Formulation of Energy Momentum Tensor for Generalized fields of dyons

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Abstract

The energy momentum tensor of generalized fields of dyons and energy momentum conservation laws of dyons has been developed in simple, compact and consistent manner. We have obtained the Maxwell's field theory of energy momentum tensor of dyons (electric and magnetic) *of electromagnetic field, Poynting vector and Poynting theorem for generalized fields of dyons in a simple, unique and consistent way.*

Keywords: Dyons, Electromagnetic fields, Energy-momentum tensor, Poynting vector.

1 Introduction

The concept of the energy-momentum tensor in classical field theory has a long history, especially in Einstein's theory of gravity [1]. The energy-momentum tensor combines the densities and flux densities of energy and momentum of the fields into one single object. The problem of giving a concise definition of this object able to provide the physically correct answer under all circumstances, for an arbitrary Lagrangian field theory on an arbitrary space-time background, has puzzled physicists for decades. The classical field theory with space-time translation invariance has a conserved energy-momentum tensor [1, 2]. The classical Lagrangian in constructing the energy-momentum tensor, like the canonical energy-momentum tensor was noticed long ago. The question is based on the Noethern theorem, according to which a field theory with space-time translation invariance has a conserved energy-momentum tensor. Belinfante [1, 2] and Rosenfield [3] who, in particular developed this strategy for Lorentz invariant field theories in flat Minkowski space-time to provide a symmetric energy-momentum tensor which, in the case of electrodynamics, is also gauge invariant and gives the physically correct expressions for the energy density and energy flux density i.e. Poynting vector as well as the momentum density and momentum flux density of the electromagnetic field. Callan et. al. [4] and Deser [5] proposed additional terms to define a energy-momentum tensor that, for dilatation invariant scalar field theories, is also traceless. The classical Lagrangian in constructing the energy-momentum tensor, like the canonical energy-momentum tensor was noticed long ago. The question is based on the Noethern theorem, according to which a field theory with space-time translation invariance has a conserved energy-momentum tensor. Gotay and Marsden [6], which also provides an extensive list of references witnessing the long and puzzling history of the subject, is an exception. Their approach is perhaps the first systematic attempt to tackle the problem from a truly geometric point of view. In a classical field theories on arbitrary space-time manifolds. Generically, space-time manifolds do not admit any isometrics or conformal isometrics at all, so there is no direct analogue of space-time translations, Lorentz transformations, nor are there any conserved quantities in the usual sense. The ordinary conservation law $\partial_\mu T^{\mu\nu} = 0$ for the energy-momentum tensor on flat space-time must be replaced by the covariant conservation law $\nabla_\mu T^{\mu\nu} = 0$ for its counterpart on curved space-time. The basic idea introduced by Gotay and Marsden [6], worked out in detail in using the modern geometric approach to general first order Lagrangian field theories and leading to an an improved energy-momentum tensor which is both symmetric and gauge invariant. The energy-momentum tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in space-time, generalizing the stress tensor of Newtonian physics [7]. It is an attribute of matter, radiation, and non-gravitational force fields. The energy-momentum tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity [1, 2]. The energy-momentum tensor is the conserved Noethern current associated with space-time translations. When gravity is negligible and using a Cartesian coordinate system for space-time, the divergence of the non-gravitational energy-momentum tensor will be zero. In other words, non-gravitational energy and momentum are conserved [8]-[10]. The subject of monopole has gathered [11] enormous potential importance in connection current grand unified theories, supersymmetric gauge theories and super strings. But unfortunately the experimental searches [12] for these elusive particles have proved fruitless as the monopoles are expected to be super heavy and their typically masses are about two orders of magnitude heavier than the super heavy X bosons mediating proton decay. However, a group of physicists [13] are now claiming that they have found indirect evidences for monopoles and now it is being speculated that magnetic monopoles may play an important role in condensed matter physics. In spite of the enormous potential importance of monopoles (dyons) and the fact that these particles have been extensively studied, there has been presented no reliable theory which is as conceptually transparent and predictably tact-able as the usual electrodynamics and the formalism necessary to describe them has been clumsy and not manifestly covariant. On the other hand, the concept of electromagnetic (EM) duality has been receiving much attention [11] in gauge theories, field theories, supersymmetry and super strings.In this paper, the energy momentum tensor of generalized fields of dyons and energy momentum conservation laws are discussed consistently for dyons. Here we have also discussed the momentum operator, Hamiltonian and Poynting vector for generalized electromagnetic fields in a manifest and consistent way.

2 Energy momentum Tensor

The energy momentum tensor is a tensor quantity in physics that describes the density and flux of energy and momentum in space-time, generalizing the stress tensor of Newtonian physics. It is an attribute of matter, radiation, and non-gravitational force fields. The energy momentum tensor is the source of the gravitational field in the Einstein field equations of general relativity, just as mass density is the source of such a field in Newtonian gravity. The energy momentum tensor involves the use of super-scripted variables which are not exponents. If the components of the position four vector are given by $x^0 = t$, $x^1 = x$,$x^2 = y$, $x^3 = z$. The energy momentum tensor is defined as the tensor $T^{\alpha\beta}$ of rank two that gives the flux of the α^{th} component of the momentum vector across a surface with constant x^β coordinate. In the theory of relativity, this momentum vector is taken as the four-momentum. In general relativity, the energy momentum tensor is symmetric [7]

$$T^{\alpha\beta} = T^{\beta\alpha}; \tag{1}$$

The energy momentum tensor is of rank two, its components can be displayed in matrix form

$$T^{\mu\nu}\begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix}; \tag{2}$$

The time component is the density of relativistic mass, i.e. the energy density divided by the speed of light squared [7, 8]. It is of special interest because it has a simple physical interpretation. In the case of a perfect fluid this component is

$$T^{00} = \rho; \tag{3}$$

for an electromagnetic field in otherwise empty space this component is given by

$$T^{00} = \left(E^2 + H^2\right); \tag{4}$$

where E and H are the electric and magnetic fields, respectively. The energy momentum tensor is the conserved Noethern current associated with space-time translations. When gravity is negligible and using a Cartesian coordinate system for space-time, the divergence of the non-gravitational energy momentum will be zero. In other words, non-gravitational energy and momentum are conserved

$$T^{\mu\nu}_{,\nu} = \partial_\nu T^{\mu\nu} = 0; \tag{5}$$

In free space and flat space-time, the electromagnetic energy momentum tensor is given by[9, 10]

$$T^{\mu\nu} = \left[F^{\mu\alpha}F^{\nu}_{\alpha} - \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right]; \tag{6}$$

where $F^{\mu\nu}$is the electromagnetic tensor. This expression is when using a metric of signature$(-,+,+,+)$. If using the metric with signature$(+,-,-,-)$, the expression for $T^{\mu\nu}$ will have opposite sign. $T^{\mu\nu}$ explicitly in matrix form [9, 10]

$$T^{\mu\nu} = \begin{bmatrix} \left(E^2 + H^2\right) & S_x & S_y & S_z \\ S_x & -\sigma_{xx} & -\sigma_{xy} & -\sigma_{xz} \\ S_y & -\sigma_{yx} & -\sigma_{yy} & -\sigma_{yz} \\ S_z & -\sigma_{zx} & -\sigma_{zx} & -\sigma_{zz} \end{bmatrix}; \tag{7}$$

where $\eta^{\mu\nu}$ is the Minkowski metric tensor of metric signature$(-,+,+,+)$, Poynting vector becomes

$$
\begin{aligned}
S &= \overrightarrow{E}\times\overrightarrow{H};\\
\sigma_{ij} &= E_iE_j + B_iB_j - \frac{1}{2}\left(E^2+B^2\right)\delta_{ij}.
\end{aligned}
\tag{8}
$$

is the Maxwell stress tensor. The flux of electromagnetic energy density is represents as [7, 8]

$$
u_{em} = \frac{1}{2}\left(E^2+H^2\right); \tag{9}
$$

3 Dualality invariance

Duality invarience is an old idea introduced a century ago in classical eletromagnetism[11] for the following Maxwell's equation in vacuum i.e.

$$
\begin{aligned}
\overrightarrow{\nabla}\cdot\overrightarrow{E} &= 0;\\
\overrightarrow{\nabla}\cdot\overrightarrow{B} &= 0;\\
\overrightarrow{\nabla}\times\overrightarrow{E} &= -\frac{\partial\overrightarrow{B}}{\partial t};\\
\overrightarrow{\nabla}\times\overrightarrow{B} &= \frac{\partial\overrightarrow{E}}{\partial t}.
\end{aligned}
\tag{10}
$$

where $\overrightarrow{E}$ and $\overrightarrow{B}$ are respectively the electric and magnetic field strength and for brevity we use natural units $c=\hbar=1$,space-time four-vector$\{x^{\mu}\}=(t,x,y,z)$ $\left\{x_{\mu}=\eta_{\mu\nu}x^{\mu}\right\}$ and $\left\{\eta_{\mu\nu}=+1,-1,-1,-1=\eta^{\mu\nu}\right\}$through out the text. Maxwell's equations (10) are invariant not only under Lorentz and conformal transformations but are also invariant under the following duality transformations,

$$
\begin{aligned}
\overrightarrow{E} &\Longrightarrow \overrightarrow{E}\cos\vartheta + \overrightarrow{B}\sin\vartheta;\\
\overrightarrow{B} &\Longrightarrow -\overrightarrow{E}\sin\vartheta + \overrightarrow{B}\cos\vartheta;
\end{aligned}
\tag{11}
$$

where$\overrightarrow{E}$ and$\overrightarrow{B}$ are respectively the the electric and magnetic field strengths. For a particular value of $\vartheta=\frac{\pi}{2}$, equations(11) reduces to

$$
\overrightarrow{E}\longmapsto\overrightarrow{B} \quad ; \quad \overrightarrow{B}\longmapsto-\overrightarrow{E}. \tag{12}
$$

which can be written as

$$\begin{pmatrix} \overrightarrow{E} \\ \overrightarrow{B} \end{pmatrix} \Longrightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \overrightarrow{E} \\ \overrightarrow{B} \end{pmatrix}. \tag{13}$$

Consequently, Maxwell's equations may be solved by introducing the concept of vector potential in either two ways[12]-[14].

Case-I : The conventional choice is being used as

$$\begin{aligned} \overrightarrow{E} &= -\frac{\partial \overrightarrow{A}}{\partial t} - \overrightarrow{\nabla}\phi; \\ \overrightarrow{B} &= \overrightarrow{\nabla} \times \overrightarrow{A}; \end{aligned} \tag{14}$$

where$\{A_\mu\} = \left(\phi, \overrightarrow{A}\right)$ is described as the four potential. So, the dual symmetric and Lorentz covariant Maxwell's equations(10) are written in as

$$\begin{aligned} \partial_\nu F^{\mu\nu} &= 0; \\ \partial_\nu \widetilde{F^{\mu\nu}} &= 0; \end{aligned} \tag{15}$$

where $F^{\mu\nu} = \partial^\nu A^\mu - \partial^\mu A^\nu = A^{\mu,\nu} - A^{\nu,\mu}$is anti-symmetric electromagnetic field tensor,$\widetilde{F^{\mu\nu}} = \frac{1}{2}\varepsilon^{\mu\nu\lambda\omega}F_{\lambda\omega}\,(\forall\mu,\nu,\lambda,\omega = 0, 1,$ is the dual of electromagnetic field tensor and $\varepsilon^{\mu\nu\lambda\omega}$is the four index Levi-Civita symbol.$\varepsilon^{\mu\nu\lambda\omega} = +1\forall\,(\mu\nu\lambda\omega = 0123)$ for cyclic permutation;$\varepsilon^{\mu\nu\lambda\omega} = -1$ for any two permutations and $\varepsilon^{\mu\nu\lambda\omega} = 0$ if any two indices are equal. Using equation(14) , we may obtain the electric and magnetic fields as the components of anti-symmetric electromagnetic field tensors $F^{\mu\nu}$ and $\widetilde{F^{\mu\nu}}$ given by

$$\begin{aligned} F^{0j} &= E_j; \quad F^{jk} = \varepsilon^{jkl}B_l \quad (\forall j, k, l = 1, 2, 3); \\ \widetilde{F^{0j}} &= B_j; \quad \widetilde{F^{jk}} = \varepsilon^{jkl}E_l \quad (\forall j, k, l = 1, 2, 3); \end{aligned} \tag{16}$$

whereε^{jkl} is three index Levi-Civita symbol and$\varepsilon^{jkl} = +1$ for cyclic,$\varepsilon^{jkl} = -1$ for anti-cyclic permutations and$\varepsilon^{jkl} = 0$ for repeated indices. The duality symmetry is lost if electric charge and current source densities enter to the conventional inhomogeneous Maxwell's equations given by

$$\overrightarrow{\nabla}\cdot\overrightarrow{E}=\rho;$$
$$\overrightarrow{\nabla}\cdot\overrightarrow{B}=0;$$
$$\overrightarrow{\nabla}\times\overrightarrow{E}=-\frac{\partial\overrightarrow{B}}{\partial t};$$
$$\overrightarrow{\nabla}\times\overrightarrow{B}=\overrightarrow{j}+\frac{\partial\overrightarrow{E}}{\partial t}; \tag{17}$$

where$\rho$and $\overrightarrow{j}$ are described as charge and current source densities which are the components of electric four-current$\{j_\mu\}=\left(\rho,\overrightarrow{j}\right)$source density. So, the covariant form of Maxwell's equation (17) is described as

$$\partial_\nu F^{\mu\nu}=j^\mu;$$
$$\partial_\nu\widetilde{F^{\mu\nu}}=0. \tag{18}$$

Here, we may see that the pair$\left(\overrightarrow{\nabla}\cdot\overrightarrow{B}=0;\ \overrightarrow{\nabla}\times\overrightarrow{E}=-\frac{\partial\overrightarrow{B}}{\partial t}\right)$ of Maxwell's equations(17) is described by $\partial_\nu\widetilde{F^{\mu\nu}}=0$ in equation(18). It has become kinematical while the dynamics is contained in another pair$\Big(\overrightarrow{\nabla}\cdot\overrightarrow{E}=\rho;\ \overrightarrow{\nabla}\times\overrightarrow{B}=$ Maxwell's equations(17) which described as$\partial_\nu F^{\mu\nu}=j^\mu$ in equation(18) and also reduces to following wave equation in the presence of Lorentz gauge condition $\partial_\mu A_\mu=0$ i.e.

$$\Box A^\mu=j^\mu; \tag{19}$$

where$\Box=\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}$ is the D' Alembertian operator. So, a particle of mass m electric charge e moving with a velocity$\{u^\nu\}$in an electromagnetic field is subjected by a Lorentz force given by

$$m\frac{d^2x_\mu}{d\tau^2}=\frac{dp_\mu}{d\tau}=f_\mu=eF_{\mu\nu}u^\nu. \tag{20}$$

where$\{\ddot{x}_\mu\}$is the four-acceleration, f_μ is four force and $p_\mu$is four momentum of a particle. Equation(20) is reduced to

$$\vec{f}=\frac{d\overrightarrow{p}}{dt}=m\frac{d^2\overrightarrow{x}}{dt^2}=e\left[\overrightarrow{E}+\overrightarrow{u}\times\overrightarrow{B}\right]. \tag{21}$$

where $\overrightarrow{p}$, $\overrightarrow{f}$,$\overrightarrow{x}$, $\overrightarrow{u}$ are respectively the three vector forms of momentum, force, displacement and velocity of a particle. Here we may observe that the Lorentz force equation of motion$(20-21)$ are also not invariant under duality transformations$(12-13)$.

Case-II : On the other hand, let us introduce[12]-[14] the another alternative way instead of equation(14) to write

$$\overrightarrow{B}=-\frac{\partial\overrightarrow{\mathscr{B}}}{\partial t}-\overrightarrow{\nabla}\varphi;$$
$$\overrightarrow{E}=-\overrightarrow{\nabla}\times\overrightarrow{\mathscr{B}}; \tag{22}$$

where a new potential$\{\mathscr{B}^{\mu}\}=\left(\varphi,\overrightarrow{\mathscr{B}}\right)$is introduced[14],[15] as an alternative to$\{A^{\mu}\}$. Thus, we see that source free (homogeneous) Maxwell's equation are same as those equations(10) but the inhomogeneous Maxwell's equation(17) are changed to

$$\overrightarrow{\nabla}\cdot\overrightarrow{E}=0;$$
$$\overrightarrow{\nabla}\cdot\overrightarrow{B}=0;$$
$$\overrightarrow{\nabla}\times\overrightarrow{B}=\frac{\partial\overrightarrow{E}}{\partial t};$$
$$\overrightarrow{\nabla}\times\overrightarrow{E}=-\overrightarrow{k}-\frac{\partial\overrightarrow{B}}{\partial t}. \tag{23}$$

subjected by the introduction of a new four current source density$\{k^{\mu}\}=\left(\rho,\,\overrightarrow{k}\right)$. In equation(23) we see that the pair $\left(\overrightarrow{\nabla}\cdot\overrightarrow{E}=0;\;\overrightarrow{\nabla}\times\overrightarrow{B}=\frac{\partial\overrightarrow{E}}{\partial t}\right)$becomes kinematical while the dynamics is contained in the second pair $\left(\overrightarrow{\nabla}\cdot\overrightarrow{B}=\rho;\;\overrightarrow{\nabla}\times\overrightarrow{E}=-\overrightarrow{k}-\frac{\partial\overrightarrow{B}}{\partial t}\right)$. Equation(23) may also be written in following covariant forms

$$\partial_{\nu}F^{\mu\nu}=0;$$
$$\partial_{\nu}\widetilde{F^{\mu\nu}}=k^{\mu}; \tag{24}$$

where$\widetilde{F^{\mu\nu}}=\partial^{\nu}\mathscr{B}^{\mu}-\partial^{\mu}\mathscr{B}^{\nu}$; $\widetilde{\widetilde{F^{\mu\nu}}}=F^{\mu\nu}$;$\{k^{\mu}\}=\left(\rho,\,\overrightarrow{k}\right)$and $\{k_{\mu}\}=\left(\rho,\,-\overrightarrow{k}\right)$.Equation(23) may also be obtained on applying the transformations(12) and(13) to equation(17) followed by following duality transformations for potential, current and antisymmetric electromagnetic field tensors as

$$A^{\mu}\longrightarrow\mathscr{B}^{\mu};\mathscr{B}^{\mu}\longrightarrow-A^{\mu}.$$
$$j^{\mu}\longrightarrow k^{\mu};k^{\mu}\longrightarrow-j^{\mu}.$$
$$F^{\mu\nu}\longrightarrow\widetilde{F^{\mu\nu}};\widetilde{F^{\mu\nu}}\longrightarrow-F^{\mu\nu}. \tag{25}$$

As such, we may identify the potential$\{\mathscr{B}_{\mu}\}=\left(\varphi,\,\overrightarrow{B}\right)$as the dual of potential$\{A^{\mu}\}$and the current$\{k^{\mu}\}=\left(\rho,\,\overrightarrow{k}\right)$as the dual of current$\{j^{\mu}\}$.Correspondingly, the differential equations(22) are identified as the dual Maxwell's equations. So, accordingly, we may develop the electrodynamics of a charged particle with the charge dual to the electric charge (i.e magnetic monopole). Applying the the electromagnetic duality to the Maxwell's equations, we may establish the connection between electric and magnetic charge (monopole)[16, 17], in the same manner as an electric charge e interacts with electric field and the dual charge (magnetic monopole) g interacts with magnetic field, as,

$$e \longrightarrow g;\; g \longrightarrow -e \tag{26}$$

where g is described as the dual electric charge (charge of magnetic monopole). Hence, we may recall the dual electrodynamics as the dynamics of pure magnetic monopole. consequently, the corresponding dynamical variables associated there in are described as the dynamical variables in the theory of magnetic monopole. So, we may write the new electromagnetic field tensor $\mathscr{F}_{\mu\nu}$ in place of $\widetilde{F^{\mu\nu}}$ as

$$\widetilde{F^{\mu\nu}} \longmapsto \mathscr{F}_{\mu\nu} = \partial_\nu \mathscr{B}_\mu - \partial_\mu \mathscr{B}_\nu\, (\mu,\, \nu = 1,\, 2,\, 3)\,; \tag{27}$$

which reproduces the following definition of magneto-electric fields of monopole as

$$\begin{aligned} \mathscr{F}_{0i} &= B^i; \\ \mathscr{F}_{ij} &= -\varepsilon_{ijk} E^k. \end{aligned} \tag{28}$$

Hence the covariant form of Maxwell's equations(21) for magnetic monopole may now be written as

$$\begin{aligned} \mathscr{F}_{\mu\nu,\nu} &= \partial^\nu \mathscr{F}_{\mu\nu} = k_\mu; \\ \widetilde{\mathscr{F}_{\mu\nu,\nu}} &= \partial^\nu \widetilde{\mathscr{F}_{\mu\nu}} = 0. \end{aligned} \tag{29}$$

where$\{k_\mu\} = \left(\rho,\, -\overrightarrow{k}\right)$is the four - current density due to the presence of the magnetic charge g. Accordingly, the wave equation(24) for pure monopole is described as

$$\Box \mathscr{B}_\mu = k_\mu; \tag{30}$$

in presence of Lorentz gauge condition $\partial_\mu \mathscr{B}^\mu = 0$. Accordingly, we may develop the classical Lagrangian formulation in order to obtain the field equation (dual Maxwell's equations) and equation of motion for the dynamics of a dual charge (magnetic monopole) interacting with electromagnetic field. So, the Lorentz force equation of motion for a dual charge (i.e magnetic monopole) may now be written from the duality equations(12) and(13) as

$$\frac{d\overrightarrow{p}}{dt}=\overrightarrow{f}=m\overrightarrow{\ddot{x}}=g\left(\overrightarrow{B}-\overrightarrow{u}\times\overrightarrow{E}\right); \tag{31}$$

where $\overrightarrow{p}=m\overrightarrow{\dot{x}}=m\overrightarrow{u}$ is the momentum, and $\overrightarrow{f}$ is a force acting on a particle of charge g, mass m and moving with the velocity $\overrightarrow{v}$ in electromagnetic fields. Equation(29) can be generalized to write it in the following four vector formulation as

$$m\frac{d^2x_\mu}{d\tau^2}=\frac{dp_\mu}{d\tau}=f_\mu=m\ddot{x}_\mu=g\mathscr{F}_{\mu\nu}u^\nu. \tag{32}$$

where$\{u_\nu\}$ is the four velocity,$\{p_\mu\}$is four momentum, f_μ is four force and$\{\ddot{x}_\mu\}$is the four-acceleration of a particle carrying the dual charge (namely magnetic monopole).

4 Maxwell field Theory for Energy momentum tensor of dyons

Maxwell's field theory of dyons (electric and magnetic) can be expressed in terms of a four-vector field $A_\mu$and C_μ, coupled to a current $j_\mu^{(e)}$and $j_\mu^{(g)}$ due to dyonic fields. The Lagrangian density [30] of dyons given by

$$\mathscr{L}=\mathscr{L}_{Max}+\mathscr{L}_{Matter}+\mathscr{L}_{int}; \tag{33}$$

where$\mathscr{L}_{Max}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{4}\widetilde{F}_{\mu\nu}\widetilde{F}^{\mu\nu}$ and $\mathscr{L}_{int}=-A_\mu j^{\mu(e)}-C_\mu j^{\mu(g)}$. The field strength tensor of dyons is defined by [30]

$$\begin{aligned}F_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu;\\ \widetilde{F}_{\mu\nu}&=\partial_\mu B_\nu-\partial_\nu B_\mu.\end{aligned} \tag{34}$$

Consider the energy momentum tensor of dyons for the Maxwell theory without a source j_μ and k_μ. The action is invariant under a translation

$$\begin{aligned}A_\mu(x)&\rightarrow A_\mu^{'}(x)=A_\mu(x)+c^\nu\partial_\nu A_\mu(x);\\ C_\mu(x)&\rightarrow C_\mu^{'}(x)=C_\mu(x)+d^\nu\partial_\nu C_\mu(x).\end{aligned} \tag{35}$$

which means that there are conserved currents of dyons. The generalized current of dyons is given by [30]

$$\begin{aligned}J^{\mu(e)}&=\frac{\partial\mathscr{L}}{\partial\left(\partial_\mu A_\rho\right)}\partial_\nu A_\rho c^\nu-\mathscr{L}c^\mu+c_\nu\Phi^{\nu\mu};\\ j^{\mu(g)}&=\frac{\partial\mathscr{L}}{\partial\left(\partial_\mu C_\rho\right)}\partial_\nu C_\rho d^\nu-\mathscr{L}d^\mu+d_\nu\Psi^{\nu\mu};\end{aligned} \tag{36}$$

where $\delta A_\sigma=\delta B_\sigma=0$ and replaced $\frac{d\delta x^\nu}{d\varepsilon}$ with $c_\nu$and d_ν, but have not specified $\Phi^{\nu\mu}$and $\Psi^{\nu\mu}$, which is only constrained to have $\partial_\mu\Phi^{\nu\mu}=\partial_\mu\Psi^{\nu\mu}=\delta\mathscr{L}=0$. Then, $\Phi^{\nu\mu}=\Psi^{\nu\mu}=0$

$$\begin{aligned}\frac{\partial\mathscr{L}}{\partial\left(\partial_\mu A_\rho\right)}&=F^{\rho\mu};\\ \frac{\partial\mathscr{L}}{\partial\left(\partial_\mu C_\rho\right)}&=\widetilde{F}^{\rho\mu};\end{aligned} \tag{37}$$

then

$$
\begin{aligned}
J^{\mu(e)} &= F^{\rho\mu}\partial_\nu A_\rho c^\nu + \frac{1}{4}c^\mu F^{\rho\sigma}F_{\rho\sigma} + c_\nu \Xi^{\nu\mu} = c^\nu \widetilde{T}^{\nu\mu};\\
j^{\mu(g)} &= \widetilde{F}^{\rho\mu}\partial_\nu C_\rho d^\nu + \frac{1}{4}d^\mu \widetilde{F}^{\rho\sigma}\widetilde{F}_{\rho\sigma} + d_\nu \Pi^{\nu\mu} = d^\nu \widetilde{S}^{\nu\mu}.
\end{aligned}
\tag{38}
$$

then the energy momentum tensor of electric and magnetic field can be written as [30]

$$
\begin{aligned}
\widetilde{\mathscr{T}}^{\nu\mu} =& F^{\rho\mu}\partial_\nu A_\rho + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma} + \Xi^{\nu\mu};\\
\widetilde{\mathscr{S}}^{\nu\mu} =& \widetilde{F}^{\rho\mu}\partial_\nu C_\rho + \frac{1}{4}g^{\mu\nu}\widetilde{F}^{\rho\sigma}\widetilde{F}_{\rho\sigma} + \Pi^{\nu\mu}.
\end{aligned}
\tag{39}
$$

While this $\widetilde{\mathscr{T}}^{\nu\mu}$and $\widetilde{\mathscr{S}}^{\nu\mu}$ is a conserved current of dyons. Then the term, there are two unpleasant features. First, it is not symmetric under $\mu \to \nu$, which we expect of the energy momentum tensor of dyons, is required for the angular momentum current of dyons $\mathscr{J}^{\mu\nu\rho} = \widetilde{\mathscr{T}}^{\mu\nu}x^\rho - \widetilde{\mathscr{T}}^{\mu\rho}x^\nu$ and $\mathscr{K}^{\mu\nu\rho} = \widetilde{\mathscr{S}}^{\mu\nu}x^\rho - \widetilde{\mathscr{S}}^{\mu\rho}x^\nu$ to be conserved, and to couple to the curvature in general relativity. Secondly, this $\widetilde{\mathscr{T}}$ and $\widetilde{\mathscr{S}}$ would not be invariant under a gauge transformation of dyons $A_\rho \to A_\rho + \partial_\rho \varphi$ and $C_\rho \to C_\rho + \partial_\rho \chi$, so it depends on unphysical degrees of freedom [30]. Then

$$
\begin{aligned}
\Xi^{\nu\mu} &= -F^{\rho\mu}\partial_\rho A^\nu;\\
\Pi^{\nu\mu} &= -\widetilde{F}^{\rho\mu}\partial_\rho C^\nu.
\end{aligned}
\tag{40}
$$

then we have

$$
\begin{aligned}
\partial_\mu \mathscr{T}^{\nu\mu} &= \partial_\mu \left(\Xi^{\nu\mu} + \Pi^{\nu\mu}\right);\\
\partial_\mu \mathscr{T}^{\nu\mu} &= -\left(\partial_\mu F^{\rho\mu}\right) - F^{\rho\mu}\partial_\rho\partial_\mu A^\nu - \left(\partial_\mu \widetilde{F}^{\rho\mu}\right) - \widetilde{F}^{\rho\mu}\partial_\rho\partial_\mu C^\nu = 0.
\end{aligned}
\tag{41}
$$

The first term from the equation of motion and the second from the antisymmetry of $F^{\rho\mu}$and $\widetilde{F}^{\rho\mu}$dotted into the symmetric $\partial_\rho\partial_\mu$. This term completes the F and $\widetilde{F}$ in $\widetilde{\mathscr{T}}$ and $\widetilde{\mathscr{S}}$, so

$$
\begin{aligned}
\mathscr{T}^{\nu\mu} &= \widetilde{\mathscr{T}}^{\nu\mu} + \widetilde{\mathscr{S}}^{\nu\mu};\\
\mathscr{T}^{\nu\mu} &= -F^{\rho\mu}F^\nu_\rho - \widetilde{F}^{\rho\mu}\widetilde{F}^\nu_\rho + \frac{1}{4}g^{\mu\nu}\left(F^{\rho\sigma}F_{\rho\sigma} + \widetilde{F}^{\rho\sigma}\widetilde{F}_{\rho\sigma}\right).
\end{aligned}
\tag{42}
$$

The terms, with $F^{j0} = E^j$, $F^{ij} = -\varepsilon_{ijk}H^k$ and $\widetilde{F^{j0}} = H^j$, $\widetilde{F^{ij}} = \varepsilon_{ijk}H^k$,

$$
\mathscr{T}^{00} = \frac{1}{2}\left(E^2 + H^2\right);
\tag{43}
$$

and

$$
\begin{aligned}
\mathscr{T}^{i0} &= \mathscr{T}^{0i} = -F^{j0}F^{ij} - \widetilde{F}^{jo}\widetilde{F}^{ij};\\
\mathscr{T}^{i0} &= \mathscr{T}^{0i} = \left(\overrightarrow{E} \times \overrightarrow{H}\right)_i.
\end{aligned}
\tag{44}
$$

References

[1] F. J. Belinfante, *Physica, **6** ;1939, 887.*

[2] F. J. Belinfante, *Physica, 7 ,1940; 449.*

[3] L. Rosenfield, *Mem. Acad. Roy. Belg. Sci., **18** ,1940, 1.*

[4] C. G. Callan, S. Coleman, R. Jackiw, *Ann. Phys.,* ***59*** *,1970, 42.*

[5] S. Deser, *Ann. Phys.,* ***59*** *,1970, 248.*

[6] M. J. Gotay, J. E. Marsden, *Contemporary Mathematics, AMS, Providence.,* ***132*** *,1992, 367.*

[7] W. Misner, K. Thorne and J. Wheeler, *San Frandisco., 1973, pp. 141-142.*

[8] W. Misner, K. Thorne and J. Wheeler, *San Frandisco (1973).*

[9] R. A. D'Inverno, *New York: Oxford University Press (1992).*

[10] W. Wyss, *Colorado, USA (2005).*

[11] D. I. Olive, "Exact Electromagnetic Duality", hep-th/9508089; L. Alvarez-Gaume and S. F. Hasan, Fortsch. Phys., 45 (1997), 159; hep-th/9701069; E. Kiritsis, "Supersymmetry and Duality in Field Theory and String Theory", hep-th /9911525; J. M. Figueroa- O ́Farril, "Electromagnetic Duality for Children", hep-th /9710082; J. A. Harvey, "Magnetic Monopoles, Duality and Supersymmetry", hep-th/9603086; A. Kapustin and E. Witten, "Electric-Magnetic Duality And The Geometric Lang lands Program", hep-th/0604151; P. D. Vecchia, "Duality in Supersymmetric Gauge theories", hep-th /9608090.

[12] P. B. Price, E. K. Shirk, W. Z. Osborne, and L. S. Pinsky, Phys. Rev. Lett., 35 (1975), 487; B. Cabrera, Phys. Rev. Lett., 48 (1982), 1378; G. Giacomelli and L. Patrizii, "Magnetic Monopole Searches", arXiv:hep-ex/0302011.

[13] S. Zhang et al, Science, 294 (2001), 823 ; 301 (2003), 1348; Phys. Rev., B69, (2004), 235206; Phys. Rev. Lett., 93, (2004), 156804 ; S. Sondhi, NATURE, 461, (2009), 888; R. Mössner and P. Schiffer, NATURE PHYSICS, 5 , (2009), 250; C. Castelnovo, R. Mössner and S. Sondhi, NATURE, 451, (2007), 42 .

[14] P. S. Bisht and O. P. S. Negi, Inter. J. Theor. Phys., 47 (2008), 3108; M. Baker, J.S.ball and F. Zachariasen, "Classical Electrodynamics with Dual Potentials", hep-th/9403169.

[15] P. C. R. Cardoso de Mello, S. Carneiro and M. C. Nemes, Phys. Lett., B384 (1996), 197; hep-th/9609218, P.D.Drummond, J. Phys. B; at Mol. Opt. Phys., 39 (2006)573; Phys. Rev. A60 (1999) , R 3331.

[16] N. Cabibbo and E. Ferrari, Nuovo Cimento, 23 (1962) 1147; A. Nisbet, Proc. Roy. Soc. London, A231 (1955) 250; D. Singleton, Am. J. Phys., 64, 452 (1996); Int. J. Theor. Phys., 34, 37 (1995); 35, 2419 (1996); K. Li and C. M. Na ́on; Mod. Phys. Lett., A16 (2001), 1671.

[17] B. S. Rajput and D. C. Joshi, Had. J., 4 (1981), 1805; Pramana (India), 15 (1980), 153; P. S. Bisht, O. P. S. Negi and B. S. Rajput, IL Nuovo Cimento, A104 (1991), 337; Prog. Theo. Phys., 85, (1991), 157. Inter. J. Theor. Phys., 32 (1993), 2099.

[18] P. S. Bisht, O. P. S. Negi and B. S. Rajput, Inter. J. Theor. Phys., 32 (1993), 2099; Shalini Bisht, P. S. Bisht and O. P. S. Negi, Nuovo Cimento, B113, (1998), 1449; H. Dehnen and O. P. S. Negi, "Electromagnetic Duality, Quaternion and Supersymmetric Gauge Theories of Dyons", hep-th /0608164.

[19] R. Parthasarathy, *The Legacy of Alladi Ramakrishanan in Mathematical Sciences,* ***4*** *, 2010, 565.*

[20] G. Compagno, R. Passante and F. Persico, *Cambridge University Press,* ***17*** *,1995, 352.*

[21] G. M. Shore and B. E. White, *Nucl. Phys.,* ***B581*** *,2000, 409.*

[22] E. Leader, *Cambridge University Press, Cambridge, UK (2001, 2005).*

[23] B. L. G. Bakker, E. Leader, and T. L. Trueman, *Phys. Rev.,* ***D70****, 2004, 114001.*

[24] A. L. Kholmetskii, O. V. Missevitch, T. Yarman, *Phys. Scr.,* ***83*** *,2011, 055406.*

[25] A. L. Kholmetskii, *Found. Phys.*, ***36*** *,2006, 715*.

[26] A. L. Kholmetskii, *Found*. Phys., **19** , 2006, 696.

[27] L. D. Landau and E. M. Lifshitz, *(Pergamon Press, New York, 1962).*

[28] L. D. Landau, L. P. Pitaevskii and E. M. Lifshitz, *(Pergamon Press, New York, 1984).*

[29] Gaurav Karnatak, P.S. Bisht, O.P.S. Negi, IJARSE., **2** , 2013, 12.

[30] Joel A. Shapiro, Stress-Energy tensor for Maxwell Theory.