abs/physics/0702191

https://archive.org/details/blato_links

https://archive.org/details/@a_blato

Appendix I

In the previous bibliography, if the velocity of the inertial reference frame S' relative to the inertial reference frame S is non-zero $(\, \mathbf{V} \neq 0 \,)$ then we have:

$$\frac{\gamma - 1}{\mathbf{V}^2} \; = \; \frac{\gamma^2}{\gamma + 1} \, \frac{1}{c^2} \qquad\qquad (\, \mathbf{V}^2 = \mathbf{V} \cdot \mathbf{V} \,)$$

where $(\, c \,)$ is the speed of light in vacuum and $\gamma \; = \; (1 - \mathbf{V} \cdot \mathbf{V} / c^2)^{-1/2}$

Appendix II

Quotient rule: $\quad \mathbf{a} \; = \; \dfrac{\mathbf{m}}{n} \quad \rightarrow \quad d\mathbf{a} \; = \; \Big[\, d\mathbf{m} \cdot n - \mathbf{m} \cdot dn \, \Big] \, \dfrac{1}{n^2}$

Triple product expansion: $\quad \mathbf{a} \times (\, \mathbf{b} \times \mathbf{c} \,) \; = \; \mathbf{b} \, (\, \mathbf{a} \cdot \mathbf{c} \,) - \mathbf{c} \, (\, \mathbf{a} \cdot \mathbf{b} \,)$

Anticommutativity: $\quad (\, \mathbf{a} \times \mathbf{b} \,) \; = \; -(\, \mathbf{b} \times \mathbf{a} \,)$