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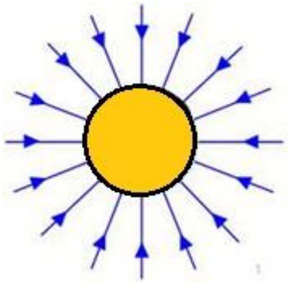
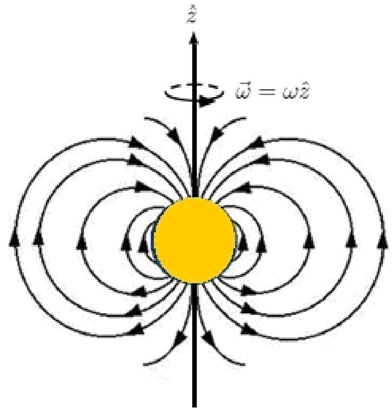
Lorentz-invariant gravitation theory.

$$\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi=4\pi\rho \qquad\qquad \frac{1}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2}-\nabla^2\vec{A}=\frac{4\pi}{c}\rho\vec{\upsilon}$$





$$ds^2=\left(1-\frac{r_s}{r}\right)c^2dt^2-\frac{dr^2}{\left(1-\frac{r_s}{r}\right)}-r^2\left(\sin^2\theta\,d\varphi^2+d\theta^2\right)+\frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\,d\varphi dt$$

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Annotation

The modern theory of gravity, which is called General Theory of Relativity (GTR or GR), was verified with sufficient accuracy and adopted as the basis for studying gravitational phenomena in modern physics. GR is the geometric theory of gravitation, in which the metric of Riemannian space-time plays the role of relativistic gravitational potential. Therefore it has certain features that make it impossible to connect it with others physics theories in which geometry plays only a supporting role. Another formal feature of general relativity is that the study and the use of its mathematical apparatus require much more time than the study of any of the branches of modern physics. This book is an attempt to build a non-geometrical version of the theory of gravitation, which is in the framework of the modern Lorentz-invariant field theory and would not cause difficulties when teaching students. A characteristic feature of the proposed theory is that it is built on the basis of the quantum field theory.

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The Lorentz-invariant theory of gravitation (LIGT) is the conditional name of the proposed theory of gravity, since Lorentz-invariance is a very important, although not the only feature of this theory.

Note that our approach was used in the past in relation to the gravitational theories that have some similarities with our theory. Therefore the results obtained by well-known scientists are widely cited in the book. However, for posing the problem and for some of the basic elements of the theory which are obtained by the author of the book, the only person responsible is the author.

NOTATIONS.

(In almost all instances, meanings will be clear from the context. The following is a list of the usual meanings of some frequently used symbols and conventions).

Mathematical signs

$\alpha, \beta, \mu, \nu ...$ - Greek indices range over 0,1,2,3

$i, j, k...$ - Latin indices range over 1,2,3

$\hat{\alpha}_\mu, \hat{\beta}$ - Dirac matrices

$\vec{A}$ - 3-dimensional vector

A^μ - 4-dimensional vector

$A^{\mu\nu}$ - Tensor components

∇ - Covariant derivative operator

∇^2 - Laplacian

$\Box$ - d'Alembertian operator $\equiv \nabla^2 - \partial^2 / \partial t^2$

Physical values

u_μ - velocity

a_μ - 4-acceleration $\equiv du_\mu / d\tau$

p_μ - 4-momentum

$T^{\mu\nu}$ - Stress-energy tensor

$F^{\mu\nu}$ - Electromagnetic field tensor

$g_{\mu\nu}$ - metric tensor of curvilinear space-time

$g^{GR}_{\mu\nu}$ - metric tensor of GR space-time

$R_{\alpha\beta\gamma\delta}$ - Riemann tensor

$R_{\alpha\beta}$ - Ricci tensor $R^\gamma{}_{\alpha\beta\delta}$

R - Ricci scalar $\equiv R^\alpha{}_\alpha$

$G_{\alpha\beta}$ - Einstein tensor

$\eta_{\mu\nu}$ - Minkowski metric

$h_{\mu\nu}$ - Metric perturbations

$\Lambda_{\mu\nu}$ - Lorentz transformation matrix

j_μ - Current density

$J^{\mu\nu}$ - Angular momentum tensor

γ_N - Newton's constant of gravitation

γ_L - Lorentz factor (L-factor)

m - mass of particle

M_S - mass of the star (Sun)

M , L - angular momentum

Abbreviations:

LIGT - Lorentz-invariant gravitation theory;

EM - electromagnetic;

EMTM - electromagnetic theory of matter;

EMTG - electromagnetic theory of gravitation;

SM - Standard Model;

QFT – quantum field theory

NQFT - nonlinear quantum field theory;

QED - quantum electrodynamics.

EHE - Einstein-Hilbert equation

HJE – Hamilton-Jacobi equation

GTR or GR - General Theory of Relativity

L-transformation - Lorentz transformation

L-invariant - Lorentz-invariant

Indexes

e - electrical

m - magnetic,

em - electromagnetic,

g - gravitational

ge - gravito-electric,

gm - gravito-magnetic

N - Newtonian

Chapter 1. Statement of problem

1.0. The place of gravitation theory in a number of other physical theories

The first classical mechanics theory was created by Newton. Two types of laws of mechanics exist: laws of motion of material bodies under the action of forces, and laws that define these forces (which are often called equations of sources). In frames of Newton's theory, his second law is the primary law of motion, while Newton’s gravitational law defines the force of gravity.

It should also be mentioned that during the further development of mechanics, numerous mathematical formulations of the original laws of Newtonian mechanics were found, which are physically almost equivalent, including the ones that use energy characteristics of the motion of bodies, rather than force.

2.0. Relativistic theories

As was revealed later, Newtonian mechanics is valid for speeds, well below the speed of light $c \cong 300000$ km/sec. Mechanics, which is valid for speeds υ from zero to the speed of light was conditionally named relativistic mechanics (detailed overview of the theory see (Pauli, 1981)). Under the condition $\upsilon << c$, Newton's laws have very high accuracy.

The definition "relativistic" is equivalent to the requirement "to be invariant under Lorentz transformations". Therefore we will use the definition of "relativistic" equally with the definition of "Lorentz-invariant (briefly "L-invariant").

In relativistic mechanics, there are also several forms of equations of motion and equations of sources. As the relativistic law of motion (including the theory of gravity) the relativistic Hamilton-Jacobi equation is often used.

2.1. How non-relativistic mechanics is related to the relativistic mechanics

Non-relativistic theories give correct predictions at speeds much less than the speed of light. The relativistic theories give exact values in the entire range of speed from $\upsilon = 0$ until the speed of light $c \approx 300000$ km/sec. The inaccuracy of non-relativistic theories compared to the relativistic can be attributed to the Lorentz factor $\gamma_L = 1/\sqrt{1-\upsilon^2/c^2}$, a factor of the Lorentz transformation (see in reference book the diagram of Lorentz-factor γ_L as a function of speed).

As seen from the graph, factor is not very different from the unit, up until the velocity of the particle reaches the 1/10 of the velocity of light (i.e., about 30000 km/c). The maximum speeds of the planets and the massive bodies on the Earth and in the solar system are: projectile - 1.5 km/s, the rocket - 10-12 km/s, meteorites - 18-25 km/s, the Earth around the Sun - 30 km/s, the Sun in the direction to the galactic center - 200 km/s, our galaxy - up to 400 km/s. Higher speed is achieved only by elementary particles in cosmic space or in accelerators, but they do not play any role in the theory of gravity.

Thus, the value υ^2/c^2 in real problems of mechanics is very small and the Lorentz factor is not very different from unit. This means that Newtonian mechanics is valid in practical applications with great accuracy.

This was already understood by one of the founders of the Lorentz-invariant physics – A. Poincare, who had warned (Poincaré, 1908):

“*I tried in a few words to give the fullest possible understanding of new ideas and explain how they were born ... In conclusion, if I may, I express a wish. Suppose that in a few years, this new theory will be tested and come out victorious from this test. Then, our school education is in serious danger: some teachers will undoubtedly want to find a place to new theories .... And then [the students] will not grasp the usual mechanics*.

Is it right to warn students that it gives only approximate results? Yes! But later! When they will be permeated by it, so to speak, to the bone, when they will be accustomed to think only with *its help, when there will not be a risk that they forget how to do this, then we can show them its borders. They will have to live with the ordinary mechanics, the only mechanic that they will apply. Whatever the success of automobilism would be, our machines will never reach those speeds where ordinary mechanics is not valid. Other mechanics is a luxury, but one can think about a luxury only when it is unable to cause harm to the necessary*."

3.0. The general theory of relativity

The modern theory of gravitation, called the general theory of relativity (GTR or GR), refers to classical mechanics. As the equation of source is considered to be the Einstein-Hilbert equation (EHE) of general theory of relativity (GTR) or (GR), which was found by these researchers almost independently and almost simultaneously (Pauli, 1981; Vizgin, 1981). As the basis for theory building, Hilbert used a variational principle. The approach of Einstein was heuristic, emanating from the experimental fact of equality of gravitational and inertial masses (note that this equivalence is also valid in nonrelativistic theories).

A very difficult question, is whether the GTR and its equation are relativistic in terms of the Lorentz invariance. Strictly speaking, it is not (Katanaev, 2013, p. 742). Einstein assumed that the general covariance of the equations of general relativity includes special relativity.

As is known, EHE is very different from other equations of mechanics, since it is based on the Riemann geometry in general system of coordinates.

Besides, GR has several disadvantages, which have not been overcome to date (Fock, 1964; Rashevskyi, 1967); Logunov, 2002). These disadvantages have been for many years the cause of searching the new theory of gravity. The L-invariant theory of gravitation is regarded as one of the basic, because it could completely eliminate the disadvantages of the GR.

Let us enumerate basic disadvantages.

1) In 1918, Schrodinger (Schroedinger, 1918) first showed that by the appropriate choice of coordinate system all components of pseudo-tensor of the energy-momentum, which in the framework of GR is the source of the gravitational field, can be turned into zero. This was confirmed by D. Hilbert and other scientists (Bauer, 1918; Fock, 1964; Logunov, (2002; Pauli, 1958;) (For more information about this issue, see chapter 2).

2) GTR has no connection with quantum field theory (i.e., with the theory of elementary particles - the smallest particles of matter, capable to produce the gravitational field). Some prominent scientists even argue that gravity is some independent object of nature, which has no connection with the rest of physics.

4.0. The scientific goals

"*The nature of time*, *space* and *reality* are to large extent dependent on our interpretation of - *Special (SRT) and General Theory of Relativity (GTR). In STR essentially two distinct interpretations exist; the "geometrical" interpretation by Einstein based on the Principle of Relativity and the Invariance of the velocity of light and, the "physical" Lorentz-Poincare* interpretation with underpinning by *rod contractions*, *clock slowing* and *light synchronization*, see e.g. (Bohm, 1965; Bell, 1987). *It can be questioned whether the Lorentz-Poincare-interpretation of STR can be continued into GTR*" (Broekaert, 2005).

It can be said that the purpose of creation of Lorentz-invariant theory of gravitation (LIGT) is to show that the Lorentz-Poincare-interpretation of STR can be continued into gravitation theory. Such a theory could allow to overcome all the shortcomings of general relativity.

Since the Hilbert-Einstein equations give proven results, obviously, we have to show that such a LIGT gives equivalent results.

Our additional goal will be to explain the features of general relativity within the framework of nongeometric physics.

For the purity of the theoretical conclusions of LIGT we will not use anywhere of ideas of GTR or of similar metric theory as the basis of our theory (this does not apply to those cases, in which we will compare the results of these theories).

In the book we shall use the CGS system of units, in particular, the system of units of Gauss, since here all units are a unified system of mechanical units.

Chapter 2. Origin of the gravitation field source

1.0. The source of gravitation in the theories of gravitation and conservation laws

1.1. The source of gravity in general relativity

Initially Einstein assumed that the source of gravity in the Hilbert-Einstein equations is symmetric energy-momentum tensor $T_{\mu\nu}$ of the Lorentz-invariant mechanics satisfying the law of energy-momentum conservation:

$$\sum_{k=0}^{3} \frac{\partial T^{ik}}{\partial x_k} = 0 , \tag{1.1}$$

which corresponds to ten integrals of motion of Lorentz-invariant mechanics. As the generalization of $T_{\mu\nu}$ in GR should be the general covariant derivative and instead (1.1) we have:

$$\nabla_\nu T^{\mu\nu} \equiv \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x_\nu} \left(\sqrt{-g} T^{\mu\nu} \right) + \Gamma^{\mu}_{\alpha\beta} T^{\alpha\beta} = 0 , \tag{1.2}$$

But, it appears that (Landau and Lifshitz, 1971) "*in this form, however, this equation does not generally express any conservation law whatever*".

As a way out of this situation Einstein's formulation of energy-momentum conservation laws in the form of a divergence involved the introduction of a pseudo-tensor quantity t^{ik} which is not a true tensor (although covariant under linear transformations).

To determine the conserved total four-momentum for a gravitational field plus the matter located in it, Einstein choose a system of coordinates of such form that at some particular point in space-time all the first derivatives of the g_{ik} vanish.

Then we can enter the value t^{ik} by the following expression:

$$(-g)\left(T^{ik} + t^{ik}\right) = \frac{\partial h^{ikl}}{\partial x^l} , \tag{1.3}$$

where $\partial h^{ikl} = \dfrac{c^4}{16\pi\gamma_N} \dfrac{\partial}{\partial x^m} \left[(-g)\left(g^{ik} g^{lm} - g^{il} g^{km} \right) \right]$.

From the definition (96.4) it follows that for the sum $T^{ik} + t^{ik}$ the equation

$$\frac{\partial}{\partial x^k} (-g)\left(T^{ik} + t^{ik}\right) = 0 , \tag{1.4}$$

is identically satisfied. This means that there is a conservation law for the quantities

$$P^i = \frac{1}{c} \int (-g)\left(T^{ik} + t^{ik}\right) dS_k , \tag{1.5}$$

In the absence of a gravitational field, in galilean coordinates, $t^{ik} = 0$, and the integral goes over into into the four-momentum of the matter. Therefore the quantity (1.5) must be identified with the total four-momentum of matter plus gravitational field. But it is obvious that this result depends on the choice of coordinates and is ambiguous.

Unfortunately, there is still no generally accepted definition of energy and momentum in GR. Attempts aimed at finding a quantity for describing distribution of energy-momentum due to matter, non-gravitational and gravitational fields only resulted in various energy-momentum complexes, which are non-tensorial under general coordinate transformations.

1.2. The source of gravity in LIGT and conservation laws

In the Lorentz-invariant mechanics, in general, the values that make up the energy-momentum tensor (see above), are used in the theory, without being recorded in the form of the tensor (Fock, 1964) (it is noteworthy that W. Fock called this tensor the mass tensor (Fock, 1964, §31)).

Note, that after being divided by the square of the speed of light, these values are identical to the mass and mass flow (in general case, densities of mass and mass flow).

Therefore following to V. Fock (Fock, 1964, §54), "in formulating Einstein's theory we shall likewise start from the assumption that the mass distribution is insular. This assumption makes it possible to impose definite limiting conditions at infinity as for Newtonian theory, and so makes the mathematical problem a determined one. Theoretically, other assumptions are also admissible".

(As mass distribution of insular character V. Fock describes *"the case that all the masses of the system studied are concentrated within some finite volume which is separated by very great distances from all other masses not forming part of the system. When these other masses are sufficiently far away One can neglect their influence on the given system of masses, which then may be treated as isolated*.")

The foregoing allows us in framework of our theory to call, for the sake of brevity, the source of gravity - "mass/energy" or simply – "mass" (meaning by this term any element of the energy-momentum tensor of given task).

Mass as a source of gravitation is called gravitational mass or gravitational charge. Currently, the origin of the gravitational mass is unknown. But we know that it is equal with great precision to inertial mass, which appears in the laws of motion in mechanics.

Thus, if we find out the origin of inertial mass/energy, we can conclude that gravitational mass/energy and gravitation field have the same origin.

The question now is what do we know about the origin of inertial mass, particularly, of the elementary particles as initial source of gravitation?

2.0. The mass theories (classical and modern views)

To state the existing views on the considered issues, we will use the works of contemporary scientists (Feynman et al, 1964; Quigg, 2007; Dawson, 1999; etc):

2.1. Classical views

"*Mass remained an essence - part of the nature of things - for more than two centuries, until J.J. Thomson* (1881), *Abraham* (1903) *and Lorentz* (1904) *sought to interpret the electron mass as electromagnetic self-energy*", (Quigg, 2007).

Theory, created by J.J. Thomson and H. Lorentz (1881 - 1926), lies entirely in the field of classical electromagnetic theory. According to this theory, the inertial mass has electromagnetic origin.

The electromagnetic origin of the mass of all elementary particles, as well as the weakness of the gravitational field compared to the electromagnetic field, allowed to O.F. Mossotti (Mossotti, 1936) to assume that the gravitational field is a residual electromagnetic field

"Wilheim Weber (1804-91) of Gottingen and Friedrich Zollner[3] (1834-82) of Leipzig developed this conception into the idea that all ponderable molecules are associations of positively and negatively charged electrical corpuscles, with the condition that the force of attraction between corpuscles of unlike sign is somewhat greater than the force of repulsion between corpuscles of like sign. If the force between two electric units of like charge at a certain distance is a dynes, and the force between a positive and a negative unit charge at the same distance is y dynes, then, taking account of the fact that a neutral atom contains as much positive as negative electric charge, it was found that $(\gamma-\alpha)/\alpha$ need only be a quantity of the order 10^{-35} in order to account for gravitation as due to the difference between α and γ" (Whittaker, 1953).

"At the meeting of the Amsterdam Academy of Sciences on 31 March 1900, Lorentz communicated a paper entitled "*Considerations on Gravitations on Gravitation*", in which he reviewed the problem as it appeared at that time" (Whittakker, 1953).

Unfortunately, attempts to apply this theory to quantum theory has not been undertaken. However, until now there was no evidence of that the inertial mass is not fully electromagnetic (Feynman et al, 1964):

"*We only wish to emphasize here the following points:*

1) the electromagnetic theory predicts the existence of an electromagnetic mass, but it also falls on its face in doing so, because it does not produce a consistent theory – and the same is true with the quantum modifications;

2) there is experimental evidence for the existence of electromagnetic mass; and

3) all these masses are roughly the same as the mass of an electron.

So we come back again to the original idea of Lorentz - may be all the mass of an electron is purely electromagnetic, maybe the whole 0.511 MeV *is due to electrodynamics. Is it or isn't it? We haven't got a theory, so we cannot say*."

As we will be convinced later, the results of modern theory of elementary particles do not contradict to the original idea of Lorentz that all the mass of an electron may be purely electromagnetic.

2.2. Modern views

The modern mass theory is the, so-called, Higgs mechanism of the Standard Model theory (SM) (Quigg, 2007; Dawson, 1999; etc).

"*Our modern conception of mass has its roots in known Einstein's conclusion: "The mass of a body is a measure of its energy content. Among the virtues of identifying mass as* $m_0=\varepsilon_0/c^2$, *where* ε_0 *designates the body's rest energy, is that mass, so understood, is a Lorentz-invariant quantity, given in any frame as* $m=\left(1/c^2\right)\sqrt{\varepsilon^2-p^2c^2}$. *But not only is Einstein's a precise definition of mass, it invites us to consider the origins of mass by coming to terms with a body's rest energy.*

We understand the mass of an atom or molecule in terms of the masses of the atomic nuclei, the mass of the electron, and small corrections for binding energy that are given by quantum electrodynamics.

Nucleon mass is an entirely different story, the very exemplar of $m_0=\varepsilon_0/c^2$. *Quantum chromodynamics (QCD), the gauge theory of the strong interactions, teaches that the dominant contribution to the nucleon mass is not the masses of the quarks that make up the nucleon, but the energy stored up in confining the quarks in a tiny volume. The masses* m_u *and* m_d *of the up and down quarks are only a few MeV each. The quarks contribute no more than 2% to the* 939*MeV mass of an isoscalar nucleon (averaging proton and neutron properties).*

Hadrons such as the proton and neutron thus represent matter of a novel kind. In contrast to macroscopic matter and beyond what we observe in atoms, molecules and nuclei, the mass of a nucleon is not equal to the sum of its constituent masses - quarks; it is, basically, a confinement energy of gluons!" (Quigg, 2007).

The Higgs mechanism, under certain assumptions, allows us to describe the generation of masses of fundamental elementary particles: intermediate bosons, leptons and quarks. But as it is mentioned above (Quigg, 2007), more than 98% of the visible mass in the Universe is composed by the non-fundamental (composite) particles: protons, neutrons and other hadrons.

Thus, the Higgs mechanism can not be used in the gravitation theory.

3.0. Electromagnetic origin of elementary particles and their interactions

Starting with quantization of Maxwell's theory of electromagnetism, physicists have made tremendous progress in understanding the basic forces and particles constituting the physical world.

Modern quantum theories of elementary particle, such as the Standard model, are quantum Yang-Mills theories. In a quantum field theory the quanta of the fields are interpreted as particles. In a Yang-Mills theory these fields have an internal symmetry: they are appear by a space-time dependant non-Abelian group transformations. These transformations are known as local gauge transformations and Yang-Mills theories are also known as non-Abelian gauge theories.

If we will proceed to the gauge theories, we will see that Maxwell's equations are a special case of the Yang-Mills equations, which describe not only electromagnetism but also the strong and weak nuclear forces. Maxwell's equations can be regarded as a classical Yang-Mills theory with gauge group U(1).

Quantum electrodynamics is an Abelian gauge theory with the symmetry group U(1) and has one gauge field, the electromagnetic four-potential, with the photon being the gauge boson. The Standard Model is a non-Abelian gauge theory with the symmetry group U(1)×SU(2)×SU(3) and has a total of twelve gauge bosons: the photon, three weak bosons and eight gluons.

For us it is important to emphasize that the Yang-Mills theory is a generalization of Maxwell's theory (Ryder, 1985).

"We have the working renormalizable theory of strong, electromagnetic and weak interactions... This is of course the Yang-Mills theory… Essentially, all that we managed to do is just to generalize quantum electrodynamics (QED). QED was invented around 1929 *and since then has never changed... Now QED is generalized and includes strong and weak interactions along with electromagnetic, quarks and neutrinos, along with electrons*" (Gell-Mann, 1985).

As we know, these theories cover all types of elementary particles: massless photons and massive leptons, bosons and hadrons.

Therefore, it can be argued that the mass of elementary particles and hence of the whole matter has electromagnetic origin. This answers the Feynman question in the above passage.

From this follows that gravitational mass/energy and gravitational field also have an electromagnetic origin. Obviously, then the theory of gravity should be some variant of the nonlinear theory of the electromagnetic field. We will present an attempt to build such a theory in the following chapters.

Note first, that we won't be to derive the equations from a Lagrangian (i.e., from least-action) formulation. A full exposition of these ideas would add too much extra length to the book. Second, everything we will do is classical. To get to the standard model or the other quantum field theories, we need to quantize the theory.

Chapter 3. The axiomatics of LIGT and its consequences

1.0. Lemma of electromagnetism

In the previous chapter of LITG, we presented evidence of the electromagnetic origin of inertial mass. Feynman noted (see above), that this statement does not contradict the experimental data.

On this basis, we state here the following lemma, which will serve as a foundation for building LIGT (let us call it conditionally "*Lemma of electromagnetism*").

Lemma of electromagnetism: ***The electromagnetic field is the basis for the origin of matter.***

From here follow a number of conclusions that are important for the theory of gravitation.

1) The equivalence of gravitational and inertial masses leads to the conclusion that gravity has an electromagnetic origin.

This conclusion is of fundamental importance for the construction of the Lorentz-invariant theory of gravitation.

2) The Lorentz-invariance of the laws of electromagnetism, determines Lorentz-invariance of the laws of gravity.

3) Elementary particles are the primary carriers of matter and its characteristics. Hence, the equation of gravitation should follow from the equations of elementary particles.

4) Matter is involved in the creation of the gravitational field as its source, without quantization of this source. Thus, the gravitational field can be regarded as a classical field, which does not require quantization. The assumed origin of this equation from quantum equations of elementary particles, is not a limitation here, because a transition exists from quantum to classical equations.

5) In the elementary particles' theory, inertial mass is associated with energy and momentum of particle by the equation:

$$\varepsilon^2 - c^2 p^2 = m_0^2 c^4 ,$$

where m_0 is the rest mass (invariant quantity). From this follows, what in general is the equivalence of mass and energy-momentum

$$m_0 = \frac{1}{c^2}\sqrt{\varepsilon^2 - c^2 p^2} ,$$

According to the above mentioned cause we can consider mass, energy and momentum as the gravitation sources.

6) Since in general case, the original equations of microcosm are nonlinear, we should assume that the gravitational equations are non-linear.

Based on formulated above Lemma of electromagnetism, we can choose the following axioms for LIGT, which do not contradict to the experimental data.

2.0. Axiomatics of LIGT

As the first and second postulates we will take the experimental facts:

1. Postulate of source: ***the source of the gravitational field is matter in the form of an island matter or a field mass/energy.***

2. Postulate of the masses' equivalence: ***the gravitational charge (mass) is proportional to the inertial mass/energy.***

3. Postulate of Mossotti -Lorentz: Postulate of Mossotti-Lorentz: the gravitational field is a residual electromagnetic field, which is remained as a result of incomplete compensation of electric and magnetic fields of different polarity.

(*Note: we do not associate this axiom with the Mossotti model which explains how this residue is formed, but have in mind the general idea that the gravitational field is a small part of the electromagnetic field, which acts attractively*).

4. The locality postulate: ***gravitational field is locally Lorentz-invariant, that is Lorentz-invariant on any infinitely small time interval and on any infinitely small distance.***

(*Note: since the EM field is itself Lorentz- invariant, this axiom can be seen as a consequence of the axiom of Mossotti-Lorentz. But classical mechanics is globally Lorentz- invariant. With* *the introduction of postulate 4 we actually emphasize that gravitation, in the general case, is not globally Lorentz-invariant*).

From these axioms the next consequences follow, proof of which may serve as a confirmation of the axioms.

Corollary 1: since the gravitational field is residual, it is much weaker than the electromagnetic field, but in the case of a neutral matter (in the electromagnetic sense), the gravitational field is decisive.

Corollary 2: the gravitational constant is determined as a portion of full electromagnetic interaction.

Corollary 3: as in the theory of electromagnetism the interaction is described by the Lorentz force, the same (or its modification) describes the theory of gravitation.

Corollary 4: the equations of massive elementary particles can be regarded as the source equations of the gravitational field.

Corollary 5: all the features of motion of matter in the gravitational field come from the electromagnetic theory, in particular, from the effects associated with the Lorentz transformations.

Corollary 6: all the characteristics of gravitational field (its energy, momentum, angular momentum, etc) have an electromagnetic origin and obey the laws of electromagnetism.

Chapter 4. The connection of electromagnetic theory and gravitation

Here, we will show that the adopted by us the Mossotti-Lorentz postulate does not contradict the existing results of physics, including general relativity.

1.0. Transition from EM field theory to gravitational field theory

In the works of Lorentz (see, e.g., (Lorentz, 1900)) it was shown in sufficient detail that in the theory of electromagnetic fields the residual electromagnetic field can actually be described. But for the specific purpose of its introduction it is easier and more convenient to use the methods of similarity theory and dimensional analysis (Sedov, 1993).

We will compare the expressions of EM theory with the parallel expressions of gravitational theory and select the correspondences between them. For the control of the conclusions we use dimensional analysis.

The main characteristic of the source field in the one and in the other theory is the expression of the interaction force or the corresponding interaction energy between the two bodies.

1.1. Gravity electrostatic (ge-) field. The transition from the Coulomb's field to the Newton's field

If we assume that gravity is generated by electric field, but quantitatively, by very small part of it (see Appendix A1), then Newton's gravitation law:

$$\vec{F}_N = \gamma_N \frac{m \cdot M}{r^2} \vec{r}^{\,0}, \tag{1.1}$$

should take the form of Coulomb's law:

$$\vec{F}_C = k_0 \frac{q \cdot Q}{r^2} \vec{r}^{\,0}, \tag{1.2}$$

where m and q are the mass and electric charge of the particle, M and Q are the mass and electric charge of the source, γ_N is Newton's gravitational constant, and the coefficient k_0 in

Gauss's units is $k_0 = 1$. In this case, the definitions of gravitational field strengths of Newton and Coulomb electric field have the form $\vec{E}_N = \frac{\vec{F}_N}{m} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0}$ and $\vec{E} = \frac{\vec{F}_C}{q} = k_0 \frac{Q}{r^2}\vec{r}^{\,0}$, respectively.

We introduce the gravitational charge q_g, corresponding to mass m (Ivanenko and Sokolov, 1949) , by means of the relation:

$$q \to q_g = \sqrt{\gamma_N}\, m, \tag{1.3}$$

In this case, Newton's law can be rewritten in the form of Coulomb's law:

$$\vec{F}_g = \frac{q_g \cdot Q_g}{r^2}\vec{r}^{\,0} = \gamma_N \frac{m \cdot M}{r^2}\vec{r}^{\,0} = \vec{F}_N, \tag{1.4}$$

where $Q_g = \sqrt{\gamma_N} M$ is the gravitational charge of source, corresponding to the mass M of the source.

From the comparison of equations (1.2) and (1.4) it follows that the dimensions of the electromagnetic and gravitational charges coincide. At the same time, a gravitational charge (1.3) has electromagnetic origin, and, hence, the corresponding mass is the inertial mass. On the other hand, the law (1.4) comprises the gravitational masses. This implies the equivalence of inertial and gravitational masses.

We introduce the g-field strength within framework of EMGT as:

$$\vec{E} \to \frac{\vec{E}_g}{\sqrt{\gamma_N}}, \tag{1.5}$$

where the tension of the Coulomb field is equal to: $\vec{E} = \frac{Q}{r^2}\vec{r}^{\,0}$. Substituting the values of gravitation theory here, we get:

$$\vec{E}_g = \sqrt{\gamma_N}\frac{Q_g}{r^2}\vec{r}^{\,0} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} = \vec{E}_N, \tag{1.6}$$

where $\vec{E}_N$ is the strength of the Newton gravitational field.

Let us introduce the scalar gravitational potential within the framework of EMTG as:

$$\varphi \to \frac{\varphi_g}{\sqrt{\gamma_N}}, \tag{1.7}$$

where the potential of the Coulomb field is: $\varphi = \frac{Q}{r}$. Substituting the values of gravitation theory here, we get:

$$\varphi_g = \sqrt{\gamma_N}\frac{Q_g}{r} = \gamma_N \frac{M}{r} = \varphi_N, \tag{1.8}$$

where φ_N is the potential of the Newton gravitational field.

The Poisson equation for the g-field can serve as test for (1.7). Indeed, for the EM field the Poisson equation can be written as:

$$\Delta\varphi = 4\pi\,\rho_e, \tag{1.9}$$

where $\rho_e = \frac{dq}{d\tau}$ is the electric charge density, $d\tau$ is the volume element. We introduce the density of gravitational charge ρ_g similarly to the electric density:

$$\rho_e \to \rho_g = \frac{dq_g}{d\tau} = \sqrt{\gamma_N}\,\rho_m\,, \tag{1.10}$$

where $\frac{dm}{d\tau} = \rho_m$ is mass density. Then, replacing the potential and the charge density in (1.9) according to (1.7) and (1.10), we obtain the Poisson equation for the gravitational potential:

$$\Delta\varphi_g = 4\pi\sqrt{\gamma_N}\,\rho_g = 4\pi\,\gamma_N\,\rho_m\,, \tag{1.11}$$

which corresponds to the Poisson equation for the Newton gravitational field.

1.2. Gravi-magnetic field (gm-field)

In this case, by analogy with electrodynamics, the existence of the variable ge-field and associated with them alternating or direct gm-fields is assumed. The existence of a similar field is confirmed by general relativity and experiments. Unfortunately, since the Newton theory does not contain an analog of magnetic field, the verification of existence of the g-magnetic field within framework of EMGT, can presently be done only by dimensional analysis. Serious confirmation should be obtained by the solution of the corresponding equations of gravitation, which will give equivalent results to the general theory of relativity.

As is known, the magnetic field is generated by the motion of electric charges or movement of an electric field. In this case, we need to obtain an expression for the magnetic field, similar to Coulomb's law for the electric field. This is the Biot-Savart–Laplace law.

For simplicity, we will consider the special case of uniform motion of a source charge Q, which create a current I (current from motion of charge q will be denoted by i). In real tasks, of course, charges and masses are divided into point (differential) values, and field calculated by integrating over a set of point charges.

Magnetic vector $\vec{H}$ that occurs when the charge Q moves at a speed $\vec{\upsilon}$ in circuit element dl, will be:

$$\vec{H} = \frac{Q[\vec{\upsilon}\times\vec{r}]}{|r|^3} = \frac{I\left[d\vec{l}\times\vec{r}\right]}{|r|^3}\,, \tag{1.12}$$

Using the gravitational charge density ρ_g according to (1.10), similarly to the electric current $i = dq/dt$ and the current density $\vec{j}$, we will define respective g-current (or current of mass) as:

$$i \to i_g = \frac{dq_g}{dt} = \rho_g\upsilon_n dS = \sqrt{\gamma_N}\,\rho_m\upsilon_n dS\,, \tag{1.13}$$

and the density of g-current of mass, as:

$$\vec{j} \to \vec{j}_g = \left(i_g/dS\right)\vec{n} = \sqrt{\gamma_N}\,\rho_m\upsilon_n\,, \tag{1.14}$$

where $\vec{\upsilon}$ is the velocity of the charge in a conductor with a cross section dS, and υ_n the projection of the velocity on the normal to dS.

If the e-charge q moves close to the e-current (or permanent magnet field $\vec{H}$), this current (or field $\vec{H}$) acts on the charge via the magnetic part of the Lorentz force F_{Lm}:

$$\vec{F}_{Lm} = q\left[\vec{\upsilon}\times\vec{H}\right] = \frac{q\cdot Q}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \frac{i\cdot I}{|r|^3}\left[d\vec{l}\times\left(d\vec{l}'\times\vec{r}\right)\right], \tag{1.15}$$

Let us introduce the strength of gm-field within framework of EMGT as:

$$\vec{H} \to \frac{\vec{H}_g}{\sqrt{\gamma_N}}\,, \tag{1.16}$$

where the magnetic field $\vec{H}$ is given by (1.12).

Substituting the corresponding physical quantities according to (1.13) and (1.16) in (1.12), we will obtain the gravi-magnetic (gm-) vector that arises when the charge $Q_g = M\sqrt{\gamma}$ moves at a speed $\vec{\upsilon}$ in an element of a circuit dl:

$$\vec{H}_g = \sqrt{\gamma_N}\,\frac{Q_g\left[\vec{\upsilon}\times\vec{r}\right]}{|r|^3} = \gamma_N\,\frac{M\left[d\vec{l}\times\vec{r}\right]}{|r|^3}, \tag{1.17}$$

or

$$\vec{H}_g = \sqrt{\gamma_N}\,\frac{I_g\left[d\vec{l}\times\vec{r}\right]}{|r|^3} = \gamma_N\,\frac{\left[d\vec{l}\times\vec{r}\right]}{|r|^3}\rho_m\upsilon_n dS\ , \tag{1.18}$$

where $\vec{r}$ is the distance between the test particle and the moving charged source or element of current I_g, which generate the gm-vector.

Using (1.15), for the gravito-magnetic Lorentz force we obtain:

$$\vec{F}_{Lgm} = \frac{q_g Q_g}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \gamma_N\,\frac{mM}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right], \tag{1.19}$$

Since the magnetic field $\vec{H}$ in the electrodynamics can be expressed via a vector potential $\vec{A}$ by the expression:

$$\vec{H} = rot\vec{A}, \tag{1.20}$$

it is useful to define the transition from the EM vector potential $\vec{A}$ to the gravitational $\vec{A}_g$. We assume that:

$$\vec{A} \to \frac{\vec{A}_g}{\sqrt{\gamma_N}}, \tag{1.21}$$

Then, using (1.16), we can rewrite (1.20) in the form:

$$\vec{H}_g = rot\vec{A}_g, \tag{1.22}$$

Expression (1.21) also satisfies the full EM expression for the electric strength vector:

$$\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \tag{1.23}$$

Using (1.5) , (1.7) and (1.21) , we obtain for g-field:

$$\vec{E}_g = -grad\varphi_g - \frac{1}{c}\frac{\partial\vec{A}_g}{\partial t}, \tag{1.23'}$$

Thus, we have shown that the basic EM quantities and equations can be associated with similar quantities and equations for the g-field.

As an illustration of the correctness of the relationship of electromagnetic and gravitoelectromagnetic quantities, we put in Appendix A2 to this chapter the table of dimensions of physical quantities, considered above.

2.0. GTR, EMG and EMGT

A lot of the solutions of general relativity are obtained in linear approximation, using the method of perturbation. It was found that the results of this linear theory may be presented in the form of Maxwell's equations. Such a representation has been called gravitoelectromagnetism, or, briefly, GEM.

2.1. Gravito-electromagnetism (EMG)

In general relativity (GR) (Overduin, 2008), “space and time are inextricably bound together. In special cases, however, it becomes feasible to perform a "3+1 split" and decompose the metric of four-dimensional spacetime into a scalar time-time component, a vector time-space component and a tensor space-space component.

When gravitational fields are weak and velocities are low compared to *c*, then this decomposition takes on a particularly compelling physical interpretation: if we call the scalar component a "gravito-electric **(ge-)** potential" and the vector one a "gravito-magnetic **(gm-)** potential", then these quantities are found to obey almost exactly the same laws as their counterparts in ordinary electromagnetism.

In other words, one can construct a "gravito-electric field" $\vec{E}_{ge}$ and a "gravito-magnetic field $\vec{H}_{gm}$, and these fields are obeyed equations that are identical to Maxwell's equations and the Lorentz force law of ordinary electrodynamics.

From symmetry considerations we can infer that the earth's **gravito-electric field** must be radial, and its **gravito-magnetic** one dipolar, as shown in the diagrams 2.1 and 2.2. below:

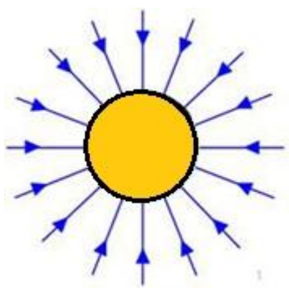


Fig.2.1. Radial gravitation field lines of Earth

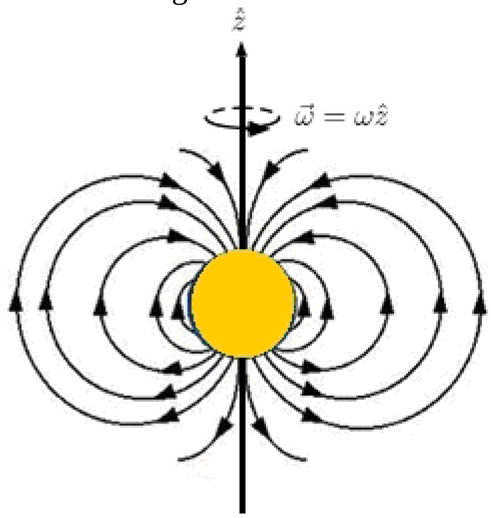


Fig. 2.2. Dipole gravitation field lines of Earth

These facts allow one to derive the main predictions of general relativity, simply by replacing the electric and magnetic fields of ordinary electrodynamics $\vec{E}$ and $\vec{H}$ by $\vec{E}_{ge}$ and $\vec{H}_{gm}$ respectively”.

The mathematical aspect of GEM theory is described in many papers (see, for example, (Forward, 1961; Wald, 1984; Ruggiero and Tartaglia, 2002; Grøn and Hervik, 2007; Mashhoon, 2008; Forrester, 2010;))

To avoid misunderstanding, it should be noted that the electromagnetic theory of gravitation (EMGT) and gravitoelectromagnetism (GEM) - are not the same (Mashhoon, 2008). GEM is an auxiliary representation of GR, which allows to physically imagine of results of the metric theory.

In contrast, EMGT is an independent theory of gravitation, which arose on the basis of the hypothesis Mossotti and then was developed by number of scientists, including O. Heaviside, H.Lorentz and others (Heaviside, 1912; Lorentz, 1900; Webster, 1912; Wilson, 1921; etc).

Appendixes:

A1. Relationship between electric and gravitational charges

It is easy to show that the gravitational field is a small fraction of the electromagnetic field.

To this corresponds the fact that the gravitational charge of the electron is less than its electric charge $e >> q_g$, where $q_g = m_e\sqrt{\gamma}$ (here $e = 4{,}8 \cdot 10^{-10}$ unit. SGSEq is electron charge (1 unit CGSEq = g$^{1/2}$sm$^{3/2}$s^{-1}), $m_e = 0{,}91 \cdot 10^{-27}$ g is electron mass, $\gamma = 6{,}67 \cdot 10^{-8}$ cm^3/g sec^2 is the gravitational constant. It is easy to see that the dimension of the gravitational charge of the electron coincides with the dimension of electric charge and its magnitude in 10^{21} times less. Indeed, $e/m_e\sqrt{\gamma} \approx 2 \cdot 10^{21}$.

For a proton (the only stable heavy particle), this value is of the order $e/m_p\sqrt{\gamma} \approx 2 \cdot 10^{18}$. The heaviest known elementary particles are the highly unstable bosons W$^{\pm}$ (mass ≈80 GeV). This is about 100 times more than the mass of the proton, giving a ratio of no less than 10^{16}.

A2. Dimensions of electromagnetic and gravi-electromagnetic quantities

For the verification of the correctness of correlations in the transition from the EM physical quantities to the gravitation quantities, the accordance of their dimensions plays an important role. The worded below list confirms that electrodynamics can be considered as the basis of mechanics.

Electromagnetic theory

e-charge	$[q]$ = g$^{1/2}$ cm$^{3/2}$ s^{-1}
e-charge density	$[\rho_e]$ = g$^{1/2}$ cm$^{-3/2}$ s^{-1}
e-current	$[i]$ = g$^{1/2}$ cm$^{3/2}$ s^{-2}
e-current density	$[j]$ = g$^{1/2}$ cm$^{-1/2}$ s^{-2}
Coulomb force	$[F_C]$ = g cm s^{-2}
Strength of e-fields	$[E]$ = g$^{1/2}$ cm$^{-1/2}$ s^{-1}
Strength of m-field	$[H]$ = g$^{1/2}$ cm$^{-1/2}$ s^{-1}
Scalar potential	$[\varphi]$ = g$^{1/2}$ cm$^{1/2}$ s^{-1}
Vector potential	$[A]$ = g$^{1/2}$ cm$^{1/2}$ s^{-1}
Field energy	$[\varepsilon_e]$ = g cm^2 s^{-2} = $[q\varphi]$
Field energy density	$[\rho_\varepsilon]$ = g cm^{-1} s^{-2} = $[E^2] = [H^2]$

Gravitation theory of Newton

Newton's force	$[F_N]$ = g cm s^{-2}
Newton's gravitational constant	$[\gamma_N]$ = g^{-1} cm^3 s^{-2} ($[\sqrt{\gamma_N}]$ = g$^{-1/2}$ cm$^{3/2}$ s^{-1})
Field strength	$[E_N]$ = cm/s^2 (acceleration)
Scalar potential	$[\varphi_N]$ = cm^2/s^2 ≡ (cm/s)^2 (velocity square)
Scalar potential	$[\varepsilon_N] = [m\varphi_N]$ = g cm^2/s^2

Electromagnetic gravitation theory (EMGT)

g-charge	$[q_g]$ = g$^{1/2}$cm$^{3/2}$s^{-1} = $[q_e] = [m\sqrt{\gamma_N}]$
g-charge density	$[\rho_g]$ = g$^{1/2}$ cm$^{-3/2}$ s^{-1}
g-current	$[i_g]$ = g$^{1/2}$ cm$^{3/2}$ s^{-2}
g-current density	$[j_g]$ = g$^{1/2}$ cm$^{-1/2}$ s^{-2}

g-force	$[F_g] =$ g cm s^{-2} $= [F_N]$
Strength of ge-fields	$[E_g] =$ cm/s^2 $= [E_N] = [E\sqrt{\gamma_N}]$
Strength of gm-field	$[H_g] =$ cm/s^2 $= [H\sqrt{\gamma_N}]$
Scalar potential of g-fields	$[\varphi_g] =$ cm^2/s^2 $= [\varphi\sqrt{\gamma_N}]$
Vector potential of g-fields	$[A_g] =$ cm^2/s^2 $= [A\sqrt{\gamma_N}]$
g-field energy	$[\varepsilon_g] =$ g cm^2/s^2 $= [m\varphi_g]$

Chapter 5. Electromagnetic base of relativistic mechanics

1.0. General principles of electromagnetic theory of matter

Under the moving masses (gravitational charges) we will understand the two interacting bodies, one of which we call the source of the gravitational field, and the other - the test particle.

Our approach to the theory of gravitation is based on a modern version of the electromagnetic theory of matter (EMTM) (Lorentz, 1916; Richardson, 1914; Becker, 1933). In framework of EMTM the mass is of electromagnetic (EM) origin. Therefore in framework of our axiomatics, the main results of EM theory are equivalent to results of the theory of gravitation.

The Maxwell EM theory was the first theory, whose properties were found to depend on the speed of the charge (in this case, electric). This theory is called the Lorentz-invariant (L-invariant) or relativistic theory. At speeds of up to one-tenth of the speed of light, these parameters are hardly different from the parameters of static objects.

This suggests that the basis of mechanics is still the classical Newtonian mechanics, and relativistic mechanics is Newton's mechanics plus minor amendments thereto.

Moreover, in the framework of EMTM it is easy to show that the calculation of corrections to the non-L-invariant theory is determined by the non-L-invariant theory. The amendments are calculated on the basis of non-L-invariant laws that take into account the changes in the parameters at high speeds. The calculation procedure is equivalent to the method of calculation which is based on perturbation theory, when the zero approximation is the non-relativistic theory. Below we show this, based on the known results presented in textbooks.

2.0. The Maxwell-Lorentz equations

2.1. The Maxwell-Lorentz equations written in terms of field strengths

The general equations of the electromagnetic theory of matter (EMTM) are formulated on the basis of Maxwell's equations, taking into account the Lorentz hypothesis. Under this hypothesis, all elementary particles (and, consequently, atoms, molecules and bodies) are composed of an electromagnetic field, which is in a concentrated ("condensed" according to Einstein) state. Since among these particles are the free EM fields (photons), they are also included in this list. At the same time, charges and currents are also determined by the electromagnetic fields. Consequently, there is only one kind of vectors, describing the field, namely the electromagnetic (EM) field strengths in vacuo $\vec{E}$ and $\vec{H}$ or equivalent quantities.

The self-consistent Maxwell-Lorentz microscopic equations are the independent fundamental field equations. The Maxwell-Lorentz equations are following four differential (or, equivalent, integral) equations for any electromagnetic medium (Jackson, 1965; Tonnelat, 1966):

$$rot\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}\,, \qquad (2.1)$$

$$rot\vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0\,, \tag{2.2}$$

$$di\upsilon\vec{E} = 4\pi\rho\,, \tag{2.3}$$

$$di\upsilon\vec{B} = 0\,, \tag{2.4}$$

where $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ are electric field vector, magnetic field vector, electric induction vector, magnetic induction vector, correspondingly; in vacuum $\vec{D} = \vec{E}$ and $\vec{B} = \vec{H}$; $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, where $\varphi, \vec{A}$ are scalar and vector potentials, correspondingly; ρ is the charge density; $\vec{j}$ is the current density: c is the speed of light.

The difference between these equations and Maxwell's equations is that $\vec{E}$ and $\vec{H}$, as well as all other quantities needed to describe a matter, refer to an arbitrarily small volume of space. In this case the equations (2.1-2.4) are called the Maxwell-Lorentz (ML) equations.

The Maxwell's macroscopic quantities $\bar{\vec{E}}$ and $\bar{\vec{H}}$ can be deduced from the microscopic quantities $\vec{E}$ and $\vec{H}$ only by averaging over space and time. This averaging and deduction of the actual Maxwell equations from (2.1-2.4) is considered in many courses on electromagnetism (Becker, 1933).

In the equations (2.1-2.4) nothing is said about how the velocity $\vec{\upsilon}$ of the charges changes over time. For this purpose the Lorentz law is used. According to Lorentz the density of force has the form:

$$\vec{f} = \rho\left(\vec{E} + \frac{1}{c}\vec{\upsilon}\times\vec{B}\right), \tag{2.5}$$

hence $fd\tau$ is a force, acting on the volume $d\tau$.

2.2. The Maxwell-Lorentz equations written in terms of field potentials

For the analysis of the field equations (2.1-2.4), it is advisable to go from fields themselves to the electromagnetic field potentials (Becker, 1933). This is done as follows: first of all, we satisfy the equation $di\upsilon\vec{H} = 0$ (2.4) by substituting:

$$\vec{H} = rot\vec{A}, \tag{2.6}$$

where vector $\vec{A}$ is named the vector potential.

Then from the equation of (2.2) it follows that $\left(\vec{E} + \partial\vec{A}/c\,\partial t\right)$ should be zero. Therefore, we demand that the value $rot\left(\vec{E} + \partial\vec{A}/c\,\partial t\right)$ is equal to the gradient of a scalar φ:

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} - grad\varphi\,, \tag{2.7}$$

where vector φ is named the scalar potential.

The vector field is uniquely determined by divergence and vorticity of this field. Until now, we determined only $rot\vec{A}$. Now we can in addition freely dispose by the divergence of the vector $\vec{A}$. We will use this in order to put

$$di\upsilon\vec{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0\,, \tag{2.8}$$

If we now substitute (2.6) and (2.7) in the two remaining equations (2.1-2.4), then with the help of (2.8), we obtain two equations for the potentials:

$$\frac{\partial^2\vec{A}}{\partial x^2}+\frac{\partial^2\vec{A}}{\partial y^2}+\frac{\partial^2\vec{A}}{\partial z^2}-\frac{\partial^2\vec{A}}{c^2\partial t^2}=-\frac{4\pi}{c}\rho\vec{\upsilon},$$
$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-\frac{\partial^2\varphi}{c^2\partial t^2}=-4\pi\rho, \qquad , \qquad (2.9)$$

2.3. The Lorentz transformation as transition from rest to motion

For their integration we use the well-known fact that the field at time t at in any point is equal to the field at time $(t-dt)$ at the point, shifted back to the segment $\vec{\upsilon}dt$. This means that for all the quantities, characterizing the field, we will again have the relation:

$$\frac{\partial\chi}{\partial t}=-(\vec{\upsilon}\cdot grad)\chi$$

where $\chi=\chi(x,y,z)$ is a function of the field.

For example, for change in time of the electric vector $\vec{E}$ we will obtain:

$$\frac{\partial\vec{E}}{\partial t}=-(\vec{\upsilon}\cdot grad)\vec{e}$$

Thus, if the velocity is parallel to the positive x-axis, in our equations for the potentials (2.9), second time derivatives are replaced by derivatives with respect to the coordinate x according to the formula

$$\frac{\partial^2}{\partial t^2}=\upsilon^2\frac{\partial^2}{\partial x^2}$$

Therefore, for the potentials $\vec{A}$ and φ we get the equation:

$$\left(1-\frac{\upsilon^2}{c^2}\right)\frac{\partial^2\vec{A}}{\partial x^2}+\frac{\partial^2\vec{A}}{\partial y^2}+\frac{\partial^2\vec{A}}{\partial z^2}=-\frac{4\pi}{c}\rho\vec{\upsilon},$$
$$\left(1-\frac{\upsilon^2}{c^2}\right)\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}=-4\pi\rho, \qquad , \qquad (2.10)$$

Note that the equations for the components of the vector potential differ from the equation for the scalar potential only by constant factor $\vec{\upsilon}/c$. Therefore, if we resolve the equation for φ, then a solution for the vector potential follows directly from it:

$$\vec{A}=\frac{\vec{\upsilon}}{c}\varphi, \qquad (2.11)$$

From this we obtain two equations:

$$rot\vec{A}=\frac{1}{c}rot(\vec{\upsilon}\varphi)=-\frac{1}{c}\vec{\upsilon}\times grad\varphi$$
$$\frac{\partial\vec{A}}{\partial t}=\frac{\vec{\upsilon}}{c}\frac{\partial\varphi}{\partial t}=-\frac{1}{c}\vec{\upsilon}(\vec{\upsilon}grad\varphi)$$

If we introduce them to the definitions (2.6) and (2.7), we get:

$$\vec{E}=-grad\varphi+\frac{1}{c}\vec{\upsilon}(\upsilon grad\varphi) \quad \text{and} \quad \vec{H}=\frac{1}{c}\vec{\upsilon}\times\vec{E}, \qquad (2.12)$$

It turns out that the relation (2.11) between the vector and scalar potentials leads to known dependence $\vec{H}=\frac{1}{2}\left[\vec{\upsilon}\times\vec{E}\right]$ between $\vec{H}$ and $\vec{E}$.

Therefore, to solve our problem, we can confine ourselves to integrating the equation (2.10) for φ. Note that this equation differs from the equation for the ordinary electrostatic potential only by constant coefficients $\left(1-\upsilon^2/c^2\right)$ at $\partial^2\varphi/\partial x^2$. So technically we can reduce our problem to a simple electrostatic problem, if instead of coordinates x, y, z, t we introduce the new coordinates x', y', z', t' using the transformation:

$$x = x'\sqrt{1-\beta^2}\ ,\ y = y',\ z = z',\ t = t', \tag{2.13}$$

where for brevity we put $\upsilon/c = \beta$. Due to this change, the functions $\varphi(x, y, z, t)$ and $\rho(x, y, z, t)$ pass to functions φ' and ρ' from x', y', z', t', so that we have the identities:

$$\begin{aligned} \rho'(x', y', z', t') &\equiv \rho\left(x'\sqrt{1-\beta^2}, y', z', t'\right), \\ \varphi'(x', y', z', t') &\equiv \varphi\left(x'\sqrt{1-\beta^2}, y', z', t'\right), \end{aligned} \tag{2.14}$$

Therefore, our equation for the potential in the primed coordinates is

$$\frac{\partial^2\varphi'}{\partial x'^2} + \frac{\partial^2\varphi'}{\partial y'^2} + \frac{\partial^2\varphi'}{\partial z'^2} = -4\pi\rho\ , \tag{2.15}$$

As such, this equation is completely identical to the equation that determines the potential of the *fixed charge system*. Therefore, its integration can produced according to the well-known theory of the electrostatic potential. We get:

$$\varphi'(x', y', z', t') = \iiint \frac{\rho'(\xi', \eta', \varsigma', t')d\xi' d\eta' d\varsigma'}{\sqrt{(x'-\xi')^2 + (y'-\eta')^2 + (z'-\xi')^2}}$$

If we again turn to the unprimed coordinates with the help of (2.13) and (2.9), we will obtain the solution of equation (2.10) for the scalar potential in the form

$$\varphi(x, y, z, t) = \iiint \frac{\rho(\xi, \eta, \varsigma, t)d\xi d\eta d\varsigma}{\sqrt{(x-\xi)^2 + (1-\beta^2)(y-\eta)^2 + (z-\xi)^2}}\ , \tag{2.16}$$

Now let us find a particular solution for the time $t = t_0$ when the electron is in the beginning of the coordinate system, and restrict ourselves to the case of the point electron, i.e., assume that the charge density is different from zero only in the immediate vicinity of the origin of coordinates $(\xi = \eta = \varsigma = 0)$. Then the integration can be done, and we get the solution:

$$\varphi(x, y, z, t_0) = \frac{\vec{E}}{\sqrt{x^2 + (1-\beta^2)(y^2 + z^2)}}\ , \tag{2.17}$$

For the purposes of brevity, we introduce for the expression that appears in the denominator instead of the distance r, the designation:

$$s = \sqrt{x^2 + (1-\beta^2)(y^2 + z^2)}, \tag{2.18}$$

Then we will be able to present the solution to our problem in the form

$$\varphi = \frac{e}{s}, A_x = \frac{e\upsilon}{sc}, A_y = A_z = 0\ , \tag{2.19}$$

Using these potentials we can calculate the field $\vec{E}$ and $\vec{H}$ by the formulas (2.6), (2.7) or (2.12), and taking into account that differentiation by time is always replaced by $-\upsilon\frac{\partial}{\partial x}$, for the electrical strength in vector form we will get:

$$\vec{E} = \left(1-\beta^2\right)\frac{e}{s^3}\vec{r}\ , \tag{2.20}$$

Further, the magnetic strength $\vec{H} = \frac{1}{c}\vec{\upsilon}\times\vec{E}$; hence:

$$H_x = 0, H_y = -\frac{e\upsilon}{c}\frac{1-\beta^2}{s^3}z, H_z = \frac{e\upsilon}{c}\frac{1-\beta^2}{s^3}y, \qquad (2.21)$$

2.3.1 *Lorentz transformations and their consequences*

Our aim (Lorentz, 1904; Lorentz, 1916; Poincaré, 1905) must again be to reduce the equations for a moving system to the form of the ordinary formulae that hold for a system at rest. It is found that the transformations needed for this purpose may be left indeterminate to a certain extent; our formulae will contain a numerical coefficient l, of which we shall provisionally assume only that it is a function of the velocity of translation υ, whose value is equal to unity for $\upsilon = 0$, and differs from 1 by an amount of the order of magnitude υ^2/c^2 for small values of the ratio υ/c.

If *x, y, z* are the coordinates of a point with respect to axes fixed in the vacuum, or, as we shall say, the "absolute" coordinates, and if the translation takes place in the direction of *OX*, the coordinates with respect to axes moving with the system, and coinciding with the fixed axes at the instant $t = 0$, will be

$$x_r = x - \upsilon t, \quad y_r = y, \quad z_r = z, \qquad (2.22)$$

Now, instead of x_r, y_r, z_r we shall introduce new independent variables differing from these "relative" coordinates by certain factors that are constant throughout the system. Putting

$$\frac{c^2}{c^2 - \upsilon^2} = \frac{1}{1-\beta^2} = \gamma_L^2, \qquad (2.23)$$

we define the new variables by the equations

$$x' = \gamma_L l x_r, \quad y' = l y_r, \quad z' = l z_r \qquad (2.24)$$

or

$$x' = \gamma_L l (x - \upsilon t)_r, \quad y' = l y_r, \quad z' = l z_r, \qquad (2.25)$$

and to these we introduce as our fourth independent variable

$$t' = \frac{l}{\gamma_L} t - \gamma_L l \frac{\upsilon}{c^2}(x - \upsilon t) = \gamma_L l \frac{\upsilon}{c^2}\left(t - \frac{\upsilon}{c^2}x\right), \qquad (2.26)$$

It was Poincaré (Poincaré, 1905) who first introduced that the real meaning of the substitution (2.25), (2.26) lies in the relation

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = l^2\left(x^2 + y^2 + z^2 - c^2 t^2\right), \qquad (2.27)$$

that can easily be verified, and from which we may infer that we shall have

$$x'^2 + y'^2 + z'^2 = c^2 t'^2, \qquad (2.28)$$

when

$$x^2 + y^2 + z^2 = c^2 t^2, \qquad (2.29)$$

This may be interpreted as follows. Let a disturbance, which is produced at the time $t = 0$ at the point $x = 0$, $y = 0$, $z = 0$ be propagated in all directions with the speed of light *c*, so that at the time t it reaches the spherical surface determined by(2.29). Then, in the system *x', y', z', t',* this same disturbance may be said to start from the point $x' = 0$, $y' = 0$, $z' = 0$, at the time $t' = 0$ and to reach the spherical surface (2.28) at the time t'. Since the radius of this sphere is *ct'*, the disturbance is propagated in the system *x', y', z', t'* as it was in the system *x, y, z, t*, with the speed c. Hence, the velocity of light is not altered by the transformation.

3.0. The static fields of Coulomb and Newton as fundamental fields with respect to fields of moving sources.

The moving source of the gravitational field is a gravitational current, i.e., the movement of gravitational charge (mass). As we have seen (see above), the transition from the fixed charge and their fields to the mobile charge and fields, and vice versa, is described by Lorentz transformations.

In the theory of gravity the transition from the non-L-invariant theory to the L-invariant theory requires, first and foremost, to find the L-invariant expression for the static Newton's law of gravity $\vec{F}_N = \gamma_N mM\vec{r}^{\,0}/r^2$, or $di\upsilon\vec{G} = 4\pi\gamma_N\rho_m$, where $\vec{G} \equiv \vec{F}/m$ (or by introducing potential φ_N through $\vec{F}_N = grad\varphi$, in the form $\vec{\nabla}^2\varphi_N = 4\pi\gamma_N\rho_m$).

According to LIGT the Newton law of gravity is a consequence of the law of the static interaction between charges of Coulomb (or of more general assertion - of Gauss theorem). This gives us an opportunity to consider the gravity problem on the basis of the electromagnetic problem that has been solved.

At first glance, here lies the contradiction. Static (non-L-invariant) Coulomb's law $\vec{F}_C = \gamma_e qQ\vec{r}^{\,0}/r^2$ (where in the CGS $\gamma_e = 1$) in the form $di\upsilon\vec{E} = 4\pi\rho_e$ or $\vec{\nabla}^2\varphi_e = 4\pi\rho_e$, is included as part in the M-L equation, which, in its totality, is of course, L-invariant.

The exit from this contradiction is somewhat unexpected. We will show below that in the transition from source шт a stationary reference frame to the same source in moving frame, new additional fields are generated, which together with the same static field, meet the requirements of the L-invariance.

Farther we assume that all the statements that we can make with respect to EM theory, are valid for the theory of gravity, taking into account the established terminology (for example, the charge in EM theory is called mass in the theory of gravity, etc).

(Farther to confirm our ideas, we will use the quotes from the book of E. Purcell (Purcell, 1985)).

3.1. Gauss's law

The flux of the electric field $\vec{E}$ through any closed surface, that is, the integral $\int \vec{E}\cdot d\vec{s}$ over the surface, equals 4π times the total charge enclosed by the surface:

$$\int \vec{E}\cdot d\vec{s} = 4\pi\sum_i q_i = 4\pi\int \rho d\tau, \qquad (3.1)$$

We call the statenlent in the box a law because it is equivalent to Coulomb's law and it could serve equally well as the basic law of electrostatic interactions, after charge and field have been defined. Gauss's law and Coulomb's law are not two independent physical laws, but the same law expressed in different ways.

This suggests that Gauss's law, rather than Coulomb's law, offers the natural way to define quantity of charge for a moving charged particle, or for a collection of moving charges.

It would be embarrassing if the value of $Q = \frac{1}{4\pi}\int\limits_{S(t)} \vec{E}\cdot d\vec{s}$ so determined depended on the size and shape of the surface S. For a stationary charge it doesn't-that is Gauss's law.

But how do we know that Gauss's law holds when charges are moving? We can take that as an experimental fact.

3.2. Invariance of charge

There is conclusive experimental evidence that the total charge in a system is not changed by the motion of the charge carriers.

This invariance of charge lends a special significance to the fact of charge quantization. It is known the fact that every elementary charged particle has a charge equal in magnitude to that of every other such particle. And this precise equality holds not only for two particles at rest with respect to one another, but for any state of relative motion.

3.3 Electric field measured in different frames of reference

If charge is to be invariant under a Lorentz transformation, the electric field $\vec{E}$ has to transform in a particular way. "Transforming $\vec{E}$" means answering a question like this: if an observer in a certain inertial frame F measures an electric field $\vec{E}$ as X volts/cm, at a given point in space and time, what field will be measured at the same space-time point by an observer in a different inertial frame F'? For a certain class of fields, we can answer this question by applying Gauss's law to some simple systems.

Gauss's law tells us that the magnitude of E' must be

$$E' = \frac{E}{\sqrt{1-\beta^2}} = \gamma_L E$$

But this conclusion holds only for fields that arise from charges stationary in F. As we shall see below, if charges in F are moving, the prediction of the electric field in F' involves knowledge of two fields in F, the electric and the magnetic.

3.4 Force on a moving charge

At some place and time in the lab frame we observe a particle carrying charge q which is moving, at that instant, with velocity $\vec{\upsilon}$ through the electrostatic field. What force appears to act on q?

Force means rate of change of momentum, so we are really asking, What is the rate of change of momentum of the particle, $d\vec{p}/dt$, at this place and time, as measured in our lab frame of reference? That is all we mean by the force on a moving particle.

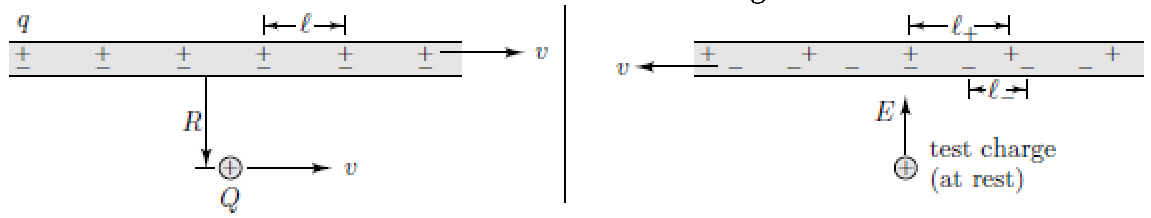
3.5 Interaction between a moving charge and other moving charges

We know that there can be a velocity-dependent force on a moving charge. That force is associated with a magnetic field, the sources of which are electric currents, that is, other charges in motion.

3.5.1 *Magnetism as a consequence of Lorentz's length contraction*

Model a current-carrying wire (Schroeder, 1999) as a line of negative charges ($-q$) at rest and a line of positive charges ($+q$) moving to the right at speed $\vec{\upsilon} = \upsilon\vec{x}^0$, where $\vec{x}^0$ is unit vector of x-axis,. The average linear separation between charges is l. Consider a "test charge" Q moving parallel to the wire, at the same speed υ (for simplicity). In the frame of the test charge it is at rest and so are the (+)-charges in the wire, but the − charges are moving to the left. According to relativity, the distance between the (−)-charges is length-contracted to $l_- = l\sqrt{1-(\upsilon/c)^2}$, while the distance between the (+)-charges is un-length-contracted to $l_+ = l\sqrt{1-(\upsilon/c)^2}$. Therefore the wire carries a net negative charge and exerts an attractive electrostatic force on the test charge. Back in the lab frame, we call this a magnetic force.

Lab Frame: Test Charge Frame:



test charge (at rest)

To calculate the strength of the force, first we find the linear charge density of the wire in the test charge frame (assuming $\upsilon << c$ for simplicity):

$$\lambda = \frac{q}{l_+} - \frac{q}{l_-} = \frac{q}{l}\left(\sqrt{1-(\upsilon/c)^2} - \frac{1}{\sqrt{1-(\upsilon/c)^2}}\right) \approx \frac{q}{l}\left[1 - \frac{1}{2}\left(\frac{\upsilon}{c}\right)^2 - 1 - \frac{1}{2}\left(\frac{\upsilon}{c}\right)^2\right] = -\frac{q}{l}\left(\frac{\upsilon}{c}\right)^2, \quad (3.2)$$

In a typical household wire $\upsilon/c \sim 10^{-13}$, so the Lorentz factor differs from 1 by only about one part in 10^{26}. This tiny amount of length contraction is still observable, because the total charge of all the moving electrons is enough to exert enormous electrostatic forces.

The same derivation can be adapted to more complicated cases where the test charge has an arbitrary velocity, in either direction. To understand the case where the test charge is moving toward or away from the wire, you need to digress to show how the electric field of a point charge in motion is weaker in front of and behind the charge but stronger in the transverse directions. (This can be derived using length contraction and some simple gedanken experiments.)

From our present vantage point (Purcell, 1985), the magnetic interaction of electric currents can be recognized as an inevitable corollary to Coulomb's law. If the postulates of relativity are valid, if electric charge is invariant, and if Coulomb's law holds, then, as we shall now show, the effects we commonly call "magnetic" are bound to occur. They will emerge as soon as we examine the electric interaction between a moving charge and other moving charges.

Two charge distributions experience Lorentz contraction of various values - this is the solution of the problem.

A more general and detailed analysis of the problem is described, for example, in the book Let us use the results of book (Purcell, 1985) to get the mathematical expression of the arising force and magnetic field (for brevity we use the notation introduced earlier $\beta = \upsilon/c$, $\gamma_L = 1/\sqrt{1-\beta^2}$)

In general case the total linear density of charge in the wire in the test charge frame, λ, can be calculated:

$$\lambda = \lambda_+ - \lambda_- = -\frac{2\lambda\gamma_L \upsilon\upsilon_0}{c^2}, \quad (3.2')$$

(the meaning of the unknown variables in (3.2') is explained below

The wire is positively charged. The use of Gauss's law (applied to the cylinder which surrounds the line) guarantees the existence of a radial electric field E'_r given by the formula for the field of any infinite line charge:

$$E'_r = \frac{2\lambda}{r} = \frac{4\lambda\gamma_L \upsilon\upsilon_0}{rc^2}, \quad (3.3)$$

Hence, the test charge q will experience a force, which is directed inwardly radially

$$F_r' = qE_r' = \frac{2q\lambda}{r} = \frac{4q\lambda\gamma_L \upsilon\upsilon_0}{rc^2}, \quad (3.4)$$

Now let's return to the lab frame. What is the magnitude of the force on the charge q as measured there? If its value is qE_r' in the rest frame of the test charge, observers in the lab frame will report a force smaller by the factor $(1/\gamma_L)$. Since $r = r'$, the force on our moving test charge, measured in the lab frame, is:

$$F_r = \frac{F_r'}{\gamma_N} = \frac{4q\lambda\upsilon\upsilon_0}{rc^2}, \quad (3.5)$$

Now $2\lambda\upsilon_0$ is just the total current I in the wire, in the lab frame, for it is the amount of charge flowing past a given point per second. We'll call current positive if it is equivalent to positive charge flowing in the positive x direction. Our current in this example is negative. Our result can be written this way:

$$F = \frac{2q\upsilon I}{rc^2} \,, \tag{3.6}$$

We have found that in the lab frame the moving test charge experiences a force in the y direction which is proportional to the current in the wire, and to the velocity of the test charge in the x direction.

If we had to analyze every system of moving charges by transforming back and forth among various coordinate systems, our task would grow both tedious and confusing. There is a better way. The overall effect of one current on another, or of a current on a moving charge, can be described completely and concisely by introducing a new field, the magnetic field.

3.6 Introduction of the magnetic field

Thus, a charge which is moving parallel to a current of other charges experiences a force perpendicular to its own velocity. We can see it happening in the deflection of the electron beam.

Let us state it again more carefully. At some instant t a particle of charge q passes the point (x, y, z) in our frame, moving with velocity υ. At that moment the force on the particle (its rate of change of momentum) is $\vec{F}$. The electric field at that time and place is known to be $\vec{E}$. Then the magnetic field at that time and place is defined as the vector $\vec{B}$ which satisfies the vector equation

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{\upsilon} \times \vec{B} \,, \tag{3.7}$$

What kind of vector should be $\vec{B}$, in order to make the equation (3.6) compatible with the equation (3.7).

For fields that vary in time and space equation (3.7) is to be understood as a local relation among the instantaneous values of $\vec{F}$, $\vec{E}$, $\vec{\upsilon}$ and $\vec{B}$. Of course, all four of these quantities must be measured in the same inertial frame.

In the case of our "test charge" in the lab frame, the electric field $\vec{E}$ was zero. With the charge q moving in the positive x direction, $\vec{\upsilon} = \upsilon\vec{x}^0$, we found that the force on it was in the negative y direction, with magnitude $2q\upsilon I/rc^2$

$$\vec{F} = -\vec{y}^0 \frac{2q\upsilon I}{rc^2} \,, \tag{3.8}$$

In this case the magnetic field must be

$$\vec{B} = \vec{z}^0 \frac{2I}{rc} \,, \tag{3.9}$$

for then equation (3.7) becomes

$$\vec{F} = \frac{q}{c}\vec{\upsilon} \times \vec{B} = \left(\vec{x}^0 \times \vec{z}\right)\frac{q\upsilon}{c}\frac{2I}{rc} = -\vec{y}^0 \frac{2q\upsilon I}{rc^2} \,, \tag{3.10}$$

in agreement with equation (3.8).

3.7 Vector potential

We found that the scalar potential function $\varphi(x, y, z)$ gave us a simple way to calculate the electrostatic field of a charge distribution. If there is some charge distribution $\rho(x, y, z)$, the potential at any point (x_1, y_1, z_1) is given by the volume integral

$$\varphi(x_1, y_1, z_1) = \int \frac{\rho(x_2, y_2, z_2)}{r_{12}} d\upsilon_2 \,, \tag{3.11}$$

The integration is extended over the whole charge distribution, and r_{12} is the magnitude of the distance from (x_2, y_2, z_2) to (x_1, y_1, z_1). The electric field $\vec{E}$ is obtained as the negative of the gradient of cp:

$$\vec{E} = -grad\varphi, \tag{3.12}$$

The same trick won't work here, because of the essentially different character of $\vec{B}$. The curl of $\vec{B}$ is not necessarily zero, so $\vec{B}$ can't, in general, be the gradient of a scalar potential. However, we know another kind of vector derivative, the curl. It turns out that we can usefully represent $\vec{B}$, not as the gradient of a scalar function but as the curl of a vector function, like this:

$$\vec{B} = rot\vec{A}, \tag{3.13}$$

By obvious analogy, we call $\vec{A}$ the vector potential. It is not obvious, at this point, why this tactic is helpful. That will have to emerge as we proceed. It is encouraging that equation (2.4) ($di\upsilon\vec{H} = 0$) is automatically satisfied, since $di\upsilon\, rot\vec{A} = 0$, for any $\vec{A}$.

In view of equation (2.1), the relation between $\vec{J}$ and $\vec{A}$ is

$$rot\left(rot\vec{A}\right) = \frac{4\pi\vec{J}}{c}. \tag{3.14}$$

Equation (3.14) , being a vector equation, is really three equations. We shall work out one of them, say the x-component equation. Among the various functions which might satisfy our requirement (3.13), let us consider as candidates only those which also have zero divergence $di\upsilon\vec{A} = 0$. Then, after a series of transformations we get from (3.14):

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = \frac{4\pi J_x}{c}, \tag{3.15}$$

Thus, we shown that the calculation of L-invariant amendments to the non-L-invariant theory is determined by the non-L-invariant theory.

Chapter 6. The equation of motion in LIGT

1.0. Equation of massive boson

Let us use the electromagnetic representation of tht Dirac equation (see in details (Kyriakos, 2003; 2004; 2009)).

More often the Dirac equation is described in the bispinor form. Entering the function:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \tag{1.1}$$

called bispinor, the Dirac equations can be written in one equation. There are two bispinor Dirac equation forms:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) + \hat{\beta}\ m_e c^2\right]\psi = 0, \tag{1.2}$$

$$\psi^+\left[\left(\hat{\alpha}_o\hat{\varepsilon} - c\hat{\vec{\alpha}}\ \hat{\vec{p}}\right) - \hat{\beta}\ m_e c^2\right] = 0, \tag{1.3}$$

which correspond to the two signs of the relativistic expression of the energy of the electron:

$$\varepsilon = \pm\sqrt{c^2\vec{p}^2 + m^2c^4}\ , \tag{1.4}$$

Here $\hat{\varepsilon} = i\hbar\frac{\partial}{\partial t}$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$ are the operators of the energy and momentum, ε, $\vec{p}$ are the electron energy and momentum, c is the light velocity, m is the electron mass, ψ is the wave function (ψ^+ is the Hermitian-conjugate wave function) named bispinor and $\left\{\hat{\alpha}_0, \hat{\vec{\alpha}}\right\}$ are the Dirac matrices. It is also known that for each sign of the equation (2.6) there are two Hermitian-conjugate Dirac equations.

In the case when, e.g., the bispinor $\psi = \psi(y)$ has the following form:

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} = \begin{pmatrix}E_x\\ E_z\\ iH_x\\ iH_z\end{pmatrix},\ \psi^+ = \begin{pmatrix}E_x & E_z & iH_x & iH_z\end{pmatrix}, \tag{1.5}$$

using (1.5), from (1.2) and (1.3) we obtain the Maxwell equations with complex currents $j = i\frac{\omega}{c}\psi$, where $\omega = \frac{mc^2}{\hbar}$.

By squaring the Dirac equation we can obtain the equation of a massive vector particle, such as a massive intermediate boson:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\Phi = \frac{m^2c^4}{\hbar^2}\Phi, \tag{1.6}$$

where Φ is a matrix, which contains the components of the wave function of an electromagnetic field $\vec{E}$, $\vec{H}$. In general this wave is a superposition of two waves with plane polarization: $\Phi_1 = \begin{pmatrix}E_x\\ iH_z\end{pmatrix}$ and $\Phi_2 = \begin{pmatrix}E_z\\ iH_x\end{pmatrix}$.

The equation (1.6) can be rewritten in the view:

$$\left[(\hat{\alpha}_o\hat{\varepsilon})^2 - c^2\left(\hat{\vec{\alpha}}\hat{\vec{p}}\right)^2\right]\Phi = m^2c^4\Phi, \tag{1.6'}$$

or

$$\left(\hat{\varepsilon}^2 - c^2\hat{\vec{p}}^2 - m^2c^4\right)\Phi = 0, \tag{1.7}$$

From equation (1.6) follows (see below) the conservation equation:

$$\varepsilon^2 - c^2\vec{p}^2 - m^2c^4 = 0, \tag{1.8}$$

Note that this equation is valid both in quantum mechanics and in classical mechanics for all particles.

Using the Compton wave length $r_C = \hbar/m_ec$, mass term in (1.1) is $m_{ph}^2c^4/\hbar^2 = 1/4r_C^2$. In other words, the equation (1.6) can be expressed as:

$$\left(\frac{\partial^2}{\partial t^2} - c^2\vec{\nabla}^2\right)\Phi = \frac{1}{4r_C^2}\Phi, \tag{1.9}$$

This equation is similar to the equation obtained by Schrödinger as the generalization of the Dirac equation on Riemannian space .

2.0. The generally covariant equation of "massive boson"

Schroedinger (Schroedinger, 1932) was the first to obtain by squaring of Dirac equation, the generally covariant equation of “massive boson”, written for the curved space:

$$\frac{1}{\sqrt{g}}\nabla_k \sqrt{g}\, g^{kl}\nabla_l - \frac{R}{4} - \frac{1}{2} f_{kl}S^{kl} = \mu^2 , \qquad (2.1)$$

Here $\mu = \frac{m_e c}{\hbar} = \frac{1}{r_C}$, R is the invariant curvature.

In the first term is easy to find a regular operator of the Klein second order equation in the Riemann geometry. In the third term on the left is recognized well-known term associated with the spin magnetic and electric moments of the electron (tensor S^{kl}).

“*To me, the second term seems to be of considerable theoretical interest. To be sure, it is much too small by many powers of ten in order to replace, say, the term on the r.h.s. For* μ *is the reciprocal Compton length, about* $10^{11} cm^{-1}$. *Yet it appears important that in the generalised theory a term is encountered at all which is equivalent to the enigmatic mass term*.”

3.0. Quantum equations of particles’ motion in the external field

The Dirac equations of electron and positron with external field are:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp \varepsilon_{ex}\right) + c\hat{\vec{\alpha}} \cdot \left(\hat{\vec{p}} \mp \vec{p}_{ex}\right) + \hat{\beta}\ m_e c^2\right]\psi = 0 , \qquad (3.1)$$

where $\hat{\varepsilon}$ and $\hat{p}$ are the energy and momentum operators, ε_{ex} and $\vec{p}_{ex}$ are the energy and momentum of external field, accordingly.

For a complete accordance with the electromagnetic theory of matter (EMTM), the energy ε_{ex} and momentum $\vec{p}_{ex}$ in the equation (3.1) must be expressed as the EM values.

As is known, the total momentum and the total energy of a charged particle in an electromagnetic field is determined by the following expressions:

$$\vec{p}_{ful} = \vec{p} + \frac{q}{c}\vec{A}, \quad \varepsilon_{ful} = \varepsilon + q\varphi , \qquad (3.2)$$

where q is charge, $\vec{p} = \frac{m\vec{\upsilon}^2}{\sqrt{1-\vec{\upsilon}^2/c^2}}$ and $\varepsilon = \frac{mc^2}{\sqrt{1-\vec{\upsilon}^2/c^2}}$ are the momentum and energy of a free particle, $\vec{\upsilon}$ is particle velocity, $\vec{p}_{ex} = \frac{q}{c}\vec{A}_{ex}$ and $\varepsilon_{ex} = q\varphi_{ex}$ are the potential momentum and energy of some external source (charged particles), obtained in the EM field.

Hence, (3.1) can be rewritten as the Dirac equation with an external EM field

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} \mp e\varphi_{ex}\right) + c\hat{\vec{\alpha}} \cdot \left(\hat{\vec{p}} \mp \frac{q}{c}\vec{A}_{ex}\right) + \hat{\beta} m_e c^2\right]\psi = 0 , \qquad (3.3)$$

The corresponding differential equations for the "massive boson" will be:

$$\left[\left(\varepsilon + q\varphi_{ex}\right)^2 - c^2\left(\vec{p} + \frac{q}{c}\vec{A}_{ex}\right)^2 - m^2c^4\right]\Phi = 0 , \qquad (3.4)$$

From this we can obtain the equations of energy-momentum conservation of a particle in an EM field:

$$\left(\varepsilon + q\varphi_{ex}\right)^2 - c^2\left(\vec{p} + \frac{q}{c}\vec{A}_{ex}\right)^2 - m^2c^4 = 0 , \qquad (3.5)$$

From the above it follows that the values $\frac{q}{c}\vec{A}_{ex}$ and $q\varphi_{ex}$ completely characterize the external field source of EM field. Below we will find the expression for the force, with the source acts on the particle.

4.0. The transition from quantum mechanical equations of motion to the motion equations of classical mechanics

There are three main methods of transition from the quantum mechanical equations of motion to the classical equations (Schiff, 1955; Levich, Myamlin and Vdovin, 1973, Landsman, 2005; Anthony, 2014): a) theorem of Ehrenfest, b) on the basis of Hamilton's canonical equations, using Poisson brackets, c) the transition from the wave equation to the Hamilton-Jacobi equation. We shall illustrate this transition based on the methods a) and b).

4.1. Ehrenfest's theorem in the case of the Lorentz-invariant quantum theory

Let us use the Lorentz-invariant quantum wave equation of "massive photon" in external EM field (6.3), obtained in the above section:

In this case (Anthony, 2014) the wave function has the form

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left[\left(\vec{p} - \frac{q}{c}\vec{A}\right)\vec{r} - (\varepsilon + q\varphi)t\right], \tag{4.1}$$

Now we want to see whether that equation gives us a description of Reality that conforms to the classical theory. To that aim we will calculate the expectation value of the rate at which a particle's linear momentum changes with the elapse of time.

Using the relativistic formula for the probability density, we have

$$\frac{d}{dt}\langle\vec{p}\rangle = \frac{i\hbar}{2mc^2}\int\left[\psi^+\left(-i\hbar\frac{d}{dt}\vec{\nabla}\right)\frac{\partial\psi}{\partial t} - \psi\left(i\hbar\frac{d}{dt}\vec{\nabla}\right)\frac{\partial\psi^+}{\partial t}\right]d\tau, \tag{4.2}$$

In that equation the operators extract the argument of the wave function and differentiate it, so we have

$$-i\hbar\frac{d}{dt}\vec{\nabla}\frac{\partial\psi}{\partial t} = \frac{\partial\psi}{\partial t}\left[\frac{d}{dt}\vec{\nabla}\left(\vec{p}\cdot\vec{r} - \frac{q}{c}\vec{A}\cdot\vec{r}\right) - \frac{d}{dt}\vec{\nabla}(\varepsilon t + q\varphi t)\right], \tag{4.3}$$

The vector variables $\vec{r}$ and $\vec{p}$ do not represent fields, but rather represent points in phase space that the particle occupies as time elapses, so we take the spatial derivatives of those variables as equal to zero. Further, if we do not want to have the complications with radiation fields, then with respect to the source of the potential fields we must take $d\varphi/dt = 0$ and $d\vec{A}/dt = 0$.

Carrying out the differentiations thus gives us:

$$\begin{aligned}-i\hbar\frac{d}{dt}\vec{\nabla}\frac{\partial\psi}{\partial t} &= \frac{\partial\psi}{\partial t}\left[q\frac{d\vec{r}}{dt}\times\left(\vec{\nabla}\times\vec{A}\right) - q\left(\frac{d}{dt}\vec{\nabla}\right)\vec{A} - \vec{\nabla}U - q\vec{\nabla}\varphi\right] = \\ &= \frac{\partial\psi}{\partial t}\left[q\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right) - q\left(\frac{d\vec{A}}{dt} - \frac{\partial\vec{A}}{\partial t}\right) - \vec{\nabla}U - q\vec{\nabla}\varphi\right]\end{aligned}, \tag{4.4}$$

Substituting that result and its complex conjugate into Equation 18 then gives us:

$$\frac{d}{dt}\langle\vec{p}\rangle = q\left\langle\vec{\upsilon}\times\left(\vec{\nabla}\times\vec{A}\right) - \frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi\right\rangle + \left\langle -\vec{\nabla}U\right\rangle, \tag{4.5}$$

which describes the Lorentz electromagnetic force plus the force due to any other static potentials of the particle interaction. Thus we gain strong evidence that the relativistic quantum theory, like its non-relativistic version, has the classical limit.

4.2. Derivation of generally covariant classical equation of motion on the base of Ehrenfest theorem

An interesting application of the theory (see chapter 4) is to establish an analogue of Ehrenfest's theorem for the Dirac equation, generalized to the Riemann geometry (Sokolov and Ivanenko, 1952; pp. 650-651). In addition to the results obtained above, by squaring of the Dirac equation, for the center of gravity of the wave packet (provided $\hbar \to 0$), we obtain the equation of relativistic mechanics of point:

$$\frac{d}{dx^4}\left(\gamma^4 p_\alpha\right) = \Gamma^\sigma_{\alpha\rho} p_\alpha + \gamma^\rho \frac{e}{c} F_{\rho\alpha}, \tag{4.6}$$

where γ^4 is the fourth Dirac matrix, γ^ρ corresponds to the particle velocity in fraction of the speed of light c, $\Gamma^\sigma_{\alpha\rho}$ is the Christoffel brackets $\{\mu\nu,\alpha\} = \Gamma^\sigma_{\alpha\rho} = \frac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x_\nu} + \frac{\partial g_{\nu\sigma}}{\partial x_\mu} + \frac{\partial g_{\mu\nu}}{\partial x_\sigma}\right)$, $F_{\rho\alpha}$ is the electromagnetic field tensor. The first term on the right of equation is the force of gravity, and the second term is the Lorentz force.

4.3. Derivation of classical Hamilton-Jacobi equation of motion on the base of quantum wave equation

The Hamilton-Jacobi equation (HJE) in the classic mechanics is usually obtained by postulating the action in the form of:

$$S = S_{free} + S_{\text{int}} + S_{ext}, \tag{4.7}$$

where S_{free} is the action of a free particle in the absence of other particles; S_{int} is the action of the interaction between the free particle and other particles; S_{ext} is the action of other particles in the absence of the first particle.

In quantum physics HJE can be obtained, if we postulate that the action is equal to phase of the de Broglie wave (as Schrödinger did for the derivation of the Schrödinger equation (Schroedinger, 1932).

The particle wave function, in general, has the form:

$$\psi = \psi_0 \exp i\theta, \tag{4.8}$$

where θ is the phase of the wave function. In the case of a free particle the wave function has the form:

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left(\varepsilon t - \vec{p}\vec{r} + \varphi_0\right), \tag{4.9}$$

Substituting this function in the equation (4.1), we obtain the law of conservation of energy and momentum for a free particle (5.3):

$$\varepsilon^2 - c^2\vec{p}^2 = m^2c^4, \tag{1.3}$$

In the case of a particle in an external field with the energy and momentum $\varepsilon_{ex}, \vec{p}_{ex}$ the wave function has the form:

$$\psi = \psi_0 \exp\frac{i}{\hbar}\left[(\vec{p} - \vec{p}_{ex})\vec{r} - (\varepsilon + \varepsilon_{ex})t + \varphi_0\right], \tag{4.10}$$

Substituting these functions in the equation (6.3), we obtain the conservation law for a particle in an external field (6.4):

$$\left(\varepsilon - \varepsilon_{ex}\right)^2 - c^2\left(\vec{p} - \vec{p}_{ex}\right)^2 = m^2c^4, \tag{3.4}$$

According to Schrödinger in case of a free particle we take:

$$S = \theta\hbar = \varepsilon t - \vec{p}\vec{r} + \varphi_0, \tag{4.11}$$

and in case of a particle in external field:

$$S = [(\vec{p} - \vec{p}_{ex})\vec{r} - (\varepsilon + \varepsilon_{ex})t + \varphi_0] \,, \tag{4.12}$$

Hence we have in the first case for the energy and momentum $\frac{\partial S}{\partial t} = \varepsilon$, $\frac{\partial S}{\partial \vec{r}} = \vec{p}$, and in the second case $\frac{\partial S}{\partial t} = \varepsilon + \varepsilon_{ex}$, $\frac{\partial S}{\partial \vec{r}} = \vec{p} - \vec{p}_{ex}$.

Substituting partial derivatives of first type in the conservation law of energy-momentum without an external field, we obtain the relativistic HJE without an external field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 - \left(\frac{\partial S}{\partial z}\right)^2 = m^2c^2, \tag{4.13}$$

Substituting second partial derivatives of second type in the conservation law of energy-momentum with an external field, we obtain the relativistic HJE with the external field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \varepsilon_{ex}\right)^2 - \left(\frac{\partial S}{\partial x} - p_{x\,ex}\right)^2 - \left(\frac{\partial S}{\partial y} - p_{y\,ex}\right)^2 - \left(\frac{\partial S}{\partial z} - p_{z\,ex}\right)^2 = m^2c^2, \tag{4.14}$$

In the case of the electromagnetic field we have:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + q\varphi\right)^2 - \left(\frac{\partial S}{\partial x} - \frac{q}{c}A_x\right)^2 - \left(\frac{\partial S}{\partial y} - \frac{q}{c}A_y\right)^2 - \left(\frac{\partial S}{\partial z} - \frac{q}{c}A_z\right)^2 = m^2c^2, \tag{4.15}$$

According to GR, the introduction of an external field in the same manner as is the case in EM theory, does not give the desired results. The theory of gravitation requires a different method, which we will analyze below.

Chapter 7. Geometry and Physic of LIGT and GTR

1.0. Introduction

Basic result of this chapter is that the math expression of interval is mutually uniquely associated with physical equations of elementary particles and LIGT.

In addition we will show that in LIGT the metric tensor has the physical meaning of the scale factor, defined by means of the Lorentz-invariant transformations.

Also the evidences will be given of that the metric tensor in general relativity should have the same meaning as in LIGT.

1.1. Geometry and Physic in the GR

According to general relativity the gravitational field is described by the metric tensor.

The practical side of the Einstein-Hilbert theory (Tonnelat, 1965/1966) is following:

"*All the predictions of general relativity follow from*: **1)** *The solution of the field equations*:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T_{\mu\nu}\,, \tag{1.1}$$

or

$$G_{\mu\nu}\left(g_{\alpha\beta}, \partial_\rho g_{\alpha\beta}, \partial^2_{\rho\sigma} g_{\alpha\beta}\right) = \chi T_{\mu\nu}(m, \vec{u}) \to g_{\alpha\beta} \,, \tag{1.1'}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$, $\chi = \frac{8\pi G}{c^4}$, $R_{\mu\nu} = \frac{\Gamma^{\lambda}_{\mu\nu}}{\partial x^{\lambda}} - \frac{\Gamma^{\nu}_{\mu\lambda}}{\partial x^{\nu}} + \Gamma^{\lambda}_{\mu\nu}\Gamma^{\alpha}_{\lambda\alpha} - \Gamma^{\alpha}_{\mu\lambda}\Gamma^{\lambda}_{\nu\alpha}$ is the Ricci curvature tensor, $\Gamma^{\lambda}_{\mu\nu}$ are the Christoffel symbols, R is the scalar curvature, γ_N is Newton's gravitational constant, c is the speed of light in vacuum and $T_{\mu\nu}$ is the stress–energy tensor.

and $g_{\mu\nu}$ is the metric tensor of Riemannian space, and

2) The law of motion in form of geodesic equation or the Hamilton-Jacobi equation for a massive body (Landau and Lifshitz, 1951):

$$g^{ik}\left(\frac{\partial S}{\partial x^i}\right)\left(\frac{\partial S}{\partial x^k}\right) + m^2c^2 = 0, \qquad (1.2)$$

The equation (1.1) allows to determine $g_{\mu\nu}$ and to put this value in (1.2).

Since the metric tensor is contained in the square of interval of Riemannian space:

$$(ds)^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}, \qquad (1.3)$$

it is often said that the purpose of solution of equation (1.1) is to find the interval (1.3).

The basis for the introduction and use of metric tensor (MT) is the interval (often they are identical). Then the question can be reformulated in a different way: how interval and MT in this composition relates to physics?

It is often said that interval in STR is a generalization of interval of Euclidean geometry on pseudo-Euclidean geometry. In turn, the interval in general relativity is a generalization of interval of pseudo-Euclidean geometry on pseudo-Riemannian geometry. But it is easy to make sure, that the introduction of interval in STR and GTR is a postulates rather than a logical conclusion. Indeed, the intervals in STR and GTR are a generalization of interval of Euclidean geometry, but the reason for the introduction of these new intervals is not geometry, but physics: *in general relativity, it is postulated that, due to the transition to the Riemann geometry, the metric tensor* $g^{\mu\nu}$ *is a function of the gravitational field -* $g^{GR}_{\mu\nu}$.

Whether this is proved by experiment, we do not know because all the experimental confirmation of general relativity are obtained for problems in the pseudo-Euclidean metric.

Another fact also raises the doubt about the necessity of introduction of Riemann's geometry into physics. As we know, all theories of physics, except the GTR, are built in a Euclidean space, although mathematically, relativistic theories can be constructed in the pseudo-Euclidean space. But there is no such theory, which needs the introduction of the Riemann geometry.

The question is, why is there such a difference and why is the external field in GTR inserted through the metric tensor?

To answer this question, we will try to find out the physical sense of the metric tensor.

2. Geometry and Physics of LIGT

Let us consider the connection of interval with physics in the case of the *pseudo-Euclidean geometry.*

A study of the literature shows that the pseudo-Euclidean coordinates and interval of the four-dimensional space-time are introduced into physics by analogy with the interval of Euclidean geometry (Landau and Lifshitz, 1973)

"*It is frequently useful for reasons of presentation to use a fictitious four-dimensional space, on the axes of which are marked three space coordinates and the time*".

2.1. Interval and square of 4-distance differential

In the Euclidean geometry in the simplest case an interval is the distance s between two points on a straight line in space, which is calculated according to the Pythagorean theorem. Since in physics trajectories are often curved lines, the Pythagorean theorem in this case is valid only for the infinitelisemal distances. Therefore, an interval is defined here as the square root of the square of the distance differential of three-dimensional space.

In the pseudo-Euclidean geometry an interval is defined as the square root of the square of the distance differential of four-dimensional space-time in form (taking into account the summation of Einstein)

$$ds = \sqrt{dx_\mu dx_\mu}\ ,$$

where $\mu = 0,1,2,3$ $dx_0 = icdt$. The square of the interval looks like:

$$(ds)^2 = (ic\, dt)^2 + (d\vec{r})^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$

Note that currently the imaginary time coordinate is rarely used (although it is by no means a mistake and has certain advantages), and the square of the interval is written as:

$$(ds)^2 = (c\, dt)^2 - (d\vec{r})^2 = c^2(dt)^2 - (dx)^2 - (dy)^2 - (dz)^2\ , \tag{2.1}$$

$$(ds)^2 = dx_\mu dx_\mu\ , \tag{2.1'}$$

where $\mu = 1,2,3,4$, and $dx_4 = cdt$. In addition, the squares of differentials are often written without parentheses: ds^2, dx^2, instead of $(ds)^2$, $(dx)^2$, etc ..

Thus, the use of characteristics of the 3-dimensional space in the case of 4-dimensional space – time is a postulate, i.e., some chosen mathematical expression, which is necessary for the construction of special relativity by Minkowski . It also follows from the fact that in nature the length of the arc in the 4- space-time is not measurable.

Therefore the question of the physical meaning of the 4-interval arises. Let's try to answer it.

2.2. Derivation of pseudo-Euclidean interval from the physical equations

The vectors of the Lorentz-invariant (i.e., relativistic) theories necessarily depend on four coordinate: one time coordinate and three space coordinates. Does these theories contain the equations, which have a sum of terms, each of which is associated with one of the four coordinates, like as in the square of the interval?

As we know, in the first time such equations in classical electrodynamics appear, and then in quantum field theory. The wave equations of these theories include a sum of terms, each of which is associated with one of the variables *t, x, y, z*. It would be logical, to seek the cause and the meaning of the appearance of 4-interval in them, instead of introducing them artificially, as did Minkowski.

Recall that our study of motion in the gravitational field is based on an inhomogeneous wave equation of the so-called "massive boson", which in mathematical notation is similar to the Klein-Gordon equation. It is an equation for the two vectors of the electric and magnetic fields.

From (2.1) we can easily obtain:

$$(ds)^2 = c^2(dt)^2 - (d\vec{r})^2 = c^2(dt)^2\left(1 - \frac{(dr/dt)^2}{c^2}\right) = c^2(dt)^2\left(1 - \frac{\upsilon^2}{c^2}\right), \tag{2.2}$$

At the same time interval is associated with proper time $d\tau$ by relation:

$$ds = c\sqrt{1-\upsilon^2/c^2}\,dt = cd\tau\ , \tag{2.3}$$

For a free material point the concept of the 4-momentum is introduced:

$$p_\mu = mcu_\mu \quad \text{or} \quad p_\mu = (p_0, p_i), \tag{2.4}$$

where $p_0 = i\dfrac{\varepsilon}{c} = \dfrac{mc}{\sqrt{1-\upsilon_i^2/c^2}}$, $p_i = \dfrac{m\upsilon_i}{\sqrt{1-\upsilon_i^2/c^2}}$, $\varepsilon == \dfrac{mc^2}{\sqrt{1-\upsilon^2/c^2}}$; u_μ is the 4-velocity.

From this:

$$\frac{\varepsilon^2}{c^2} - p_i^2 = m^2c^2 \quad \text{or} \quad \varepsilon^2 - c^2 p_i^2 = m^2 c^4 , \qquad (2.5)$$

where the energy and momentum is rewritten for convenience as follows: $p_0 \equiv \varepsilon = mc^2\gamma_L$, $p_i = m\upsilon_i\gamma_L = m(dx_i/dt)\gamma_L$ (where $\gamma_L = 1\big/\sqrt{1-\upsilon^2/c^2}$ and $\gamma_L^{-1} = \sqrt{1-\upsilon^2/c^2}$ are the Lorentz factor and antifactor, respectively). Hence, in the Cartesian coordinate system:

$$\frac{\varepsilon^2}{c^2} - p_x^2 - p_y^2 - p_z^2 = m^2c^2 , \qquad (2.5')$$

Since $p_i = m\upsilon_i\gamma_L = m(dx_i/dt)\gamma_L$, a $\varepsilon = mc^2\gamma_L$, this relation can be rewritten as:

$$\gamma_L^2 c^2 (dt)^2 - \gamma_L^2 (dx)^2 - \gamma_L^2 (dy)^2 - \gamma_L^2 (dz)^2 = c^2 (dt)^2 , \qquad (2.6)$$

Multiplying it by γ_L^{-2} , we get:

$$c^2 (dt)^2 \gamma_L^{-2} = c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 , \qquad (2.7)$$

Since (see above (2.2)) we got $c^2 (dt)^2 \gamma_L^{-2} = c^2 (dt)^2 \left(1 - \upsilon^2/c^2\right) = (ds)^2$, the expression (2.7) can be written as square of a 4-interval:

$$(ds)^2 = c^2 (dt)^2 - (dx)^2 - (dy)^2 - (dz)^2 , \qquad (2.1')$$

In general case of use in Euclidean space of any other, than the Cartesian, coordinate system for recording of the relation (2.5'), particularly, the orthogonal curvilinear coordinates, this interval takes the form:

$$(ds)^2 = g^{\mu\nu} dx_\mu dx_\mu , \qquad (2.8)$$

where $g^{\mu\nu}$ is a so-called metric tensor, whose elements take into account the changes in the projections of the segments of the trajectory of the body on the coordinate axes, at the transition from the Cartesian coordinate system to any other. In a Cartesian system, all elements $g^{\mu\nu}$ are equal to unities.

Obviously, if we go in the opposite direction, we can obtain the equation (2.5') from the square of the interval. This implies, firstly, that these equations - (2.1) and (2.5') - closely bind the massive elementary particles physics and geometry. Secondly, the equation of "massive photon" is derived from Maxwell's equations of a massless photon as a result of his self-interaction of fields (chapter 2).

This non-linearity of a self-acting fields of the “massive photon” does not mean transition from Euclidean to some new geometry. From this it follows that (2.1) is not a metric of pseudo-Euclidean geometry, but it is a metric of Euclidean geometry that describes the Lorentz-invariant field equations. The only change in the geometry, which we can observe in this case is the transition from rectilinear to curvilinear geometry.

In addition, another link between the interval (2.1) and the physical equation is detected. As we have shown in chapters 6, using the Schrödinger definition of action ($p_\mu = \partial S/\partial x_\mu$), from the equation (2.5') it is easy obtain Lorentz-invariant Hamilton-Jacobi equation in general view. For this it is enough to write the equation (2.5') in a form, suitable for any of the Euclidean coordinate system:

$$g_{\mu\nu} p^\mu p^\nu = m^2c^2 , \qquad (2.9)$$

where, we recall, $g_{\mu\nu}$ is the metric tensor of geometrical space, but not of the gravitational space-time of general relativity (in other words, in this case the tensor $g_{\mu\nu}$ does not include the physical characteristics of the field). In this case the Hamilton-Jacobi equation of free particles obtains the form:

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^{\mu}}\right)\left(\frac{\partial S}{\partial x^{\nu}}\right)-m^2c^2=0\,, \qquad (2.10)$$

Thus, we conclude that the three equations (2.1) (2.5) and (2.10) are closely bonded to each other and, in fact, follow from one differential equation. From this follows that the interval (2.1) within a relativistic physics is the physical law, and not a geometric relation.

3. The physical sense of the metric tensor of curvilinear coordinates' system of the Euclidean geometry

Recall the transition from Cartesian's system of coordinates to the generalized coordinate system (Korn and Korn, 1968). Let us introduce a new set of coordinates q_1, q_2, q_3, so that among x, y, z and q_1, q_2, q_3 there are some relations:

$$x=x(q_1,q_2,.q_3),\ y=y(q_1,q_2,.q_3),\ z=z(q_1,q_2,q_3), \qquad (3.1)$$

The differentials are then

$$dx=\frac{\partial x}{\partial q_1}dq_1+\frac{\partial x}{\partial q_2}dq_2+\frac{\partial x}{\partial q_3}dq_3\,, \qquad (3.2)$$

and the same for dy and dz.

In Cartesian coordinates the measure of distance, or metric, in a given coordinate system is the arc length ds, which is defined by

$$ds^2=dx^2+dy^2+dz^2\,, \qquad (3.3)$$

In general, taking into account (3.2), from (3.3) we obtain

$$ds^2=g_{11}dq_1{}^2+g_{12}dq_1dq_2+\ldots=\sum_{ij}g_{ij}dq_idq_j\,, \qquad (3.4)$$

where g_{ij} is the metric tensor. Thus in orthogonal system we can write

$$ds^2=(H_1dq_1)^2+(H_2dq_2)^2+(H_3dq_3)^2\,, \qquad (3.5)$$

where the H_i's are

$$H_i=\sqrt{\left(\frac{\partial x}{\partial q_i}\right)^2+\left(\frac{\partial y}{\partial q_i}\right)^2+\left(\frac{\partial z}{\partial q_i}\right)^2}\,, \qquad (3.6)$$

are called Lame coefficients or scale factors, and are 1 for Cartesian coordinates.

Thus, the metric tensor, recorded in coordinates q_i, is a diagonal matrix whose diagonal contains the squares of Lame coefficients:

For example, in the case of spherical coordinates, the bond of spherical coordinates with Cartesian is given by:

$$x=r\sin\theta\cos\varphi,\ \ y=r\sin\theta\sin\varphi,\ \ z=r\cos\theta, \qquad (3.7)$$

The Lame coefficients in this case are equal to: $H_r=1,\ H_\theta=r,\ H_\varphi=r\sin\theta$, and the square of the differential of arc (interval) is:

$$ds^2=dr^2+r^2d\theta^2+r^2\sin^2\theta d\varphi^2\,, \qquad (3.8)$$

Since the metric tensor is determined by means of Lame coefficients, let us recall the geometric meaning of the latter: *the Lame coefficients show how many units of length are* *contained in the unit of length of coordinates of the given point, and used to transform vectors when transition from one system to another takes place.*

This means that the metric tensor in Euclidean geometry defines rescaling of three coordinates r, θ, φ, and in the pseudo-Euclidean or pseudo-Riemannian geometry it determines rescaling of four coordinates t, r, θ, φ.

As we will show in the following chapter, from the solution of the Kepler problem within LIGT, the relativistic corrections within LIGT correspond to changes of scales t and r, caused by the Lorentz-invariant effects (time dilation and Lorentz-Fitzgerald length contraction). In the next article, we will show that the same thing occurs in problems of a moving source. Note that this is the case for problems of stationary and moving source.

Thus, we conclude that relationships (1.2) and (1.3) have metric tensor $g_{\mu\nu}$ as a factor that takes into account the change of scales of time and distance due to relativistic effects associated with motion of bodies.

From the foregoing analysis follows that by regular way the interval of a 4-space-time can be obtained only for the pseudo-Euclidean space, as a variant of the physical law of motion of elementary particles.

Since there is no other law of motion for massive particles, we can assume that the hypothesis of Einstein that the gravitational field is created by the curvature of space-time, which requires a transition to a pseudo-Riemannian geometry, needs considerable adjustment.

Chapter 8 . The equivalence principle and metric tensor of LIGT

1.0. Equivalence of inertial and gravitational masses and its consequences

Interpretation of the equivalence of inertial and gravitational masses by Einstein led him to assertion that the theory of gravity can not be a Lorentz-invariant theory, but it should be a general relativistic theory in a Riemannian – non-flat - space-time. At the same time a characteristic feature of the Lorentz-invariant theory is a flat space-time. Can we solve this contradiction between our approach and the approach of general relativity?

1.1. Is GTR an L-invariant theory?

It is known that general relativity is considered a relativistic theory, but it is not a L-invariant theory (Katanaev, 2013, pp. 742)

«*Lorentz metric satisfies the Einstein's vacuum equations* . [But] *"in GTR is postulated that space-time metric is not a Lorentz metric, and is found as a solution of Einstein's equations. Thus, the space-time is a pseudo-Riemanian manifold with metric of a special type that satisfies the Einstein equations*."

The general relativity principle, according to Einstein's hypothesis, should be a generalization of the Lorentz-invariance of the special relativity theory. As such principle, Einstein proclaimed the requirement of general covariance. As is known, most physicists - see, e.g., Hilbert, Synge, 1960; Fock, Logunov (Polak, 1959; Fock, 1964; Logunov, 2002) - do not consider the general covariance to be equivalent with some type of relativity, which generalizes the Lorentz-invariance. This follows from the fact that any Lorentz-invariant theory can always be written in covariant form.

Thus, the absence of such a generalization makes the Lorentz- invariance a basic requirement for any relativistic theory. The real space of such theories is Euclidian (or, conditionally, taking into account time, it is pseudo-Euclidian). Obviously, this is also valid for the gravitation theory. Hence, the Riemannian space is not a real space, but a mathematical model. Indeed, the assertion that the real space is Riemannian is not supported by theory or experiment.

Let us analyze the possibility of describing the gravitational interaction without the involvement of a Riemannian space.

1.2. Einstein interpretation of the equivalence of inertial and gravitational masses in building a theory of gravitation

Let us see first of all how in modern physics is described the transition of Einstein from Euclidean space to Riemann, the starting point of which was the principle of equivalence.

«*Universal gravitation does not fit into the framework of uniform Galilean space. The deepest reason for this fact was given by Einstein. It is that not only the inertial mass, but also the gravitational mass of a body depends on its energy. It proved possible to base a theory of universal gravitation on the idea of abandoning the uniformity of space as a wholef and attributing to space only a certain kind of uniformity in the infinitesimal. Mathematically, this meant abandoning Euclidean, or rather pseudo-Euclidean, geometry in favour of the geometry of Biemann* » (Fock, 1964) .

Is it possible to give a different interpretation of the equivalence of gravitational and inertial mass?

Let us begin with the formulation of the principle of equivalence which Einstein gave himself:

"*A little reflection will show that* ***the law of the equality of the inertial and gravitational mass is equivalent to the assertion that the acceleration imparted to a body by a gravitational field is independent of the nature of the body****. For Newton's equation of motion in a gravitational field, written out in full, it is:*

(Inertial mass) .(Acceleration) =(Intensity of the gravitational field) .(Gravitational mass).

It is only when there is numerical equality between the inertial and gravitational mass that the acceleration is independent of the nature of the body" (Einstein, 2005).

Let us consider the mathematical basis of the principle of Einstein's equivalence and try to give this mathematics another form.

As we can see, Einstein relied on the Newtonian law of motion of a particle with inertial mass m_{in} in a gravitational field of source with a mass M :

$$m_{in}\frac{d\vec{\upsilon}}{dt}=\gamma_N\frac{m_{gr}M}{r^2}\vec{r}^{\,0} , \qquad (1.1)$$

where m_{gr} is gravitational mass. Since $m_{in}=m_{gr}=m$, then dividing (1.1) by m we obtain in the case of gravitation the movement equation of the form:

$$\frac{d\vec{\upsilon}}{dt}=\gamma_N\frac{M}{r^2}\vec{r}^{\,0} , \qquad (1.1')$$

where acceleration is on the left and the Newton force per unit mass is on the right.

It is easy to see that this equation is the mathematical expression of Einstein's abovementioned principle of equivalence: the power (ie, action) of Newton's gravity exerted on the unit mass (i.e., local point mass), coincides with the acceleration of the moving body in this field (i.e., with the force of inertia acting per unit mass).

Our second question was whether it is possible to give another explanation to this principle.

1.3. Interpretation of the equivalence of inertial and gravitational masses in framework of LIGT

In the GTR we assume that in the (pseudo-) Euclidean reference frame, "*the noninertial frames possessed spatial and temporal inhomogeneities that show up as inertial forces, that depend on the specific characteristics of the reference frame. Obviously, the inertial forces have to have a noticeable effect on the physical processes in these reference frames*" (Vladimirov et al, 1987).

In this case, this heterogeneity does not generate the real Riemannian space-time (although, following the example of Jacobi, this heterogeneity can be displayed mathematically as a Riemannian space).

Indeed, this heterogeneity can be considered as inhomogeneity of the field in space and time of the real Euclidean space and time, but not as heterogeneity of spacetime itself. In accordance with this our interpretation of the principle of equivalence is as follows.

The gravitational field, and not the space and/or time, sets the variable speed of body motion. Therefore, if the field depends on space and time, it is not necessary to bind the body velocity with time and space; it is enough to relate this speed with field itself.

Thus, all we need is to describe the action of force on the movement of the body, to find this relation between field and speed.

It appears, that based on the same mathematics, we can actually find this connection. As is known, the equation (1.1 ') can be represented in the energy form. For this let us rewrite the Newton's motion law in the form:

$$d\vec{\upsilon} = \gamma_N \frac{M}{r^2}\vec{r}^{\,0} dt\,, \tag{1.2}$$

Multiplying the left and right hand side of equation (1.2) on the speed $\vec{\upsilon}$, and taking into account that $\vec{\upsilon} = d\vec{r}/dt$ and $d\vec{r}/r^2 = -d(1/r)$, we have from (1.2) after integration:

$$\frac{\upsilon^2}{2} + \gamma_N \frac{M}{r} = const\,, \tag{1.3}$$

where $\upsilon^2/2 = \varepsilon_\kappa/m$ is the kinetic energy of the moving particle per unit mass, and $\gamma_N M/r = \varepsilon_{pot}/m$ is the potential energy of a particle per unit mass at a given point of the gravitational field.

Thus, taking into account the postulate of equivalence and the expression for the potential of the gravitational field $\varphi_N = \gamma_N M/r$, we obtain from (1.1), the relationship between the velocity of the particle and potential of the gravitational field at the position of the particle:

$$\upsilon^2 = 2\varphi_N + const\,, \tag{1.4}$$

If at the initial moment a particle was at rest, and the motion is only carried out via the potential energy outlay, then during the whole period of motion *const* = 0. For example, this occurs when the reference frame, that is related to the observer, falls freely to the center of gravity source along the radius (*radial infall*) from infinity, where it had a zero velocity. In this case, we have:

$$\upsilon^2 = 2\varphi_N = \frac{2\gamma_N M}{r}\,, \tag{1.5}$$

Thus, as a mathematical consequence of Newton's theory of gravity, we have received another interpretation of the fact of the equality of inertial and gravitational mass. Following the example of Einstein's equivalence principle, it can be expressed as follows: *the potential of the gravitational field is equivalent to the square of the velocity of the motion of particles in this field.*

In addition, (see chapter 4) the electromagnetic basis of gravitational equations allows one to write the vector potential of the gravitational field through the scalar potential.

2.0. Peculiarities of metric tensor of LIGT

As is known, Einstein came to the metric tensor of the pseudo-Riemannian space on the basis of Einstein’s equivalence principle.

Above we have given a different interpretation of the equivalence of masses. Now we can try to obtain an expression for the metric tensor of L-invariant theory of gravitation.

As we mentioned (Chapter 7), in differential geometry, the metric tensor elements are equal to the squares of the Lame scale coefficients. The Lame coefficients indicate how many units of length are contained in the unit coordinates in a given point and are used to transform vectors in the transition from one coordinate system to another.

At the same time, in the framework of GR, these two coordinate systems represent basically two dissimilar geometric coordinate systems from a number of well-known rectangular, oblique, or any other coordinates.

In contrast, in the L-invariant transformation is examined the transition between two identical from geometric point of view, coordinate systems, which are attached to two reference frames moving relative to each other. Moreover, it was found that a simultaneously this transition requires to take into account the transformation of time.

It is clear that geometric transformation may not affect the final results of the solution of physical problems. In our case it is about physical transformation of trajectory and time of the particle motion. We proved this by showing that the square of the arc element (interval) in this case is a consequence of the well-known relation between the energy, momentum and mass of the moving particle. Thus, these changes are purely physical. They contribute to the correction of physical problems non-relativistic physics.

However, conditionally this interval can be seen as a geometric object that generates a pseudo-Euclidean geometry, which has in addition to three spatial coordinates, one time coordinate (as it is done in 4-Minkowski's geometry). From this geometrical point of view, coordinates and time undergo the change of the scales. These changes can be considered, along with changes of coordinates that take place during the transition between two different coordinate systems. But we should not forget that from the physical point of view it is a completely different transformations and changes of scales.

3.0. Calculation of metric tensor of LIGT

The linear arc element in the 3-dimensional mechanics is expressed through Lame's scale factors in the form of linear elements:

$$ds = \sum_{i=1}^{3} h_i dx_i = h_1 dx_1 + h_2 dx_2 + h_3 dx_3 , \tag{3.1}$$

where $x_i = \vec{r} = (x_1, x_2, x_3)$, $i = 1, 2, 3$. . In a Cartesian coordinate system $x_i = \vec{r} = (x, y, z)$, and all the Lame coefficients equal to one.

In the L-invariant mechanics it is impossible to enter the *line* element of the arc since the physical equation, from which follows the magnitude of the arc, connects the squares of the energy, momentum and mass, and not the first degrees of these values. The exact expression is obtained in the form of the square of length of arc element, which is often referred to simply as an interval. In the 4-geometry it is of the form:

$$(ds)^2 = \sum_{\mu-0}^{3} (h_\mu dx_\mu)^2 = (h_0 dx_0)^2 + (h_1 dx_1)^2 + (h_2 dx_2)^2 + (h_3 dx_3)^2 , \tag{3.2}$$

or, taking into account that $\lambda_{\mu\mu} = h_\mu h_\mu$, we receive from (3.2) the form:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu} (dx_\mu)^2 = \lambda_{00} (dx_0)^2 + \lambda_{11} (dx_1)^2 + \lambda_{22} (dx_2)^2 + \lambda_{33} (dx_3)^2 , \tag{3.2'}$$

where $x_\mu = (ict, \vec{r}) = (ict, x_i) = (x_0, x_i)$ $\mu = 0, 1, 2, 3$, $\lambda_{\mu\mu}$ is metric tensor in LIGT.

3.1. The Lorentz-Fitzgerald length contraction and time dilation as a change of the scales of coordinates of space and time in LIGT

Using the definition of the metric tensor in LITG given above, let us calculate it in the simplest case. Consider (Pauli, 1958) Lorentz transformation in the transition from the coordinate system *K* to K', which is currently moving at a speed υ along the axis x. In this case only the coordinate x and time t undergo transformations.

The Lorentz effects of length contraction and time dilation are the simplest consequences of the Lorentz transformation formulae, and thus also of the two basic assumptions of SRT.

$$x = \frac{x' - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}\,,\; y = y',\; z = z',\; t = \frac{t' - \frac{\upsilon}{c^2}x'}{\sqrt{1-\upsilon^2/c^2}}\;, \tag{3.3}$$

The transformation which is the inverse of (1) can be obtained by replacing υ by $-\upsilon$:

$$x = \frac{x' + \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}\,,\; y = y',\; z = z',\; t = \frac{t' + \frac{\upsilon}{c^2}x'}{\sqrt{1-\upsilon^2/c^2}}\;, \tag{3.3a}$$

Take a rod lying along the x-axis, at rest in reference system K'. The position coordinates of its ends, x_1' and x_2' are thus independent of t' and $x_2' - x_1' = l_0$ is the rest lengtl of the rod. On the other hand, we might determine the length of the rod in system K' in the following way. We find x_1 and x_2 as functions of t. Then the distance between the two points which coincide simultaneously with the end points of the rod in system K will be called the length l of the rod in the moving system: $x_2(t) - x_1(t) = l$

Since these positions are not taken up simultaneously in system K', it cannot be expected that l equals l_0. In fact, it follows from (3.3) :

$$x_2' = \frac{x_2(t) - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}\,; \quad x_1' = \frac{x_1(t) - \upsilon t'}{\sqrt{1-\upsilon^2/c^2}}$$

for infinitesimal time intervals of length dx has form $dx' = \dfrac{dx}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of **coordinates** (denote it as k_x^L) will be equal to:

$$k_x^L = \frac{dx'}{dx} = \frac{1}{\sqrt{1-\upsilon^2/c^2}} = \gamma_L\,, \tag{3.4}$$

where $\gamma_L = 1\big/\sqrt{1-\upsilon^2/c^2}$ is the Lorentz factor.

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{xx} = \frac{dx'}{dx}\frac{dx'}{dx} = (\gamma_L)^2\,, \tag{3.5}$$

The rod is therefore contracted in the ratio $\sqrt{1-\upsilon^2/c^2} : 1$, as was already assumed by Lorentz. It therefore follows that the Lorentz contraction is not a property of a single measuring rod taken by itself, but is a reciprocal relation between two such rods moving relatively to each other, and this relation is in principle observable.

Analogously, the **time** scale is changed by the motion. Let us again consider a clock which is at rest in K'. The time t' which it indicates in x' is its proper time, τ and we can put its coordinate x' equal to zero. It then follows from (3.3a) that $t = \dfrac{\tau}{\sqrt{1-\upsilon^2/c^2}}$, which for infinitesimal time intervals dt give: $dt = \dfrac{dt'}{\sqrt{1-\upsilon^2/c^2}}$.

From here the scaling factor of the Lorentz transformation of time (denote it as k_t^L) will be equal to:

$$k_t^L = \frac{dt'}{dt} = \sqrt{1-\upsilon^2/c^2} = \gamma_L^{-1}, \tag{3.6}$$

The corresponding element λ_{xx} of the metric tensor of the Lorentz transformation will be:

$$\lambda_{tt} = \frac{dt'}{dt}\frac{dt'}{dt} = (\gamma_L)^{-2}, \tag{3.7}$$

Measured in the time scale of K, therefore, a clock moving with velocity υ will lag behind one at rest in K in the ratio $\sqrt{1-\upsilon^2/c^2}:1$. While this consequence' of the Lorentz transformation was already implicitly contained in Lorentz's and Poincare's results, it received its first clear statement only by Einstein.

Then, in framework of LITG the square interval will be as follows:

$$(ds)^2 = \sum_{\mu-0}^{3} \lambda_{\mu\mu}\eta_{\mu\mu}(dx_\mu)^2 = \lambda_{00}\eta_{00}(dx_0)^2 + \lambda_{11}\eta_{11}(dx_1)^2 + \lambda_{22}\eta_{22}(dx_2)^2 + \lambda_{33}\eta_{33}(dx_3)^2, \tag{3.8}$$

where $\eta_{\mu\mu}$ is the geometric metric tensor in LIGT (tensor of pseudo-Euclidian space); $\lambda_{\mu\mu}$ is the physical metric tensor in LIGT. Using the values $\lambda_{00} = \lambda_{tt}$ and $\lambda_{11} = \lambda_{xx}$, according to (3.5) and (3.7), we obtain in the Cartesian system of coordinates:

$$(ds)^2 = -(\gamma_L)^{-2}(dt)^2 + (\gamma_L)^2(dx)^2 + (dy)^2 + (dz)^2, \tag{3.9}$$

4.0. Relation between Lorentz factor and characteristics of the Newton gravitational field

The main characteristic of the Lorentz transformation is the Lorentz factor γ_L: $\gamma_L = 1/\sqrt{1-\beta^2}$ (where $\vec{\beta} = \vec{\upsilon}/c$), which is determined by the speed of motion of the body $\vec{\upsilon} = \vec{\upsilon}(\vec{r},t)$. The vector of speed of the particle motion can be considered as its main component, by which its trajectory, acceleration and some other quantities are determined.

On the base of our interpretation of the principle of equivalence of mass, we found relation between field and speed.

In the case of Newton's theory, probably the first, that found this relationship was E.A.Milne (Milne, 1934). Later, independently, and from an other primary bases, this was also done by Arnold Sommerfeld assistant - Wilhelm Lenz. He took advantage of this connection to find a solution to the Kepler problem, which coincides with the results of the Schwarzschild-Droste solution of Einstein-Hilbert equation (Sommerfeld, 1952). Below we will expand this relationship to the case of the Lorentz-invariant mechanics, to obtain the next approximations in the form of a power series.

Using (1.5) it is easy to find an expression for the Lorentz-factor due to the gravitational field of Newton:

$$\gamma_L = \frac{1}{\sqrt{1-2\varphi_g/c^2}} \quad \text{or} \quad \gamma_L = \frac{1}{\sqrt{1-r_S/r}}, \tag{4.1}$$

where $r_S = \dfrac{2\gamma_N M}{c^2}$ is the, so-called, Schwarzschild radius.

Taking into account (4.1) it is easy to see that (3.9) corresponds to the Schwarzschild-Droste solution:

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^4, \qquad (4.2)$$

5.0. Relation among the Lorentz factor and characteristics of the gravitational field, taking into account the Lorentz-invariant generalization of mechanics

Within the framework of the Lorentz-invariant theory, the particle mass is a function of velocity and position in the field: $m = m(\vec{r}, \vec{\upsilon})$, and the kinetic energy ε_κ is entered by the following expression:

$$\varepsilon_\kappa = \varepsilon_f - m_0c^2 = m_0c^2\gamma_L - m_0c^2 = m_0c^2(\gamma_L - 1), \qquad (5.1)$$

where c is the speed of light, m_0 means the particle rest mass, and ε_f is full energy of particle.

Since $\upsilon < c$, the expressions, containing δ, can be expanded to Maclaurin series (we take here into account only 4 terms):

$$\gamma_L = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots, \qquad (5.2)$$

$$\gamma_L^{-1} = 1 - \frac{1}{2}\beta^2 - \frac{1}{8}\beta^4 - \frac{1}{16}\beta^6 - \frac{5}{128}\beta^8 + \ldots, \qquad (5.3)$$

Thus we can obtain for energy and momentum the following expressions:

$$\varepsilon_f = m_0c^2 + m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\}, \qquad (5.4)$$

$$\begin{aligned}\varepsilon_\kappa &= \varepsilon_f - m_0c^2 = m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\} = \\ &= \frac{1}{2}m_0\upsilon^2 + \frac{3}{8}m_0\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16}m_0\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128}m_0\left(\frac{\upsilon^8}{c^6}\right) + \ldots\end{aligned}, \qquad (5.5)$$

At $\beta << 1$ we obtain from (5.7)-(5.9) as first approximation the non-relativistic expressions:

$$\varepsilon_f \approx m_0c^2 + \frac{1}{2}m_0\upsilon^2; \quad \varepsilon_k \approx \frac{1}{2}m_0\upsilon^2; \qquad (5.6)$$

According to Newton's theory of gravity, the potential energy of a particle is equal to $\varepsilon_N = m_0\varphi_N = \gamma_N m_0 M/r$. A change of the speed of the particle is accompanied by a change in its kinetic energy (5.1):

$$\begin{aligned}\frac{\gamma_N m_0 M}{r} &= m_0\varphi_N = \varepsilon_k = \frac{1}{2}m_0\upsilon^2 + \frac{3}{8}m_0\left(\frac{\upsilon^4}{c^2}\right) + \frac{5}{16}m_0\left(\frac{\upsilon^6}{c^4}\right) + \frac{35}{128}m_0\left(\frac{\upsilon^8}{c^6}\right) + \ldots = \\ &= m_0c^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\}\end{aligned}, \qquad (5.7)$$

In the case of sufficiently small velocities, we obtain a first approximation, $\gamma_N M/r \approx \varphi'_N = \upsilon^2/2$ or; $2\gamma_N M/r \approx 2\varphi'_N = \upsilon^2$, from where:

$$\frac{2\gamma_N M}{c^2 r} = \frac{r_S}{r} \approx \frac{2\varphi'_N}{c^2} = \frac{\upsilon^2}{c^2} = \beta^2, \qquad (5.8)$$

where $r_S = 2\gamma_N M/c^2$ is called the gravitational radius of the body of mass M (Schwarzschild radius).. Using the following term of the expansion in (5.7) it is possible to clarify the relation between φ_N and r_S to obtain a second approximation:

$$\frac{2\gamma_N M}{c^2 r} \approx \frac{2\varphi''_N}{c^2} = \frac{\upsilon^2}{c^2} + \frac{3}{4}\frac{\upsilon^4}{c^4} = \frac{2\varphi'_g}{c^2} + \frac{3}{4}\left(\frac{2\varphi'_g}{c^2}\right)^2 = \frac{r_S}{r} + \frac{3}{4}\frac{r_S^2}{r^2}, \qquad (5.9)$$

6.0. Calculation of gravitation field potentials

Thus, according to our results, it is sufficient to calculate the gravitational field of the potentials to be able to enter the L-invariant amendments to the gravitational theory of Newton.

In particular, for the calculation of the metric tensor elements of LIGT we use the potential of the Newtonian gravitation theory φ_N . It is remarkable that this non-relativistic potential gives the relativistic corrections to the solution of the Kepler problem.

Since LITG is based on electromagnetic theory, this calculation is not difficult. We only briefly recall the results of this approach, adequately set out in the chapter 5.

As we have seen, the Maxwell-Lorentz equations can be written in potentials in the form of equations of the electromagnetic field propagation. Using $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, in the case of the Lorentz condition $\frac{1}{c}\cdot\frac{\partial\varphi}{\partial t} + \nabla\vec{A} = 0$, we have:

$$\frac{1}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = 4\pi\rho , \qquad (6.1)$$

$$\frac{1}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2} - \nabla^2\vec{A} = \frac{4\pi}{c}\rho\vec{\upsilon} , \qquad (6.2)$$

From a mathematical point of view, these equations are the d'Alembert non-homogeneous equation. Its solution is known (see chapter 5, equation (2.16)). Additionally, it turns out that the vector potential associates with the scalar potential by expression:

$$\vec{A} = \varphi\frac{\vec{\upsilon}}{c}, \qquad (6.3)$$

In this case, the main characteristics of the electromagnetic vector field are the scalar and vector potentials φ and $\vec{A}$, respectively, or the 4-potential $A_\mu = \left(\frac{\varphi}{c}, -\vec{A}\right)$. ($A_\mu = \frac{\varphi}{c^2}u_\mu$, where $u_\mu = dx_\mu/d\tau$ is 4-speed , dx_μ is 4-movement , τ is the proper time of the particle).

If the system contains a set of particles, each of which generates its own potential, then the potentials φ and $\vec{A}$ of the system of particles depend mainly on the general system parameters – the dimensions of the system, the total charge, etc. It is very important that the calculation of the system potential is defined by the superposition principle, i.e., by summation of the potentials of all the particles. Thus we can determine all the main characteristics of the system's electromagneticfield with the help of the 4-potential.

But before we will find the 4- potential of the system, we need to determine the potentials of a single particle. As it is known, it is the centrally symmetric potential of Newton that defines the field of a point particle. As we have seen (see chapter 5), the calculation of the potential of a system of point sources - i.e., of a body with known charge density requires the integration of the potential of al l particles over the volume of the body.

Turning to gravitation, it can be expected that in the relativistic theory of gravity, along with the scalar potential φ_g there should be also a vector potential $\vec{A}_g$. There is also no doubt that the relationship (6.3) is valid in the case of the gravitational field. General relativity confirms this, and a great achievement for GR was to demonstrate this characteristic.

Thus, the solutions of equations (6.1) and (6.2) allow us to calculate both Lorentz factor and relativistic amendments to Newton's theory of gravitation.

Summary

As we have shown (Chapter 7), the square of the interval, which in SRT and GTR is considered as a geometry object, in the physics of elementary particles and within LIGT is a mathematical notation of the Lorentz-invariant energy-momentum conservation law.

That is why the Lorentz transformation can be found formally as a group of transformations preserving invariant the squared interval.

In the generalized system of coordinates this quadratic form contains the metric tensor, in which elements take into account the change of the coordinate scale in the transition from one coordinate system to another, differing in the geometrical sense.

In the presence of a gravitational field this quadratic form contains a metric tensor, in which the amendment of changing the scale of coordinates derived due to the effects of the Lorentz transformation at the transition from the moving to stationary system or vice versa, is taken into account. But for all that, these coordinate systems are the same in terms of geometry.

As we have shown, this tensor is identical to the one obtained from the solution of equations of general relativity. Thus there is no need to interpret this interval as belonging to a Riemann space. It may be written in any (including rectangular) coordinate system.

Chapter 9. Solution of the Kepler problem in the framework of LIGT

In present chapter, based on results of previous Chapter 8 , we consider the solution of the Kepler problem, i.e., the solution of the problem of motion of a body of little mass in a centrally symmetric gravitational field of a stationary source of great mass. It is shown that this solution coincides with that obtained in GR.

As the motion equation of LITG we use the Hamilton-Jacobi equation (Chapter 6). According to Chapter 6 , the equation of motion of Hamilton-Jacobi has a one-to-one connection with the square of the interval (square of arc element of trajectory) in framework of LITG. Therefore, as we will show below, it is not necessarily to find an appropriate interval to write the corresponding Hamilton-Jacobi equation for particle motion in gravitation field.

1.0. Effects of Lorentz transformation

A consequence of the previously adopted axiomatics (chapter 3) of Lorentz-invariant gravitation theory (LIGT) is the assertion that all features of the motion of matter in the gravitational field owed their origin to effects associated with the Lorentz transformations. This means that the amendments to Newton's gravitation theory must follow from considering of these effects.

Effects, that owe their existence to the Lorentz transformations are discussed in many textbooks devoted to the EM theory or SRT (Pauli, 1981; Becker, 2013; et al.).

1.1. The transition from Newtonian mechanics to the Lorentz-invariant mechanics

Let us try (Becker, 2013) to alter the Newtonian equations so that they satisfy the Lorentz transformations. We begin by considering the motion of a particle in a given force field (e.g., electromagnetic or gravitational). Newtonian equations of motion read as follows:

$$m\frac{d\vec{\upsilon}}{dt} = \vec{F}_L , \tag{1.1}$$

where $\vec{F}_L$ is, e.g., the Lorentz force :

$$\vec{F}_L = q\vec{E} - \frac{q}{c}\vec{\upsilon}\times\vec{H} , \tag{1.2}$$

Now we will try to give this equation the Lorentz-invariant form. Obviously, the Lorentz-invariant version of the equation (1.1) instead of the classical time t must contain the proper time $\tilde{t}$:

$$m\frac{d\vec{\upsilon}}{d\tilde{t}} = \vec{F}_L , \tag{1.1'}$$

In order to find this version of the equation, we replace in (1.1') its proper time in line with the ratio for the Lorentz time dilation $d\tilde{t} = dt\sqrt{1-\beta^2}$ on $dt\sqrt{1-\beta^2}$:

$$m_0\frac{d}{dt}\frac{\vec{\upsilon}}{\sqrt{1-\beta^2}} = q\vec{E} - \frac{q}{c}\vec{\upsilon}\times\vec{H} , \tag{1.3}$$

As is known, the equation (1.3) is the Lorentz-invariant equation of motion of a charged particle in an EM field.

Below we will consistently apply this method to obtain the relativistic equations of gravitation in the form of Hamilton-Jacobi equations.

2.0. Solution of the Kepler problem in the framework of LIGT

Two of the most important effects from the point of view of mechanics that arise due to the Lorentz transformations, are the Lorentzian time dilation and contraction of lengths:

$$d\tilde{t} = dt\sqrt{1-\beta^2} , \quad d\tilde{r} = \frac{dr}{\sqrt{1-\beta^2}} , \tag{2.1}$$

where, as shown previously, $\beta^2 = r_S/r$, and r_S is the Schwarzschild radius.

The free particle motion is described by the Hamilton-Jacobi equation (Landau and Lifshitz, 1971):

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\vec{\nabla}S\right)^2 = m^2c^2 , \tag{2.2}$$

In a spherical coordinate system (taking into account both relativistic effects) it takes the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - \left(\frac{\partial S}{\partial \tilde{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - \frac{1}{r^2}\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2 , \tag{2.3}$$

where $\tilde{t}$ and $\tilde{r}$ are measured in a fixed coordinate system associated with a stationary spherical mass M .

We will start with the account of the first effect

2.1. The equation of motion of a particle in a gravitational field, taking into account the relativistic effect of time dilation

Taking into account that the motion of a particle around the source occurs in the plane, we define this plane by condition $\theta = \pi/2$. In this case, the equation (2.3) takes the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - \left(\frac{\partial S}{\partial \tilde{\vec{r}}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2 , \qquad (2.4)$$

Taking into account only the transformation of time $d\tilde{t} = dt\sqrt{1-\beta^2}$, equation (2.4) can be rewritten as follows:

$$\frac{1}{1-\beta^2}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 , \qquad (2.5)$$

Substituting $1-\beta^2 = 1 - r_s/r$, we obtain:

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 , \qquad (2.6)$$

Let us simplify this equation, taking into account the expansion $1/(1-x) = 1 + x + x^2 + ... + x^n$ for $x << 1$. Since for the actual sizes of the planets and Sun and the distances between them, value $r_s/r << 1$, we can be limited by first two terms of the expansion. At the same time $\frac{1}{1-r_s/r} \cong 1 + r_s/r$, and the equation (2.4) takes the form:

$$\left(1+\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(\frac{\partial S}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 , \qquad (2.7)$$

We will show that L-invariant time dilation leads to the appearance of Newton's gravitational field.

2.1.1 *Newton's approximation*

Let us present this equation to the non-relativistic mind, using the transformation $S = S' - mc^2 t$ (Landau and Lifshits, 1971):

$$\left(\frac{\partial S}{\partial t}\right)^2 = \left(\frac{\partial S'}{\partial t}\right)^2 - 2mc^2\frac{\partial S'}{\partial t} + m^2c^4 .$$

Substituting this in (7), we find

$$\left(1+\frac{r_s}{r}\right)\left[\left(\frac{\partial S'}{\partial t}\right)^2 - 2mc^2\frac{\partial S'}{\partial t} + m^2c^4\right] - c^2\left(\frac{\partial S'}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^4 .$$

Expanding the brackets, we obtain:

$$\left(\frac{\partial S'}{\partial t}\right)^2 - 2mc^2\frac{\partial S'}{\partial t} + \frac{r_s}{r}\left(\frac{\partial S'}{\partial t}\right)^2 - 2mc^2\frac{r_s}{r}\frac{\partial S'}{\partial t} + \frac{r_s}{r}m^2c^4 - c^2\left(\frac{\partial S'}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = 0 .$$

Dividing this equation by $2mc^2$, we find:

$$\frac{1}{2mc^2}\left(\frac{\partial S'}{\partial t}\right)^2 - \frac{\partial S'}{\partial t} + \frac{1}{2mc^2}\frac{r_s}{r}\left(\frac{\partial S'}{\partial t}\right)^2 - \frac{r_s}{r}\frac{\partial S'}{\partial t} + \frac{1}{2}\frac{r_s}{r}mc^2 - \frac{1}{2m}\left(\frac{\partial S'}{\partial \vec{r}}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = 0 , \qquad (2.8)$$

Taking into account that $r_s = \frac{2\gamma M}{c^2}$, we obtain $\frac{1}{2}\frac{r_s}{r}mc^2 = \frac{\gamma m M}{r} = m\varphi_N = -U$, where U is the energy of the gravitational field in the Newtonian theory. In the non-relativistic case we put $c \to \infty$. Furthermore, for real distances r of the body movement around source with Schwarzschild radius r_s, we have $\frac{r_s}{r} << 1$ and $\frac{r_s}{r}\frac{\partial S'}{\partial t} << \frac{\partial S'}{\partial t}$, and then we can ignore the term $\frac{r_s}{r}\frac{\partial S'}{\partial t}$.

In the limit as $c \to \infty$, equation (2.8) goes over into the classical Hamilton-Jacobi equation for Newton gravitation field:

$$\frac{\partial S'}{\partial t} + \frac{1}{2m}\left(\frac{\partial S'}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = -U\,, \tag{2.9}$$

As is known, the solution of this problem leads to a closed elliptical (not precession) satellite orbit around the spherical central body.

From this it follows that the inclusion only of Lorentz time dilation into the free Hamilton-Jacobi equation leads to the Kepler problem in non-relativistic theory of gravitation.

Note also that equation (2.9) is a consequence of the L-invariant HJE with the Newton potential field:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + U\right)^2 - \left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - \frac{1}{r^2}\frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2\,, \tag{2.10}$$

Thus, the equations (2.6), (2.9) and (2.10) are equivalent from point of view of their results.

2.2. The equation of motion of a particle in a gravitational field with the Lorentz time dilation and length contraction

Now in order to take into account the length contraction effect along with the effect of time dilation, we will use the Hamilton-Jacobi equation (2.3) in form:

$$\left(\frac{\partial S}{\partial \tilde{t}}\right)^2 - c^2\left(\frac{\partial S}{\partial \tilde{r}}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^2\,, \tag{2.11}$$

Substituting in (2.11) not only $d\tilde{t} = dt\sqrt{1-\beta^2}$, but also $d\tilde{r} = dr\big/\sqrt{1-\beta^2}$, we obtain:

$$\frac{1}{1-\beta^2}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(1-\beta^2\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^4\,, \tag{2.12}$$

Taking into account that in our theory $1-\beta^2 = 1 - r_s/r$, we obtain from (2.12) the well-known Hamilton-Jacobi equation for general relativity in the case of the Schwarzschild-Droste metric (Schwarzschild, 1916; Droste, 1917):

$$\frac{1}{1-\frac{r_s}{r}}\left(\frac{\partial S}{\partial t}\right)^2 - c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2 - \frac{c^2}{r^2}\left[\left(\frac{\partial S}{\partial \theta}\right)^2 + \frac{1}{\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2\right] = m^2c^4\,, \tag{2.13}$$

We found above that the term $\frac{1}{1-r_s/r}\left(\frac{\partial S'}{\partial t}\right)^2$ (which contains the Lorentz time dilation effect) in the classical approximation leads to the equation of motion with Newton's gravitational energy.

From this it follows that the precession of the orbit ensure the introduction of an additional term $c^2\left(1-\frac{r_s}{r}\right)\left(\frac{\partial S}{\partial r}\right)^2$.

As is known, the Kepler problem solution, based on this equation, gives an additional term in the energy, which is missing in Newton's theory:

$$U(r) = -\frac{\gamma_N m M_s}{r} + \frac{M^2}{2mr^2} - \frac{\gamma_N M_s M^2}{c^2 m r^3}\,, \qquad (2.14)$$

which is responsible for the precession of the orbit of a body, rotating around a spherically symmetric stationary center. From the above analysis it follows that the appearance of this term is provided by Lorentz effect of the length contraction.

As is well known (Landau and Lifshitz, 1971), the solutions of this equation disclose three well-known effects of general relativity, well confirmed by experiment: the precession of Mercury's orbit, the curvature of the trajectory of a ray of light in the gravitational field of a centrally symmetric source and the gravitational frequency shift of EM waves.

Chapter 10 . The solution of non-cosmological problems in framework of LIGT

In this chapter in the framework of LIGT we consider the problems arising in the description of the test particle motion in a gravitational field not only of a stationary source, but also of a moving source. We will show that the solution for the moving body is connected with the solution for the fixed body on the basis of the Lorentz transformations.

We have seen that in the framework of LITG, as well as in GR, the metric tensor defines the calculation of relativistic amendments. The distinction between GR and LITG lies in the difference of metric tensors in the first and the second cases.

Here, we will use the Lenz approach for obtaining the corresponding square of the interval, which is also convenient for solution of other tasks of the gravitation theory within the framework LITG.

Assume that only one compact spherically symmetric mass exists in the Universe and that space-time is asymptotically characterized (at the spatial infinity) by the pseudo-Euclidean metric (the square of infinitesimal interval):

$$(ds)^2 = ds_\mu ds^\nu = H_\mu H_\nu x^\mu x^\nu \;, \qquad (1.1)$$

where $\mu,\nu = 0,1,2,3$, H_μ are the Lamé coefficients or scale factors (conditionally accepting here $H_0 = c$, where c is velocity of light), and $H_\mu H_\nu = g_{\mu\nu}$ (the summation is done over μ,ν).

In spherical coordinates it can be given as

$$(ds')^2 = (cdt)^2 - (d\vec{r})^2 - r^2 d\theta^2 - r^2 \sin^2\theta (d\varphi)^2\,, \qquad (1.2)$$

It is easier to operate on linear differential forms (linear element of interval) rather than on separate components of square of interval. These are defined as

$$ds_\mu = H_\mu x^\mu \;, \qquad (1.3)$$

In our case

$$ds_0' = cdt,\; ds_1' = dr,\; ds_2' = rd\theta,\; ds_3' = r\sin\theta d\varphi\,, \qquad (1.4)$$

so that for a distant observer, the coordinates t, r, θ, φ have the sense of standard spherical coordinates (and can for instance be measured by standard methods).

1.1. The Lenz solution of the Kepler problem

It turns out that it is sufficient to take into account the two Lorentz effects: time dilation and lengthening distances. We present the W. Lenz slution, following literally to A. Somerfield, who published it in his book (Sommerfeld, 1952)

"We will show, that this equivalence principle suffices for the elementary calculation of the $g_{\mu\nu}$ in a specific case (on the basis of an unpublished paper of W. Lenz, 1944).

Consider a centrally symmetric gravitational field, e.g. that of the sun, of mass M, which may be regarded as at rest. Let a reference frame (box) K' fall in a radial direction toward M. Since it falls freely, K' is not aware of gravitation (as the consequence of the equivalence principle, i.e., $m_{grav} = m_{inert}$) and therefore carries continuously with itself the Euclidean metric valid at infinity ∞. Let the coordinates measured within it be x_∞ (longitudinal, i.e. in the direction of motion), y_∞, z_∞ (transversal), and t_∞. K' arrives at the distance r from the sun with the velocity υ. υ and r are to be measured in the reference frame K of the sun, which is subject to gravitation. In it we use as coordinates r, ϑ, φ, and t. Between K' and K there exist the relations of the special Lorentz transformation, where K' plays the role of the system "moving" with the velocity $\upsilon = \beta c$, K that of the system "at rest".

Since the time and space scales are essentially the basis for the frame relative to which the measurements are done the freely falling basis carried by the observer from infinity is related to the basis of reference frame (the one the observer passes at a given instant) 8s follows:

$$ds'_0 = dt\sqrt{1-\beta^2} \quad \text{(Lorentz dilatation),} \tag{1.5}$$

$$ds'_1 = \frac{dr}{\sqrt{1-\beta^2}} \quad \text{(Lorentz contraction),} \tag{1.5'}$$

$$\left.\begin{aligned} ds'_2 &= r\,d\vartheta \\ ds'_3 &= r\sin\vartheta\, d\varphi \end{aligned}\right\} \text{(Invariance of the transversal lengths)} \tag{1.5''}$$

Hence the Euclidean world line element

$$ds^2 = ds'^2_0 + ds'^2_1 + ds'^2_2 + ds'^2_3, \tag{1.6}$$

passes over into

$$ds^2 = -c^2 dt^2 (1-\beta^2) + \frac{dr^2}{1-\beta^2} + r^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right), \tag{1.7}$$

The factor $\left(1-\beta^2\right)$, which occurs here twice, is meaningful so far only in connection with our specific box experiment. In order to determine its meaning in the system of the sun we write down the energy equation for K, as interpreted by an observer on K. Let m be the mass of K', m_0 its rest mass . The equation then is:

$$\left(m - m_0\right)c^2 - \frac{\gamma_N mM}{r} = 0 \;, \tag{1.8}$$

At the left we have the sum of the kinetic energy and of the (negative) potential energy of gravitation, i.e.. T + V = 0. The energy constant on the right was to be put equal to zero since at infinity ∞ $m = m_0$ and $r = \infty$. We have computed the potential energy from the Newtonian law, which we shall consider as a first approximation. We divide (1.8) by mc^2 and obtain then, since $m = m_0\sqrt{1-\beta^2}$:

$$1-\sqrt{1-\beta^2}=\frac{\alpha}{r},\ \ \alpha=\frac{GM}{c^2}=\frac{\kappa M}{8\pi}, \tag{1.9}$$

where κ is the Einstein constant. It follows from (1.9) that

$$\sqrt{1-\beta^2}=1-\frac{\alpha}{r},\quad 1-\beta^2\approx 1-\frac{2\alpha}{r}, \tag{1.10}$$

From (1.7) we have:

$$ds^2=-c^2dt^2\left(1-\frac{2\alpha}{r}\right)+\frac{dr^2}{1-\frac{2\alpha}{r}}+r^2\left(d\vartheta^2+\sin^2\vartheta d\varphi^2\right), \tag{1.11}$$

This is the line element derived by K. Schwarzschild from the GTR equation. In Eddington's presentation the 40 components $\Gamma^{\sigma}_{\mu\nu}$ of the gravitational field are computed and (1.11) is shown to be the exact solution of the ten equations contained in the GTR equation. For the single point mass it is completely described by the four coefficients of the line element (1.11) and the vanishing of the remaining $g_{\mu\nu}$".

The difference between the Lenz approach and GTR is that in the first is not used the hypothesis about geometrical origin of gravitation.

Perhaps the only book, in which the authors, to obtain the results of GTR, have used the Lenz approach, is the review of the problems of gravitation in book (Vladimirov et al, 1987)

In this book, along with the Schwarzschild solution, by means of Lenz method are obtained the solutions of Lense-Thirring and Kerr for the metric around a rotating body, and solutions of Reissner-Nordstrom and Kerr-Newman, when this source has an electric charge.

These solutions we will present below. To begin with it is worth discussing some general properties of rotation (Vladimirov et al, 1987).

1.2. Gravitational fields around rotating source

In order to describe the rotation of a rigid body an angular velocity Ω is introduced in addition to the conventional (linear) velocity V of a point of the body, because the angular velocity is constant for a rigid rotation, whereas the linear velocity of any point of the body is proportional to the distance between the point and the axis of rotation.

The relationship between angular and linear velocities in cylindrical coordinates is

$$V=\Omega\rho, \tag{1.12}$$

and in spherical coordinates (here $\rho=r\sin\theta$).

$$V=\Omega r\sin\theta, \tag{1.13}$$

However, a body can rotate not as a rigid one (for example, Jupiter's atmosphere rotates with different angular velocities at different latitudes as a result having different periods of rotation). The rotation period is related to angular velocity thus: $T=2\pi/\Omega$. Hence the angular velocity may depend on position (coordinates) of point.

A reference frame may be rotating, too; though a rigid body rotation is even less natural for such a system than a rotation with different angular velocities at different points. Also, if a reference frame extended to infinity could rotate as a rigid body, that is, with a constant angular velocity Ω, then a linear velocity at a finite distance from its axis (on a cylinder $\rho=c/\Omega$) would reach the velocity of light c, and outside of this "light cylinder" would surpass it. Obviously, this kind of reference frame is impossible to simulate for any material bodies, therefore, the angular velocity of any realistic reference frame must change with distance from the axis. The slowdown must not be less than inversely proportional to that distance. But there should be a domain, well within the light cylinder, where the reference frame would rotate as a rigid body.

A rotating physical body possesses an angular momentum L as a conserved characteristic, which in certain respects is related to energy and momentum, which are also subject to the conservation laws.

In Newton's theory, mass (or energy, divided by the velocity of light squared) is the source of a gravitational field, while linear and angular momenta have no such a role. In the GR, however, a gravitational field is generated by a combination of distributions of energy, and linear and angular momenta, and the stress, too. Let us examine, e.g., the angular momentum of an infinitely thin ring (which, however, has a finite mass), rotating around its axis. This angular momentum is a vector which is directed along the axis of rotation and has an absolute value of

$$L = M_r VR = M_r R^2 \Omega = I\Omega \ , \tag{1.14}$$

where M_r is the mass of the ring, V is its linear velocity, Ω is its angular velocity, R is the radius of the ring and $I = M_r R^2$ is moment of inertia.

1.2.1. *The satellite motion around rotational Earth*

In the real case, we have to evaluate the effect of rotation of the Earth to the satellite and to show that it is associated with the angular moment of the Earth.

Here we will use the work of R. Forward (Forward, 1961), who, following to the work of Moeller (Moeller, 1952), presented an analogy between electromagnetism and gravitation, which allows calculation of various gravitational forces by considering the equivalent electromagnetic problem.

When the analogy is carried out and all the constants are evaluated, we obtain an isomorphism between the gravitational and the electromagnetic quantities.

First we need to know the gravi-rotational field of the earth. From Smythe (Smythe, 1950) we find an expression for the external magnetic field produced by a ring current i at a latitude $\theta = \alpha$ on a spherical shell of radius R. By transforming the magnetic quantities in gravitational quantities, we obtain an expression for the gravi-rotational field of a rotating massive ring with mass current i_m:

$$P_\theta = \frac{-\eta\, i_m \sin\alpha}{2R} \sum_{n=1}^{\infty} \frac{1}{(n+1)} \left(\frac{R}{r}\right)^{n+2} \cdot P_n^1(\cos\alpha) P_n^1(\cos\theta),$$

Since it is assumed that superposition is valid, we can construct the gravi-rotational field of a solid spinning body by integrating over the volume:

$$P_\theta = \frac{-\eta\, \Omega \sin\theta}{8\pi\, r^2} \int_V \left[\mu(\alpha, R) R^2 \sin^2\alpha\right] R^2 \sin\alpha \, d\alpha \, d\phi \, dR + higher\ multipoles,$$

Since $r\sin\theta$ is the distance from the axis of rotation to the mass element, we see that the integral is merely the **moment of inerti**a I of the body (Earth). Thus, in general the rotational field of any rotation body is approximately:

$$P_\theta = \frac{-\eta\, I\Omega \sin\theta}{8\pi\, r^2} = \frac{-\eta L \sin\theta}{8\pi\, r^2},$$

Similarly, it can be show that:

$$P_r = \frac{-\eta\, I\Omega \cos\theta}{4\pi\, r^2} = \frac{-\eta L \cos\theta}{4\pi\, r^2},$$

But the Forward approach does not allow to compare the results of his calculation with metrics Lense-Thirring and Kerr. That is why we will try to obtain a metric which describes the gravitational field around the rotating ring using a technique like the above W. Lenz technique (Vladimirov et all, 1987).

1.3. The Lense-Thirring metrics in framework of LIGT

To account for rotation effects (Vladimirov et al, 1987) using the equivalence principle, we start from a rotating reference frame (it rotates not as a rigid body, but so that at large distances the effect of the rotation weakens; the nature of the frame rotation will be examined at the final stage of this analysis). In this rotating frame, we let a box with an observer (test particle) fall towards the gravitating centre and in the box we take into account the slowing-down of the clock and the contraction of the scales in the direction of the fall. Assume that the box falls radially in the rotating frame. Then, we shall get back to the initial, non-rotating reference frame and consider the result.

We begin with the Euclidean space-time in which we introduce spherical coordinates in a non-rotating frame; we assume the basis is, thus relative to it the flat space-time metric will be

$$(ds')^2 = (cdt)^2 - (d\vec{r})^2 - r^2 (d\theta)^2 - r^2 \sin^2\theta (d\varphi)^2, \qquad (1.15)$$

A transition to a non-uniformly rotating reference frame is done by locally applying Lorentz transformations so that every point has its own speed of motion directed towards an increasing angle φ. The absolute value of this velocity is a function V which depends, generally speaking, on the coordinates r and θ: $V = V(r,\theta)$.

Such a local Lorentz transformation is not equivalent to the transformation of the coordinates in the domain studied (in practice this domain is the whole of space) but is limited only to the transformation of the basis at each point.

Thus, we have:

$$\begin{aligned} d\tilde{s}_0 &= \left(ds'_0 - \frac{V}{c} ds'_3 \right) \Big/ \sqrt{1 - V^2/c^2}\,,\ d\tilde{s}_1 = ds'_1 \\ d\tilde{s}_2 &= ds'_2\,,\ d\tilde{s}_3 = \left(ds'_3 - \frac{V}{c} ds_0' \right) \Big/ \sqrt{1 - V^2/c^2} \end{aligned}, \qquad (1.16)$$

Since the motion is assumed to be slow, we will henceforth ignore the value V^2/c^2 in comparison with unity.

Now let the box with the observer be released from infinity. In this case we can write a new basis in which time has slowed down, and the lengths in radial direction have shortened. This is equivalent to the substitution of the ds'_0 in (1.5) by the basis linear elements from (1.16)

$$\begin{aligned} & ds''_0 = d\tilde{s}_0 \sqrt{1 - \upsilon^2/c^2}\,,\ ds''_1 = d\tilde{s}_1 \Big/ \sqrt{1 - \upsilon^2/c^2}\,, \\ & ds''_2 = d\tilde{s}_2\,,\ ds''_3 = d\tilde{s}_3 \end{aligned} \qquad (1.17)$$

Thus, we have assumed that the observer makes his measurements in the rotating frame and notices the relativistic changes in his observations. (We can not neglect by the value of υ^2/c^2 in comparison with unity; see the derivation of the Schwarzschild metric).

Now let us do the reverse transformation to the non-rotating reference frame by applying Lorentz transformations (inverse to (1.16)) to the basis (1.17):

$$\begin{aligned} ds_0 &= ds''_0 + \frac{V}{c} ds''_3\,, ds_1 = ds''_1\,, \\ ds_2 &= ds''_2\,, ds_3 = ds''_3 + \frac{V}{c} ds''_0 \end{aligned}, \qquad (1.18)$$

We now insert into (1.18) the ds''_μ $\omega'^{(\alpha)}$ basis, which is expressed in terms of the $d\tilde{s}_\mu$ from (1.17), and then write this expression in terms of the ds'_μ from (1.16), after a few manipulations, we obtain:

$$ds^2 = \left(1 - \frac{\upsilon^2}{c^2}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{\upsilon^2}{c^2}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{2V\upsilon^2}{c^2} r \sin^2\theta\, d\varphi dt, \qquad (1.19)$$

The principle of correspondence with Newton's theory gives $\upsilon^2 = 2\gamma_N M_s / r$.

It remains to clear out the dependence V from r and θ. On the one hand, according to equation (1.13), $V = \Omega r \sin\theta$. However, it is clear that the reference frame can not rotate as a solid body. Therefore, the angular velocity Ω must be a function of the point. Since the reason for the existence of this velocity is eventually the rotation of central mass, we can assume that it decreases in all directions away from the center. For a rough estimate, it can be assumed that Ω depends only on r. Then from (1.14) :

$$L = M_r \Omega(R) R^2, \qquad (1.20)$$

(because the ring lies in a plane $\theta = \pi/2$). If we now require that the field does not depend on the choice of the radius of the ring, but only on its angular momentum, it is natural to take for a function Ω the expression

$$\Omega = (L/M_r) r^{-2}, \qquad (1.21)$$

Let us introduce the notation for "parameter Kerr" $a = L/M_r$, so that

$$V = \frac{a \sin\theta}{r}, \qquad (1.22)$$

The substitution of the values V and expression for υ^2/c^2 into the formula (1.19) finally gives the metric of Lense-Thirring:

$$ds^2 = \left(1 - \frac{r_s}{r}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r} \sin^2\theta\, d\varphi dt, \qquad (1.23)$$

$r_s = \dfrac{2\gamma_N M_s}{c^2}$ is gravitational radius, M_s is a mass of central body (a field source), $a = L/M_s$ represents the angular momentum of the source per unit mass; more precisely stated, it is the projection of the angular momentum three-vector on the direction of the rotation axis, divided by the mass).

The obtained metric is the approximate metric in the sense that the dimensionless quantities $km/c^2 r$ and $a/c\,r$ are considered as small values of first order, and we have neglected their higher degrees. But at the beginning, for simplicity we have made the assumption about a coordinate system as the normal spherical coordinate system, which is, of course, not suitable for a rotating body because its gravitational field should have the symmetry of an oblate spheroid.

1.4. The Kerr metric in framework of LIGT

Now (Vladimirov et al, 1987), let us try to obtain a metric which describes the gravitational field around the rotating ring using a technique like the one we used above for the Lense-Thirring metric.

To do this, at first we must pass to the ellipsoidal coordinates, and secondly, use Newtonian potential source (ring). If in accordance with what has been said we minimally modify the formula (1.22) without discarding any terms (of the type V^2/c^2 in (1.18), we can directly come to the exact Kerr metric.

We will begin with (1.16):

$$d\tilde{s}_0 = \left(ds'_0 - \frac{V}{c} ds'_3 \right) \Big/ \sqrt{1 - V^2/c^2}\,,\ d\tilde{s}_1 = ds'_1$$
$$d\tilde{s}_2 = ds'_2\,,\ d\tilde{s}_3 = \left(ds'_3 - \frac{V}{c} ds'_0 \right) \Big/ \sqrt{1 - V^2/c^2}\,, \qquad (1.16)$$

and then pass to (1.17)

$$ds''_0 = d\tilde{s}_0 \sqrt{1 - \upsilon^2/c^2}\,,\ ds''_1 = d\tilde{s}_1 \Big/ \sqrt{1 - \upsilon^2/c^2}\,,$$
$$ds''_2 = d\tilde{s}_2\,,\ ds''_3 = d\tilde{s}_3\,, \qquad (1.17)$$

Thus, we have assumed that the observer makes his measurements in the rotating frame and notices the relativistic changes in his observations. Now let us do the reverse transformation to the nonrotating reference frame by applying Lorentz transformations (inverse to (1.16)) to the basis (1.17):

$$ds_0 = \left(ds''_0 + \frac{V}{c} ds''_3 \right) \Big/ \sqrt{1 - V^2/c^2}\,, ds_1 = ds''_1\,,$$
$$ds_2 = ds''_2\,, ds_3 = \left(ds''_3 + \frac{V}{c} ds''_0 \right) \Big/ \sqrt{1 - V^2/c^2}\,, \qquad (1.24)$$

We now insert into (1.24) the basis ds''_μ, which is expressed in terms of the $d\tilde{s}_\mu$ from (1.17), and then write this expression in terms of the ds'_μ from (1.16). We postulate, as we did previously, that the resulting basis (1.24) remains orthonormalized. A few manipulations yield:

$$ds^2 = \left(1 - \frac{\upsilon^2}{c^2 - V^2} \right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{\upsilon^2}{c^2} \right)} - r^2 d\theta^2 - \left(1 - \frac{\upsilon^2 V^2}{c^2 - V^2} \right) r^2 \sin^2\theta\, d\varphi^2 +$$
$$+ \frac{2V\upsilon^2}{c^2 - V^2} r \sin^2\theta\, d\varphi dt\,, \qquad (1.25)$$

Here the Newton's potential φ_N represents a solution of the Laplace equation, though under the new symmetry, that is rotational and not spherical. Therefore it is now worth considering oblate spheroidal coordinates in flat space. These coordinates, ρ, θ, and φ are defined as

$$x + iy = (\rho + ia) e^{i\varphi} \sin\varphi, z = \rho\cos\theta$$
$$\frac{x^2 + y^2}{\varphi^2 + a^2} + \frac{z^2}{\rho^2} = 1, r = \sqrt{x^2 + y^2 + z^2}\,, \qquad (1.26)$$

We know that $\Delta(1/r) = 0$ when $r \neq 0$, and this equality holds under any translation of coordinates. Let this translation be purely imaginary and directed along the z axis, i.e., $x \to x, y \to y,$ and $z \to z - ia/c$. Then we easily find that $cr \to \left(c^2 r^2 - a^2 - 2iacz \right)^{1/2} = c\rho - ia\cos\theta$. From here the expression for Newton's potential follows,

$$\varphi_N = -\frac{\gamma_N M_s}{c^2 r} \to \varphi_N = \frac{\gamma_N M_s}{c} \mathrm{Re} \frac{1}{c\rho - ia\cos\theta} = \frac{\gamma_N M_s \rho}{c^2 \rho^2 + a^2 \cos^2\theta}\,, \qquad (1.27)$$

since the Laplace equation is satisfied simultaneously by both the real and imaginary parts of the potential. Hence we can get with the help of $\upsilon^2 = 2\gamma_N M_s / r = 2\varphi_N$:

$$\frac{\upsilon^2}{c^2 - V^2} = \frac{\gamma_N M_s \rho}{c^2 \rho^2 + a^2 \cos^2\theta}\,, \qquad (1.28)$$

We determine the velocity V using a model of rotating ring of some radius ρ_0 for the source of the Kerr field, this ring being stationary relative to the rotating reference frame (1.17).

On the one hand, $V = \Omega\left(x^2 + y^2\right)^{1/2} = \left(\rho^2 + a^2/c^2\right)^{1/2}\Omega\sin\theta$ corresponds to the relation (1.13) . On the other hand, it is clear that the reference frame cannot rotate as a rigid body, otherwise the frame wouldn"t be extensible beyond the light cylinder as we dropped our box from infinity. Therefore the angular velocity Ω has also to be a function of position.

The ring lies naturally in the equatorial plane, so that its angular momentum is

$$L = mV\sqrt{\left(\rho^2 + a^2/c^2\right)}, \quad \left(\rho = \rho_0, \theta = \frac{\pi}{2}\right)$$

We now introduce an important hypothesis which establishes a connection between the angular momentum and the Kerr parameter *a,* which is also a characteristic for spheroidal coordinates (1.26), namely we put $a = L/M_s$. These last three statements yield

$$\Omega\left(\rho = \rho_0, \quad \theta = \pi/2 = c^2 a/\left(c^2\rho_0^2 + a^2\right)\right)$$

If we now add a second hypothesis, that the field is independent of the choice of the ring radius (depending only on its angular momentum), then naturally we can get for Ω:

$$\Omega = c^2 a/\left(c^2\rho^2 + a^2\right)$$

and finally

$$V = ca\left(c^2\rho^2 + a^2\right)^{-1/2}\sin\theta, \tag{1.29}$$

It only remains for us to choose the expression for a basis ds'_μ which would correspond to the assumed rotational symmetry (i.e., to the oblique spheroidal coordinates). We may substitute the coordinates x, y and z from (1.26) into the pseudo-Euclidean squared interval, $ds^2 = cdt^2 - dx^2 - dy^2 - dz^2$, hence getting a quadratic form with a non-diagonal term. This term, which contains $d\rho d\varphi$, can be excluded by a simple change of the azimuth angle: $d\varphi \to d\varphi + ca\left(c^2\rho^2 + a^2\right)^{-1} d\rho$ thus leading to a diagonal quadratic form. If now the square roots of the separate summands are taken, we get the final form of the initial basis ds'_μ:

$$\begin{aligned} &ds'_0 = c^2 dt, \quad ds'_1 = \sqrt{\left(c^2\rho^2 + a^2\cos^2\theta\right)/\left(c^2\rho^2 + a^2\right)}d\rho, \\ &ds'_2 = \sqrt{\rho^2 + a^2\cos\theta/c^2}\,d\theta, \\ &ds'_3 = \sqrt{\rho^2 + a^2/c^2}\,\sin\theta d\varphi, \end{aligned} \tag{1.30}$$

A mere substitution of these expressions into (1.25) yields the standard form of the Kerr metric in terms of the Boyer-Lindquist coordinates,

$$\begin{aligned} ds^2 = &\left(1 - \frac{r_s r}{\rho^2}\right)c^2 dt^2 - \frac{\rho^2}{\Delta}dr^2 - \rho^2 d\theta^2 - \\ &-\left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2}\sin^2\theta\right)\sin^2\theta\, d\varphi^2 + \frac{2r_s r a}{\rho^2}\sin^2\theta\, d\varphi dt \end{aligned} \tag{1.31}$$

where we have introduced the notation

$$\Delta = r^2 - r_s r + a^2, \quad \rho^2 = r^2 + a^2\cos^2\theta, \quad a = L/M_S, \tag{1.32}$$

The resulting metric is a solution of Einstein's gravitational field equations, and the method does give some hint as to how to understand the Kerr metric and its sources, and it lets us look at the structure of the latter.

If we assume in the calculations that $(V/c)^2 << 1$, thus dropping the corresponding terms in (1.16) and (1.24). This is the assumption of slow rotation (more exactly, of the smallness of L, the angular momentum of the source) and it leads to $V = a\sin\theta/r$ instead of (1.29). Thus instead of the Kerr metric (1.31) we will get the approximate the Lense-Thirring metric:

$$ds^2 = \left(1-\frac{r_s}{r}\right)c^2dt^2 - \frac{dr^2}{\left(1-\frac{r_s}{r}\right)} - r^2\left(\sin^2\theta\, d\varphi^2 + d\theta^2\right) + \frac{4\gamma_N M_s a}{c^2 r}\sin^2\theta\, d\varphi dt,$$

(we have written in it r instead of ρ and taken into account the approximate sense of the expressions).

1.5. The Reissner-Nordstroem metric in framework of LIGT

Besides the Schwarzschild (non-rotating) and Kerr (rotating) black holes, which have no electric charge, we also have exact solutions of Einstein's equations when the source has an electric charge. These solutions are referred to as the Reissner-Nordstroem and Kerr-Newman metrics..

The Reissner-Nordstrem and Kerr-Newman black holes are mostly of academic interest, but they are important for the theory.

The Reissner-Nordstroem metric can be "derived" (Vladimirov et al, 1987) using the same technique we used for the Schwarzschild metric. The only difference between the two is that the Newtonian potential for a point mass should be replaced by a solution of the Poisson equation for a distributed source,

$$\Delta\varphi_N = 4\pi\gamma_N \rho_m, \qquad (1.33)$$

The point mass remains at the origin and yields the same potential ($-\gamma_N m/r$), but the electrostatic source has a trick of its own. In Newton's theory, the ρ_m on the right-hand side of equation (1.33) is usually interpreted as the density of mass (or energy, since from special relativity mass and energy are equivalent). That was the case, however, only for non-relativistic matter, whereas an electromagnetic, or even an electrostatic field is always relativistic though it might appear at rest. It can be rigorously shown that for such a field we have to take instead of $\rho_m c^2$ *double* the energy density

$$\Delta\varphi_N = 8\pi\gamma_N\, w/c^2, \qquad (1.34)$$

The density of the energy of a Coulomb electrostatic field (i.e. of $E = q/r^2$) is

$$w = q^2/8\pi r^4, \qquad (1.35)$$

and the Laplace operator Δ in a spherically symmetric case (when $\varphi_N \neq \varphi_N(\varphi,\theta)$) takes the form

$$\Delta\varphi_N = \frac{1}{r}\frac{d^2}{dr^2}\left(r\varphi_N\right), \qquad (1.36)$$

Bearing this in mind, we have, as a complete solution of equation (1.34), the Newtonian potential

$$\varphi_N = -\gamma_N m/r + \gamma_N q^2/2c^2r^2, \qquad (1.37)$$

which enters the 00-component of the metric tensor in the form:

$$g_{00} = 1 + 2\varphi_N/c^2 = 1 - \gamma_N m/r + \gamma_N q^2/2c^2r^2, \qquad (1.38)$$

Hence, by doing exactly what we did in above sections, we finally obtain the Reissner-Nordstroem field in the form:

$$ds^2 = \left(1 - \frac{r_s}{r} + \frac{\gamma_N q^2}{c^4 r^2}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r} + \frac{\gamma_N q^2}{c^4 r^2}\right)} - r^2 \left(\sin^2 \theta \, d\varphi^2 + d\theta^2\right), \tag{1.39}$$

Thus, within the framework of the non-geometric gravitational theory - LITG – we have obtained in framework of LIGT all the exact non-cosmological solutions of the equations of GR, which were verified experimentally. The objective of our next chapter will be to receive in framework of LIGT the cosmological solutions of the equations of GR.

Chapter 11 . The cosmological solutions in the framework of LIGT

1.0 Cosmological solutions of GR

All solutions of the equations of General Relativity concerning the movement of single massive bodies relative to each other (planets, stars, etc.) and which are tested experimentally, were obtained by us within the framework of LIGT in the previous chapters.

In addition to non-cosmological solutions exist solutions that are interpreted as cosmological, that is, related to the entire Universe.

At the moment, as a tested solution is considered the solution, obtained by means of the postulates of the homogeneity and anisotropy of Universe, jointly with the results of general relativity and thermodynamics.

The question of the legality of such description of the Universe that contains, along with an almost infinite number of stars, planets and smaller bodies also an almost infinite number of other objects (microwave cosmic background, gases, dust, supernovae, neutron and many other types of stars, different types of galaxies and so forth.), will be left outside the limits of this chapter. Also, we will not consider the contribution of electromagnetic field (in particular, its lower state - physical vacuum) and elementary particles, although their presence in the universe is primary. Thus, according to the Hans Alfven theory (Alfven, 1942; Alfven and Arrhenius, 1976) (for which he received the Nobel Prize), electric and magnetic fields play a crucial role in the formation of the solar and other star systems.

Let us only note that direct experimental proofs of correctness of cosmological postulates and solutions do not exist (Baryshev, 1995). However, under the current cosmological paradigm are accepted interpretations of observational data, which was recognized as confirmation of abovementioned solutions.

2.0. Formulation of the problem in LIGT

It is obvious that if we want to fully confirm the equivalence of general relativity and LIGT, it seems necessary to obtain the corresponding cosmological solution in framework of LIGT. The present article will be dedicated to this subject. At the same time, our paper bears a feature which the Chapter 10 also bore. We have practically no need to present this solution since it has long been known, and is even taken into consideration at the pedagogical level.

The basis upon which the solution of Friedman is built (Dullemond et al . 2011, Ch. 4) are the two postulates mentioned above about the state of the universe. Besides that, it was proven by Robertson and Walker that the only one choice of metric exists, that satisfies these postulates.

Basic cosmological solutions of general relativity (for three types of curvature of space-time Universe) were obtained by Friedman (1922). Their derivation is reported in numerous textbooks, lectures and monographs; See, for example. (Bogorodsky, 1971; Dullemond et al. 2011, Ch. 4.).

2.2. The Robertson-Walker Universe metric in framework of LIGT

Since Newton's equation is a first approximation of the equations of gravitation LIGT, you can expect that the results of Friedman's (at least to a first approximation) can be derived from Newton's theory of gravitation.

Such solutions were indeed found in 1934 (Milne, 1934; McCrea and Milne, 1934). Moreover, it appears that these solutions are the same as the solutions of Fridman. Later they were refined (Milne, 1948; Krogdahl, 2004).

"*A Lorentz-invariant cosmology based on E. A. Milne's Kinematic Relativity is shown to be capable of describing and accounting for all relativistic features of a world model without space-time curvature. It further implies the non-existence of black holes and the cosmological constant. The controversy over the value of the Hubble constant is resolved as is the recent conclusion that the universe's expansion is accelerating. "Dark matter" and "dark energy" are possibly identified and accounted for as well*" (Krogdahl, 2004).

A modern formulation of this solution in Russian can be found, for example, in the presentation of the expert in the field of general relativity, academician Ya.B.Zeldovich; see Appendix I to the book (Weinberg, 2000), p. 190, titled "The classical non-relativistic cosmology" , who note here:

"All the calculations could have been made not only in the nineteenth century, but also in the eighteenth century".

The lecture 2 from the modern cosmology course ((Dullemond et al. 2011, Ch. 2) is dedicated to this subject.

Chapter 12. Quantization of gravitation theory

Numerous attempts to quantize general relativity, which are continued for almost a century, have not led to a positive result.In GR the quantization is only possible in the linearized theory, for example, in the form of GEM. But there are also some difficulties.

Is there a quantum LITG and, if so, how to build it?

2.0. Statement of the problem within LITG

"*Instead of imagining space-time as being warped by mass and energy, one can speak of a classical spin-2 graviton field in flat space-time that generates gravitation. Although we don't know yet how to quantize this field, we can think of it in a way similar to how we think of electromagnetism being mediated by photons. And just as a* $1/r^2$ *Coulomb force generates magnetism when the finite speed of the mediating photon is taken into account, a* $1/r^2$ *Newtonian gravitational force generates "gravito-magnetism" when the finite speed of the mediating graviton is taken into account. Magnetism is fundamentally an electric-force effect, and gravity must have some analogous "magnetic" force, meaning a gravitational force proportional and perpendicular to the velocity of a test mass. Einstein showed that gravity should be non-linear, so we know that the graviton should self-interact. General relativity also implies that the graviton should be spin* 2. *The self-interaction and spin*-2 *bring us all the way to the EM equivalent of general relativity. But it may be that in most of the Universe (barring black holes, supernovae, et cetera), all you really need to know about gravitation is the electromagnetic-analogue* (Forrester, 2010).

3.0. Quantization of LIGT

In LIGT the problem of quantization of gravity is set differently than in the GR. We will show below that the quantization of LIGT is possible in principle, but does not have a sense, because in this case the classical equations of gravity coincide with the quantum ones, similar to what occurs in the quantum theory of electromagnetic field. This is facilitated by the fact that within LIGT gravity is the residual EM field.

Recall that, according to GR, the source (charge) of the gravitational field is the mass/energy. Its peculiarity lies in the fact that it has almost a sufficiently strong field only if its value is much larger than the mass/energy of the elementary particles (let's call this gravitational charge "effective"). Therefore, because of its value, it can be difficult to characterize by means of the quantum parameters of an elementary particle.

In addition, gravitational charge may have angular momentum (let us say, spin), but its quantization also does not make sense because of the magnitude of the effective charge. Therefore, from this point of view, we can not attribute the gravitational charge either to bosons or to fermions. At the same time it has the property of bosons: the superposition of individual masses-energies is possible and creates a new gravitational charge as the sum of mass-energy. In addition, as part of the GEM the gravitational radiation field is considered as composed of bosons - gravitons: particles with spin 2.

If we leave aside the value of the spin of the graviton, all this corresponds to the consequences of LIGT. Since we can conditionally say that the basis of LIGT is EM theory of the "massive photon" (see chapter 2), then we can assume that it can be the basis of the quantum theory of gravity. To some extent this is true. But such a theory is almost meaningless because of the size of the effective gravitational charge.

However, these quantum equations can be used because they coincide with the classical ones (as is the case for all bosons). This means that, having classical equations of gravity, we are, in fact, already using quantum equations of gravity:

"…*in the situation in which we can have very many particles in exactly the same state, there is possible a new physical interpretation of the wave functions. The charge density and the electric current can be calculated directly from the wave functions and the wave functions take on a physical meaning which extends into classical, macroscopic situations.*

Something similar can happen with neutral particles. *When we have the wave function of a single photon, it is the amplitude to find a photon somewhere… There is an equation for the photon wave function analogous to the Schrödinger equation for the electron.* ***The photon equation is just the same as Maxwell's equations for the electromagnetic field,*** *and the wave function is the same as the vector potential* ***A***. *The wave function turns out to be just the vector potential. The quantum physics is the same thing as the classical physics because photons are noninteracting Bose particles and many of them can be in the same state — as you know, they like to be in the same state. The moment that you have billions in the same state (that is, in the same electromagnetic wave), you can measure the wave function, which is the vector potential, directly.*

Now the trouble with the electron is that you cannot put more than one in the same state" (Feynman, Leighton and Sands, 1964).

According to our approach, the words of Feynman in bold, can be attributed to gravitation after some adjustments:

"***The graviton equation is just the same as Maxwell's equations for the gravitation field*** "

Therefore, it is obvious that the quantization of gravity has no practical value.

In general relativity the existence of the graviton and the value of its spin is uncertainty, since the quantum GR does not exist. If we will consider the linear approximation of GR (e.g., GEM) as reliable enough, then, because the graviton is a boson, it makes no sense to speak about its spin; in this case it is enough to speak about the classical gravitational waves.

From a formal point of view we can accept the existence of the graviton in LITG. Moreover, it can be assumed that the graviton should have spin 2, not 1 as a photon, since neutral waves can only be radiated by a system of quadrupole gravitational charges (see. Ivanenko and Sokolov, 1949). But, as we know, it does not make sense to quantize all the waves. In particular, the quantization of low-energy (long) EM waves does not make sense. The energy of gravitational waves in many orders of magnitude is lower, than of EM waves. As it is impossible to prove the existence of the graviton experimentally, it hardly makes any sense to discuss further.

(Note that, as we know, (Akhiezer and Berestetskii. 1965; Fermi, 1950; 1951), equally with the wave function of the electromagnetic field in the form of the vector potential, the wave function in form of the vectors of the EM field can be used, as this is accepted in NQFT).

Closing notes

This concludes our presentation of LIGT itself. It would be interesting to analyze the question of whether the Hilbert-Einstein's general relativity has some advantages over non-geometrical approach, besides the fascinating mathematical interpretation that goes beyond the usual physics. Some thoughts on this matter will be set out in independent articles.

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