So Feynman Was Right

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The curious relationship between the Planck Length and the Bohr Radius enables one to write down the mass of the electron in terms of the Planck Mass, the fine structure constant and π. Building on the relationship between the GUT scale and the mass of the electron, the fine structure constant is shown to be related to both π and e, as Richard Feynman surmised.

The Bohr Radius $a_0 = 0.52917721067(12) \times 10^{-10}$ m [1] and the Planck Length $l_P = 1.616229(38) \times 10^{-35}$ m [1] are related through

$$a_0 \sim \left(\frac{\pi}{2}\right)^{125} l_P \tag{1}$$

More precisely, the exponent in (1) is 125.00058(5). On the basis of (1), for the mass of the electron, $m_e = 0.5109989461(31)$ MeV [1], one may write

$$m_e \sim \alpha^{-1}.\left(\frac{\pi}{2}\right)^{-125} m_P \tag{2}$$

where the fine structure constant $\alpha = 1/137.035999139(31)$ [1] and the Planck Mass $m_P = 1.220910(29) \times 10^{19}$ GeV [1]. More precisely, the second exponent in (2) is 125.00058(5).

The quantity $(\pi/2)^{125}$ in (1) and (2) arises in the Planck Model [2, 3], in which various length, and mass/energy, scales of particle physics coincide with levels in sequences that ascend, and descend, from Planck scale in geometric progression, and may originate in the geometry of spacetime [4]. Particles occupy the levels, isospin doublets actually taking up a symmetrical arrangement about a level [5]. The quarks, in doublets, occupy levels, in Sequences 2 and 3[a], whose level-numbers[b] are multiples of 5 [6]. Two such levels, in Sequences 1 and 3, are the domains of the most fundamental of particles: the electron, of mass $\sim \pi^{-45}\, m_P$, and the up quark, now on its own, of mass $\sim e^{-50}\, m_P$ [6]. Two other such levels, of length scale, are of equal size to the Bohr Radius $[(\pi/2)^{125}\, l_P]$ and the pion charge radius $[(\pi/2)^{100}\, l_P]$, respectively [2].

Higgs field coupling constants are incorporated in the Planck Model [3]. One finds that the mass characteristic of the up-down quark doublet is related to the vacuum expectation value, $\upsilon = 246$ GeV, through

[a] Sequences 1, 2 and 3 descend from the Planck Mass with common ratio 1/π, 2/π and 1/e, respectively.
[b] Levels number from Planck scale (n = 0).

$$(m_u m_d)^{½}.\,c^2 \sim \left(\frac{\pi}{2}\right)^{-25}.\,\upsilon \tag{3}$$

(3) suggests a doublet mass of ~ 3.1 MeV, while Particle Data Group [7] central values of quark mass evaluation suggest the doublet mass, defined as $(m_u m_d)^{½}$, is 3.2 MeV.

Clearly, powers of π^5, $(\pi/2)^5$ and e^5, especially $(\pi/2)^{25}$, are factors, or coupling constants, relating prominent scales.

The up and top quarks have been found to be arranged symmetrically with the electron within the level structure [6]. Accordingly, their masses are given by

$$m_u \sim \alpha\left(\frac{\pi}{2}\right)^{-100} m_P \tag{4}$$

which suggests the up quark mass is ~2.18 MeV, and

$$m_t \sim \alpha\left(\frac{\pi}{2}\right)^{-75} m_P \tag{5}$$

which suggests the top quark mass is ~174.1 GeV.

A GUT scale of 2.1×10^{16} GeV [8] is related to the mass of the electron through

$$m_{GUT} \sim \left(\frac{\pi}{2}\right)^{100} m_e \tag{6}$$

Exact equality in (6) suggests the GUT scale is 2.09×10^{16} GeV. A GUT scale with this value, the mass of the electron and the Hartree Energy $E_h = \alpha^2 m_e c^2$, a fundamental electromagnetic scale of value 27.21138202(17) eV [1], are shown in Sequence 2 of the Planck Model in Figure 1. The GUT scale and the Hartree Energy are found to be related through

$$m_{GUT} \sim e^{55}\, E_h/c^2 \tag{7}$$

The precise value of the exponent in (7), on the basis of exact equality in (6), is 54.99902(2). It seems that nature chooses the value of α so that the exponent (55) is yet another multiple of 5.

We now see, from Figure 1, that

$$\alpha^2 \sim \left(\frac{\pi}{2}\right)^{100}.\,e^{-55} \tag{8}$$

Exact equality in (8) suggests α = 1/137.121.

Of the fine structure constant α, Richard Feynman wrote, 'Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms?' [9]

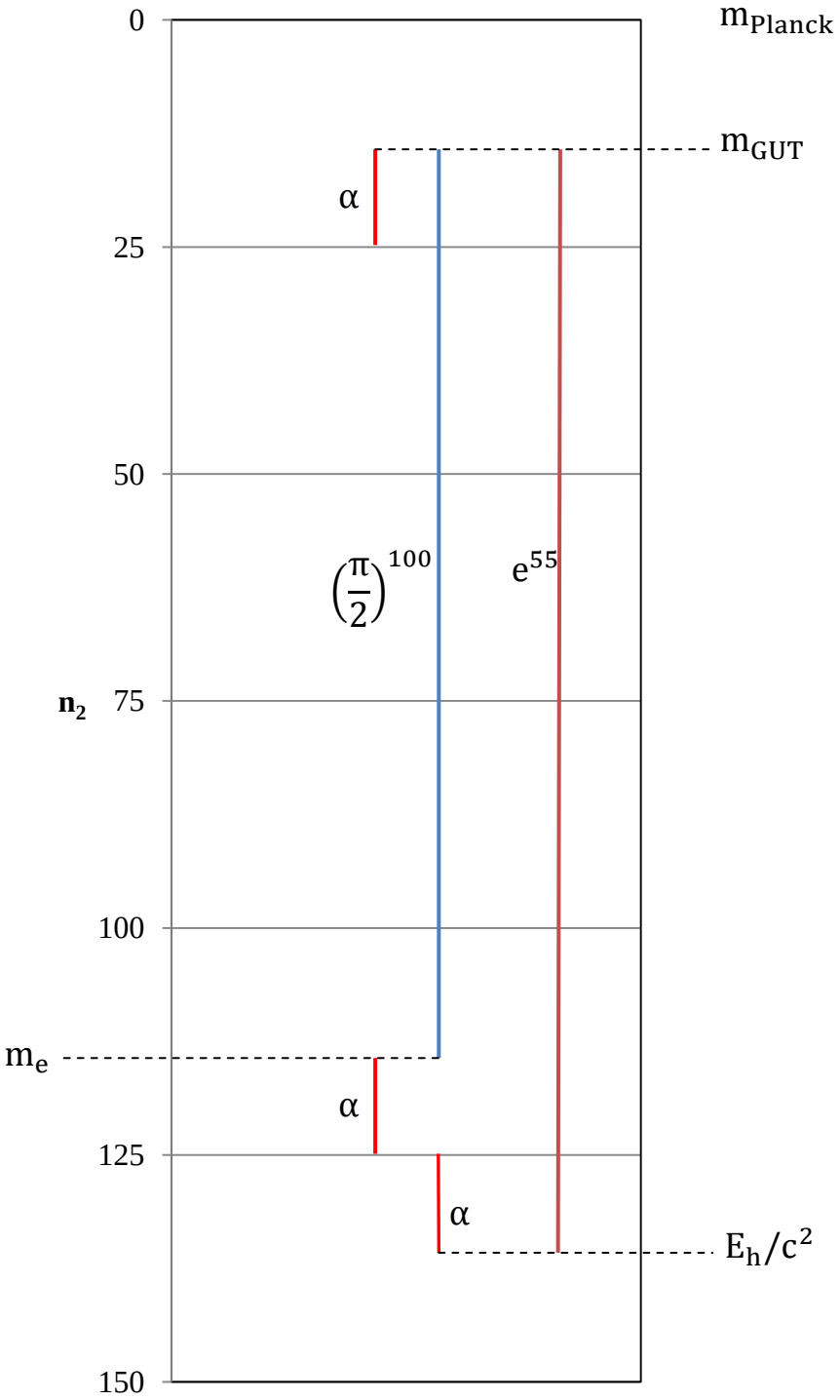

Figure 1: Some scale factors of the Planck Model, shown in Sequence 2, which descends from the Planck Mass with common ratio 2/π.
The red bars join scales related by a factor α, the fine structure constant. The blue bar joins two scales related by a factor $(\pi/2)^{100}$. The purple bar joins two scales related by a factor e^{55}.

References

1. CODATA 2014
2. B. F. Riley, *The scales of atomic and particle physics in relation to Planck scale*, viXra:1305.0061
3. B. F. Riley, *The Planck Model*, viXra:1311.0053
4. B. F. Riley, *Standard model scales from warped extra dimensions*, arXiv:0809.0111
5. B. F. Riley, *Particle mass levels*, viXra:0909.0006
6. B. F. Riley, *Elegant and accurate quark mass equations –evidence for quantum gravity*, viXra:1611.0132
7. C. Patrignani, (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)
8. R. Blumenhagen, *Gauge coupling unification in F-theory GUT models*, Phys. Rev. Lett. 102:071601, 2009; arXiv:0812.0248
9. R. P. Feynman, *QED: The strange story of light and matter*, Princeton University Press, 1985