Sets, Formulas and Electors

Max Null, Sergey Belov

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Abstract

This article is a mathematical experiment with the sets and the formulas. We consider new elements which are called the electors. The elector has the properties of the sets and the formulas.

1. Introduction

In this article we consider an example of some complete lattice of electors.

We consider the properties of the mathematical 0. We can consider 0 as the mathematical object of the arithmetic model and the constant name of the arithmetic signature, i.e. as a formula object. We distinguish between these concepts in math logic, but we do not distinguish these concepts in other mathematics (math analyze such as). Thus we can write $0 \in N$ (0 as the object in math analyze) and $0 < 1$ (0 as the element of formula in math analyze). With the point of view of the set theory, formula $0 \in N$ is a just formula too, but we do not consider the mathematical foundations. Thus, with the point of view math analyze, element 0 has the object properties and formula element properties. This approach we can associate with the idea of the middle way of the Eastern philosophy. This approach we can associate with the idea of invariant properties (in common sense) of the West philosophy.

The main idea is following. There are math objects: sets, predicate and so on. There are formula elements: variables, constants, formulas and so on. There is the space of objects. There is the space of formulas. We try to construct the common space, i.e. objects of this space has the properties of the sets and the formulas. New elements we define as electors. Our tradition objects and formulas we will interpret as some objects of the new electors space.

We begin our study with an emptiness. We have two mathematical elements which are associated with the emptiness: $\emptyset = \{\}$ and x. We can consider $\{\}$ as a object element and x as a formula element.

2. Electors

We consider some basic set $M \neq \emptyset$ as the source of the object elements. We consider some variable names $x, y, z, ...$ with index and without index. We define the scope of variable x as $Rang(x)$.

We define electors as follows:

1) if x is a variable and $Rang(x) \subseteq M$ then x is an elector;

2) if x is a variable and $Rang(x)$ is a set of electors then x is an elector;

3) if X is a set of electors then X is an elector.

Thus, we can consider electors as a complex variables.

We see that any subset $A \subset M$ we can consider as a some elector

$$X = \{x_a | Rang(x_a) = \{a\}\}.$$

We define $E(M)$ as a set of all electors with the base set M. We see that any elector of $E(M)$ can be constructed in a finite number of steps. We define $E_0(M)$ as a set of all electors with the base set M of the finite type, i.e. in 2) we assume that $Rang(x)$ is finite and in 3) we assume that X is finite.

Let $s \in E(M)$. We define $V(s)$ as the set of all variables of s. We define $V_0(s)$ as the set of all variables of s that $Rang(s) \subseteq M$. We define $E^*(s)$ as the set of all sub-electors of s.

We define the set $S(M)$ by the induction.

1) $S_0 = M$,

2) if S_n is defined then $S_{n+1} = P(S_n)$,

3) $S(M) = S_0 \cup S_1 \cup S_2 \cup ...$

We define the initialization of elector s. The initialization of elector s is the function $I : E^*(s) \to S(M)$ with the following properties.

Let $e \in E^*(s)$.

1) if $e \in V_0(s)$ then $I(e) \in Rang(e)$;

2) if $e \in V(s) \setminus V_0(s)$ then $I(e) = I(e')$, where $e' \in Rang(e)$,

3) if $e \in E^*(s) \setminus (V(s) \cup V_0(s))$ (i.e. e is the set of selectors) then $I(e) = \{I(e') | e' \in e\}$.

We see that this definition is excess.

We have that $I(s) \in S(M)$.

We define $W(s)$ as the set of all interpretations of elector s, i.e. $W(s) \subseteq S(M)$.

Let $m \in S(M)$.

We define $Ob(m) = \{p \in M | \text{there is a finite secuence}\, p \in ... \in m\}$.

Let $s \in E(M)$.

We define

$$Inv(s) = \{Ob(m) | m \in W(s)\}.$$

We define the relation $\preceq$ on $E(M)$. Let $e_1, e_2 \in E(M)$. We define $e_1 \preceq e_2 \Leftrightarrow$ for any $S_1 \in Inv(e_1)$ we have that there is $S_2 \in Inv(e_2)$ that $S_1 \subseteq S_2$.

We have that $\chi = (E(M), \preceq)$ is a complete lattice.

We see that

1) $inf(E(M)) = [x]$, where $Rang(x) = \emptyset$,

2) $sup(E(M)) = [\{x_a | Rang(x_a) = \{a\}\}]$.

2. Formula interpretation of electors

We assume that M is finite. We consider $E_0(M)$. We can consider M as a set of proposition variables. We define formula interpretation I_0 by the recursive procedure.

Let $e \in E_0(M)$.

1) if e is a variable and $Rang(e) \subseteq M$ then $I_0(e) = p_1 \vee ... \vee p_n$, where $\{p_1, ...p_n\} = Rang(e)$;

2) if e is a variable and $Rang(e) = \{e_1, ..., e_m\}$ then $I_0(e) = I_0(e_1) \vee ... \vee I(e_m)$;

3) if $e = \{e_1, ..., e_m\}$, i.e. e is a set of electors then $I_0(e) = I_0(e_1) \wedge ... \wedge I_0(e_m)$;

We see that electors of $E_0(M)$ and formulas with proposition variables from M without negation have a one to one correspondence.

Let $I_0(e)$ be equivalent to disjunctive normal formula

$$(p_1^{(1)} \wedge ... \wedge p_{n_1}^{(1)}) \vee ... \vee (p_1^{(m)} \wedge ... \wedge p_{n_m}^{(m)}).$$

We have that

$$Inv(e) = \{\{p_1^{(i)}, ..., p_{n_i}^{(i)}\} | 1 \leq i \leq m\}.$$

Thus, the relation $\preceq$ on electors corresponds to the deduction of the corresponding formulas, i.e. $e_1 \preceq e_2 \Leftrightarrow I_0(e_2) \rightarrow I_0(e_1)$ is deducted from the axiom of the proposition calculus.

3. Equivalence classes interpretation of electors

We can consider the object interpretation of electors.

Let x is variable. We use the brackets $[...]_x$ to denote the equivalence class corresponding to variable x. The base set is $Rang(x)$. All elements of this class are equivalence. If $a_1, a_2 \in Rang(x)$ then $a_1 \sim a_2$, i.e. we can denote corresponding class as $[a_1]_x = [a_2]_x = [a_1, a_2]_x$.

When variable is not essential we will denote equivalence classes as $[...]_0, [...]_i$ and so on, i.e. with indexes. If we enumerate equivalence class as $[a_1, a_2], [b_1, ..., b_n]$ then we will omit class indexes, because class is defined by own elements.

We can define interpretation of electors I_1 as follows:

1) if e is a variable and $Rang(x) \subseteq M$ then $I_1(e) = [a_1, ..., a_n]$, where $\{a_1, ..., a_n\} = Rang(x)$;

2) if e is a variable and $Rang(e)$ is a set of electors then $I_1(e) = [I(e_1), ..., I(e_n)]$, where $\{e_1, ..., e_n\} = Rang(e)$;

3) if e is a set of electors then $I_1(e) = \{I_1(e_1), ..., I_1(e_n)\}$, where $e = \{e_1, ..., e_n\}$.

How can we understand $e = \{e_1, e_2\}$ and $e' = [e_1, e_2]$?

We can consider e_1, e_2 as a difference elements in the context of the element e. We can consider e_1, e_2 as a non-difference or identical elements in the context of the element e'.

Thus we can consider elements e, e' with the point of view of the choice. We can choice any element of $\{e_1, e_2\}$ as we want to do it. We can choice e_1 or we can choice e_2. We do not difference elements of $[e_1, e_2]$, i.e. we can choice one element (e_1 or e_2) of $[e_1, e_2]$ randomly.

In the ordinary sense, $\{e_1, e_2\}$ is the open box and we choice with the light. $[e_1, e_2]$ is the open box and we choice without the light.

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REFERENCES

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