The gravitation theory review

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Annotation

The most advanced theory of gravitation is so far Einstein's general relativity theory. Some of its drawbacks are the source of attempts to construct a new theory of gravitation. The main drawback of general relativity theory is its incompatibility with the modern quantum theory of elementary particles, which does not allow to create a unified theory of matter. In the general relativity the cause of the gravitational field is the curvature of space-time, which is not a material object. The question of, how non-material space-time can cause a material gravitational field is still here unanswered. Different subdisciplines of classical physics generated different ways of approaching the problem of gravitation. The emergence of special relativity further increased the number of possible approaches and created new requirements that all approaches had to come to terms with. In this book we will survey various alternative approaches to the problem of gravitation pursued around the turn of the last century and try to assess their potential for integrating the contemporary knowledge of gravitation. Here different contributions are made to the discussion about the fundamentals of different approaches to gravitation and their advantages and drawbacks.

NOTATIONS.

(In almost all instances, meanings will be clear from the context. The following is a list of the usual meanings of some frequently used symbols and conventions).

Mathematical signs	$\square$ - d'Alembertian, operator $\equiv \nabla^2 - \partial^2/\partial t^2$
$\alpha, \beta, \mu, \nu ...$ - Greek indices range over 0,1,2,3 and represent space-time coordinates, components, etc.	$g_{\mu\nu}$ - metric tensor of curvilinear space-time
$i, j, k...$ - Latin indices range over 1,2,3 and represent coordinates etc. in 3- dimensional space	$g_{\mu\nu}^{GR}$ - metric tensor of GR space-time
$\hat{\alpha}_\mu, \hat{\beta}$ - Dirac matrices	$R_{\alpha\beta\gamma\delta}$ - Riemann tensor
$\vec{A}$ - 3-dimensional vector	$R_{\alpha\beta}$ - Ricci tensor $R^{\gamma}{}_{\alpha\beta\delta}$
A^μ - 4-dimensional vector	R - Ricci scalar $\equiv R^{\alpha}{}_{\alpha}$
$A^{\mu\nu}$ - Tensor components	$G_{\alpha\beta}$ - Einstein tensor
∇ - Covariant derivative operator	$\eta_{\mu\nu}$ - Minkowski metric
$\nabla^2 \equiv \Delta$ - Laplacian	$h_{\mu\nu}$ - Metric perturbations
	$\Lambda_{\mu\nu}$ - Lorentz transformation matrix
Physical values	j_μ - Current density
u_μ - velocity	$J^{\mu\nu}$ - Angular momentum tensor
a_μ - 4-acceleration $\equiv du_\mu/d\tau$	γ_N - Newton's constant of gravitation
p_μ - 4-momentum	γ_L - Lorentz factor (L-factor)
$T^{\mu\nu}$ - Stress-energy tensor	m - mass of particle
$F^{\mu\nu}$ - Electromagnetic field tensor	M_S - mass of the star (Sun)
	$\vec{J}$, $\vec{L}$ - angular momentum
Abbreviations:	NTEP - nonlinear theory of elementary particles;
LIGT - Lorentz-invariant gravitation theory;	QED - quantum electrodynamics.
EM - electromagnetic;	HJE - Hamilton-Jacobi equation
EMTM - electromagnetic theory of matter;	SR or STR - Special Ttheory of Relativity
EMTG - electromagnetic theory of gravitation;	GR or GTR - General Theory of Relativity
SM - Standard Model;	L-transformation - Lorentz transformation
NQFT – nonlinear quantum field theory	L-invariant - Lorentz-invariant
Indexes	g - gravitational, within the framework of EMTG
e - electrical	ge - gravito-electric,
m - magnetic,	gm - gravito-magnetic
em - electromagnetic,	N - Newtonian

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Part 1. The existing approaches to derivation of GR equation

1.0. Formulation of the problem

The present paper is devoted to the review of known approaches of A.Einstein and D.Hilbert to the derivation of the equations of GR, and also of new approaches on the base of the Yang-Mills covariant generalization of Maxwell-Lorenz equations.

But we also set the additional task: to comprehend physically the derivation of the Hilbert-Einstein equation. The Hilbert method is mathematically formal and does not allow to understand the physical meaning of the results. On the other hand, in the framework of Einstein's method, the analogy with Newton's gravitational theory was used. Thus, we can hope that here it will be easier to achieve such a goal.

2.0. Hilbert’s and Einstein's approaches to derivation of equations of general relativity

The issue of building the relativistic (in sense of SR) theory of gravitation (Vizgin, 1981) was first raised in 1905 by Henri Poincare in his work "On the dynamics of electron» (Poiancare, 1905). In this article, Poincare systematically developed the L-invariant theory of gravitation, which satisfied the basic requirements of the program of construction of the L-invariant physics. Unfortunately, the path, which Poincare chose, did not found followers. Nevertheless, the idea of building the relativistic theory of gravitation captured the imagination of many well-known theorists.

As is well known, the derivation of the equations of general relativity was implemented simultaneously time, but in other way, by Einstein and Hilbert: “*At the same time as Einstein, and independently, Hilbert, formulated the generally covariant field equations*” (Pauli, 1958)

The first method (of Einstein) was based on heuristic search of generalization of Poisson's equation on the basis of mathematics of pseudo-Rimannian geometry.

The second method (of Hilbert) was mathematically quite successive and based on the axioms of the variational approach in constructing physical theories.

The remarkable and significant is that Hilbert used the variational approach on the base of a unified non-linear (*non-quantum*) electromagnetic theory of matter of Gustav Mie: “[The Hilbert] *presentation, though, would not seem to be acceptable to physicists, for two reasons. First, the existence of a variational principle is introduced as an axiom. Secondly, of more importance, the field equations are not derived for an arbitrary system of matter, but are specifically based on Mie's theory of matter*” (Pauli, 1958).

The approaches of Einstein and Hilbert are set forth in the fundamental works of these authors, as well as in numerous books and articles. Therefore we will list only the foundations and results of these approaches. (see collection of papers of classics of the theory of relativity: (Hsu et al (editors), 2005).

3.0. Heuristic Einstein's approach to derivation of GR equation

From a formal point of view (Bogorodsky, 1971) the Riemann geometrical space can be defined as field of the metric tensor $g_{\mu\nu}$ in 4-dimensional continuum in which the distance between the infinitely close points is found by means of the main quadratic forms:

$$ds^2 = g_{11}dq_1^{\ 2} + g_{12}dq_1dq_2 + \ldots = g_{\alpha\beta}dx^\alpha dx^\beta\,, \qquad (1.3.1)$$

and the angle θ between the two linear elements - by formula:

$$\cos\theta = \frac{g_{\alpha\beta} dx^{\alpha} dx^{\beta}}{ds\delta s} , \tag{1.3.2}$$

It is believed that the geometry of Riemannian space is completely determined by the basic quadratic form.

The Riemann geometry covers a wide class of spaces and includes Euclidean geometry as a simple special case.

In GR, it is postulated that the gravitational field is defined by the metric tensor $g_{\mu\nu}^{GR}$ which contains the characteristics of gravitation fiedl.

3.1. The derivation of RTG equation

In GR (Bogorodsky, 1971), the problem is posed by analogy with Newton's gravity equation in the form of the Poisson equation. Thus, it is necessary to find a theoretical relationship between the metric field $g_{\mu\nu}$, determined by the geometry of space-time, and the mass distribution, which causes this gravitational field.

Let us formulate the basic principles which the field equations must satisfy in accordance with the Einstein theory.

Principle A. The principle of general covariance, where the tensor equation, set in one reference frame, has the same form at the transition to any other reference frame.

Principle B. The field equations are sought in the form of relation between the metric tensor and the energy-momentum tensor.

Principle C. The law of conservation of energy-momentum tensor must be applied, limiting the form of the field equations.

Principle D. The correspondence principle requires that the GR in the non-relativistic limit was the gravitation theory of Newton.

The field equations that satisfy the above principles can be sought in the form

$$X_{\mu\nu} = -\kappa T_{\mu\nu} \ , \tag{1.3.3}$$

where $X_{\mu\nu}$ is the symmetric tensor of the second order, consisting of the metric tensor and satisfy relationship $X^{\alpha}_{\mu\nu\alpha} = 0$; κ is a constant. The tensor $X_{\mu\nu}$ must ensure the principle D; it necessary for the harmonization of the field equations with the results of observations.

Einstein came to solution of this problem heuristically, forming the left side (1.3.3) from $g_{\mu\nu}$ by different ways. But it can show that the derivation of the field equations, meeting all above the requirements, is an uncertain task. Therefore, further the form of the field equation was limited to an additional principle.

Principle E. Tensor $X_{\mu\nu}$ must be a linear function relative to the second derivative component of $g_{\mu\nu}$ and does not depend on the derivatives of higher orders.

This principle, taken by analogy with the Poisson equation, is purely a mathematical constraint, which does not get a physical justification. Meanwhile, it is very important and allows us to determine the form of the tensor $X_{\mu\nu}$ up to two arbitrary constants.

Note that if this additional requirement is declined, the field equations of general relativity will appear ambiguous, since they can not be inferred from the principles A-D.

For the first time (Ebner, 2006), Einstein proposes (manifestly) fully covariant field equations for the gravitational field:

$$R_{\mu\nu} = -\kappa T_{\mu\nu} \ , \tag{1.3.4}$$

where $X_{\mu\nu} = R_{\mu\nu}$ is the Ricci tensor (the minus sign on the right-hand side of (1.3.4) is conventional, depending on a sign in the definition of the Ricci-tensor).

Conservation of energy and momentum in flat Minkowski space of special relativity leads to the equation

$$\nabla_{\nu} T^{\mu\nu} = 0 \tag{1.3.5}$$

But Einstein was dissatisfied with equations (1.3.4), because $\nabla_{\nu} R^{\mu\nu} \neq 0$.

By virtue of the Bianchi identities (Synge, 1960)

$$\nabla_{\eta} R_{\lambda\mu\nu\kappa} + \nabla_{\kappa} R_{\lambda\mu\eta\nu} + \nabla_{\nu} R_{\lambda\mu\kappa\eta} = 0, \tag{1.3.6}$$

where $R_{\mu\nu\alpha\beta}$ or $R^{\rho}_{\sigma\mu\nu}$ is Riemann tensor $R^{\rho}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$, the tensor

$$G_{\mu\nu} = \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right), \tag{1.3.7}$$

satisfies the four conservation equations (or identities)

$$\nabla_{\nu} G_{\mu\nu} = 0, \tag{1.3.8}$$

These equations are important physically in connection with the conservation of momentum and energy.

Hilbert had derived these conservation equations (1.3.8) independently from Bianchi.

Which of the components of the curvature should be related to the sources was at first obscure, until Einstein discovered that the tensor now called after him, a slight rearrangement of the Ricci tensor, satisfies four laws of continuity, with a formal structure exactly like that of the energy-stress tensor. Accordingly, he postulated that these two tensors are proportional to each other (Bergmann, 1968).

Using (1.3.5) and (1.3.7) and postulating $G_{\mu\nu} = \kappa T_{\mu\nu}$, Einstein arrives at the final equations of General Relativity including the trace term:

$$\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = -\kappa T_{\mu\nu} \quad \text{or} \quad R_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right), \tag{1.3.9}$$

To be in agreement with Newton's theory in the limit of weak fields one must have:

$$\kappa = 8\pi\gamma_N c^{-4} \tag{1.3.10}$$

where γ_N is Newton's constant of gravitation, and c is the velocity of light.

In the above reasoning of Einstein there is some non-consistency, which was discovered when Einstein was still alive.

Actually, the right side of the equation of general relativity is the energy-momentum tensor only in the framework of STR. When we try to generalize this value within the curved Riemann space-time, we obtain a tensor, whose covariant divergence can not be interpreted as the law of conservation of energy and momentum.

For a formal introduction of a full 4-momentum of the gravitational field it is necessary to supplement the resulting tensor with summand $t^{\alpha\beta}$, which is not a tensor. Therefore such a tensor is called pseudo-tensor. But the introduction of a pseudo-tensor makes the theory uncertain, since at different coordinate transformations, the gravitational field (if it is determined by the metric tensor) may disappear or appear out of nothing. The set of quantities $t^{\alpha\beta}$ is called the energy-momentum pseudo-tensor of the gravitational field.

3.2. Pseudo-tensor of energy-momentum and the energy-momentum conservation law

Indeed (Landau and Lifshitz, 1975), "by choosing a coordinate system which is inertial in a given volume element, we can make all the $t^{\alpha\beta}$ vanish at any point in space-time (since then all the Christoffel symbols Γ^{i}_{ikl} vanish). On the other hand, we can get values of the $t^{\alpha\beta}$ different from zero in flat space, i.e. in the absence of a gravitational field, if we simply use curvilinear coordinates instead of Cartesian. Thus, in any case, it has no meaning to speak of a definite localization of the energy of the gravitational field in space. If the tensor $T_{\mu\nu}$ is zero at some world point, then this is the case for any reference system, so that we may say that at this point there is no matter or electromagnetic field. On the other hand, from the vanishing of a pseudo-tensor at some point in one reference system it does not at all follow that this is so for another reference system, so that it is meaningless to talk of whether or not there is gravitational energy at a given place. This corresponds completely to the fact that by a suitable choice of coordinates, we can "annihilate" the gravitational field in a given volume element, in which case, from what has been said, the pseudotensor $t^{\alpha\beta}$ also vanishes in this volume element".

4.0. Hilbert's approach to derivation of GR equation

Hilbert's derivation (Ebner, 2006) of the Einstein equations are essentially based on variational approach with the corresponding choice of invariants.

Hilbert was the first to have identified the Ricci scalar R as the correct Lagrangian density and therefore the finding of equation by well known variational procedures was only a trivial exercise

4.1. Derivation of the field equation from the variational principle

The total action (Sazhin, 2001) for the system "gravitational field plus matter, as a source of gravity", is the sum of two terms: the action for the gravitational field S_g and the action for the matter S_{matt}. The full field equations are obtained as the sum of the variations of action for the field and of action for the matter:

$$\delta S_g + \delta S_{matt} = 0 \text{ or } \delta S_g = -\delta S_{matt}, \tag{1.4.1}$$

Let us consider firstly the action for the gravitational field. This action must be represented as some scalar. That scalar density is only a value $\sqrt{-g}R$, formed from the scalar of curvature. Thus, the action S_g of the gravitational field can be represented as:

$$S_g = -\kappa \int R\sqrt{-g}d^4x \, , \tag{1.4.2}$$

where κ is some gravitational constant.

By taking from $\sqrt{-g}R$ the derivative of Euler-Lagrange, we can obtain a gravitational field equation. But the direct calculations are very time-consuming. At the present time, to simplify them, usually two properties of the scalar curvature are used.

The first simplification (Landau and Lifshitz, 1975) is based on the fact that in the scalar curvature the second derivatives of the metric tensor are included linearly. This allows us to allocate a total divergence, which does not affect the equations of motion.

In addition, according to Ostragradski theorem a total divergence can be transformed into an integral over a three-dimensional hypersurface. When variations are calculated, this term will be zero, since, by definition, a variation on the hypersurface, which surrounds a volume, is equal to zero. Final variation of the gravitational action takes the form:

$$\delta S_g = -\kappa \, \delta \int G\sqrt{-g}d^4x$$

where the value $G = g^{\mu\nu}\left(\Gamma^{\sigma}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\lambda}_{\sigma\lambda}\right)$ determines the action of the gravitational field.

The derivative of the Euler - Lagrange from the value $\sqrt{-g}R$ is defined as:

$$\frac{\delta\left(\sqrt{-g}G\right)}{\delta g_{\mu\nu}} = \frac{\delta\left(\sqrt{-g}G\right)}{\partial g_{\mu\nu}} - \frac{\partial}{\partial x^{\alpha}}\frac{\delta\left(\sqrt{-g}G\right)}{\partial g_{\mu\nu,\alpha}}$$

After calculations and simplifications we can obtain relativistic field equations in the general covariant form, in empty space, that is, without a source:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \quad , \tag{1.4.3}$$

The full equation must include the source of the gravitational field. Variation from S_{matt} by the metric tensor is called the energy - momentum tensor:

$$\frac{\delta S_{matt}}{\delta g_{\mu\nu}} = \frac{1}{2c} \sqrt{-g} T^{\mu\nu} \quad , \tag{1.4.4}$$

Finally, the gravitational field equations in general relativity have the form:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi\gamma_N}{c^4} T_{\mu\nu} \quad , \tag{1.4.5}$$

Hilbert (Ebner, 2006) was first who obtained the following three formulae:

$$L = R + L_{matt} , \tag{1.4.6}$$

$$\delta \int (R + L_{matt}) \sqrt{g} d\tau = 0 , \tag{1.4.7}$$

$$\sqrt{g} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + \frac{\partial L_{matt} \sqrt{g}}{\partial g^{\mu\nu}} = 0 , \tag{1.4.8}$$

The equation (1.4.8) are the correct field equations of General Relativity, including the trace term $-\frac{1}{2} R g_{\mu\nu}$. The minus sign under the square root is absent giving the imaginary unit *i* which drops out of the equation. $R_{\mu\nu}$ is the Ricci curvature tensor. R is the Ricci scalar. (1.4.7) is Hamilton's principle of least action. L is the Lagrangian of the theory, which by (1.4.6) is the sum of the Lagrangian R of the gravitational field and the Lagrangian L_{matt} of everything else (matter, radiation).

As is well known in Special Relativity Theory (SRT), given a Lagrangian L_{matt} of matter (or radiation), its energy-momentum tensor is

$$T_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\partial L_{matt} \sqrt{g}}{\partial g^{\mu\nu}} , \tag{1.4.9}$$

The gravitation constant κ does not occur in (1.4.6-4.8) as it can be put to unity while choosing suitable units.

Note again that Hilbert use the Lagrange density $L_{matt} = L_{em}$, where L_{em} is an electromagnetic Lagrangian density, later specialized to that one of Mie's nonlinear electron theory.

5.0. The variational approach to the derivation of the gravitation equation on the basis of covariant generalization of Maxwell-Lorentz equation

"The generalization of the Maxwell theory is the theory of the Yang-Mills fields or non-Abelian gauge fields. Its equations are nonlinear. In contrast to this, the equations of Maxwell are linear, in other words, Abelian" (Y. Nambu).

Let us also underline that the gauge-invariant Yang-Mills theory is a covariant generalization of the Maxwell-Lorentz theory. Moreover, the Yang-Mills theory contains a generalization of the Dirac electron equation.

Along with the Yang-Mills fields and EM field, the gravitational field also relates to the gauge fields. In this case the role of gauge fields is played by the Christoffel symbols $\Gamma^{\rho}_{\mu\nu}$. The Yang-Mills fields, like the gravitational field, allow a geometric interpretation. (Gauge fields, 1985).

Since the appearance of the Yang-Mills theory, there have been many attempts to build a gravitation theory based on this theory; i.e., practically, in the framework of the electromagnetic theory of gravitation.

There have been encouraging results, but the final solution to the problem has not been reached.

5.1. The Yang-Mills theory

Since 1954 (Nielsen, 2007), one of the most important guiding principles in physics has been that our description of the world should be based on a special type of classical field theory known as a Yang-Mills theory. With the exception of gravitation, all the important theories of modern physics are quantized versions of Yang-Mills theories. These include quantum electrodynamics, the electroweak theory of Salam and Weinberg, the standard model of particle physics.

It is non difficult to show (Ryder, 1996) "how the electromagnetic field arises naturally by demanding invariance of the action under gauge transformation of the second kind, i.e. under local (x -dependent) rotations in the internal space of the complex ψ field, when the Lagrangian has a symmetry O(2) or U(1). Mathematically it is expressed in replacement of the simple derivatives on the covariant derivatives.

The generalization of this result on a case of 3D-space is the Yang-Mills field. The simplest generalisation is to SU(2). This group, as well as the more complicated ones which are considered in physics, is non-Abelian, so what we are studying is the subject of *non-Abelian gauge fields*.

The problem is that we are performing a different 'isorotation' at each point in space, which we may express by saying that the 'axes' in isospace are oriented differently at each point. The reason $d\psi/dx$ is not covariant is that $d\psi(x)$ and $d\psi(x+dx)=d\psi(x)+d\psi$ are measured in different co-ordinate systems. To form a properly covariant derivative, we should make the parallel transport in isospace, illustrated in Fig. 1.

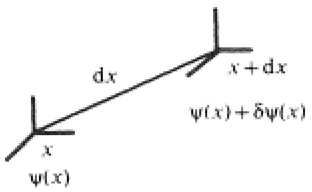

Fig. 1. $\delta\psi$ is defined by parallel transport.

Our strategy now is to proceed as far as possible in analogy with the case of electromagnetism.

Let us examine the rotations of a certain field vector $\vec{F}$ about the 3 axis in the internal symmetry space through an infinitely small angle $\vec{\varphi}$. The meaning of this angle is that $|\vec{\varphi}|$ is the angle of rotation, and $\vec{\varphi}/|\vec{\varphi}|$ is the axis of rotation. Then transition from the initial position of the vector to the final one will be determined by the transformation:

$$\vec{F} \rightarrow \vec{F}^{'} = \vec{F} - \vec{\varphi} \times \vec{F} , \qquad (1.5.1)$$

We have then a gauge transformation of the first kind, and is, of course, effectively three equations.

$$\delta\vec{F} = \vec{F}^{'} - \vec{F} = -\vec{\varphi} \times \vec{F} , \qquad (1.5.2)$$

In contrast to electrodynamics the present case is more complicated, however, and this is directly traceable to the fact that in the present case the rotations form the group SO(3) which is non-Abelian. Its non-Abelian nature is responsible for that fact that $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$: the vector product is not commutative. It will be seen below how this complicates matters. These complications have direct physical consequences.

First note that (1.5.2) is an instruction to perform a rotation in the internal space of $\vec{F}$ through the same angle $\vec{\varphi}$ at all points in space-time. We modify this to the more reasonable demand that

$\vec{\varphi}$ depends on x^{μ} (i.e. that the same relationships are also valid in the four-dimensional space). We then have

$$\vec{\varphi} = \vec{\varphi}\left(x^{\mu}\right), \tag{1.5.3}$$

In this case:

$$\delta(\partial_{\mu}\vec{F}) = \partial_{\mu}\vec{F}^{'} - \partial_{\mu}\vec{F} = -\partial_{\mu}\vec{\varphi}\times\vec{F} - \vec{\varphi}\times\partial_{\mu}\vec{F}\,, \tag{1.5.4}$$

Expressed in words, $\partial_{\mu}\vec{F}$ does not transform covariantly, like $\vec{F}$ does. We must construct a 'covariant derivative'.

This will involve introducing a gauge potential analogous to electromagnetic potential A_{μ}

$$\delta\vec{W}_{\mu} = -\vec{\varphi}\times\vec{W}_{\mu} + \frac{1}{g}\partial_{\mu}\vec{\varphi}\,, \tag{1.5.5}$$

We then write the covariant derivative of the vector $\vec{F}$ as

$$D_{\mu}\vec{F} = \partial_{\mu}\vec{F} + g\vec{W}_{\mu}\times\vec{F}\,, \tag{1.5.6}$$

What is the analogue of the field strength $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ Let us call it $\vec{W}_{\mu\nu}$. Unlike $F_{\mu\nu}$, which is a scalar under SO(2), $\vec{W}_{\mu\nu}$ will be a vector under SO(3)

$$\vec{W}_{\mu\nu} = \partial_{\mu}\vec{W}_{\nu} - \partial_{\nu}\vec{W}_{\mu} + g\vec{W}_{\mu}\times\vec{W}_{\nu}\,, \tag{1.5.7}$$

The field strength $\vec{W}_{\mu\nu}$ is a vector, so $\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu}$ is a scalar and will appear in the Lagrangian, which is, therefore,

$$L = (D_{\mu}\vec{F})\cdot(D^{\mu}\vec{F}) - m^{2}\vec{F}\cdot\vec{F} - \frac{1}{4}\vec{W}_{\mu\nu}\cdot W^{\mu\nu}\,, \tag{1.5.8}$$

The equations of motion are obtained by functional variation of this Lagrangian in the usual way from the Euler-Lagrange equation

$$D^{\nu}\vec{W}_{\mu\nu} = g(D_{\mu}\vec{F})\times\vec{F} \equiv g\vec{J}_{\mu}\,, \tag{1.5.9}$$

This equation is analogous to Maxwell's equation for the 4-current,

The non-Abelian generalisation of equations, which are analogous to the homogeneous Maxwell equations:

$$\partial_{\lambda}F_{\mu\nu} + \partial_{\mu}F_{\nu\lambda} + \partial_{\nu}F_{\lambda\mu} = 0\,, \tag{1.5.10}$$

is

$$D_{\lambda}\vec{W}_{\mu\nu} + D_{\mu}W_{\nu\lambda} + D_{\nu}W_{\lambda\mu} = 0\,, \tag{1.5.11}$$

(end of the citation)

5.2. Basic discrepancy between GRT and Yang-Mills theory

According to Einstein, as a result of the equivalence principle, gravitation can be eliminated from consideration, if we in each point of space-time pass to the appropriate local inertial reference frame moving freely in the gravitational field ("the local elevator"). If we want to pass from this elevator to a nearby point of space-time, it is necessary to make a local Lorentz transformation.

In parallel to this, in the field theory of Yang-Mills t is postulated that the theory must be invariant under the rotation of phase of the wave function at each point in space-time, i.e., with respect to local gauge transformations (see above, introduction of tensor $G_{\mu\nu}$ in the theory). We can assume that to each world-point corresponds its unobserved gauge phase. The phase rotation in each point will be compensated by the addition of the phase gradient to the gauge field. As a result, we can not directly compare vector A_μ at different points, and the usual derivative loses its meaning.

To overcome this difficulty, we artificially introduce an extended (covariant) derivative, which contains a compensating term. This additionally rotates the phase at comparing the fields in two world points that are near one an other.

The emergent additional phase rotation, in the Yang-Mills theory is similar to local Lorentz transformations (since the Lorentz transformation group describes the rotations in the 4-space). Thus, in this sense, GR resembles a non-Abelian gauge theory of the Lorentz group *SO*(1,3), and the tensor $G_{\mu\nu}$ can be identified with the curvature in the charge space.

Unfortunately, this analogy is limited, since the coincidence of the form of the Riemann tensor and field tensor is not full. The difficulty is that the Lagrangian of Hilbert's gravitational field

$$L_g = -\frac{2}{\kappa^2}\sqrt{g}R, \qquad (1.5.12)$$

is linear in $R^{\mu}_{\nu\rho\sigma}$, while Yang-Mills Lagrangian is quadratic in $G_{\mu\nu}$.

$$L_{YM} = -\frac{1}{4g^2}F^{\alpha}_{\mu\nu}F^{\alpha\mu\nu}, \qquad (1.5.13)$$

For this reason, this analogy is incomplete and does not allow the direct transfer of results from one theory to another. Nonetheless, in many cases both theories yield the same results.

5.3. Overcoming of the RGT difficulties through alternative formulation of gravitation theory

Einsteinian general relativity theory (Minkevich, 2008) is the base of modern theory of gravitational interaction, relativistic cosmology and astrophysics. At the same time GR possesses certain principal difficulties.

5.3.1. *The energy-momentum conservation law*

(This difficulty we noted above in the overview of GR).

As it is known, the local gauge invariance principle is the basis of modern theories of fundamental physical interactions. From physical point of view, this principle establishes the correspondence between important conserving physical quantities, connected to the Noether theorem.

There are, by now, many theoretical research works done in trying to put the Yang-Mills theory to be a candidate as a gauge model of gravitation for the Poincare group. If we accept the arguments favouring this theory then Einstein's equations can be derived by a different method than they arise from a dynamical equation for the curvature field in a particular case.

Similar to Yang-Mills fields the gravitational Lagrangian of these theories has to include various invariants quadratic in the curvature and torsion tensors.

5.3.2 *Dark energy and non-baryonic dark matter*

Other principal problem of GR is connected with explanation of cosmological and astrophysical observations. To explain observational cosmological and astrophysical data in the framework of GR it is necessary to suppose that approximately 96% energy in the Universe is related to some hypothetical kinds of gravitating matter – dark energy and non-baryonic dark matter, and only 4% energy is related to usual gravitating matter, from which galaxies are built.

5.3.3. *Cosmological singularity and quantizing of the gravitation field*

One of the most principal cosmological problems remains the problem of cosmological singularity (PCS):

Because in the frame of GR there is not restrictions on admissible values of energy density, and the energy density can reaches the Planckian scale, according to opinion of many physicists the solution of PCS has to be connected with quantum gravitation theory.

Support for this point of view (Debney, G. et al, 1978) has come from the proof that Yang-Mills formulation of the gravitational dynamics is as a quantum theory renorrnalizable.

A number of regular cosmological solutions was obtained in the frame of candidates to quantum gravitation theory - string theory/M-theory and loop quantum gravity.

It is also known that Lagrangian of the Einstein theory $L_0 = R$ is not invariant with respect to a change of the units measuring the interval.

5.3.4. ***Unification of all fields and interactions***

The four fundamental interactions in physics (Yang and Yeung, 2013) are described by two different disciplines. The gravitational interaction follows the curved space-time approach of the GTR laid down by Einstein in which the dynamical variable is the metric tensor. In one's turn, the electroweak and strong interactions follow the local gauge vector boson approach pioneered by Yang and Mills in which the dynamical variables are the wave function of the vector bosons. Both disciplines give spectacular success in terms of experiments and physical observations, despite of the fact that they look very different.

5.4. The Yang-Mills theory as the gravitation theory

Analogies between the Yang Mills theory at the classical level and GR, under their common geometrical basic setting, have long been noticed.

There were many attempts with the purpose to solve indicated problems of GR. We will discuss briefly the most known from them.

Some scientists try to visualize gauge vector boson interactions as geometrical manifestations in a higher dimensional manifold with our spacetime as a four dimensional sub-manifold. Other scientists try to consider the geometrical gravitation theory in the form of a local gauge theory. But all of these ideas are met with difficulties in one way or the other.

Especially not meaningfull are attempts to use the square of the curvature tensor of a different rank for obtaining the Hilbert-Einstein equations.

Before proceeding to the construction of generalization, we should mention one important property of the gravitational equations of Einstein-type: the vanishing of the 4-divergence from the left and right sides of the gravitational equation $\nabla_\nu G^{\mu\nu} = 0$ and $\nabla_\nu T^{\mu\nu} = 0$, respectively. This consequence is a consequence of the energy-momentum conservation law and must be carried out necessarily. In other words, it is a prerequisite for constructing of gravitation equation. Some quadratic Lagrangians do not satisfy this requirement.

It is known (Folomeshkin, 1971) that the standard external Schwarzschild solution satisfies the equations of the quadratic Lagrangians theory. A conclusion is likely to be made that with respect to its experimental consequences the gravitation theory with quadratic Lagrangians

$$L_1 = R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu}, \quad L_1 = R_{\alpha\beta} R^{\alpha\beta}, \quad L_1 = R^2$$

is equivalent to the usual formulation of General Relativity.

In addition, using of the quadratic form Lagrangian in the theory of gravitational waves can produce interesting results.

Moreover in this direction it managed to show with a good approximation that every vacuum solution of Yang-Mills gravity is a solution of vacuum Einstein gravity, and conversely.

The variational methods (Baskal, 1997) implemented on a **quadratic** Yang-Mills type Lagrangian yield two sets of equations interpreted as the field equations and the energy-momentum tensor for the gravitational field. A covariant principle is imposed on the energy-momentum tensor to represent the radiation field. A generalized parallel plain wave (pp-wave) metric is found to simultaneously satisfy both the field equations and the radiation principle.

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Part 2. The existing approaches to gravitation theory

Statement of the Problem

The most advanced theory of gravitation is so far Einstein's general relativity theory (GRT). A number of its drowbacks lie in the basis of attempts to construct a new theory of gravitation (Fock, 1964; Logunov and Mestvirishvili, 1984; Logunov, 2002; and others)

GRT does not explain the equality of the inert and active gravitational masses, and gives no unique prediction for gravitational effects, since this depents on the choice of the coordinate system.. It does not contain the usual conservation laws of energy–momentum and of angular momentum of matter. “Einstein offered the principle of general covariance as the fundamental physical principle of his general theory of relativity and as responsible for extending the principle of relativity to accelerated motion. This view was disputed almost immediately with the counter-claim that the principle was no relativity principle and was physically vacuous..” (Norton, 1993)

But the main drawback of general relativity theory is its incompatibility with the modern quantum theory of elementary particles (quantum field theory), which does not allow to create a unified theory of matter.

The general relativity theory is based on the idea of a Riemannian geometry of space-time, and gravitation is described by the metric tensor $g_{\mu\nu}$ of space-time.

Creating the GTR, Einstein postulated that gravitation is a property of certain four-dimensional spacetime, which, in the language of mathematics, is called curvature. The conclusions from his theory were quantitatively confirmed for weak gravitational fields in the solar system and allowed to qualitatively explain a number of astrophysical and space observations. On the other hand, all the other fundamental interactions are described by representations of the physical fields in non-curved pseudo-Euclidian (Minkowski) space. At the same time they do not have the first shortage of general relativity theory, specified above.

After the appearance of GRT, there have been attempts to geometrize the quantum theory, i.e. to write quantum equations in geometric form. This was done for the theory of electron, and then it turned out that the theory of hadrons - Yang-Mills theory – can also be written in a geometrized form. Unfortunately, this did not lead to unification of all physical theories.

In our time the question arose, if it is possible to do the opposite: to write the equations of gravitation in the form of the equations of elementary particles. It turned out that this area provides much more encouraging results (Logunov, and other authors). But these theories of gravitation, also, have drawbacks.

In this article, unless specifically stated another or if there can not be misunderstandings, is used a system of units of Gauss; to describe the electrodynamic quantities is used as a subscript letter 'e', and to indicate the gravitation theory quantities - the letter 'g'.

1.0. Newton's gravitational theory

Newton's gravitational theory is based on two Newton's laws (Woan, 2000).

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1) on Newton's gravitation law:

$$\vec{F}_g = \gamma_N \frac{m_1 m_2}{r^2} \vec{r}^{\,0}, \tag{2.1.1}$$

Where m_1 and m_2 are the interacting masses, r is the distance between them, $\vec{r}^{\,0}$ is a unit vector, γ_N is the gravitational constant. For γ_N in vacuum currently accepted value is ~ 6,67×10^{-11} м³/(кг·с²) = 6.67x 10^{-8} CGS units.

2) on Newton's second law:

$$\vec{F} = \frac{d\vec{p}}{dt}, \qquad (2.1.2)$$

where $\vec{p}$ is momentum of the body $\vec{p} = m\vec{\upsilon}$, t is time. The equation, corresponding to this law, is called the *equations of motion of a material point*. Here $\upsilon << c$ and m is constant.

Often other forms of Newton's law of gravitation are used also. Using the mass density ρ_m, Gauss' law for gravitation in differential form can be used to obtain the corresponding Poisson equation for gravitation. Gauss' law for gravitation is:

$$\vec{\nabla}\cdot\vec{G} = -4\pi\,\gamma_N \rho_m , \qquad (2.1.3)$$

and since the gravitational field $\vec{G}$ is conservative, it can be expressed in terms of a scalar potential φ_g: $\vec{G} = -\vec{\nabla}\varphi_g$; substituting this expression into Gauss' law, we obtains Poisson's equation for gravitation:

$$\vec{\nabla}^2\varphi_g \equiv \Delta\varphi_g = 4\pi\,\gamma_N \rho_m , \qquad (2.1.4)$$

where Δ is the Laplace operator (in the case of a point particle located at a point $\vec{r} = 0$ should be substituted in place of ρ_m a point function $\delta(\vec{r})$) .

The Poisson equation is a special case of the inhomogeneous wave equation:

$$\frac{\partial^2 u}{\partial t^2} - \upsilon^2 \Delta u = f , \qquad (2.1.5)$$

where $f = f(x, t)$ is some given function of the external effects (external forces), c is constant.

In special relativity, electromagnetism and wave theory the operator $\Box = \Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ has name the d'Alembert operator, also called the d'Alembertian or the wave operator. In Minkowski space in standard coordinates (t, x, y, z) it has the forms:

$$\Box = \partial^{\mu}\partial_{\mu} = g^{\mu\nu}\partial_{\mu}\partial_{\nu} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta , \qquad (2.1.6)$$

Sometimes this operator is written with the opposite sign.

In general curvilinear coordinates, the d'Alembert operator can be written as:

$$\Box u \equiv \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{\nu}}\left(\sqrt{-g}\,g^{\mu\nu}\frac{\partial u}{\partial x^{\nu}}\right), \qquad (2.1.7)$$

where g is the determinant of a matrix $\left\| g_{\mu\nu} \right\|$, which is composed of the coefficients of the metric tensor $g_{\mu\nu}$.

Here is the inverse Minkowski metric with $g_{11} = g_{22} = g_{33} = -1$, $g_{\mu\nu} = 0$, for $\mu \neq \nu$. Note that the μ and ν summation indices range from 0 to 3. We have assumed units such that the speed of light $c = 1$. Some authors also use the negative metric signature of [− + + +] with $\eta_{00} = -1,\ \eta_{11} = \eta_{22} = \eta_{33} = 1$.

Lorentz transformations leave the Minkowski metric invariant, so the d'Alembertian is a Lorentz scalar. The above coordinate expressions remain valid for the standard coordinates in every inertial frame.

Disadvantages of Newton's gravitation theory: 1) it is not relativistic, and 2) the force of gravitation tends to infinity at $r = 0$.

2.0. The Lorentz-invariant electrodynamics gravitation theories

At the end of the 19th century, many tried to combine Newton's force law with the established laws of electrodynamics, like those of Wilhelm Eduard Weber, Carl Friedrich Gauß, Bernhard Riemann and James Clerk Maxwell. Those theories are not invalidated by Laplace's critique, because although they are based on finite propagation speeds, they contain additional terms which maintain the stability of the planetary system.

In 1900 Hendrik Lorentz tried to explain gravitation on the basis of electromagnetic ether theory and the Maxwell equations. After proposing (and rejecting) a Le Sage type model, he assumed like Ottaviano Fabrizio Mossotti and Johann Karl Friedrich Zöllner that the attraction of opposite charged particles is stronger than the repulsion of equal charged particles. The resulting net force is exactly what is known as universal gravitation, in which the speed of gravitation is that of light.

Henri Poincaré argued in 1904 that a propagation speed of gravitation which is greater than c would contradict the concept of local time (based on synchronization by light signals) and the principle of relativity. Poincaré calculated that changes in the gravitational field can propagate with the speed of light if it is presupposed that such a theory is based on the Lorentz transformation.

The contributions presented in this section follows the ideas of Heaviside (1893), Lorentz (1900), Brillouin and Lucas (1966), Brillouin (1970), Carstoiu (1969) , Webster (1912). Wilson (1921) and others.

These authors emphasized the startling similarity between electrostatics and equations of a static (nonrelativistic) gravitation field (gravistatics). In order to discuss non-static (relativistic) problems, authors assume the existence of a second gravitational field, which is called sometimes the gravitational vortex. Both fields are supposed to be coupled, in a first approximation, by equations similar to Maxwell's equations.

So, conditionally speaking, to go to the gravitational theory is sufficient, all the equations of classical electrodynamics, to rewrite, indicating by subscript '*g*' that they belong to a gravitational theory, and indicating also compliance with the parameters adopted in the theory of gravitation of Newton and Einstein. In such a Lorentz invariant theory, Lorentz and others managed to get in his time all the effects of Einstein's gravitational theory. But, unfortunately, the accuracy of the results was low. Some recent results (see below the A. Logunov theory) give hope for improvement of this theory.

At first we enumerate the forms of electrodynamics, which may be of interest to construct a theory of gravitation (recalling that the EM field equations - Maxwell's equations - are relativistic equations).

2.1. Electrodynamic forms used in relativistic theories of gravitation (Tonnelat, 1966)

2.1.1. *The experimental laws. Coulomb’s law*

“The law for action at a distance between two charged particles was experimentally established by Coulomb. Like the Newtonian force of gravitation, the force exerted between two particles of charges q and q' is inversely proportional to the square of the distance separating the two particles. In vacuum, its magnitude is

$$\vec{F} = \frac{1}{\varepsilon}\frac{q_1 \cdot q_2}{r^2} \cdot \bar{r}_0 , \qquad (2.2.1)$$

The vector $\vec{E} = \frac{\vec{F}}{q}$ is called the field strength (field density, field force, field intensity also) of electric field. Then $\vec{E} = -grad\varphi$. The value of the constant ε_0 depends solely on the system of units, in which the particle's charge is expressed (In the Gaussian system of units ε is a dimensionless quantity).

This type of a law (2.1) can always be expressed as a function of the scalar:

$$\varphi = \frac{1}{\varepsilon_0} \frac{q_2}{r^2}, \tag{2.2.2}$$

called the potential associated with the charge q. Then from (2.1) we have

$$\vec{F} = -q_1 \cdot grad\varphi, \tag{2.2.3}$$

or as force dencity

$$\vec{f} = \rho\vec{E} = -\rho \cdot grad\varphi, \tag{2.2.4}$$

2.1.2. *The experimental law of Biot and Savart*

A constant current results from a uniform succession of charges.

$$i = \rho\upsilon_n dS, \tag{2.2.5}$$

where $\vec{\upsilon}$ is velocity of charge in wire with cross-section dS and υ_n the projection of that velocity on the normal to dS. A current density according to (2.2.5) is:

$$\vec{j} = (i/dS)\vec{n} = \rho\vec{\upsilon}, \tag{2.2.6}$$

If a charge q moves in the neighbourhood of an electric current or permanent magnet, one finds that this charge is acted on by a force

$$\vec{F} = q[\vec{\upsilon} \times \vec{B}], \tag{2.2.7}$$

The magnetic induction $\vec{B}$ is itself produced by the uniform motion of a charge q' of velocity $\vec{\upsilon}'$ in a circuit-element dl'. Its value is given by Biot and Savart's experimental law:

$$\vec{B} = \frac{q'[\vec{\upsilon} \times \vec{r}]}{|r|^3} = \frac{i'[d\vec{l}' \times \vec{r}']}{|r|^3}, \tag{2.2.8}$$

Whence

$$\vec{F} = \frac{qq'}{|r|^3}[\vec{\upsilon} \times (\vec{\upsilon}' \times \vec{r})] = \frac{ii'}{|r|^3}[d\vec{l} \times (d\vec{l}' \times \vec{r})], \tag{2.2.9}$$

$\vec{r}$ being the distance between the test particle and the charge (or current-element) creating the magnetic induction $\vec{B}$.

2.1.3. *The basic equations*

The self-consistent Maxwell-Lorentz microscopic equations are the independent fundamental field equations. The Maxwell-Lorentz equations are following four differential (or, equivalent, integral) equations for any electromagnetic medium (Jackson, 1999; Tonnelat, 1959):

$$rot\vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j}, \tag{2.2.10}$$

$$rot\vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0, \tag{2.2.11}$$

$$di\upsilon\vec{E} = 4\pi\rho, \tag{2.2.12}$$

$$di\upsilon\vec{B} = 0, \tag{2.2.13}$$

Here

$$\vec{D} = \varepsilon\vec{E}, \ \vec{B} = \mu\vec{H}, \ \vec{j} = \sigma_c\vec{E}, \ \vec{E} = -grad\,\varphi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \ \vec{B} = rot\vec{A}$$

where $\vec{E}, \vec{H}, \vec{D}, \vec{B}$ (electric field, magnetic field, electric induction, magnetic induction, correspondingly) are the electromagnetic field vectors (field strength, field density, field force, field intensity, also), $\varphi, \vec{A}$ are scalar and vector potentials, correspondingly; ρ is the charge density; $\vec{j}$ is the current density. $\varepsilon, \mu, \sigma_c$ are permittivity, permeability, conductivity, correspondingly; in Gauss system units $\varepsilon, \mu, \sigma_c$ are dimensionless coefficients; in medium $\varepsilon = \varepsilon(\vec{r}, t)$, $\mu = \mu(\vec{r}, t)$, $\sigma_c = \sigma_c(\vec{r}, t)$; in vacuum $\varepsilon = 1$, $\mu = 1$, $\sigma_c = 0$. c is the speed of light in medium (in vacuum $c = c_0 = 3 \cdot 10^8\ m/\sec$).

Values $\vec{j}$ and ρ (or in 4-vector form $j_\mu = \{\vec{j}, i\rho\}$, where $\mu = 1,2,3,4$) in these equations should be considered as functions (more precisely, functionals) of strength $\vec{E}$ and $\vec{H}$ of the same fields, which these charges and currents substantially define: $j_\mu = j_\mu(\vec{E}, \vec{H})$.

As is known, the Maxwell-Lorentz theory predict the existence of electromagnetic waves.

2.1.4. *Lorentz force equation*

$$F = q\left(\vec{E} + \frac{1}{c}\vec{\upsilon} \times \vec{B}\right), \qquad (2.2.14)$$

2.1.5. *The energy of the electromagnetic field*

The energy density of the electromagnetic field is equal to

$$u = \frac{1}{8\pi}\left(\vec{E}\vec{D} + \vec{H}\vec{B}\right) = \frac{1}{8\pi}\left(\varepsilon E^2 + \mu H^2\right), \qquad (2.2.15)$$

2.1.6. *Poynting vector and momentum of the electromagnetic field*

Poynting vector

$$\vec{S}_P = \frac{1}{4\pi}\left[\vec{E} \times \vec{H}\right], \qquad (2.2.16)$$

The momentum density g

$$\vec{g} = \frac{1}{4\pi c}\left[\vec{E} \times \vec{H}\right] = \frac{1}{c} S_P, \qquad (2.2.17)$$

Poynting's theorem (the continuity equation of the electromagnetic field)

$$\frac{1}{c}\frac{\partial u}{\partial t} + di\upsilon S_P = -\frac{1}{c}\vec{j} \cdot \vec{E}, \qquad (2.2.18)$$

2.1.7. *Equations for the propagation of fields* (Tonnelat, 1966)

One of the greatest successes of Maxwell's theory was the prediction of the existence of electromagnetic waves with a propagation velocity that could be known theoretically and measured.

The equations of the propagation of the fields has the following form:

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \Delta\vec{E} = -\frac{4\pi\sigma_c}{c^2}\mu\frac{\partial \vec{E}}{\partial t}, \qquad (2.2.19)$$

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} - \Delta\vec{H} = -\frac{4\pi\sigma_c}{c^2}\mu\frac{\partial \vec{H}}{\partial t}, \qquad (2.2.20)$$

Each of the quantities $\vec{E}$ and $\vec{H}$ thus satisfies an equation of propagation of the following form:

$$\frac{\varepsilon\mu}{c^2}\frac{\partial^2 \vec{a}}{\partial t^2} - \Delta\vec{a} = -\frac{4\pi\sigma_c}{c^2}\mu\frac{\partial \vec{a}}{\partial t}, \qquad (2.2.21)$$

By means of potentials:

Using $\vec{E} = -grad\varphi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$, $\vec{B} = rot\vec{A}$, from basic equations, in the case of Lorentz condition $\frac{\varepsilon\mu}{c}\cdot\frac{\partial\varphi}{\partial t} + \nabla\vec{A} = 0$, we can obtain:

$$\frac{\varepsilon\mu}{c^2}\cdot\frac{\partial^2\varphi}{\partial t^2} - \nabla^2\varphi = \frac{4\pi}{\varepsilon}\rho, \tag{2.2.22}$$

$$\frac{\varepsilon\mu}{c^2}\cdot\frac{\partial^2\vec{A}}{\partial t^2} - \nabla^2\vec{A} = \frac{4\pi}{c}\mu\,\vec{j}, \tag{2.2.22}$$

2.2. The equations of the Lorentz-invariant electrodynamic theory of gravitation

2.2.1. *The Two Gravitational Fields in electrodynamic form*

Gravy-electric (g-electric) field

Based on a comparison of Newton's law of gravitation with Coulomb's law, we introduce the intensity of the gravitational electric (g-electric) field.

If we introduce the gravitational charge (g-charge) (Ivanenko and Sokolov, 1949; pp. 420, 422)

$$q_g = \sqrt{\gamma_N}\,m, \tag{2.2.23}$$

and, accordingly, the intensity of the static gravitational field $\vec{E}_g = \frac{\vec{F}_g}{q_g}$, the law of gravitation Newton can be rewritten in the form of Coulomb's law:

$$\vec{F}_g = \frac{q_{g1}\cdot q_{g2}}{r^2}\vec{r}^{\,0}, \tag{2.2.24}$$

Obviously, in this case, the dimension of the electric q_e and gravitational charges q_g are the same . Assuming q_{g1} is a test charge, the strength of the gravitational field (g-electric field) created by the mass m_2 will be equal: $\vec{E}_{g2} = \frac{\vec{F}_g}{q_{g1}} = \frac{q_{g2}}{r^2}\vec{r}^{\,0}$; hence, Newton's law of gravitation can be written as: $\vec{F} = q_{g1}\vec{E}_{g\,2}$. Using the Poisson equation

$$\vec{\nabla}^2\varphi_g \equiv \Delta\varphi_g = 4\pi\,\gamma_N\rho_m, \tag{2.2.25}$$

we can (Ivanenko and Sokolov, 1949; pp. 430) rewrite the Poisson equation by introducing the normalization $\varphi' = \frac{\varphi}{\sqrt{\gamma_N}}$ and the density of the gravitational charge. Multiplying $\rho_m = \frac{dm}{d\tau}$ by $\sqrt{\gamma_N}$ and using $m' = \sqrt{\gamma_N}\cdot m$, we obtain $\sqrt{\gamma_N}\rho_m = \frac{d\sqrt{\gamma_N}m}{d\tau} = \frac{dm'}{d\tau} = \rho'_m$. Then the Poisson equation takes the following form:

$$\vec{\nabla}^2\varphi'_g \equiv \Delta\varphi'_g = 4\pi\rho'_m, \tag{2.2.26}$$

There is currently no rigorous proof of the possibility of reducing the gravitational field to the electromagnetic, but we can show that the converse is true. Indeed, each field corresponds to the matter, which has some mass. Since the electrostatic field, e.g., of electron, has the electrostatic energy density, which is much higher than the energy density of gravitational field, therefore, mater of electron "consists" mainly of the electromagnetic field.

To this the fact corresponds that the gravitational charge of the electron is less than its electric charge $e >> q_g$, where $q_g = m_e\sqrt{\gamma_N}$ (here $e = 4{,}8\cdot 10^{-10}$ unit. SGSEq is electron charge (1 unit CGSEq = $g^{1/2}sm^{3/2}s^{-1}$), $m_e = 0{,}91\cdot 10^{-27}$ g is electron mass, $\gamma = 6{,}67\cdot 10^{-8}$ $cm^3/g\ sec^2$ is the gravitational constant. It is easy to see that the dimension of the gravitational charge of the electron coincides with the dimension of electric charge and its magnitude in 10^{21} times less. Indeed, the $e/m_e\sqrt{\gamma_N} \approx 2\cdot 10^{21}$.

If we assume that gravitation is generated by an electric field, but quantitatively very small part of it, then the law of gravitation must be written similarly to the Coulomb law: $F_g = \frac{q_{g1}\cdot q_{g2}}{r^2}$. However, in electromagnetic theory m_e is the inertial (electromagnetic) mass of the electron. Using the relation $q_g = m_e\sqrt{\gamma_N}$, we obtain Newton's law of gravitation: $F_g = \gamma_N \frac{m_{e1}\cdot m_{e2}}{r^2}$, where m_e plays the role of gravitational mass. Thus, the equivalence of the masses n be here considered as consequence of the hypothesis of an electromagnetic origin of gravitation.

Gravy-magnetic (g-magnetic) field

Now, using the Biot and Savart law, by analogy with the g-electric field strength of gravitation, we introduce the magnetic field.

In this case, by analogy with the introduction of stationary g-electric field, postulated the existence of an alternating electric field of and the related g-magnetic field. We introduce this field, introducing artificially electromagnetic quantities the subscript '*g*'. As a result, we obtain:

A constant g-current results from a uniform succession of g-charges:

$$i_g = \rho_g \upsilon_n dS, \tag{2.2.27}$$

where $\vec{\upsilon}$ is velocity of charge in wire with cross-section dS and υ_n the projection of that velocity on the normal to dS. A current density according to (2.1) is:

$$\vec{j}_g = (i/dS)\vec{n} = \rho_g\vec{\upsilon}, \tag{2.2.28}$$

If a g-charge q_g moves in the neighbourhood of an electric current or permanent magnet, one finds that this charge is acted on by a magnetic Lorentz force

$$\vec{F}_{Lg} = q_g\left[\vec{\upsilon}\times\vec{B}_g\right], \tag{2.2.29}$$

The g-magnetic induction $\vec{B}_g$ is itself produced by the uniform motion of a g-charge q'_g of velocity $\vec{\upsilon}'$ in a circuit-element dl'. Its value is given by Biot and Savart's experimental law:

$$\vec{B}_g = \frac{q'_g\left[\vec{\upsilon}\times\vec{r}\right]}{|r|^3} = \frac{i'_g\left[d\vec{l}'\times\vec{r}'\right]}{|r|^3}, \tag{2.2.30}$$

Whence

$$\vec{F}_{Lg} = \frac{q_g q'_g}{|r|^3}\left[\vec{\upsilon}\times\left(\vec{\upsilon}'\times\vec{r}\right)\right] = \frac{i_g i'_g}{|r|^3}\left[d\vec{l}\times\left(d\vec{l}'\times\vec{r}\right)\right], \tag{2.2.31}$$

$\vec{r}$ being the distance between the test particle and the charge (or current-element) creating the magnetic induction $\vec{B}$.

Thus two new gravitation vectors, which are the gravitational analogues of the vectors of electrodynamics, are defined: $\vec{E}_g$ and $\vec{B}_g$.

2.2.2. *The relativistic gravitational equations*

Rewriting Maxwell's equations in gravitational form, we obtain

$$\begin{cases} rot\vec{B}_g - \dfrac{1}{c}\dfrac{\partial \vec{E}_g}{\partial t} = \dfrac{4\pi}{c}\vec{j}_g \\ rot\vec{E}_g + \dfrac{1}{c}\dfrac{\partial \vec{B}_g}{\partial t} = 0 \\ di\upsilon\vec{E}_g = 4\pi\rho_g \\ di\upsilon\vec{B}_g = 0 \end{cases}, \qquad (2.2.32)$$

where $\vec{D}_g = \varepsilon_g \vec{E}_g$, $\vec{B}_g = \mu_g \vec{H}_g$, $\vec{j}_g = \sigma_g \vec{E}_g$, $\vec{E}_g = -grad\,\varphi_g - \dfrac{1}{c}\dfrac{\partial \vec{A}_g}{\partial t}$, $\vec{B}_g = rot\vec{A}_g$,

Etc. (see all other equations and relations, which we quoted above in the review of the results of electromagnetic theory being obtained with help of '*g*' subscript).

2.3. Другой вариант построения электромагнитной теории гравитации

2.3. Another option of the construction of the electromagnetic theory of gravitation

The "Maxwellization" of gravitational field equations can be carried out in a slightly different way, virtually identical to the above, but by different mathematical formulation (Brillouin and Lucas, 1966), (Brillouin, 1970), and (Carstoiu, 1969). We will follow the book (Brillouin, 1972), keeping the notation and system of units.

We have already noted the similarity between equation of electrostatics and equation of static gravitation field $\vec{G}$. In order to study non-static problems, Carstoiu (Carstoiu, 1969) assumed the existence of a second gravitational field, which is called the gravitational vortex $\vec{\Omega}$. Both fields, $\vec{G}$ and $\vec{\Omega}$ are supposed to be coupled, in a first approximation, by equations similar to Maxwell's equations, so that in vacuo the field propagation velocity υ_g equal to the light velocity c_0; and by c is designated the variable velocity of light: $c \le c_0 \cong 3 \cdot 10^8$ meters/sec.

Coulomb's law for charges Q_1 and Q_2, dielectric power ζ is given by:

$$\vec{f}_C = \frac{Q_1 \cdot Q_2}{\xi \cdot r^2} \cdot \vec{r}^{\,0}, \qquad (2.2.33)$$

Newton's law for masses M_1 and M_2, Newton's constant *G* is written:

$$\vec{f}_N = -\gamma_N \frac{M_1 \cdot M_2}{r^2} \cdot \vec{r}^{\,0}, \qquad (2.2.34).$$

The notation $\vec{r}^{\,0}$ represents a unit vector in the direction $\vec{r}$. Both formulas (2.2.33) and (2.2.34) are identical if we assume:

$$\xi = -\frac{1}{\gamma_N} = -1{,}5 \cdot 10^7, \qquad (2.2.35)$$

As is well known, Maxwell's equations contain two constants: the dielectric constant ξ and the permeability μ, related by the condition: $\xi \cdot \mu \cdot c^2 = 1$, thus yielding the velocity c for wave propagation.

Accordingly, Carstoiu introduces two gravitation constants ξ_g and μ_g. Let us take for the ξ_g the value we selected in (2.2.33):

$$\xi_g = -\frac{1}{\gamma_N}, \qquad (2.2.36)$$

This leads to selecting:

$$\mu_g = -\frac{\gamma_N}{c^2}\,, \qquad (2.2.37)$$

in order to satisfy condition $\xi \cdot \mu \cdot c^2 = 1$. Rewriting Maxwell's equation, Carstoiu obtains:

$$\left.\begin{aligned} rot\,\vec{F} &= -\frac{\partial\vec{\Omega}}{\partial t},\, div\vec{F} = -\gamma_N \cdot \rho_g \\ rot\,\vec{\Omega} &= \frac{1}{c^2}\cdot\frac{\partial\vec{F}}{\partial t} - \frac{\gamma_N}{c^2}\cdot\vec{J}_g,\, div\vec{\Omega} = 0 \end{aligned}\right\},\qquad (2.2.38)$$

where ρ_g is the mass density, $\vec{J}_g$ the gravitational current, and $\vec{\Omega}$ the gravitational vortex. Obviously, the Carstoiu's vortex $\vec{\Omega}$ is nothing other than the the magnetic force.

Then Carstoiu discusses the possible role of his gravitational vortex on the stability of rotating masses and a variety of problems in cosmogony.

3.0. General relativity

The practical side of the Hilbert-Einstein theory (Tonnelat, 1965/1966) is following:

"*All the predictions of general relativity follow from the field equations*:

$$S_{\mu\nu}\left(g_{\alpha\beta}, \partial_\rho g_{\alpha\beta}, \partial^2_{\rho\sigma} g_{\alpha\beta}\right) = \chi T_{\mu\nu}(m,\vec{u}) \rightarrow g_{\alpha\beta}\,, \qquad (2.3.1)$$

where $S_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, $\chi = \frac{8\pi G}{c^4}$, $R_{\mu\nu} = \frac{\Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} - \frac{\Gamma^\nu_{\mu\lambda}}{\partial x^\nu} + \Gamma^\lambda_{\mu\nu}\Gamma^\alpha_{\lambda\alpha} - \Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\alpha}$, $\Gamma^\lambda_{\mu\nu}$ *are the Christoffel symbols and* $g_{\mu\nu}$ *is an element of the metric tensor of Riemannian space), and*

2) The law of motion (geodesic equation) for a massless body (photon):

$$\delta\int ds = 0, \qquad (2.3.2')"$$

or the Hamilton-Jacobi equation for a massive body (Landau and Lifshitz, 1951):

$$g^{ik}\left(\frac{\partial S}{\partial x^i}\right)\left(\frac{\partial S}{\partial y^k}\right) + m^2c^2 = 0\,, \qquad (2.3.2'')$$

The equation (2.3.1) allows to determine $g_{\mu\nu}$ and to put this value in (2.3.2).

3.1. Approximate solutions of the equations of general relativity

Various authors, based on reasonable hypotheses, within the framework of Lorentz-invariant approach received some results of general relativity by more simple way. Next, we consider some of these.

3.1.1. *Putilov approach* (Putilov and Fabrikant, 1960; Kropotkin, 1971).

"**Two interpretations of the law of proportionality of mass and energy and refinement of Newton's gravitational law**. According to Einstein's theory the gravitational field is a manifestation of the curvature of space-time. According to these ideas quantities, characterizing the gravitational field (intensity, potential, gravitational energy) have only geometric meaning, in contrast to the analogous quantities, characterizing the electromagnetic field. Adhering to the concept, adopted in general relativity, the value ε in law $\varepsilon = mc^2$ need to takes the totality of all forms of energy, except the gravitation energy. In line with this, a body mass, moving without acceleration in the gravitational field, must be also considered to be unchanged.

But it is permissible (i.e. not lead to any contradictions, but on the contrary, in many cases it is even more comfortable), a different interpretation of the law $\varepsilon = mc^2$, but as a completely universal, covering all forms of energy, including energy of gravitation. In the case of such interpretation of the law $\varepsilon = mc^2$ is necessary to consider the gravitational change in body mass Δm, determined by the relation $\Delta m \cdot c^2 = m\varphi$, where φ is the potential energy of gravitation."

The question arises, why so sharply distinguished Einstein gravitation among other types of fields? Apparently, in the other case it were not perfectly to pass to the idea of equivalence of linear acceleration and gravitation, which is the basis of general relativity. Indeed, if the energy of gravitation is equivalent to any other type of energy, the equivalence principle must be observed in relation to other fields, which does not allow to geometrize the gravitation. However, if we reject the geometrization of gravitation, then the need to integrate the energy of gravitation in mc^2 is almost inevitable because of the equivalence of all forms of energy. And if the Einstein's general relativity not existed, such a train of thought would be, obviously, the dominant long time ago. "

In the Putilov approach the law $E = mc^2$ adopted as fundamental. Together with the energy conservation law it is the simplest way, which gives results, very near to results in the general relativity theory.

In the gravitational field the body masses (from $(m - m_0)c^2 = m\varphi$)):

$$m = \frac{m_0}{1 - \dfrac{\varphi}{c^2}} \approx 1 + \frac{\varphi}{c^2}, \qquad (2.3.3)$$

The speed of light

$$c_\varphi = c\left(1 + \frac{\varphi}{c^2}\right) \quad , \qquad (2.3.4)$$

The refractive index of the vacuum:

$$n = \frac{c}{c_\varphi} = \frac{1}{1 + \dfrac{\varphi}{c^2}} \approx 1 - \frac{\varphi}{c^2}, \qquad (2.3.5)$$

The rate of processes is characterized by time (if $-\varphi << c^2$):

$$t \approx t_0\left(1 - \frac{\varphi}{c^2}\right),$$

The force of gravitation

$$f = \frac{\gamma_N mM}{r^2}\left(1 - 2\frac{\gamma_N M}{rc^2}\right) = \frac{\gamma_N mM}{r^2}\left(1 - \frac{2\alpha}{r}\right), \qquad (2.3.6)$$

where $\alpha = \dfrac{\gamma_N M}{c^2}$

Based on these results, the formula exactly coinciding with Einstein's formula for the red shift in the gravitational field, half deflection of light, some, but not a sufficient displacement of the perihelion of Mercury are obtained . The inaccuracy of last results, apparently, associated with the stationarity of solutions of Putilov.

Taking into account the finite speed of propagation of the field (see Baranov, 1970) we obtain a doubling of the result for the deflection of light of Putilov. Refinement of results for the Mercury was not made; and difficult here is not that such a clarification can not be made, but that such a clarification might be product with a number of ways.

Putilov formula can be rewritten in another way. At low speeds, when $\upsilon << c$:

$$(m - m_0)c^2 \approx \frac{m\upsilon^2}{2} = m\varphi, \qquad (2.3.7)$$

from here, dividing by c^2, we obtain:

$$\left(\frac{\upsilon}{c}\right)^2 \equiv \beta^2 = 2\frac{\varphi}{c^2} = \frac{2\gamma_N M}{c^2 r} = \frac{2\alpha}{r}\,, \tag{2.3.8}$$

And, therefore, the force of gravity in the framework of this approach can also be written as:

$$f = \gamma_N \frac{mM}{r^2}\left(1-\beta^2\right) = \gamma_N \frac{mM}{r^2}\left(1-\frac{2\varphi}{c^2}\right)$$

From the equations of general relativity in the linear approximation of weak fields, i.e. in the pseudoeuclid metric, we obtain the equation of d'Alembert with nonzero right-hand side (Pauli, 1981; Fock, 1964) :

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \Delta\varphi = 4\pi\gamma_N \rho\,, \tag{2.3.9}$$

where $\rho = W/c^2$ is mass density, defined in terms of energy density W, φ is the Newtonian potential associated with g_{00} according to the equation:

$$g_{00} = 1 - \frac{2\gamma M}{c^2 r} \equiv 1 - \frac{2\varphi}{c^2}\,, \tag{2.3.10}$$

Brillouin was consistent with the opinions, similar to Putilov view point (Brillouin, 1970).

3.1.2. ***W. Lenz approach*** (Sommerfeld, 1952)

The best results in the Lorentz-invariant approach was obtained by Wilhelm Lenz (as presented by Sommerfeld)

"*However, Einstein was the first to interpret $m_{grav} = m_{inert}$ in the final form gravitation = inertia (= world curvature).*

We shall show that this equivalence principle suffices for the elementary calculation of the $g_{\mu\nu}$ in a specific case (on the basis of an unpublished paper of W. Lenz, 1944), i.e. to solve a problem which was formulated generally in Eq. of Einstein

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa\tau_{\mu\nu}\,, \tag{2.3.11}$$

but was postponed as being too difficult.

Consider a centrally symmetric gravitational field, e.g. that of the sun, of mass M, which may be regarded as at rest. Let a box K_∞ fall in a radial direction toward M. Since it falls freely, K_∞ is not aware of gravitation (as the consequence of $m_{grav} = m_{inert}$) and therefore carries continuously with itself the Euclidean metric valid at infinity ∞. Let the coordinates measured within it be x_∞ (longitudinal, i.e. in the direction of motion), y_∞ , z_∞ (transversal), and t_∞. K_∞ arrives at the distance r from the sun with the velocity υ. υ and r are to be measured in the system K of the sun, which is subject to gravitation. In it we use as coordinates r, ϑ, φ, and t. Between K_∞ and K there exist the relations of the special Lorentz transformation, where K_∞ plays the role of the system "moving" with the velocity $\upsilon = \beta c$, K that of the system "at rest".

The relations are

$dx_\infty = \dfrac{dr}{\sqrt{1-\beta^2}}$ *(Lorentz contraction),*

$dt_\infty = dt\sqrt{1-\beta^2}$ *(Einstein dilatation),*

$$\left.\begin{aligned} dy_\infty &= r\,d\vartheta \\ dz_\infty &= r\sin\vartheta\, d\varphi \end{aligned}\right\}\ \textit{(Invariance of the transversal lengths)}$$

Hence the Euclidean world line element

$$ds^2 = dx_\infty^2 + dy_\infty^2 + dz_\infty^2 + dt_\infty^2, \qquad (2.3.12)$$

passes over into

$$ds^2 = \frac{dr^2}{1-\beta^2} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) - c^2 dt^2 (1-\beta^2), \qquad (2.3.12')$$

The factor $\left(1-\beta^2\right)$*, which occurs here twice, is meaningful so far only in connection with our specific box experiment. In order to determine its meaning in the system of the sun we write down the energy equation for* K*, as interpreted by an observer on* K*. Let* m *be the mass of* K_∞*,* m_0 *its rest mass . The equation then is:*

$$\left(m - m_0\right)c^2 - \frac{\gamma_N mM}{r} = 0, \qquad (2.3.13)$$

At the left we have the sum of the kinetic energy and of the (negative) potential energy of gravitation, i.e.. $T + V = 0$ (This assumption is clearly equivalent to the Putilov hypothesis on the massiveness gravitational field – A.K).). *The energy constant on the right was to be put equal to zero since at infinity* ∞ $m = m_0$ *and* $r = \infty$. *We have computed the potential energy from the Newtonian law, which we shall consider as a first approximation. We divide* (2.3.13) *by* mc^2 *and obtain then, since* $m = m_0\sqrt{1-\beta^2}$ *:*

$$1 - \sqrt{1-\beta^2} = \frac{\alpha}{r}, \quad \alpha = \frac{\gamma_N M}{c^2} = \frac{\kappa M}{8\pi}, \qquad (2.3.14)$$

where κ is the Einstein constant. It follows from (2.3.14) that

$$\sqrt{1-\beta^2} = 1 - \frac{\alpha}{r}, \quad 1-\beta^2 \approx 1 - \frac{2\alpha}{r}, \qquad (2.3.15)$$

(If we use the approach of Putilov, then (2.3.15) can be obtained from (2.3.13) easier from: $\left(m - m_0\right)c^2 \approx \frac{m\upsilon^2}{2}$ - A.K.). From (2.3.12') we have:

$$ds^2 = \frac{dr^2}{1-\frac{2\alpha}{r}} + r^2\left(d\vartheta^2 + \sin^2\vartheta d\varphi^2\right) - c^2 dt^2\left(1 - \frac{2\alpha}{r}\right), \qquad (2.3.16)$$

This is the line element derived by K. Schwarzschild from Einstein's equation (2.3.11). In Eddington's presentation the 40 components $\Gamma_{\mu\nu}^{\sigma}$ of the gravitational field are computed and (2.3.16) is shown to be the exact solution of the ten equations contained in (2.3.11). Our derivation claims only to yield an approximation, since it utilises the Newtonian law as first approximation and 'neglects, in the second Eq. (2.3.15), the term $\left(\alpha/r\right)^2$; nevertheless, our result is, as shown by Schwarzschild and Eddington, exact in the sense of Einstein's theory.

It might be asked at this point: What is the relativistically exact formulation of the Newtonian law? The question is wrongly put if a vector law is meant hereby. The gravitational field is not a vector field, but has a much more complex tensor character. For the single point mass it is completely described by the four coefficients of the line element (2.3.16) and the vanishing of the remaining $g_{\mu\nu}$".

Perhaps the only book, in which the authors to present concepts of the theory of general relativity, using the Lenz approach, is the review of the current status of the problems of gravitation in book (Vladimirov, Mickevich and Khorsky, 1984)

In this book, along with the Schwarzschild solution, by means of Lenz method are obtained the solution of J. Lense and H. Thirring for the metric around a rotating body, and its refinement:

solution of the R. Kerr (p. 77 et seq.). In addition to these solutions are obtained in the same way (p. 135 and following) the exact solutions of general relativity in the presence of electromagnetic fields, when the source has an electric charge. In the presence of rotation this leads to the appearance, along with an electric dipole, a magnetic dipole field (solutions of Reissner-Nordstrom and Kerr-Newman).

It is easy to see that the difference between the approaches, which are discussed above, and the GTR is that in the firsts are not used a Riemannian space and the hypothesis about the geometrical origin of gravitation. The question arises whether there are methods of introducing in such theories of Riemann geometry? This procedure is conventionally called a "geometrization" of the equations. It turns out the geometrization of equations is already possible in the nonrelativistic limit.

4.0. Geometrization of the equations of motion in the nonrelativistic limit

(Buchholz, 1972; Encyclpedia of mathematics, 2011)

The **action of Hamilton** has the form:

$$S = \int_0^t L dt, \tag{2.4.1}$$

where $L = T - V$ is the Lagrange function, T is kinetic energy, V is the potential energy, and $T + V = H = const$ is condition of energy conservation.

Function:

$$W = \int_0^t 2T dt = \int_0^t \sum_{i=1}^{N} m_i \upsilon_i^2 dt, \tag{2.4.2}$$

is called the **Lagrange action** and is associated with function S with relation:

$$S = W - Ht_1, \tag{2.4.3}$$

Because $\upsilon_i dt = dl_i$, where dl_i is an element of arc of the trajectory of the system with the index i, then: $\upsilon_i^2 dt = \upsilon_i dl_i$, and therefore:

$$W = \int_A^B \sum_{i=1}^{N} m_i \upsilon_i \, dl_i, \tag{2.4.4}$$

where the integration is performed along arcs of trajectories of system points from the configuration A to B. Equation (2.4.4) gives an expression for the action in the form of Maupertuis.

The form of Jacobi. Expression of the action in the form of Jacobi differs because in it with the energy integral the time is eliminated. We have:

$$2E = \sum_{i=1}^{N} m_i \upsilon_i^2 = \sum_{i=1}^{N} m_i \left(\frac{dl_i}{dt} \right)^2;$$

In addition, the energy integral gives:

$$2T = 2(U + h), \tag{2.4.5}$$

From above we have:

$$dt = \frac{\sqrt{\sum_{i=1}^{N} m_i dl_i^2}}{\sqrt{2(U + h)}}, \tag{2.4.6}$$

Substituting now (2.4.5) and (2.4.6) in the expression of W in the Lagrange form, we obtain:

$$W = \int_A^B \sqrt{2(h - V)} \cdot \sqrt{\sum_{i=1}^{N} m_i dl_i^2}, \tag{2.4.7}$$

This expression has a geometrical meaning because time and speed in it are excluded. If we introduce independent coordinates and express $x_1, x_2, ..., x_{3N}$ as a function of generalized coordinates $q_1, q_2, ..., q_n$, then:

$$\sum_{i=1}^{N} m_i ds_i^2 = \sum_{i=1}^{3N} m_i dx_i^2 = \sum_{i,j=1}^{n} a_{ij} dq_i dq_j \ , \qquad (2.4.8)$$

and the expression in the form of Jacobi takes the form:

$$W = \int_A^B \sqrt{2(h-V)} \cdot \sqrt{\sum_{i,j=1}^{n} a_{ij} dq_i dq_j} \ , \qquad (2.4.7')$$

In the special case, when the external forces do not act on the system, $V = const$, and therefore by (2.4.7) and (2.4.7') we can express the action W, neglecting the constant factor, as follows:

$$W = \int_A^B \sqrt{\sum_{i,j=1}^{n} a_{ij} dq_i dq_j} \ , \qquad (2.4.8)$$

Interpreting the system movement as a movement of a point in space of n measurements, for which the element of arc (the fundamental metric form) has the expression:

$$d\sigma^2 = \sum_{i,j=1}^{n} a_{ij} dq_i dq_j \ , \qquad (2.4.9)$$

we can write

$$W = \int_A^B d\sigma \, , \qquad (2.4.10)$$

Thus, the problem of determining the inertial motion of a holonomic system is reduced to finding the minimum of the integral (2.4.10), i.e. to the problem of geodesic lines. The principle of stationary action in this case can be expressed as follows: *holonomic system moves inertially so that the point, representing the system in the corresponding space, moves along a geodesic line of this space*.

To the problem of geodesic lines we can also reduce the motion of holonomic system in the case, when it moves in a potential field, which is determined by a function $u = -V$. Indeed, since $V = V(q)$, then:

$$2(h-V)\sum_{i,j=1}^{n} a_{ij} dq_i dq_j = \sum_{i,j=1}^{n} b_{ij} dq_i dq_j \ , \qquad (2.4.11)$$

where, $b_{ij} = 2(h-V)a_{ij}$; therefore

$$W = \int_A^B \sum_{i,j=1}^{n} b_{ij} dq_i dq_j = \int_A^B d\sigma' , \qquad (2.4.12)$$

where

$$d\sigma' = \sqrt{\sum_{i,j=1}^{n} b_{ij} dq_i dq_j} \ , \qquad (2.4.13)$$

Thus, the motion of a holonomic system under the action of potential forces can always be regarded as the inertial motion in a Riemannian space, whose metric is determined by the fundamentally metric form (2.4.12)."

These arguments prove, in fact, that in such a quasi-Riemannian form we can write all of the classic potential fields. For our purposes it is necessary to generalize these calculations to the case of pseudo-Euclidean geometry.

As an example of writing the equations of the Lorentz-invariant theory in the Riemann geometry can be considered a relativistic theory of gravitation (RTG) of A.A. Logunov and co-workers (see, for example, (Logunov and Mestvirishvili, 1984; Vlasov, Logunov and Mestvirishvili, 1984, Logunov, 2002)

5.0. The relativistic theory of gravitation (RTG) of Logunov (Logunov and Mestvirishvili, 1984; Vlasov, Logunov and Mestvirishvili, 1984, Logunov, 2002)

As A. Logunov and his collaborators shows, the predictions of general relativity theory (GRT) are not unambiguous. For some of the effects such ambiguity displays itself in first order terms in powers of the gravitational constant *G*, for other – in second order ones. The absence both the energy-momentum and angular momentum conservation laws for matter and gravitation field, taken together, and besides its incapacity to give simple definite predictions for gravitational effects leads inevitably to the refusal from GRT as the physical theory.

The relativistic theory of gravitation (RTG) is constructed uniquely on the basis of the special principle of relativity and the principle of geometrization. The gravitational field being regarded as a physical field in the spirit of Faraday and Maxwell, possessing energy, momentum, and spin 2 and 0. The source of the gravitational field is the total conserved energy-momentum tensor of the matter and the gravitational field in Minkowski space. Conservation laws hold rigorously for the energy, momentum, and angular momentum of the matter and the gravitational field. The theory explains all the existing gravitational experiments. By virtue of the geometrization principle, the Riemannian space has a field origin in the theory, arising as an effective force space through the action of the gravitational field on the matter. The theory gives a prediction of exceptional power - the Universe is not closed, merely "flat." It follows from this that in the Universe there must be "hidden mass" in some form of matter.

In short

In an arbitrary reference system the interval assumes the form

$$d\sigma^2 = \gamma_{\mu\nu}(x) dx^\mu dx^\nu ,$$

where $\gamma_{\mu\nu}(x)$ is the metric tensor of Minkowski space.

Any physical field in Minkowski space is characterized by the density of the energy-momentum tensor $\tau_{\mu\nu}$, which is a general universal characteristic of all forms of matter that satisfies both local and integral conservation laws.

Owing to gravitation being universal, it would be natural to assume the conserved density of the energy-momentum tensor of all fields of matter, $\tau^{\mu\nu}$, to be the source of the gravitational field.

Further, we shall take advantage of the analogy with electrodynamics, in which the conserved density of the charged vector current serves as the source of the electromagnetic field, while the field itself is described by the density of the vector potential $\tilde{A}^\nu$:

$$\tilde{A}^\nu = (\tilde{\varphi}, \tilde{A}), \tag{2.5.1}$$

Here and further we shall always deal with the densities of scalar and tensor quantities defined in accordance with the rule $\tilde{\varphi} = \sqrt{-\gamma}\varphi,\ \tilde{A} = \sqrt{-\gamma}A,\ \ \gamma = \det(\gamma_{\mu\nu})$.

In the absence of gravitation, Maxwell’s equations will have the following form in arbitrary coordinates:

$$\gamma^{\alpha\beta} D_\alpha D_\beta A^\nu + \mu^2 A^\nu = 4\pi\, j^\nu , \tag{2.5.2}$$

$$D_\beta A^\nu = 0 , \tag{2.5.3}$$

where D_μ represents the covariant derivative in Minkowski space. Here, for generalization we have introduced the parameter μ, which, in the system of units $\hbar = c = 1$ is the photon rest mass.

According to D. Hilbert, the density of the energy-momentum tensor $\tau^{\mu\nu}$ is expressed via the scalar density of the Lagrangian L as follows:

$$\tau^{\mu\nu} = -2\frac{\delta L}{\delta\gamma_{\mu\nu}} \ , \qquad (2.5.4)$$

Here $\tau^{\mu\nu}$ is the total conserved density of the energy-momentum tensor for all the fields of matter.

Since we have decided to consider the conserved density of the energy-momentum $\tau^{\mu\nu}$ to be the source of the gravitational field, it is natural to consider the gravitational field a tensor field and to describe it by the density of the symmetric tensor $\varphi_{\mu\nu}$:

$$\tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\phi^{\mu\nu}, \qquad (2.5.5)$$

and in complete analogy with Maxwell's electrodynamics the equations for the gravitational field can be written in the form

$$\gamma^{\alpha\beta} D_\alpha D_\beta \tilde{\phi}^{\mu\nu} + m^2 \tilde{\phi}^{\mu\nu} = \lambda\,\tau^{\mu\nu}, \qquad (2.5.6)$$

$$D_\mu \tilde{\phi}^{\mu\nu} = 0\,, \qquad (2.5.7)$$

Here $m = m_g c/\hbar$, m_g is the graviton rest mass, λ is a certain constant which, in accordance with the principle of correspondence to Newton's law of gravitation, should be equal to 16π. Equation (2.5.7) excludes spins 1 and 0′, only retaining those polarizational properties of the field, that correspond to spins 2 and 0.

In more details

Equations (2.5.6) and (2.5.7), which we formally declared the equations of gravitation by analogy with electrodynamics, must be derived from the principle of least action, since only in this case we will have an explicit expression for the density of the energy-momentum tensor of the gravitational field and of the fields of matter. But, to this end it is necessary to construct the density of the Lagrangian of matter and of the gravitational field.

The density of the energy-momentum tensor of matter $\tau^{\mu\nu}$ consists of the density of the energy-momentum tensor of the gravitational field $\tau_g^{\mu\nu}$, and of the energy-momentum tensor of matter, $\tau_M^{\mu\nu}$. We understand matter to comprise all the fields of matter, with the exception of the gravitational field $\tau^{\mu\nu} = \tau_g^{\mu\nu} + \tau_M^{\mu\nu}$. The interaction between the gravitational field and matter is taken into account in the density of the energy-momentum tensor of matter $\tau_M^{\mu\nu}$.

An interesting picture arises consisting in that the motion of matter in Minkowski space with the metric $\gamma^{\mu\nu}$ under the influence of the gravitational field $\varphi^{\mu\nu}$ is identical to the motion of matter in effective Riemannian space with the metric $g^{\mu\nu}$. We term such interaction of the gravitational field with matter the *geometrization principle*.

Geometrization principle is carried out by "connecting" of the gravitational field $\phi^{\mu\nu}$ to the metric tensor $\gamma^{\mu\nu}$ of Minkowski space in the Lagrangian density of matter according to the rule:

$$L_M\left(\tilde{\gamma}_{\mu\nu}, \phi_A\right) \to L_M\left(\tilde{g}_{\mu\nu}, \phi_A\right), \qquad (2.5.8)$$

$$\tilde{g}^{\mu\nu} = \sqrt{-\gamma} g^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu},\ \tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu},\ \tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\phi^{\mu\nu} \ , \qquad (2.5.9)$$

where with ϕ_A are designated the fields of substance.

The geometrization principle is a consequence of the initial assumption that a universal characteristic of matter - the density of the energy-momentum tensor - serves as the source of the gravitational field. Such a density structure of the Lagrangian of matter indicates that a unique possibility is realized for the gravitational field to be attached inside the Lagrangian density of matter directly to the density of the tensor $\tilde{\gamma}^{\mu\nu}$.

For description of the effective Riemannian space the density of the Lagrangian L assumes the form

$$L = L_g(\gamma_{\mu\nu}, \tilde{\phi}^{\mu\nu}) + L_M(\gamma_{\mu\nu}, \tilde{\phi}^{\mu\nu}, \phi_A), \qquad (2.5.10)$$

At the same time, from the previous arguments we arrive at the important conclusion that the density of the Lagrangian of matter, L, has the form

$$L = L_g(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}) + L_M(\tilde{g}^{\mu\nu}, \phi_A), \qquad (2.5.11)$$

As substance we mean all forms. According to the principle of geometrization the motion of matter under the influence of the gravitational field $\phi^{\mu\nu}$ in Minkowski space with the metric $\gamma^{\mu\nu}$ is identical to the free movement in the effective Riemannian space with metric $g^{\mu\nu}$. The metric tensor $\gamma^{\mu\nu}$ of Minkowski space and the tensor of the gravitational field $\phi^{\mu\nu}$ in this space are the primary concepts, and a Riemannian space and its metric - secondary.

Built on the basis of the gauge principle the Lagrangian of the free gravitational field in general will have the form:

$$L_g = \frac{1}{16\pi}\tilde{g}^{\mu\nu}\left(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}\right) - \frac{m^2}{16\pi}\left[\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - (-g)^{1/2} - (-\gamma)^{1/2}\right], \qquad (2.5.12)$$

where $G^{\lambda}_{\mu\nu}$ is the tensor of rank three: $G^{\lambda}_{\mu\nu} = \frac{1}{2}g^{\lambda\sigma}\left(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}\right)$

Its corresponding dynamic equations for the free gravitational field can be written in the form:

$$J^{\mu\nu} - m^2\tilde{\phi}^{\mu\nu} = -16\pi\tau_g^{\mu\nu}, \qquad (2.5.13)$$

or

$$R^{\mu\nu} - \frac{m^2}{2}\left(g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}\right) = 0, \qquad (2.5.14)$$

where $J^{\mu\nu} = D_\alpha D_\beta\left(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu} + \gamma^{\alpha\nu}\tilde{g}^{\beta\mu} - \gamma^{\alpha\beta}\tilde{g}^{\mu\nu} - \gamma^{\mu\nu}\tilde{g}^{\alpha\beta}\right)$ and $R^{\mu\nu}$ is the Ricci tensor: $R^{\mu\nu} = D_\lambda G^{\lambda}_{\mu\nu} - D_\mu G^{\lambda}_{\nu\lambda} + G^{\sigma}_{\mu\nu}G^{\lambda}_{\sigma\lambda} - G^{\sigma}_{\mu\lambda}G^{\lambda}_{\nu\sigma}$

Based on (2.5.11) a complete system of equations for the gravitational field will have the form:

$$R^{\mu\nu} - \frac{m^2}{2}\left(g^{\mu\nu} - g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta}\right) = \frac{8\pi}{(-g)^{1/2}}\left(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T\right), \qquad (2.5.15)$$

$$D_\mu\tilde{g}^{\mu\nu} = 0, \qquad (2.5.16)$$

or in a slightly different form:

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\phi}^{\mu\nu} + m^2\tilde{\phi}^{\mu\nu} = 16\pi\,\tau^{\mu\nu}, \qquad (2.5.15a)$$

$$D_\mu\tilde{\phi}^{\mu\nu} = 0, \qquad (2.5.16a)$$

where $T^{\mu\nu} = -2\frac{\delta L_M}{\delta g_{\mu\nu}}$ is the energy-momentum tensor of substance and $\tau^{\mu\nu} = -2\frac{\delta L}{\delta\gamma_{\mu\nu}}$ is the total tensor of substance and gravitational field in Minkowski space.

In calculations of effects in the gravitational field of the Sun one usually takes as the idealized model of the Sun a static spherically symmetric body of radius R⊙. The general form of the metric of Riemannian space in an inertial reference system in spherical coordinates is

$$ds^2 = U(r)\left(dx^0\right)^2 - V(r)(dr)^2 - W^2(r)\left[(d\vartheta)^2 + \sin^2\Theta\left(d\varphi^2\right)\right], \qquad (2.5.17)$$

In the absence of a gravitational field the metric has the form

$$d\sigma^2 = \left(dx^0\right)^2 - \left(dr\right)^2 - r^2\left[\left(d\vartheta\right)^2 + \sin^2\Theta\left(d\varphi\right)^2\right], \tag{2.5.17a}$$

Substituting (2.5.17) and (2.5.17a) into equations (2.5.15) and (2.5.16) we precisely obtain the external solution for the Sun:

$$ds^2 = \frac{r-\gamma M}{r+\gamma M}\left(dx^0\right)^2 - \frac{r-\gamma M}{r+\gamma M}\left(dr\right)^2 - \left(r+\gamma M\right)^2\left[\left(d\vartheta\right)^2 + \sin^2\Theta\left(d\varphi^2\right)\right], \tag{2.5.18}$$

where M is mass of the Sun, and γ is Newton gravitation constant.

It can easy shown that from the RTG equations (2.5.15) and (2.5.16) follow directly the equations of motion for matter,

$$\nabla_\nu T_{\mu\nu} = 0\,, \tag{2.5.19}$$

Hence it is easy to obtain the equations of motion for a test body in a static gravitational field.

Thus, from the requirement that the density of the energy-momentum tensor of matter be the source of the gravitational field it follows in a natural way that the motion of matter should take place in effective Riemannian space. This assertion has the character of a theorem.

Hence it becomes clear, why Riemannian space arose, instead of some other. Precisely this circumstance provides us with the possibility of formulating the gauge group.

The effective Riemannian space is literally of a field origin, owing to the presence of the gravitational field. Thus, the reason that the effective space is Riemannian, and not any other, lies in the hypothesis that a universal conserved quantity — the density of the energy-momentum tensor — is the source of gravitation. We shall explain this fundamental property of gravitational forces by comparing them with the electromagnetic forces.

6.0. The geometrization of Dirac electron (Goenner, 2004)

V. Fock (Fock, 1929, 1929a) in May 1929 and later in the year wrote several papers on the subject of "geometrizing" Dirac's equation:

The main aim of theory is to establish the fact that the Dirac equation is perfectly compatible with the notion of a Riemann space. The geometric method introduced by Einstein in the macroscopic physics turns out to be applicable to the microscopic physics; a semi-vector and the Dirac wave equation are as fundamental notions as a vector and the d'Alembert equation.

In the described theory, a connection between electromagnetic and gravitational quantities is established via the notion of a semi-vector. The vector-potential finds its place in Riemann geometry and one does not need to introduce the remote parallelism (Einstein, 1928). It seems that theory could serve as the basis for a future theory, including maybe the quantization of gravitation, which would unify the electricity and the gravitation theory.

In another paper (Fock und Iwanenko, 1929), Fock and Ivanenko took a first step towards showing that Dirac's equation can also be written in a generally covariant form.

6.1. Connection of the Dirac matrices with the metric tensor (Fock und Iwanenko, 1929)

After discovering of the Dirac electron equation, the base was introduced for the geometrization of quantum theory in the spirit of general relativity. The linearity of relativistic operator of Hamilton suggests that we can get instead of a quadratic basic form of the metrics, a linear expression:

$$ds = \sum_{\nu=1}^{4}\gamma_\nu dx_\nu\ , \tag{2.6.1}$$

which may be taken as a geometrical operator of the line element ds. This assumption makes it possible geometrical interpretation of the Dirac equation. Consideration may be transferred to general relativity, and reveals the relationship of the variables matrices γ_ν, introduced by Tetrode, with Einstein's four-component values $h_{\nu\alpha}$.

Divide the expression (1) at the differential of proper time $d\tau$. As result we obtain a "linear-geometric" expression for the vector 4-speed:

$$\frac{ds}{d\tau} = \sum_{\nu=1} \gamma_\nu \upsilon_\nu \,, \tag{2.6.2}$$

where υ_ν is a 4-speed. The eigenvalues of the "geometrical" operators υ_ν are equal $\pm c$. Turning now from "linear geometry" to "quantum geometry", we want to present the components υ_ν as a quantum-mechanical operator:

$$\upsilon_\nu = \frac{1}{m}\left(\hat{p}_\nu + \frac{e}{c}A_\nu\right) = \frac{1}{m}\left(\frac{\hbar}{i}\frac{\partial}{\partial x_\nu} + \frac{e}{c}A_\nu\right), \tag{2.6.3}$$

In this case "quantum-geometric" operator (3) becomes:

$$\frac{d\hat{s}}{d\tau} = \frac{1}{m}\sum_{\nu=1}^{4} \gamma_\nu \left(\hat{p}_\nu + \frac{e}{c}A_\nu\right), \tag{2.6.4}$$

The corresponding classical quantity of 4-speed is equal to c. Equating the corresponding operator (2.6.4) to the c and acting with them on the wave function (semi-vector) ψ:

$$\frac{1}{m}\sum_{\nu=1}^{4} \gamma_\nu \left(p_\nu + \frac{e}{c}A_\nu\right)\psi = c\psi \,, \tag{2.6.5}$$

we obtain the Dirac wave equation. The passage to (5) is purely formal. Physically, a satisfactory interpretation of the wave equation is obtained by introducing the concept of statistical expectation. In fact, the integral

$$\int_{t_1}^{t_2} dt \iiint \overline{\psi} \left\{ \frac{1}{m}\sum_{\nu=1}^{4} \gamma_\nu \left(p_\nu + \frac{e}{c}A_\nu\right) - c \right\} \psi \; dxdydz \,, \tag{2.6.6}$$

whose variation according to Darwin (Darwin, 1928) gives the Dirac equation.

Our observations can also carry over to the general relativity. The "linear geometry" ds of the formula (2.6.1) is also valid if we as γ_ν understand the covariant matrixes, presented by Tetrode (Tetrode, 1928). The conditions of the Dirac equations for the four four-row matrices:

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\delta_\mu^\nu \quad (\delta_\mu^\nu = 1 \text{ for } \mu = \nu \text{ and } = 0 \text{ for } \mu \neq \nu),$$

we replace by

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2g_{\mu\nu},$$

where $g_{\mu\nu}$ are the components of the metric fundamental tensor. As in the classical theory, they should always be of "ordinary" sizes, i.e., they should be different from the identity matrix only by a factor and are therefore compatible with all matrices from the group γ_μ. According to (2.6.2) γ_μ are the same as $g_{\mu\nu}$ in general space-time functions, while it could be sooner considered as constants.

Noteworthily is the connection of γ_ν with the Ricci four-dimension values ("Vierbeingroessen"), which Einstein referred to as $h_{\nu\alpha}$.

We can put namely

$$\gamma_\nu = \sum_\alpha h_{\nu\alpha} \gamma_\alpha^0 \,, \tag{2.6.7}$$

where γ_α^0 denote the constant Dirac matrices. Then we remain in the area of geometry and consider the dx_ν as commuting quantities, so we have the replacement of (2.6.7) into (2.6.1) and squaring

$$ds^2 = \sum_\alpha \left(\sum_\nu h_{\nu\alpha} dx_\nu \right)^2, \tag{2.6.8}$$

in keeping with Einstein's formula. Analogically γ_ν expresses itself through h_α^ν:

$$\gamma^\nu = \sum_\alpha h_\alpha^\nu \gamma_\alpha^0, \tag{2.6.9}$$

Conversely from (2.6.7) or (2.6.9) we can also express $h_{\nu\alpha}$ or h_α^ν by γ_ν or γ^ν:

$$h_{\nu\alpha} = \frac{1}{2}\left(\gamma_\nu \gamma_\alpha^0 + \gamma_\alpha^0 \gamma_\nu\right), \tag{2.6.10}$$

$$h_\alpha^\nu = \frac{1}{2}\left(\gamma_\alpha^0 + \gamma_\alpha^0 \gamma^\nu\right), \tag{2.6.11}$$

Demanded by Einstein the rotation invariance of equations corresponds to the fact that only up to a canonical transformation are fixed.

The close relationship of γ_ν with $h_{\nu\alpha}$ allows to be noted that between remote areas - gravitation and quantum theory - there is a bridge in the form of "linear geometry".

6.2. Geometrization of Dirac theory (Goenner, 2004)

"In a subsequent note in the Reports of the Parisian Academy, Fock and Ivanenko introduced Dirac's 4-spinors under Landau's name "*semi-vector*" and defined their parallel transport with the help of Ricci's coefficients. In modern parlance, by introducing a covariant derivative for the spinors, they in fact already obtained the "*gauge-covariant*" derivative

$$\nabla_k \psi = \left(\frac{\partial}{\partial x^k} - \frac{i}{\hbar}\frac{e}{c} A_k \right)\psi, \tag{2.6.12}$$

Thus $\delta\psi = \frac{i}{\hbar}\frac{e}{c} A_k dx^k \psi$ is interpreted in the sense of Weyl:

"*Thus, it is in the law for the transport of a semi-vector that Weyl's differential linear form must appear*."

In order that gauge-invariance results, ψ must transform with a factor of norm 1, innocuous for observation, i.e., $\psi \to \exp\left(\frac{i}{\hbar}\frac{e}{c}\sigma\right)$ if $A_k \to A_k + \frac{\partial \sigma}{\partial x^k}$. Another note and extended presentations in both a French and a German physics journal by Fock alone followed suit. In the first paper Fock defined an asymmetric matter tensor for the spinor field,

$$T^j{}_k = \frac{ch}{2\pi i}\left[\bar{\psi}\gamma^j \left(\frac{\partial \psi}{\partial x^k} - \Gamma_k \psi\right) - \frac{1}{2}\nabla_k \left(\bar{\psi}\gamma^j \psi\right)\right], \tag{2.6.13}$$

where $\Gamma_k = \sum_i e_l \alpha_l h_{kl} C_l$ is related to the matrix-valued spin connection in the expression for the parallel transport of a half-vector:

$$\delta\psi = \sum_l e_l C_l ds_l \psi, \tag{2.6.14}$$

The covariant derivative then is $D_k = \frac{\partial}{\partial x^k} - \Gamma_k$. Fock made clear that the covariant formulation of Dirac's equation did not need the special geometry of Einstein's theory of distant parallelism:

“By help of the concept of parallel transport of a semi-vector, Dirac’s equations will be written in a generally invariant form. [...] The appearance of the 4-potential φ_l besides the Ricci-coefficients γ_{ikl} in the expression for parallel transport, on the one hand provides a simple reason for the emergence of the term $p_l - \frac{e}{c}\varphi_l$ in the wave equation and, on the other, shows that the potentials φ_l have a place of their own in the geometrical world-view, contrary to Einstein’s opinion; they need not be functions of the γ_{ikl}.”

6.3. The Dirac electron in gravitation field (Schroedinger, 1932)

“*The joining of Dirac’s theory of the electron with general relativity has been undertaken repeatedly, such as by Wigner, Tetrode, Fock, Weyl, Zaycoff, Podolsky. Most authors introduce an orthogonal frame of axes at every event, and, relative to it, numerically specialised Dirac-matrices. This procedure makes it a little difficult to find out whether Einstein’s idea concerning teleparallelism, to which authors sometimes refer, really plays a role, or whether there is no dependence on it. To me, a fundamental advantage seems to be that the entire formalism can be built up by pure operator calculus, without consideration of the* ψ *-function*.” (this paragraph translated from (Goenner, 2004)).

The operator $\gamma^k \nabla_k$, which can be called "gradient", is invariant. The generalized Dirac equation, following from this, is:

$$\gamma^k \nabla_k \psi = \mu\psi , \tag{2.6.15}$$

where $\mu = \frac{mc}{\hbar} = \frac{1}{r_C}$, where r_C is the Compton wavelength of an electron.

We now want to square the Dirac equation to compare the results with those obtained from the special theory. (For brevity, we omit ψ):

$$\gamma^k \nabla_k \gamma^l \nabla_l = \mu^2 , \tag{2.6.16}$$

Replace the first two factors by the relation:

$$\gamma^k \nabla_k - \gamma^k \nabla_k = -\Gamma^k_{k\mu}\gamma^\mu = -\frac{\partial \lg \sqrt{g}}{\partial x_\mu}\gamma^\mu , \tag{2.6.17}$$

From the relations $\gamma_i\gamma_k + \gamma_k\gamma_i = 2g_{ik}$ and $S^{\mu\nu} = \frac{1}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$ follows $\gamma^k\gamma^l = g^{kl} + S^{kl}$.

Then we obtain:

$$\nabla_k \left(g^{kl} + S^{kl}\right)\nabla_l + \frac{\partial \lg \sqrt{g}}{\partial x_\mu}\gamma^\mu\gamma^l\nabla_l = \mu^2 , \tag{2.6.18}$$

Then, having made a series of transformations, we obtain:

$$\frac{1}{\sqrt{g}}\nabla_k \sqrt{g} g^{kl}\nabla_l - \frac{1}{2}S^{kl}\Phi_{kl} = \mu^2 , \tag{2.6.19}$$

where $\Phi_{kl} = \nabla_l\nabla_k - \nabla_k\nabla_l$. This expression can be expressed as follows:

$$\Phi_{kl} = -\frac{1}{4}R_{kl,\mu\nu}S^{\mu\nu} + f_{kl}\cdot 1 , \tag{2.6.20}$$

where $R_{kl,\mu\nu}$ is a symmetric Riemann tensor, and f_{kl} is free multiplier, which multiple to unit.

On the other hand:

$$\Phi_{kl} = \frac{\partial \Gamma_l}{\partial x_k} - \frac{\partial \Gamma_k}{\partial x_l} + \Gamma_l \Gamma_k - \Gamma_k \Gamma_l , \tag{2.6.21}$$

where Γ_l are four independent from time and space matrixes. Thus there is an invariant $\frac{1}{8} R_{kl,\mu\nu} S^{kl} S^{\mu\nu} \equiv \frac{1}{16} R_{kl,\mu\nu} \left(S^{kl} S^{\mu\nu} + S^{\mu\nu} S^{kl} \right)$. Making some calculations and transformations, we obtain finally:

$$\frac{1}{8} R_{kl,\mu\nu} S^{kl} S^{\mu\nu} = -\frac{1}{4} g^{k\mu} g^{l\nu} R_{kl,\mu\nu} = -\frac{R}{4} , \tag{2.6.22}$$

where R is an invariant curvature. Using (2.6.22) and substituting in (2.6.19) Φ_{kl} according with (2.6.20), we finally obtain:

$$\frac{1}{\sqrt{g}} \nabla_k \sqrt{g} g^{kl} \nabla_l - \frac{R}{4} - \frac{1}{2} f_{kl} S^{kl} = \mu^2 , \tag{2.6.23}$$

In the first term is easy to find a regular operator of the Klein second order equation in the Riemann geometry. In the third term on the left is recognized well-known term associated with the spin magnetic and electric moments of the electron (tensor S^{kl}).

"*To me, the second term seems to be of considerable theoretical interest. To be sure, it is much too small by many powers of ten in order to replace, say, the term on the r.h.s. For* μ *is the reciprocal Compton length, about* $10^{11} cm^{-1}$. *Yet it appears important that in the generalised theory a term is encountered at all which is equivalent to the enigmatic mass term*." (this paragraph translated from (Goenner, 2004)).

7.0. Peculiarities of unified theories of electromagnetism and gravity (Tonnelat, 1966)

The connection between the field and its sources has been and continues to be one of the most difficult problems that have to be resolved by electrodynamics, both in the classical and quantum realms.

Among the many theories that have been put forward concerning this problem, one can distinguish *dualistic* and, contrariwise, *non- dualistic* theories.

a) The dualistic theories assume that the particles, which are the sources of the field, constitute entities that, along with their various characteristics, mass and charge, remain fundamentally different from the field itself.

b) On the other hand, non-dualistic theories assume that the sources of the field are not essentially different in nature from the field itself.

7.1. Nonlinearity and the characteristics of a pure field theory

Electrodynamics of G. Mie and the Born-Infeld on the one hand, and the theory of gravitation on the other hand, have one thing in common - non-linearity. From the results obtained in the general theory of relativity (GRT), it follows that such a nonlinearity is a necessary, but not sufficient, condition to the motion of singularities can be deduced from the field equations themselves.

7.1.1. *Вывод уравнений движения.*

A. Derivation of the equations of motion. Classical Electrodynamics is based on Maxwell's equations, which are partial differential equations of first order. These equations are linear.

With the help of Maxwell's equations alone, the additional condition, which characterizes the interaction of two particles, can not be obtained, i.e., it is impossible to obtain Coulomb's law, because from the linear theory, this condition does not arise with the necessity.

Thus, the field equations can not bring such additional condition, and it should apply independently. That way, we arrive at an expression for the Lorentz force.

If, in contrast to the previous case the field equations are nonlinear, then this makes it possible to derive from the themselves field equations the equations of motion of the particles.

7.1.2. *Reduction of the field sources to the field itself.*

So it seems that a unified theory can achieve any sort of internal cohesion in physics only if it is doubly unified, assuring the synthesis

a) of the electrodynamic field and the gravitational field, so that classical electrodynamics is subsumed,

b) of the generalized field thus defined and of the particles - the motion of the particles being deducible from the properties of the generalized field.

"A coherent field theory", writes Einstein, "requires that all its elements be continuous... And from this requirement arises the fact that the material particle has no place as a basic concept in a field theory. Thus, even apart from the fact that it does not include gravitation, Maxwell's theory cannot be considered a complete theory."

«So it becomes interesting to compare Einstein's conclusions with those that could be drawn from, for example, the electrodynamics of Born-Infeld.

In both cases there is an attempt made to reduce the sources of the field to the field itself (that is, they are both non-dualistic theories).

In both cases non-linear equations are employed.

The non-linearity of the field equations is a necessary condition for obtaining the equations of motion. It does not constitute, however, a sufficient condition for achieving that objective. The electrodynamics of Born-Infeld, for example, cannot, on its own, come up with Coulomb's law. On the other hand, the law motion for charged particles will be obtained if the equations of the gravitational field contain, as expressed in the right-hand member, the contribution of the electromagnetic field».

The G. Mie theory (and its special case: the Born-Infeld theory) are purely electromagnetic theory. In a series of articles in "Prespacetime Journal" we have shown that in the Nonlinear Theory of Elementary Particles (NTEP) all the results of Mie and Born-Infeld theory remain in force and, therefore, from this point of view NTEP can be the basis for the relativistic theory of gravitation.

Moreover, as shown in the NTEP (Kyriakos, 2011), the Lagrangian of the Dirac electron equation is a quantum analogue of the Lagrangian of the nonlinear theory of G. Mie. As we know, in the QED the Dirac equation allows us to derive Coulomb's law, and allows to deduce the motion equation of macroscopic bodies (Ehrenfest theorem). Below, we consider the briefly description of these results.

8.0. The "nonlinear" features of relativistic quantum theory of the Dirac electron

8.1. The Coulomb equation (Dirac, 1978)

“There is one further development of this theory that I (i.e. Dirac) would like to point out here, as it is very neat and satisfying. In the first place, one obtains the Hamiltonian not actually in terms of the electromagnetic field-vector variables $\vec{E},\vec{H}$, which we had in the discussion, but rather in terms of the four-potential components A_μ, $\mu = 0, 1, 2, 3$. Now, the Hamiltonian… represents the energy of a field of pure radiation, with only transverse electromagnetic waves. As long as we are dealing only with transverse waves, we cannot bring in the Coulomb interactions between particles. To bring them in, we have to introduce longitudinal electromagnetic waves and include them in the potentials A_μ.

The longitudinal waves can be eliminated by means of a mathematical transformation. We like to eliminate them, because they are only rather remotely connected with experiment. The result of this elimination is to give some new variables, replacing the old operators, for the emission and absorption of electrons. These new variables have quite a simple physical meaning. Each new variable refers to the emission of an electron together with the Coulomb field around it, not just a bare electron. Similarly, each rj annihilates an electron and its Coulomb field.

We thus get a new version of the theory, in which the electron is always accompanied by the Coulomb field around it. Whenever an electron is emitted, the Coulomb field around it is simultaneously emitted, forming a kind of dressing for the electron. Similarly, when an electron is absorbed, the Coulomb field around it is simultaneously absorbed. This is, of course, very sensible physically, but it also means a rather big departure from relativistic ideas. For, if you have a moving electron, then the Coulomb field around it is not spherically symmetrical. Yet it is the spherically symmetric Coulomb field that has to be emitted here together with the electron.

Now, when we do make this transformation which results in eliminating the longitudinal electromagnetic waves, we get a new term appearing in the Hamiltonian. This new term is just the Coulomb energy of interaction between all the charged particles:

$$\sum_{(1,2)} \frac{e_1 e_2}{r_{12}}, \tag{2.8.1}$$

where the summation extends over all pairs of particles existing in the state of the system with which we are concerned. This term appears automatically when we make the transformation of the elimination of the longitudinal waves".

For details, see, e.g., (Sokolov and Ivanenko, 1952, p.143).

8.2. The equation of motion

An interesting application of the theory (Sokolov and Ivanenko, 1952; pp. 650-651) is to establish an analogue of Ehrenfest's theorem for the Dirac equation, generalized to the Riemann geometry. In addition to the results obtained above (see section), by squaring of the Dirac equation, for the center of gravity of the wave packet (provided $\hbar \to 0$), we obtain the equation of relativistic mechanics of point:

$$\frac{d}{dx^4}\left(\gamma^4 p_\alpha\right) = \Gamma^\sigma_{\alpha\rho} p_\alpha + \gamma^\rho \frac{e}{c} F_{\rho\alpha}, \tag{2.8.2}$$

where γ^4 is the fourth Dirac matrix, γ^ρ corresponds to the particle velocity in fraction of the speed of light c, $\Gamma^\sigma_{\alpha\rho}$ is the Christoffel brackets $\{\mu\nu,\alpha\} = \Gamma^\sigma_{\alpha\rho} = \frac{1}{2}\left(\frac{\partial g_{\mu\sigma}}{\partial x_\nu} + \frac{\partial g_{\nu\sigma}}{\partial x_\mu} + \frac{\partial g_{\mu\nu}}{\partial x_\sigma}\right)$, $F_{\rho\alpha}$ is the electromagnetic field tensor. The first term on the right of equation is the force of gravity, and the second term is the Lorentz force.

Conclusion

The continuation of this article will be devoted to the construction of a gravitational theory in the framework of NTEP.

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Part 3. The nature of pre-spacetime and its geometrization

> "Einstein's theory of gravitation may be compared with thermodynamics. Both start with a few fundamental principles or axioms based on experience and proceed by logical deductions without further appeal to facts. Neither attempts any explanation of the nature of the phenomena considered.
>
> The kinetic theory of gases is an example of a different type of theory. A definite mechanism is assumed for a gas and the properties of the mechanism are worked out and compared with the known properties of gases.
>
> If the axioms of the first type of theory are true, the theory itself must be true, unless the reasoning is at fault; both types of theory are to be tested by comparison of the results with observation...
>
> If we regard Einstein's geometry of orbits as geometry of space, we may say that the body describes an ellipse because the space and time around a heavy particle are not straight but curved.
>
> This does not seem to be an explanation any more than Newton's force. Why should a particle describe a curve in a curved space, and, anyhow, what is the meaning of curved space?
>
> The idea of space is obtained by experience; it cannot be explained… The conception of a curvature or distortion of empty space is meaningless, since only material bodies can be measured. This difficulty may be removed by supposing that space is not empty, but filled with a medium". (Wilson, 1921).

According to current assumptions such a medium is the physical vacuum - the material medium, which is the background of the existence of elementary particles.

1.0. Statement of problems

1. Einstein's theory is a theory of the macrocosm. The basis of this theory, which is conventionally called general relativity, is the notion of space-time. On one hand, the gravitational field is here a consequence of the curvature of space-time, which generates a pseudo-Riemannian space-time. However, the gravitational field is declared as cause of the curvature of space-time and the appearance of a pseudo-Riemannian space-time. Obviously, the simultaneous existence of these two statements does allow neither to determine the cause of gravitation, nor to give a definition of the space-time.

In general relativity, as a representative of pseudo-Riemannian space-time is defined the mathematical object, which called metric tensor. However, the elements of the metric tensor are relative dimensionless quantities, i.e., non-material objects and do not correspond to any material objects of nature. On the other hand, the gravitational field is a material field. In general relativity, the question how can non-material space-time produce a material object, is also not explained.

Indeed, despite these oddities, the GTR is tested well enough, at least for weak gravitational fields. Then, we must answer the question, of what is there in nature - physical, material, that is mathematically described as space and time, and how does it relate to the conversion of matter fields.

This the (yet unknown) physical body we will call **pre-spacetime**. This article is devoted to the elucidation of its nature and connection with the geometric space-time.

To avoid controversy, we will specify the initial definitions of the concepts adopted in modern physical theory.

1.1. Some definitions

1. (PhED, 1965): "**Space and time** are common forms of coordination of material objects and their states.

Space is a set of relations, expressing the coordination of coexisting objects: their location relative to each other and relative magnitude (distance and orientation).

Time is a set of relations that express the coordination of successive states (events): their sequence and duration. "

The abovementioned coordination is given by the reference frame system and linked to them coordinate systems.

2. (TD, 1989): "**The reference frame system** is a real or conditionally solid body, which is connected with the coordinate system, equipped with a clock and used to determine the position in space of the physical objects (particles, bodies, etc.) at different time points. Often as frame of reference is understood the system of coordinates, equipped with a clock".

Under the conditionally solid body is meant a system of rigid bodies, whose positions relative to each other are the same. For example, in astronomy, such a system for the solar system is the Sun (as the center of the system) and two or three stars in the Universe, which are with very good approximation fixed relatively to each other (for example, the Sun, the Pole Star, etc.)

3. (PhED, 1962): "**Coordinates of point** are the numbers that define its position on a surface in space."

The system of coordinates (or reference system) is a method to specify uniquely the coordinates of the point. Usually, the coordinate system is defined by means of a point called the center of coordinate system, and one or more lines associated with this point and with each other. In the simplest case straight lines are selected. But there are also countless other selections of lines.

As straight lines trajectory are taken the trajectories of the rays of light (i.e., photon trajectories) at infinite distance from the material bodies (i.e., in very weak fields). Therefore, a straight-line (Cartesian) coordinate system plays in physics the role of the standard for all other coordinate systems.

The unambiguity of the coordinate system setting in a physical theory is determined by its binding to a real or notional reference frame system.

2.0. Pre-spacetime in terms of quantum field theory and general relativity

At this stage of research we give a naive definition of a pre-spacetime, as volume containing the moving material bodies and fields. This allows us to enter the relationships that define the space and time according to the above definitions. According to the objectives of the study, we, first of all, should answer two questions.

What is a pre-spacetime of Universe, i.e., a volume containing all the bodies, which we feel by our sense organs and the experimental devices?

If we remove all material bodies and fields of the Universe, will it be empty, or will it be filled with something?

At the times of K. Maxwell, J.J. Thomson and H. Lorentz, scientists thought that this pre-spacetime of the Universe is not empty, and it is filled with some medium - electromagnetic ether.

(Lodge,1909): "Introduction. The problem of the constitution of the Ether, and of the way in which portions of it are modified to form the atoms or other constituent units of ordinary matter, has not yet been solved… Meanwhile there are few physicists who will dissent from Clerk-Maxwel's penultimate sentence in the article "Ether," of which the beginning has already been quoted:

"Whatever difficulties we may have in forming a consistent idea of the constitution of the ether, there can be no doubt that the interplanetary and interstellar spaces are not empty, but are occupied by a material substance or body, which is certainly the largest, and probably the most uniform body of which we have any knowledge."

What do we have in this regard in modern physics?

According to the prominent Russian Academician A. Migdal (Migdal, 1982) "The history of the ether provided us with a good example of intertwining of old and new ideas.

In the XIX century ether was endowed with contradictory properties to explain the laws of propagation of light in vacuum and in moving bodies. The special theory of relativity resolved all the contradictions of ether. Moreover, the need has disappeared in the very concept of the ether. However, it later emerged that emptiness - former "ether" - is not only the carrier of electromagnetic waves; the continuous oscillations of the electromagnetic field ("zero-point oscillations"), creation and annihilation of electrons and positrons, protons and antiprotons and, in general, of all elementary particles occur in it. If e. g. two protons collide, these "virtual" particles can become real: from "emptiness" the bundle of particles is occur in it.

It appears that emptiness is a very complex physical object. In fact, physicists have returned to the concept of "ether", but without the controversy. The old concept was not taken from the archive - it arose again in the process of science development. We call the new ether "vacuum" or "physical emptiness." But the history of the ether did not end here.

The theory of relativity is based on the assumption that in our world there is no selected coordinate system and therefore there is no absolute velocity; we observe only the relative motion. However, the selected coordinate system was introduced in our universe with the discovery of cosmic microwave background (CMB). This is a system, in which the CMB photons' distribution over velocities is spherically symmetric (as the gas particles in a stationary box).

In the "new ether" there is absolute velocity. However, the conclusions of the theory of relativity are conserved with enormous precision in accordance with the "correspondence principle".

Indeed, the cosmic microwave background (CMB) has temperature anisotropy (see image 2.1)

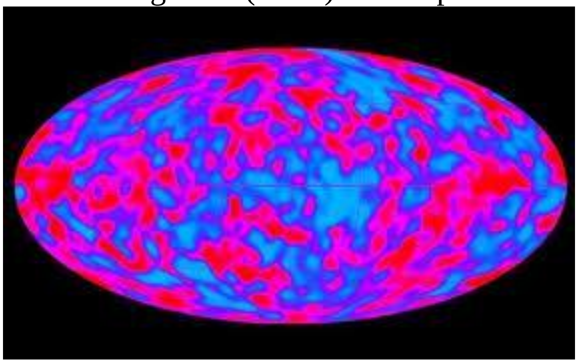

Fig. 2.1

This anisotropy allows us to introduce a single coordinate system for the entire Universe (roughly speaking, it allows us to fasten it to the inhomogeneities of the temperature distribution of microwave radiation). In this sense this system can be called "absolute". However, since the electromagnetic ether obeys the Lorentz transformations, as emphasized Oliver Lodge and Gustav Mie, all inertial coordinate systems are equal here, and the “absolute” frame of reference is one of them. Hence, indeed, the principle of relativity in such "absolute" space is not violated.

These conclusions of A. Migdal certainly do not contradict the fact that he wrote the creator of the theory of relativity, Einstein.

(Einstein, 1920): “More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny ether. We may assume the existence of an ether,; only we must give up ascribing a definite state of motion to it, i.e. we must by abstraction take from it the last mechanical characteristic which Lorentz had still left it. We shall see later that this point of view is justified by the results of the general theory of relativity...

The special theory of relativity forbids us to assume the ether to consist of particles observable through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether...

But on the other hand there is a weighty argument to be adduced in favor of the ether hypothesis. To deny the ether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view...

Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an ether. According to the general theory of relativity space without ether is unthinkable; for in such space there not only wonld be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense. But this ether may not be thought of as endowed with the quality characteristic of ponderable inertia, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it... “

Ether, about which Einstein speaks, was conceived as a continuous medium of unknown origin, whose properties are such that its fluctuations are electromagnetic waves (light, etc.). This ether is known as "electromagnetic ether" (briefly, "EM-ether"). Since the detection of the Lorentz transformations and then the construction of special relativity theory, Einstein pointed out that the existence of EM-ether does not contradict the theory of relativity (contrary to the mechanical ether of pre-Maxwellian times). Thus, Gustav Mie, who is known as the creator of the first unified theory of gravitation and electromagnetism, who based all his research on the special theory of relativity, emphasized this feature of the EM-ether (Mie, 1925): "EM ether, contrary to matter, can not itself be noticeable, since it is homogeneous and does not have any irregularities, which can be measurable."

Unfortunately, the mathematical description of the ether in classical physics has not been developed. But in the framework of quantum electrodynamics (QED) has been shown that the quantization of the electromagnetic field discloses the existence of medium like ether, which was named “physical vacuum". Comparison shows that the physical vacuum is equivalent to quantized electromagnetic ether (Kyriakos, 2010); (for details see below the analysis of this problem by Zel'dovich et al.).

But, the theory of physical vacuum has not yet been developed, (Dirac, 1957).: "Until now, there are significant difficulties with the description of the physical vacuum ..."). Many well-known physicists have noted that the theory of vacuum is necessary to complete our knowledge about the world.

3.0. Space-time in general relativity

3.1. The cosmological constant (Zel'dovich and Novikov, 1971)

The general requirements usually placed on the equations of the theory of gravitation permit one to write a variational principle with the action in the form

$$S = -mc\int ds - \frac{c^3}{16\pi\gamma}\left[\int RdV + \int 2\Lambda dV\right], \tag{3.3.1}$$

(where V is the four-dimensional volume)

´The corresponding field equations have the form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = \frac{\kappa}{c^2}\tau_{\mu\nu}, \tag{3.3.2}$$

Here Λ is the so-called cosmological constant, and quantities that are proportional to it $(\Lambda dV, \Lambda g_{\mu\nu})$ are called cosmological terms. These field equations obviously satisfy the condition of local Lorentz-invariance and include the equations of motion in the same sense as the equations without Λ do $\tau^k_{i;k} = 0$, as before.

Einstein initially selected Λ in such a manner as to obtain a stationary cosmological solution with a mean density $\tau^0_0 = \rho c^2 = const$ that is different from zero; for this, it is necessary that

$\Lambda = \frac{8\pi\gamma\rho}{3c^2}$. After the discovery of the cosmological redshift, Einstein preferred the equations with $\Lambda = 0$.

Both stationary and nonstationary cosmological solutions with $\Lambda \neq 0$ were investigated in detail before 1930; however, until 1967 there were no observational indications as to the necessity or even desirability of introducing Λ. Since 1967, observational data on quasars have suggested that A might not actually be zero, but might instead have a value of the order of $\Lambda \approx 10^{-55}$ cm^{-2} .

At present this hypothesis is not at all proved; in fact, it encounters difficulties in explaining the quasar observations. However, in the course of the discussions it has become apparent that the simplest assumption of Λ = 0, while not refuted, has not been distinctly and uniquely proved.

How can the physical meaning of the cosmological constant be understood? Why, in fact, is it interesting for physics as a whole?

One approach was prompted by the dimensions of $[\Lambda] = cm^{-2}$. In this approach one views Λ as the curvature of empty space. But the theory of gravitation links the curvature to the energy, momentum, and pressure of matter. Putting the terms with Λ onto the right side of the field equation, we obtain

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi\gamma}{c^4} \tau_{\mu\nu} - g_{\mu\nu}\Lambda , \qquad (3.3.3)$$

The assumption that $\Lambda \neq 0$ means that empty space creates a gravitational field identical with that in the theory with Λ = 0, but with matter filling all of space-time with a mass density $\rho_\Lambda = \frac{c^2\Lambda}{8\pi\gamma}$, an energy density $u_\Lambda = \frac{c^4\Lambda}{8\pi\gamma}$, and a pressure $P_\Lambda = -u_\Lambda$.

In this sense one can speak about the energy density and the pressure (stress tensor) of the vacuum.

Notice that our assumptions about P_Λ and u_Λ were formulated in such a manner that the relativistic invariance of the theory was not broken; P_Λ and u_Λ are the same in all coordinate systems moving relative to each other (Lorentz- transformed).

These quantities do not make themselves felt either in elementary-particle experiments or in atomic and molecular physics: the vacuum energy of the vessel in which an experiment takes place plays the role of a constant term which can be canceled in the law of energy conservation.

The only sort of phenomenon in which P_Λ and u_Λ manifest themselves is a gravitational one. In this case P_Λ and u_Λ "work" not only in empty space; they are, as is clear from the formulae (3.3.2), full and equal members of the field equations even when normal matter is present.

A practical measurement of the influence of P_Λ and u_Λ is not possible, either in laboratory experiments or in observations of planetary motions in the solar system or stellar motions in the Galaxy.

Let us dwell on the nature of Λ. One can take the viewpoint that a certain Λ and corresponding ρ_Λ, u_Λ and P_Λ are universal constants that do not require any further explanation. A different viewpoint is also conceivable: assume that in some zeroth-order approximation $\Lambda = \rho_\Lambda = u_\Lambda = P_\Lambda =$ 0. Higher-order values that are different from zero and characterize the vacuum might be derivable from considerations of the theory of elementary particles. There is no such derivation at the present time, any more than there is a theory proving that Λ = 0. Below we present some ideas on how to create a theory of Λ.

3.2. Physical vacuum (Zel'dovich ,1981)

“The vacuum is space in which there are no elementary particles. But does a vacuum theory exist? Can one say something meaningful about vacuum, i. e., about space containing nothing?

The present-day rich and complicated picture of the vacuum arises as a logical consequence of experiments and theories. One cannot say that the vacuum, i. e., empty space, is devoid of all properties "by definition".

We define vacuum as space without any particles. Such a definition coincides with the condition of a minimum of the energy density in the given volume of space. If the energy *E* of some region of space is greater than the minimal value ε_{min} for this region, then ε can be represented as the sum $\varepsilon_{min}+\Delta$, and the addition Δ can be regarded as the energy of the field or particles present in the given volume. Hence, a state with $\varepsilon > \varepsilon_{min}$ should not be called "vacuum.

"But the actual properties of the "minimal" state which is called the "vacuum" are dictated by the laws of physics, and we cannot insist that the minimum be zero or that the simplest possible situation be as simple as we wish.

The theory of electromagnetism leads to the conclusion that besides the static electric field surrounding charges there also exist specific solutions in the form of fields propagating freely in space and describing electromagnetic waves (radio waves, light, X- rays, gamma rays).

The modern view of electromagnetic waves emphasizes their similarity to a mechanical oscillator, i. e., a mass on a spring.

If one writes down the corresponding equations (which we shall not), it is found that the magnetic field plays the part of the spring, i. e., the energy of the magnetic field is analogous to the deformation of a spring, depending on the departure from the equilibrium position. The energy of the electric field is the analog of the kinetic energy of a moving particle. Thus, each definite oscillation mode of the electromagnetic field is analogous to the mechanical vibration of a mass on a spring.

Thus, in classical (but not quantum !) theory the vacuum concept is indeed rather simple — there is neither field nor energy.

Now quantum mechanics appears on the scene. The momentum and the coordinate of the mass cannot have definite values simultaneously. Applied to the electromagnetic field, this means (tot the magnetic and electric field cannot vanish simultaneously.

Quantum mechanics predicts that the possible values of the total energy of an oscillator are $\varepsilon_n=(n + 1/2)h\nu$ with arbitrary integral *n,* where *h is* Planck's constant and *v* is the oscillator frequency. Thus, one can have only states with an energy value in the sequence:

$n = 0,\ \varepsilon_0 = (1/2)\,h\nu,\ \ n = 1,\ \varepsilon_1 = (3/2)\,h\nu,\ \ n = 2,\ \ \varepsilon_2=(5/2)h\nu,...$

If an oscillator can exchange energy with other objects, then it gives up or receives energy only in definite portions, which are multiples of $h\nu$.

In a transition $n = 1 \to n = 0$, the oscillator gives up $h\nu$; in a transition $n = 0 \to n = 2$, it acquires $2h\nu$, etc. But here we wish to draw attention to the mysterious "halves," i. e., the value $(1/2)h\nu$ of the oscillator groundstate energy.

Experiments with atoms and molecules confirm the presence of the "halves." It is impossible to use quantum mechanics and avoid this result.

By analogy, one could readily believe that the application of quantum theory to the electromagnetic field will necessarily lead to a similar result. Indeed, the electric field and the magnetic field cannot vanish simultaneously; the electromagnetic energy density cannot vanish. One can pose the question of the minimum of the energy in the same way that one can speak about the lowest (ground) state of an oscillator. It is clear however that this minimum is not zero.

To get any further, we must now make more precise what we mean by the modes of the electromagnetic waves and consider what are the quantities that occur in the expressions relating to electromagnetic wave. It is important that the appropriate variables, i. e., the analogs of the position and velocity of the mass, are not the magnetic and electric field at one point of space; for Maxwell's equations contain derivatives with respect to the spatial coordinates, and the evolution of fields at a given point depends on the values of the fields at other points of space.

This circumstance makes it necessary to consider individual waves, which are independent of each other.

Why is the possibility of describing the solution in the form of a system of independent equations for individual oscillators so important? One answer, seen immediumtely in the 19th century, is that if a set of particular solutions is known it is possible to construct a solution to the problem with arbitrary initial conditions. For we are concerned with a linear equation, and any sum of particular solutions is also a solution.

Different initial conditions give a different set of quantities a_n and φ_n in the general expression $y = \sum a_n \cos(\omega_n t + \varphi_n) \sin \pi n x/l$.. There is however a deeper reason for using solutions of this type.

The point is that these solutions can be numbered and ordered. They can be arranged in a sequence with increasing value of the frequency. One can find the number of solutions with frequency less than a definite value or in a given interval of frequencies. In particular, for electromagnetic radiation in volume V the number of such solutions is $dN = V(8\pi\nu^2 d\nu/c^3)$.

It is here understood that the frequency ν is such that the corresponding wavelength $\lambda = c/\nu$ is less than the linear dimension of the container $d \sim V^{1/3}$, and we consider an interval $d\nu$ that is not too narrow, so that $dN = 8\pi(V/\lambda^3)d\nu/\nu \gg 1$ (despite $d\nu/\nu \ll 1$). Accordingly, the total number of solutions with frequency less than the given ν (per unit volume) is $n = (8\pi/3)(\nu/c)^3 = 8\pi/3\lambda^3$."'

For a string, bell, and so forth there is a physical restriction, namely, the minimal wavelength of the vibrations cannot be less than the distance between the atoms. But in vacuum there is no definite minimal wavelength! Accelerator experiments study photons with an energy of about 10^{10} eV, and their wavelength is $\lambda \approx 10^{-14}$cm.

In cosmic rays, we observe photons of even higher energy and shorter wavelength. But more important is the argument of relativistic invariance: There is not and cannot be a limit to the photon energy or wavelength because these are quantities that depend on the motion of the observer. For an oncoming observer, the energy will be higher, the wavelength shorter.

The vacuum has an infinite number of vibration modes, or, more precisely, an infinite number of vibrations per unit volume of the vacuum. Theory must take into account this fact and must be able to overcome the difficulties— computational and conceptual, i. e., "physical" associated with this fact.

3.3. Vacuum energy density

We now turn to the above assertion ($\varepsilon_0 = (1/2)h\nu$) follows from quantum theory. In granting a modest $0.5h\nu$ to each individual wave, we soon discover that when all the waves are taken together they give an infinite energy density. If we were to restrict ourselves to a definite maximal frequency ν_m, we would obtain a result of the $u = a\int_0^{\nu_m} \frac{1}{2} h\nu \cdot \nu^2 d\nu = (ah/8)\nu_m^4$, where u is the energy density and a is a constant ($a = kc^{-3}$, where c is the velocity of light and k is a number of order unity). In the limit $\nu_m \to \infty$, the value of e also tends to infinity. If we set $\nu_m = \infty$ directly, we obtain a divergent integral.

This is the well-known divergence problem, the socalled "ultraviolet catastrophe" of quantum electrodynamics or, rather, it is part of this problem.2) And there is no simple escape; one cannot ignore or simply reject the problem. The nonvanishing fields in the absence of photons (the fields corresponding to the "halves" (½)$h\nu$ for all possible ν) are observed, and they modify the motion of electrons in atoms. The famous Lamb-Retherford experiment confirms this.

We note also a phenomenon associated with the idea of the zero-point energy - the (Casimir and Polder, 1948; Casimir, 1948; Barash and Ginzburg, 1975).

Casimir calculated a more subtle effect, namely, he found the dependence of the zero-point energy on the mutual position of the bodies, for example, on the distance between the plates of an uncharged capacitor. But the derivative of the energy with respect to the displacement is the force acting in the direction of the displacement. This quantity is finite and the corresponding integrals converge.

However, such a favorable situation does not occur in all phenomena, and the density of the zero-point energy does not always cancel

The most important manifestation of the nonzero vacuum energy density could be its influence on the gravitational force field and on the gravitational potential.

The theory of gravitation contains the energy density of a body, including the energy density of the vacuum within the body and the surrounding space.

In this case, we are not speaking of energy differences, which could be zero. At first glance, we face an ineluctable contradiction. In principle, the contradiction could perhaps be avoided by taking into account the contribution of other particles. We shall merely emphasize here that this fundamental possibility has not yet been realized by modern science quantitatively and exactly!

More generally, the positive contribution of bosons could in principle be compensated by the negative contribution of the fermions.

Nevertheless, the most important theoretical question— that of the vacuum energy density— remains unanswered. Only astronomy gives definite strong restrictions (see Weinberg, 1980).

How would a finite energy density be manifested? In relativity theory, it is necessary that this energy density be the same for any observer. This leads to the condition that the pressure (tension) is the same in all directions and equal to $p = -u$, where p is the pressure and u is the energy density of the vacuum. As early as 1917, Einstein considered the possibility that the vacuum energy density could be nonzero. He used a different terminology and introduced the "cosmological constant" Λ , which is proportional to u. This name emphasized that such an energy density would have its strongest influence on cosmological phenomena.

4.0 On the possibility of calculating the gravitation constant from elementary-particle theory (Zel'dovich and Novikov, 1971)

As in Newtonian theory, so also in general relativity the gravitation constant γ is considered a universal constant to be determined by experiment. Neither general relativity nor Newtonian theory attempts to express γ in terms of other more simple quantities.

However, such an attempt has been made by Sakharov (Sakharov, 1967). This attempt is described below. So far this attempt has not led to any concrete results. Sakharov's formula for G contains another unknown quantity. Nonetheless, the novelty of Sakharov's approach to this deep problem of principle by itself justifies our discussing it.

Sakharov's starting point is the typical GTR viewpoint, which connects gravity with the concept of spacetime curvature. The essence of GTR is contained in the expression for the action

$$S = -mc\int ds - \frac{c^3}{16\pi\gamma}\int RdV \ , \tag{3.4.1}$$

The first term in this equation is a sum over the trajectories of all particles.

By varying the trajectories of the particles in a space of fixed metric, we obtain the law of particle motion from the extremal condition on S. In curved space the action associated with a trajectory depends on the curvature, so that the first term in equation (3.4.1) takes into account the influence of the gravitational field on the particles' motion.

By varying the metric in the expression for S we obtain the gravitational field equations. Symbolically,

$$\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{2c}\tau_{\mu\nu} - \frac{c^3}{16\pi\gamma}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 0, \tag{3.4.2}$$

The first term in this expression comes from the first term in equation (3.4.1); the second arises from the curvature integral, i.e., from the second term in equation (3.4.1).

Roughly speaking, the first term describes the force with which the particles curve space. Such a description is in accordance with the principle of the equality of action and reaction; there is a reciprocity between the action of the curvature on the motion of the particles and the action of the particles on the curvature. This reciprocity manifests itself in the fact that both effects are obtained from one expression, $-mc\int ds$, where ds is calculated in Riemannian spacetime.

The terms containing R and $R_{\mu\nu}$ in formulae (3.4.1) and (3.4.2) can be interpreted as describing the elasticity of space - i.e., the "attempt" of space to remain flat, the resistance of space to being curved.

The constant c $\dfrac{c^3}{16\pi\gamma}$ associated with the elasticity of the vacuum is the quantity which we wish to calculate. This general-relativistic constant is huge in magnitude (recall that in Newtonian theory we dealt with the inverse of it, which was very small). So that we may deal with a dimensionless quantity, let us think about this elasticity of the vacuum in the following manner: The mass *m* of an elementary particle spread over its Compton wavelength, $\hbar$/mc, creates a very small curvature of space because the inelasticity of the vacuum, which resists the curvature, is so large.

Let us remind ourselves that here and in the discussion that follows, the curvature of space and general relativity are considered from a non-quantum, classical, deterministic viewpoint; whereas the motion of elementary particles through this classical curved space is governed by quantum-mechanical laws. We have not yet overstepped the bounds of conventional general-relativity theory; at most, we have only added decorative words to it.

Now we turn to the goal of Sakharov's discussion, the attempt to calculate the "elasticity" of the vacuum, $\dfrac{c^3}{16\pi\gamma}$.

The vacuum elasticity depends on the effects of the curvature of space on the quantum-mechanical motion of particles. One can say in this sense that the goal is to obtain the second term in equation (3.4.1) from the curved-space laws of motion, i.e., from the first term in equation (3.4.1), as written out for quantized particles. Recall that we are concerned here with virtual particles which are created alone (e.g., photons) or in pairs (e^-e^+) , and with the vacuum, i.e., with space in which there are no real particles.

It follows from the general considerations of relativistic invariance and from dimensional considerations that the correction to the action must depend on the invariant R and must diverge. This means that a finite answer can be obtained only if we suppose that quantum theory breaks down above a certain cutoff momentum po.

The correction to the action is

$$\frac{kp_0}{\hbar}\int RdV,$$

where k is a dimensionless numerical factor of the order of unity (We cannot even discuss the exact value of k today because we have no concrete picture of what happens for momenta equal to or larger than p_0. Even the sign of k is not clear). It is assumed that quantum effects alone completely determine the vacuum elasticity. Put the other way around, Sakharov presumes that the gravitation constant, which we usually find from experiment, can be calculated, at least in principle, from the condition

$$\begin{aligned}&\frac{c^3}{16\pi\gamma}\int RdV \equiv \frac{kp_0}{\hbar}\int RdV, \gamma = k'\frac{c^3L^2}{\hbar}\\ &L = \frac{\hbar}{p_0}, k' = \frac{1}{16\pi k}\end{aligned}, \qquad (3.4.3)$$

The details of the calculations can be found in the original work of Sakharov. In order to obtain the observed value of the elasticity, one must put the cut-off momentum equal to an enormously large value-a value corresponding to the mass 10^{-5} g. Since we do not know of any elementary particle with such a mass, our theory for calculating one quantity, $\dfrac{c^3}{16\pi\gamma}$, containing the unknown γ depends upon another unknown quantity, p_0.

Thus, the formula remains the same as it always was:

$$m_g^2c^2/\hbar = c^3/\gamma, \qquad (3.4.4)$$

Our new progress lies in the realization that this formula should be read from right to left as a definition of γ.

Notice, stepping beyond the bounds of gravitation theory, that Sakharov's ideas can probably be extended to electrodynamics and to the theory of weak interactions (Zel'dovich 1967). One usually writes the action in electrodynamics as

$$S = -mc\int ds - \frac{e}{c}\int A_\mu dx^\mu - \frac{1}{8\pi}\int \left(E^2 - H^2\right) dV\,, \tag{3.4.5}$$

where the first term is the action for free, charged particles; the second is the action for the interaction between the charged particles and the electromagnetic field; and the third is the action of the free electromagnetic field. Sakharov's train of thought suggests that we take the first two terms as primary and obtain the third (the action of the field) as a result of quan turn vacuum corrections, just as we did for the term $-\int RdV$ in the theory of gravity. In the remarkable work of Landau and Pomeranchuk (1955) one can find justifications for such an approach.

One can introduce a similar viewpoint into the theory of the intermediumte charged W -bosons, which characterize the weak interactions according to $n = p + W^-, W^- = e^- + \bar{\nu}$.

In this case the theory leads to the conclusion that the mass of a photon must be zero, in contrast to the mass of a *W*- boson. One and the same value of p_0 (under certain assumptions about the spectrum of the fermions, masses) gives the correct magnitudes for the gravitation constant, for the charge of an elementary particle, and for the weak-interaction constant.

5.0. Geometry of empty space and non-empty curved space

Let us consider the behaviour of the vacuum in the case of non-empty curved space.

What happens to the empty space when the material body is entered it?

The result (Zel'dovich, 1981) “can be interpreted as a change in the gravitational constant: $\gamma m + 0.01\gamma m = \gamma \cdot 1.01 m = \gamma' m$. It is γ' that we observe and measure, so that for the whole of macroscopic physics the "old" unobservable value of γ is unimportant, and, these problems being interrelated, we do not face the problem of establishing what is the real contribution of the vacuum, which we took above arbitrarily, for illustration, to be 0.01.

A similar procedure was used for the first time in the fifties in quantum electrodynamics in connection with the electric charge: a free charge produces a vacuum polarization charge, perturbing the motion of charged particles (electrons, etc.) in the Dirac sea of states with negative energy, which are everywhere present in the vacuum. With each electric charge there is associated a transformation $e \to e'$ like the transformation $\gamma \to \gamma'$ for the gravitational constant. This procedure is called "charge renormalization."

It can be carried out even if the ratio e/e' is infinite. The theoreticians developed schemes for calculating all observable effects using only the observed value e'. However, it is necessary to emphasize a difference between electrodynamics and the theory of gravitation. In electrodynamics, one can study the interaction of two elementary particles at a very short distance.

We can study the gravitational interaction only at the macroscopic level, and therefore the experimental investigation of the gravitational vacuum polarization is at present outside the scope of the possible. We must content ourselves with an analysis of the theoretical conclusions.»

From astrophysical data we know that the vacuum energy in flat space is very small. The part of the vacuum energy in curved space proportional to the energy of the matter producing the curvature is manifested in a change in the previously unknown gravitational constant, and in this sense is unobservable.

The notion of a possible change in the gravitational constant due to vacuum polarization does not change the form of the equations but changes their meaning. Sakharov has conjectured that the gravitational constant is entirely determined by vacuum polarization.

The equations of gravitation can be perspicuously interpreted as a manifestation of the elasticity of space-time”.

6.0. The “bending” of space-time by electromagnetic field

As we know, the light bending (Kim and Lee, 2011a) by a massive object is one of the prominent features of the general relativity and is a useful tool in astrophysics through the gravitational lensing. A question may be raised whether there is an electrodynamic version of the bending: that is, whether an electric charge can bend light toward, or outward of it..

At classical level the linearity of the electrodynamics precludes bending of light, and therefore any bending must involve a nonlinear interaction from quantum corrections. The Euler-Heisenberg interaction that arises from the box diagram (for example see http://www.hep.ucl.ac.uk/opal/images/gg-scat.gif) in quantum electrodynamics can provide such a nonlinear interaction.

6.1. Extremely strong electromagnetic field

To associate this task with the task of bending of the light beam in general relativity, we will briefly dwell on the concept of the extreme - critical - electromagnetic field.

It is known that the electromagnetic wave field is massless. In this sense, it can not cause a gravitational field (however, according to general relativity, the electromagnetic wave is subject to the gravitational field). On the other hand, as we know, during the passage of electromagnetic waves in the electric field of a heavy elementary particle the photoproduction of massive particles takes place, in particular, of the electron and positron. It is clear that these particles possess a gravitational field. The question is what should be the value of the EM field for the production of massive particles to take place.

As we know (Ternov and Dorofeev, 1994), according to the uncertainty relation $\Delta t \cdot \Delta\varepsilon \geq \hbar$ short-time violation of the energy conservation law is possible, and the virtual electron-positron pair can be created from vacuum, which can exist for a period of time $\tau = \hbar/\Delta\varepsilon \cong \hbar/mc^2$. During this time, the particles can disperse to a distance of no more than $\Delta r = c\tau = \hbar/mc$, i.e., to a distance of order of the Compton wavelength $\lambda = \hbar/mc^2 \sim 10^{-10}$ cm. This is the so-called quantum electron radius, which characterizes the region of possible spatial localization of the electron in quantum theory. (The other - equivalent - interpretation of the uncertainty relations and the following from they conclusions is given in the article (Kyriakos, 2011a)

Now, if the external electric field can produce work ~ mc^2 on an electron at a distance Δr, the pair creation from vacuum is a real process. For this event the field value must be of the order of the critical value E_c:

$$e_0 E_c \,\hbar/mc = mc^2,\ E_c = m^2c^3/e_0\hbar$$

Under these conditions, the vacuum becomes unstable. E_c is the critical value of external field, at which it becomes possible two, conditionally speaking, phase transitions: on the one hand, massless particle (photon) is transformed into a massive particle; on the other hand, the neutral particle is converted into charged particles. If we talk about the fields of the particles, the critical value of the external field is a field, in which the particle has simultaneously an electric and gravitational fields.

Correspondingly to the above considered critical electric field, there is also a critical magnetic field, if the rotational energy of the electron $\hbar\Omega$ (where $\Omega = e_0H/mc$ is the cyclotron frequency) is equal to the electron rest energy mc^2: $\hbar\Omega = \hbar e_0 H_c \,\hbar/mc = mc^2,\ H_c = m^2c^3/e_0\hbar$ = 4,414 x 10^{13} Oersted.

However, due to the gyromagnetic properties, the magnetic field does not produce work (the Lorentz force is perpendicular to the particle trajectory). For this reason, even in case of excitation by the critical field, the vacuum remains stable. This represents a particular interest for the study of processes in such an extreme field.

Problems of development of quantum electrodynamics in a strong electromagnetic field in the last years have been the focus of attention. During the development of this area it was necessary to push the boundaries of the study of physical phenomena in the region of values of the fields, which were unavailable in the past.

6.2. Vacuum polarization by an external field in quantum electrodynamics

The problem of electrodynamics with a strong electromagnetic field goes back to the early works of Heisenberg and Euler (Heisenberg and Euler, 1936) (see also review (Kyriakos, 2011b)), dedicated to the calculation of vacuum polarization effects, and Sauter (Sauter, 1931), who analyzed the known "Klein paradox" associated with the production of electron-positron pairs in a strong electric field. As this was first pointed by Dirac, even if the external field does not lead to the production of pairs, it affects the electrons and positrons of vacuum, producing a redistribution of charges of vacuum and changing its energy (vacuum polarization).

This phenomenon is due to the fact that under the influence of the external field the energy levels of vacuum electrons are shifted, comparatively with the energy levels of vacuum in the absence of an external field. This change in energy leads to a change in the equations of electromagnetic field, and the Lagrange function is also changed. In this case the Maxwell function:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{H}^2\right), \tag{3.6.1}$$

is now the first term in the expansion of the full function in powers of constant, characterizing the interaction of electrons with the vacuum field:

$L = L_M + L_1 + \ldots$,

where

$$L_1 = \frac{1}{8\pi^2}\int s^{-1} ds e^{-im^2 s}\left[(es)^2 f_2 ctg(esH) - 1 + \frac{2}{3}\left((es)^2 f_1\right)\right], \tag{3.6.2}$$

and f_1 and f_2 are well-known invariants of the electromagnetic field:

$$f_1 = \frac{1}{2H_c^2} e^{\mu\nu\sigma\rho} F_{\mu\nu} F^{\mu\nu} = \frac{\vec{H}^2 - \vec{E}^2}{H_c^2}$$

$$f_2 = \frac{1}{8H_c^2} e^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} = \frac{\vec{H}\vec{E}}{H_c^2}$$

From here follows in particular that the correction to the Lagrangian does not depend on the parameters of a plane electromagnetic wave, i.e., the external field of plane waves does not polarize the vacuum.

From equation (3.6.2) in the special case of weak fields ($\frac{H}{H_c} \ll 1,\ \frac{E}{E_c} \ll 1$) Heisenberg and Euler found:

$$L_1 = \frac{1}{360\pi^2 H_c^2}\left\{\left(\vec{H}^2 - \vec{E}^2\right)^2 + 7\left(\vec{E}\vec{H}\right)^2\right\}, \tag{3.6.3}$$

These are the first members of the corrections in the expansion in powers of f_1 and f_2, characterizing the energy shift of the classical electromagnetic field. In the other limiting case of extremely strong field from formula (3.6.2), with logarithmic accuracy we can find that

$$\frac{H}{H_c} \ll 1, E = 0, L_1 = \frac{e^2}{24\pi^2} H^2 \ln\frac{H}{H_c}\ , \tag{3.6.4a}$$

$$\frac{E}{E_c} << 1, H = 0, L_1 = -\frac{e^2}{24\pi^2} E^2 \ln\frac{E}{E_c} + i\frac{1}{8\pi^3} e^2 E^2 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n\pi\frac{E_c}{E}} , \tag{3.6.4b}$$

(Amendment for correction of a higher order see (Ritus, 1975)).

Note that the complexity of L_1 means the quasi-stationarity of the vacuum state. The reason of violations of the stationarity is the possibility of the real production of electron-positron pairs from the vacuum when subjected to a strong field.

In the paper (Kim and Lee, 2011a) it is shown that an electric charge bends light toward it through the Euler-Heisenberg interaction, and compute the bending angle and trajectory of light in a Coulombic field. The bending of light by Euler-Heisenberg interaction is not new and has been investigated by several authors, particularly on astronomical objects. For instance, (Denisov, et al., 2001) studied light bending in the dipole magnetic field of a neutron star and (De Lorenci, et al., 2001) studied the light bending by a charged black hole (see also (Kim and Lee, 2011b)). . We develop a simple geometric way of computing the bending angle and trajectory based on the Snell's law.

The box diagram of quantum electrodynamics gives rise to a low energy effective Lagrangian of Euler-Heisenberg (1936)

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2 \hbar^3}{90 m^4 c} \left\{ \left(F_{\mu\nu} F^{\mu\nu}\right)^2 + \frac{7}{4} \left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right)^2 \right\}, \tag{3.6.5}$$

In the presence of a background electric field the nonlinear interaction modifies the dispersion relation for the electromagnetic wave and results in a modified speed of light that reads

$$\frac{\upsilon}{c} = 1 - \frac{a\alpha^2 \hbar^3}{90 m^4 c} \left(\vec{\upsilon}^0 \times \vec{E}\right), \tag{3.6.6}$$

where $\vec{\upsilon}^0$ denotes the unit vector in the direction of propagation. Because the speed of light depends on the field strength the light bends in presence of a nonuniform field. The bending can be studied in geometric optics by noting that the index of refraction of the background field is given in leading order by

$$n = 1 + \frac{a\alpha^2 \hbar^3}{45 m^4 c^5} \left(\vec{\upsilon}^0 \times \vec{E}\right)^2, \tag{3.6.7}$$

The infinitesimal bending of photon trajectory over $\delta\vec{r}$ can be obtained from the Snell's law…

The comparing of the result with the cross section for Compton scattering shows that the bending effect can easily dominate the Compton scattering, hence may be observable. Nevertheless, the observation may require extreme precision.

6.3. «Electromagnetic" pre-spacetime

Many works are devoted to the study of effects of light propagation in extremely strong electromagnetic fields (see references in the above-mentioned articles). They lead to the following conclusions:

1. The external field of plane waves does not polarize the vacuum.
2. Under the influence of electromagnetic field of sources, the vacuum changes its properties, i.e., is polarized.
3. Because of this polarization the bending of light rays takes place, similar to the one that occurs in a gravitational field.
4. The bending under the action of the electromagnetic field is bigger, than the bending by meansof the gravitational field at comparable energy field densities.
5. The latter does not contradict our assumption that the gravitational field is a residual part of the electromagnetic field.
6. It follows that the vacuum polarization (i.e., pre-spacetime) under the action of the electromagnetic field changes the geometry of space-time.

7.0. The nature of pre-spacetime

All the existing field theories (theory of gravity, electromagnetic classical theory and quantum field theory) lead to the following interpretations of existing experimental and theoretical knowledge:

The characteristics of the physical vacuum (PhV) play the role of the basis for constructing a geometric space-time (pre-spacetime).

PhV without material particles and fields is unexcited. This means that in the absence of interaction of PhV lies in a basic steady state.

Introduction of material bodies and fields transforms PhV into an excited state. This means that inside the material body and around it, the PhV loses homogeneity and stationarity. In other words it can be said that PhV is perturbed.

The intensity and magnitude of the perturbation of PhV depends on the properties of material bodies and fields, and above all, of their energy-momentum.

For a charged particle this perturbation is an electromagnetic perturbation. The intensity of the perturbation is determined by the electric charge, and in the first place, it obeys Coulomb's law. Due to the fact that this force is strong, the perturbation is very intense. However, since the particle size is small, the perturbation area of PhV is small. It is assumed that in the Universe there are large charged bodies - neutron stars, black holes, etc. and the electromagnetic fields of these objects cover a large area. But this is not experimentally proven.

Most of the large bodies of the Universe are neutral. All of the bodies - both charged and neutral - excite the gravitational field. Its intensity is determined by the gravitational charge, in fact, by mass. Therefore, the gravitational perturbation of PhV from the elementary particle is neglected. But the perturbation of PhV from the planets and stars are very intense, and extend for a considerable distance. Therefore, it is considered that the main source of perturbation of PhV in interstellar space is gravity.

Due to the perturbation of PhV, the individual particles (bodies) interact with each other. This interaction occurs as follows: PhV perturbation caused by a single particle is superimposed on the PhV perturbation caused by another particle, and thus they change their condition and movement.

If the perturbation of PhV is relatively small, then the interaction obeys the superposition principle, and it is described by linear equations. If the perturbation is large, the principle of superposition is not valid and the interaction is described by nonlinear equations. But, fortunately, it turned out that perturbation can often be represented as the sum of interactions, whose intensity decreases rapidly. Because of this, it is possible to consider this sum of interactions in accordance with a procedure, called "perturbation theory".

8.0. Geometrization of pre-spacetime

Thus, we can say that prespacetime, i.e., a basis for the introduction of mathematical space and time, is connected to the PhV and its perturbations. Our analysis allows us to answer the question: how the procedure, which we call geometrization of prespacetime, takes place. To visualize this process, we introduce the concept of the refractive index of physical vacuum.

According to modern concepts, the physical vacuum is a continuous medium consisting of an unknown primary matter (pre-matter). It is described mathematically as some pre-field consisting of a set of oscillators. Using for clarity the analogy with optical media, we can characterize this medium by specific refractive index of PhV. In this case, when we talk about some external fields of bodies (electromagnetic, strong, weak, or gravitational), we can represent the interaction of bodies as follows: one body changes the refractive index of the medium, and on other body changes its state and movement in accordance with the value of this index, and vice versa.

A ray of light is a line on which we build coordinate lines in empty space. In PhV in the non-excited state, the light ray propagates in straight lines. This basic steady state of PhV corresponds to the pseudo-Euclidean geometry, in which the Lorentz transformations take place. If in a limited volume, physical vacuum is excited, i.e., its homogeneity is violated, the light ray will propagate in this part of PhV in curve line. Being in this place, we can not build a rectilinear coordinate system. But we can (in a good approximation) build a rectilinear coordinate system in a place, located away from the excited volume of PhV, and then interpolate it in the region of excitation. For example, we could not detect the bending of light rays in the gravitational field of the Sun without taking into account the motion of light rays in the absence of the Sun.

Thus, any curvilinearity is disclosed with respect to the main reference frame, built in straight lines-rays. Due to this it is disclosed that in this volume of the PhV there is a curvature of light rays, and hence there is a field of force, and matter, which generates it. In this sense, the curvilinearity is not an evidence that the space-time geometry is Riemannian geometry. But we can declare it as Riemannian, if we consider only the excited part of PhV. It is clear that the Riemannian geometry is a way of describing the geometry of the excited PhV.

Therefore, when in general relativity it is mentioned that space-time is Riemannian, it is purely a conditional expression. Space and time - by themselves - are some of the mathematical relationships that are bound to PhV. They can not bend. This is the material carrier of this relationship - PhV, which is bent (or rather, turned into a new excited state). This bending is detected by comparing the trajectories of photons in a given location relative to the unexcited PhV. All the changes of time and space occur due to changes in the properties of a given piece of PhV. The changes can be seen as global, if we do not go beyond the excited volume of PhV. But at the same time, they are only local with respect to the total volume of PhV in the Universe.

The space, expressed by the lengths and direction of the lines, varies due to different types of PhV deformations. Time is measured by the frequency of the processes occurring in the bodies of deformed PhV, and therefore differs by time for the bodies in non-deformed PhV. And, as is clear in this case the changes in space and time are interrelated.

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Part 4. Optical-mechanical analogy and the particle-wave duality in the theory of gravity

1.0. Introduction. Statement of subject

The general problem of our research is the construction of a theory of gravity based on the achievements of the Hilbert-Einstein gravitation theory that would save all the achievements of the last, but would lack theirs drawbacks (see (Logunov, 2006; Kyriakos, 2012a)).

Because of the extremely low values of gravity force for the masses of elementary particles, gravity applies to macrophenomena and must be described by classical physics. On the other hand, a body mass is the sum of the masses of the elementary particles that make up this body. In the modern theory and the nonlinear theory of elementary particles (NTEP) (see articles in the "Prespacetime Journal") the production of mass of elementary particles is described by a special mechanism. However, the mechanics of elementary particles is the wave mechanics. At the same time, the mechanics of the motion of bodies in a gravitational field is the mechanics of macroscopic bodies and thus the classical mechanics. In this case what is the connection between them, from the physical point of view and from the point of view of mathematical description?

From a physical point of view, the relationship between the wave theory and mechanics of particles, in the form of optical-mechanical analogy of equations (recall that optics is the science of electromagnetic waves of the light range) was first discovered by R. Hamilton

Based on the study of the general laws of motion of classical mechanics, Hamilton came to the conclusion that **motion along a trajectory of a massive particle in a field corresponds to the motion of a massless particle - photon - along the ray of light in a medium with a variable index of refraction.**

In the absence of the field or at constant refractive index of medium, the one and the other particles move in a straight line. In another words, at free motion a particle moves in a Euclidean space, but at non-free motion it moves in a curved space, which, in general is a Riemannian space.

Mathematically the equation of motion of a material body is the Hamilton-Jacobi equation (HJE). In this way, as we mentioned in the review (Kyriakos, 2012a), it is used in GRT (Landau and Lifshitz, 1980). Hamilton noted that this equation also describes the motion of light rays. The dual use of this equation is namely the mathematical expression of the optical-mechanical analogy.

But this analogy was not complete. Conventional optics is divided into wave and geometrical (ray) optics. The geometrical optics is the limited case of wave optics at very high frequencies of waves. Namely in this limit, the analogy between the equations of motion of the particle and the equation of the light beam is detected . But this analogy is not related to the connection of the equations of motion of a particle with the wave equation of light; so it has long been considered nothing more than an interesting mathematical conclusion.

De Broglie was the first who pointed out the possibility of an extended analogy, at least, for elementary particles. Based on Lorentz transformations he has shown that the elementary particles (such as electron) must have wave properties. In other words, de Broglie showed that the analogy between light and a material particle is complete: light and particle have both wave and particle properties (which is called "wave-particle dualism"). De Broglie showed mathematically that the motion of an electron in a hydrogen atom can be regarded as a movement of the waves according to Fermat's principle for light waves. The existence of the wave properties of electrons has been confirmed by experiment and raised questions about the mathematical description of the motion of the electron as a wave.

As is known, the equation of motion - the wave equation of motion of the electron (for non-relativistic speeds) was found by Schrödinger (Schroedinger, 1982).. The most important for us are the two pointsont which Schrödinger relied (see Schrodinger. First and second posts):

1) Schrödinger derived his equation from HJE, postulating the proportionality of the HJE main function (“action”) with the phase of the electron wave.

2) Schrödinger noted the link of the spatial interval with the action, following from HJE .

Later, the link of HJE, as a ray equation, with the theory of gravity was investigated by V. Fock (Fock, 1964). The connection between the theory of gravity with the wave equation was not discussed, since it concerns macrobodies, which practically do not have wave properties. However, the full optical-mechanical analogy raises the question of the relation of the body motion in the gravitational field with the wave equation and we come back to this question later.

In his book V. Fock showed that a Lorentz transformation should be considered as invariant transformations of HJE. Using this approach, he pointed out that the space-time interval in the pseudo-Euclidean space is determined by the HJE main function, i.e. by action.

Unfortunately, the relationship of HJE with the space-time interval in the case of a curved space-time (in particular, in the pseudo-Riemannian space) not considered by V.Fock. This problem has not been solved and is one of the subjects to study. To take some steps in this direction, let us consider briefly the results of the above studies in terms of our problem.

2.0. Action function and its physical meaning.

2.1. Classical mechanics

The notion of function as "action" (the name given by Leibniz) was the result of the work of many famous scientists for nearly 200 years (Polak, 1959). The task in which the need occur for such a function is called the problem of functional extremum. The problem can be briefly stated as follows:

1) It is needed to find the equation of motion of a massive particle under the action of any forces between two points *A* and *B*. Clearly, the number of possible paths of motion between *A* and *B* can be infinite. But it was verified experimentally that the particle moves according to the Newton's law (or one of its equivalents), only along one definite path. Therefore, the question arose:

2) is there a physical value $S(x, y, z, t)$ that determines the choice from a set of trajectories, only the trajectory that corresponds to the task.

For some tasks was found, the function $p \cdot \Delta s$, where p is momentum of the body, and Δs is the element of path, so that the function $S' = \sum_{\Delta s \to 0} p \cdot \Delta s = \int_A^B p \cdot ds$ must have the extremum value (maximum or minimum) for real trajectory. Function S was called "action." For other problems was found function $T\Delta t$, where T is the kinetic energy of the particle, Δt is time traffic, so that action $S'' = \sum_{\Delta t \to 0} T \cdot \Delta t = \int_{t1}^{t2} T \cdot dt$. Later, for a wide range of mechanics problems was found the function $L\Delta t = (T - V)\Delta t$, where V is the potential energy, and $L = T - V$ is called the Lagrange function, so that action $S''' = \int (T - V) dt$ (note the fact that the action includes products "momentum x distance" and "energy x time".

The real trajectory of motion is determined by the variation of the action is equal to zero: $\delta S = 0$. The consequence of this equation is the Euler-Lagrange equation. But Hamilton found that the equation of motion can be written in the form of Hamilton-Jacobi (HJE) by means of action function . Its physical meaning is found in quantum mechanics - from the wave theory of the electron (a question, which we will discuss later).

Next, look at specific forms HJE for different physical problems.

2.1.1. *Relativistic and non-relativistic Hamilton-Jacobi equation* (Landau and Lifshitz, 1980)

We can say that at the free movement (i.e. in absence of field) the particles move in a pseudo-Euclidean space, and in the case of motion in the field - in a curved space (which generally is a Riemannian space).

In the absence of the field (in the case of a photon it is equivalent to the presence of constant refractive index of the medium), the massless (photon) and massive (electron) particles move in a straight line.

In the case of massless particles – the photon - the equation of the wave front of light is the homogenous Hamilton-Jacobi equation (HJE) and has the form:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\left(\frac{\partial S}{\partial x}\right)^2-\left(\frac{\partial S}{\partial y}\right)^2-\left(\frac{\partial S}{\partial z}\right)^2=0, \qquad (4.2.1)$$

which is an equation of first order and second degree. As is well known (Landau and Lifshitz, Field Theory), the action S is associated with the momentum $\vec{p}$ and Hamiltonian $\hat{H}$ (total energy) by the relations: $\vec{p}=\frac{\partial S}{\partial \vec{r}}$ (i.e. $\frac{\partial S}{\partial t}=\varepsilon$, $\frac{\partial S}{\partial x}=p_x$, $\frac{\partial S}{\partial y}=p_y$, $\frac{\partial S}{\partial z}=p_z$) and $\hat{H}=-\frac{\partial S}{\partial t}$. In this case $\hat{H}=\hat{H}(q_{1,}\dots,q_n;p_{1,}\dots,p_n;t)$, and $S=S(q_{1,}\dots,q_n,t)$. The relationship $\hat{H}=-\frac{\partial S}{\partial t}$, considering $\vec{p}=\frac{\partial S}{\partial \vec{r}}$, actually is the HJE.

Considering the action as a phase of the wave, we see that the wave vector plays in geometrical optics the role of the particle's momentum in mechanics, and frequency plays the role of the Hamiltonian, i.e. of the total energy of the particle.

This equation can be written in linearized form as:

$$\frac{\partial S}{\partial t}=c\sqrt{\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2}, \qquad (4.2.2)$$

One can assume that the motion of a photon in an external field with the energy-momentum ε_{ex}, $\vec{p}_{ex}$, is described by HJE of type:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}+\varepsilon_{ex}\right)^2-\left(\frac{\partial S}{\partial x}-p_{x\,ex}\right)^2-\left(\frac{\partial S}{\partial y}-p_{y\,\mathrm{ex}}\right)^2-\left(\frac{\partial S}{\partial z}-p_{z\,\mathrm{ex}}\right)^2=0, \qquad (4.2.3)$$

(note that ε_{ex}, $\vec{p}_{ex}$ can represent any field: electromagnetic, gravitational, etc.). This corresponds to the motion of a photon in an inhomogeneous medium with a refractive index that depends on coordinates and time. Obviously, the trajectory will not be a straight line, but a curve.

In the case of the free motion of a massive relativistic particle with mass m (such as an electron without external field) it is easy to obtain the relativistic HJE:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\left(\frac{\partial S}{\partial x}\right)^2-\left(\frac{\partial S}{\partial y}\right)^2-\left(\frac{\partial S}{\partial z}\right)^2=m^2c^2, \qquad (4.2.4)$$

which can be written more concisely in vector form $\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2-\left(\vec{\nabla}S\right)^2=m^2c^2$ or in covariant form $g^{ik}\left(\frac{\partial S}{\partial x^i}\right)\left(\frac{\partial S}{\partial y^k}\right)+m^2c^2=0$.

In the case of the motion of a massive particle in an external field with the energy-momentum ε_{ex}, $\vec{p}_{ex}$, HJE is:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}+\varepsilon_{ex}\right)^2-\left(\frac{\partial S}{\partial x}-p_{x\,ex}\right)^2-\left(\frac{\partial S}{\partial y}-p_{y\,\mathrm{ex}}\right)^2-\left(\frac{\partial S}{\partial z}-p_{z\,\mathrm{ex}}\right)^2=m^2c^2, \qquad (4.2.5)$$

In the nonrelativistic case, this equation in Cartesian coordinate system is:

$$\frac{\partial S}{\partial t}+\sum\frac{1}{2m}\left\{\left(\frac{\partial S}{\partial x}\right)^2+\left(\frac{\partial S}{\partial y}\right)^2+\left(\frac{\partial S}{\partial z}\right)^2\right\}=-V, \qquad (4.2.6)$$

2.2. Quantum mechanics

Schrödinger as the starting point of his search of the electron wave equation took the HJE equation. The latter is not a wave equation, but, as it appears, it is closely connected with it. Consider this relationship according to Schrödinger.

2.2.1. *The geometric representation of the particle motion as wave and as particles*

In his Nobel lecture Schrödinger described the basic idea of wave mechanics as follows (Schrödinger, 1933): “I would define the present state of our knowledge as follows. The ray or the particle path corresponds to a *longitudinal* relationship of the propagation process (i.e. *in the direction* of propagation), the wave surface on the other hand to a *transversal* relationship (i.e. *norma1* to it). *Both* relationships are without doubt real; one is proved by photographed particle paths, the other by interference experiments. To combine both in a uniform system has proved impossible so far. Only in extreme cases does either the transversal, shell-shaped or the radial, longitudinal relationship predominate to such an extent that we *think* we can make do with the wave theory alone or with the particle theory alone”.

“According to the wave theory of light, the light rays, strictly speaking, have only fictitious significance. They are not the physical paths of some particles of light, but are a mathematical device, the so-called orthogonal trajectories of wave surfaces, imaginary guide lines as it were, which point in the direction normal to the wave surface in which the latter advances (cf. Fig. 1 which shows the simplest case of concentric spherical wave surfaces and accordingly rectilinear rays, whereas Fig. 2 illustrates the case of curved rays)”.

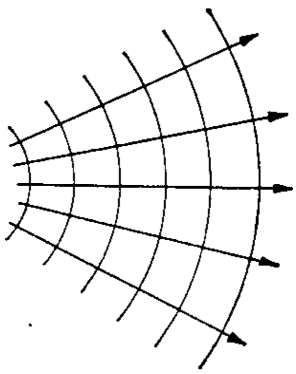

Fig. 1

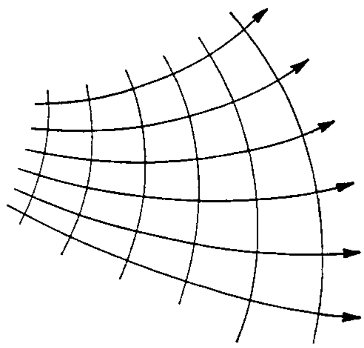

Fig. 2

Thus, on the one hand we have the wave pattern of motion (Schrödinger, Nobel lecture), which results in the following conclusion (Schrödinger, 1933):

“We identify the area of interference, the diffraction halo, with the atom; we assert that the atom in reality is merely the diffraction phenomenon of an electron wave captured us it were by the nucleus of the atom. It is no longer a matter of chance that the size of the atom and the wavelength are of the same order of magnitude: it is a matter of course”.

On the other hand, we have the motion of the electron as a particle. But the differentiation of these movements is a difficult task (Schroedinger, 1982, second part):

It is clear that then the " system path " in the sense of classical mechanics, *i.e.* the path of the point of exact phase agreement, will completely lose its prerogative, because there exists a whole continuum of points before, behind, and near the particular point, in which there is almost as complete phase agreement, and which describe totally different " paths ". In other words, the wave group not only fills the whole path domain all at once but also stretches far beyond it in all directions.

In *this* sense do I interpret the "phase waves" which, according to de Broglie, accompany the path of the electron; in the sense, therefore, that no special meaning is to be attached to the electronic path itself (at any rate, in the interior of the atom), and still less to the position of the electron on its path. And in this sense I explain the conviction, increasingly evident to-day, *firstly*, that real meaning has to be denied to the *phase* of electronic motions in the atom ; *secondly*, that we can never assert that the electron at a definite instant is to be found on *any definite one* of the quantum paths, specialised by the quantum conditions; and *thirdly*, that the true laws of quantum mechanics do not consist of definite rules for the *single path*, but that in these laws the elements of the whole manifold of paths of a system are bound together by equations, so that apparently a certain reciprocal action exists between the different paths.

It is not incomprehensible that a careful analysis of the experimentally known quantities should lead to assertions of this kind, if the experimentally known facts are the outcome of such a structure of the real process as is here represented. All these assertions systematically contribute to the relinquishing of the ideas of "place of the electron" and "path of the electron". If these are not given up, contradictions remain. This contradiction has been so strongly felt that it has even been doubted whether what goes on in the atom could ever be described within the scheme of space and time. From the philosophical standpoint…

I would consider a conclusive decision, in this sense as equivalent to a complete surrender. For we cannot really alter our manner of thinking in space and time, and what we cannot comprehend within it we cannot understand at all. There *are* such things — but I do not believe that atomic structure is one of them. From our standpoint, however, there is no reason for such doubt, although or rather *because* its appearance is extraordinarily comprehensible. So might a person versed in geometrical optics, after many attempts to explain diffraction phenomena by means of the idea of the ray (trustworthy for his macroscopic optics), which always came to nothing, at last think that the *Laws of Geometry* are not applicable to diffraction, since he continually finds that light rays, which he imagines as *rectilinear* and *independent* of each other, now suddenly show, even in homogeneous media, the most remarkable *curvatures*, and obviously *mutually influence* one another».

From this analysis follows that the particle motion as a wave and the particle motion as a material body determine one another. It can be assumed that a particle in physical vacuum at each infinitesimal step of motion selects its direction, by "touching" with waves the state of physical vacuum around himself.

2.2.2. *Relationship between the differential equation of particle motion as mechanical body and as corresponding wave*

The Hamiltonian analogy between mechanics and optics in quantum mechanics, Schrödinger considered in detail (Schroedinger, 1982, first and second parts):

"Let us throw more light on the *general* correspondence which exists between the Hamilton-Jacobi differential equation of a mechanical problem and the "allied" *wave equation*.

The *inner* connection between Hamilton's theory and the process of wave propagation is anything but a new idea. It was not only well known to Hamilton, but it also served him as the starting-point for his theory of mechanics, which grew out of his *Optics of Non-homogeneous Media*. Hamilton's variation principle can be shown to correspond to Format's *Principle* for a wave propagation in configuration space (q-space), and the Hamilton-Jacobi equation expresses Huygens' *Principle* for this wave propagation. Unfortunately this powerful and momentous conception of Hamilton is deprived, in most modem reproductions, of its beautiful raiment as a superfluous accessory, in favour of a more colourless representation of the analytical correspondence".

On this basis, Schroedinger derived his famous wave equation of the electron.

Let us consider the derivation of the Schrödinger wave equation as the steady state equation of atom (Stanyukovich, Kolesnikov et al., 1968).

For definiteness we consider a hydrogen atom, which consists of one proton (assuming it is immobile) and the electron moving around it (assuming it is a point with coordinates x, y, z). Let the total energy of the system be equal to $\varepsilon = const$. The task is to identify the values ε, which make the system stable.

We assume that the instantaneous (momentary) value of the total energy of the system is given by:

$$\varepsilon_m = T + V = \frac{\vec{p}^2}{2m} + V(x, y, z) = \frac{1}{2m}\left(p_x^2 + p_y^2 + p_z^2\right) - \frac{e^2}{r}, \tag{4.2.7}$$

Substituting the values of the projections of momentum in the action, we find

$$\varepsilon_m = \frac{1}{2m}\left\{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2\right\} - \frac{e^2}{r}, \tag{4.2.7'}$$

We will now define the difference at evry point x, y, z:

$$\Delta\varepsilon(x, y, z) = \varepsilon_m - \varepsilon = \frac{1}{2m}\left\{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2\right\} - \frac{e^2}{r} - \varepsilon, \tag{4.2.8}$$

Let us integrate this difference over all possible values of the coordinates:

$$\Delta\tilde{\varepsilon} = \int \Delta\varepsilon(x, y, z)dxdydz = \int\left\{\frac{1}{2m}\left[\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2\right] - \frac{e^2}{r} - \varepsilon\right\}dxdydz, \tag{4.2.9}$$

It is natural now to assume that the stable movements of our system meet the minimum value of the integral $\Delta\tilde{\varepsilon}$ as a functional of the function $S(x, y, z)$. Varying $\Delta\varepsilon$ with respect to S and equating the variation to zero, we obtain:

$$\delta(\Delta\tilde{\varepsilon}) = \int\left(\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}\right)\delta S\, dxdydz = 0, \tag{4.2.10}$$

From this follows the condition for the stability of motion of our system (condition of Lyapunov-Chetaev)

$$\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2} = \Delta S = 0, \tag{4.2.11}$$

This equation, however, does not contains the value of the constant ε, which makes the motion of the system stable. Therefore, Schrödinger considered transformation of the desired function

$$S(x, y, z) = A \ln \psi(x, y, z), \tag{4.2.12}$$

As result of the transformation we obtain:

$$\Delta\varepsilon(x, y, z) = \frac{A^2}{2m\psi^2}\left\{\left(\frac{\partial \psi}{\partial x}\right)^2 + \left(\frac{\partial \psi}{\partial y}\right)^2 + \left(\frac{\partial \psi}{\partial z}\right)^2\right\} - \frac{e^2}{r} - \varepsilon, \tag{4.2.13}$$

Next Schrödinger considered another integral

$$\langle \Delta\varepsilon \rangle = \int \Delta\varepsilon(x,y,z)\psi^2(x,y,z)\,dxdydz\,, \qquad (4.2.4)$$

Varying it on ψ and setting the result to zero, Schrödinger received the wave equation:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{A^2}\left(\varepsilon + \frac{e^2}{r}\right)\psi = 0\,, \qquad (4.2.15)$$

where was found that $A = -i\hbar$

Note that equation (4.2.11) is equivalent to (4.2.15) if we use (4.2.12) in the form $S(x,y,z) = -i\hbar \ln\psi(x,y,z)$. It is easy to show this by direct substitution.

2.2.3. *The geometric representation of the Schrödinger (non-relativistic) equation* (Schroedinger, 1982, second part, p.14, etc).

"Let us consider the general problem of conservative systems in classical mechanics. The Hamilton-Jacobi equation runs

$$\frac{\partial S}{\partial t} + T\left(q_k, \frac{\partial S}{\partial q_k}\right) + V(q_k) = 0\,, \qquad (4.2.16)$$

where S *is* the action function, i.e. the time integral of the Lagrange function *T* - *V* along a path of the system as a function of the end points and the time, q_k is a representative position co-ordinate; *T* is the kinetic energy as function of the *q's* and momenta, being a quadratic form of the latter, for which, as prescribed, the partial derivatives of *S* with respect to the *q's* $\frac{\partial S}{\partial q_k}$ are written. V is the potential energy….

Suppose that a function S has been found. Then this function can be clearly represented for every definite *t*, if the family of surfaces $S = const$, be described in q-space and to each member a value of S be ascribed….

Let the value S_0 be given in Fig. 3 to an arbitrary surface.

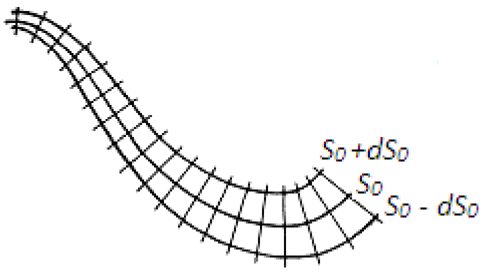

Fig. 3

In order to find the surface $S_0 + dS_0$, take side of the given surface as the positive one, the *step*

$$ds = \frac{dS_0}{\sqrt{2(\varepsilon - V)}}\,, \qquad (4.2.17)$$

The locus of the end points of the steps is the surface $S_0 + dS_0$. Similarly, the family of surfaces may be constructed successively on both sides.

Now it is seen that our system of surfaces $S = const$, can be conceived as the system of wave surfaces of a progressive but stationary wave motion in *q*-space.

The function of action S plays the part of the *phase* of our wave system. The Hamilton-Jacobi equation is the expression of Huygens' principle.

We may sum up that S denotes, apart from a universal constant 1/*h*, the phase angle of the wave function.

We have thus shown: *The point of phase agreement for certain infinitesimal manifolds of wave systems, containing* n *parameters, moves according to the same laws as the image point of the mechanical system*".

2.2.4. *The description of the electron motion by means the Hamilton-Jacobi equations* (Landau and Lifshitz, 1980, p.100)

"We consider the motion of a particle with mass and charge in the field produced by a second charge e'; we assume that the mass of this second charge is so large that it can be considered as fixed. Then our problem becomes the study of the motion of a charge e in a centrally symmetric electric field with potential $\varphi = e'/r$. The total energy of the particle is equal to

$$\varepsilon = c\sqrt{p^2 + m^2c^2} + \frac{\alpha}{r}, \tag{4.2.18}$$

where $\alpha = ee'$. If we use polar coordinates in the plane of motion of the particle, then as we know from mechanics,

$$p^2 = \frac{M^2}{r^2} + p_r^2, \tag{4.2.19}$$

Where p_r is the radial component of the momentum, and M is the constant angular momentum of the particle. Then

$$\varepsilon = c\sqrt{\frac{M^2}{r^2} + p_r^2 + m^2c^2} + \frac{\alpha}{r}, \tag{4.2.20}$$

A complete determination of the motion of a charge in a Coulomb field starts most conveniently from the Hamilton- Jacobi equation. We choose polar coordinates r, φ in the plane of the motion. The Hamilton-Jacobi equation has the form

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t} + \frac{\alpha}{r}\right)^2 - \left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \varphi}\right)^2 = m^2c^2, \tag{4.2.21}$$

We seek an S of the form

$$S = -\varepsilon t + M\varphi + f(r), \tag{4.2.22}$$

where ε, M are the constant energy and angular momentum of the moving particle. The result is

$$S = -\varepsilon t + M\varphi + \int \sqrt{\frac{1}{c^2}\left(\varepsilon - \frac{\alpha}{r}\right)^2 - \frac{M^2}{r^2} - m^2c^2} \cdot dr, \tag{4.2.22}$$

The trajectory is determined by the equation $\dfrac{\partial S}{\partial M} = const$. Integration of (4.2.22) leads to the trajectory. The integration constant is contained in the arbitrary choice of the reference line for measurement of the angle φ". (In detail see the book)

2.2.5. *Waves and particles: dualism of description*

We will write down some of the results of the Schrödinger research (including generalizations made later.)

1. The connection between the path and the wave equations is the connection between the movement of the wave surface and the motion of a point of surface along a certain line.

2. The motion of a particle as a wave, is described by the wave equation. The motion of a particle as a massive body (ray or trajectory) is described by the equation of motion of the point of the wave front - HJE.

3. The main function of HJE is a function $S(x, y, z, t)$, called "action."

4. Function $S(x, y, z, t)$ is instantaneously a function, which describes the surface of the wave front, or, equivalently, the surface of constant value of the phase of the wave.

5. In terms of the vector field theory, the function $S(x, y, z, t)$ can be regarded as a vector, the direction of which at each point is determined by the normal to the surface $\vec{S} = S(x, y, z, t)\vec{n}$.

6. With a accuracy up to the Planck constant, the action S is the phase of the wave $\Phi = \Phi_o e^{-i(\omega t - \vec{k}\,\vec{r})} = \Phi_0 e^{-\frac{i}{\hbar}(\varepsilon t - \vec{p}\,\vec{r})}$, i.e. $S = \hbar(\omega t - \vec{k}\,\vec{r}) = (\varepsilon t - \vec{p}\,\vec{r})$. In the case of electron $\frac{\Phi}{\Phi_0} = \psi$.

7. From this follows the transformation $S(x, y, z, t) = i\hbar \ln \frac{\Phi(x, y, z, t)}{\Phi_0}$, by which Schrödinger went from HJE to the wave equation.

8. The trajectory (ray) of the point of the surface $S(x, y, z, t)$ can be considered as the radius vector $\vec{r} = s\vec{n}$, where s is the path traveled by the set point $\vec{r}_0$.

9. Denote the phase of the wave with letter theta: $\vartheta = \frac{1}{\hbar}(\varepsilon t - \vec{p}\vec{r}) = (\omega t - \vec{k}\vec{r})$. It is obvious that $\vartheta = \frac{S}{\hbar} = i \ln \psi$. Using the equation of energy conservation $\varepsilon^2 - c^2 p^2 = m^2 c^4$, we can get the wave front equation for a massive particle:

$$\left(\frac{1}{c}\frac{\partial \vartheta}{\partial t}\right)^2 - (grad\,\vartheta)^2 = \frac{m^2 c^4}{\hbar^2}.$$

For massless particles (EM waves), when $m = 0$, this equation is:

$$\left(\frac{1}{c}\frac{\partial \vartheta}{\partial t}\right)^2 - (grad\,\vartheta)^2 = 0.$$

Characteristically, the values $\hbar \cdot \frac{\partial \vartheta}{\partial t} = \varepsilon$ and $\hbar \cdot grad\,\vartheta = -\vec{p}$ constitute a 4-vector. This indicates that the four-dimensional world of the theory of relativity is a consequence of the wave origin of the material particles.

10. From dimensional analysis follows that HJE is described by integral physical values, and the wave equation - by differential physical values. Specifically, the action has the dimension of angular momentum = product of momentum on the path = product of the energy on the time. According to NTEP the square of the wave function is, in absolute terms, the dimensions of density of energy-momentum (energy / volume of space) = pressure or tension (force / area), and in relative terms, the square of wave function is the probability density (dimensionless).

3.0. Wave and a particle in the gravitational theory (Fock, 1964).

3.1. The Law of Propagation of an Electromagnetic Wave Front

“The laws of propagation of light in empty space are thoroughly understood. They find their expression in the well-known equations of Maxwell

However, we are not interested in the general case of light propagation, but only in the propagation of a signal advancing with maximum speed, i.e. the propagation of a wave front. Ahead of the front of the wave all components of the field vanish. Behind it some of them are different from zero. Therefore, some of the field components must be discontinuous at the front.

On the other hand, given the field on some surface moving in space, the derivatives of the field on the surface are, in general, determined by Maxwell's equations.

Such a surface is called a characteristic surface or, briefly, a characteristic. Thus, discontinuities of the field can occur only on a characteristic, but since there must certainly be discontinuities at a wave front, such a front is clearly a characteristic.

Let us determine the equation of a characteristic for the system of Maxwell's equations.

Let the value of the field be given for those points and instants whose coordinates are related by the equation

$$t = \frac{1}{c} f(x, y, z), \tag{4.3.1}$$

In particular, if $f = 0$ this amounts to stating initial conditions. Equation (4.3.1) may be looked upon as the equation of a certain hypersurface in the four-dimensional space-time manifold. When $(gradf)^2 > 1$ the same equation can be considered as the equation of an ordinary surface moving through space. Assume that on the hypersurface (4.3.1) the values of a certain function u are given

$$u\left(x, y, z, \frac{f}{c}\right) = u_0(x, y, z), \qquad (4.3.2)”$$

3.2. The photon trajectory (equations for rays)

“The equation describing the propagation of a wave front can be written in the **linear** form

$$\frac{\partial S}{\partial t} = c\sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2}\ , \tag{4.3.3}$$

(for definiteness we have chosen the plus sign before the square root).

In mechanics S play the role of the action function and the derivatives $\vec{p} = \frac{\partial S}{\partial \vec{r}}$ - the momenta , p_x, p_y, p_z. Corresponding to the Hamiltonian we have here the expression $\hat{H} = -\frac{\partial S}{\partial t}$ or

$$\hat{H} = c\sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2}\ , \tag{4.3.4}$$

To the trajectories of mechanics there correspond light rays. The equations for them are analogous to Hamilton's equations. They can be written

$$\frac{dx}{dt} = \frac{\partial \hat{H}}{\left(\frac{\partial S}{\partial x}\right)} = c\frac{\left(\frac{\partial S}{\partial x}\right)}{\sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2}}, \text{ etc. }, \tag{4.3.5}$$

$$\frac{d\left(\frac{\partial S}{\partial x}\right)}{dt} = -\frac{\partial \hat{H}}{\partial x}, \text{ etc }, \tag{4.3.6}$$

Equation (4.3.6) shows that the quantities $\left(\frac{\partial S}{\partial x}\right), \left(\frac{\partial S}{\partial y}\right), \left(\frac{\partial S}{\partial z}\right)$ are constant along a ray, though they can, of course, vary from one ray to another. Therefore the rays will be straight

$$x - x_0 = c \frac{\left(\frac{\partial S}{\partial x}\right)}{\sqrt{\left(\frac{\partial S}{\partial x}\right)^2 + \left(\frac{\partial S}{\partial y}\right)^2 + \left(\frac{\partial S}{\partial z}\right)^2}} (t - t_0), \quad \text{etc.,} \tag{4.3.7}$$

which, according to mechanical-optical analogy, is the equation of motion of a point along the ray.

If the sign of S, and hence of $\left(\frac{\partial S}{\partial x}\right), \left(\frac{\partial S}{\partial y}\right), \left(\frac{\partial S}{\partial z}\right)$, is changed, the direction of the ray is reversed ; the sign must be chosen according to the given sense of direction of the ray".

3.3. Connection of action with space-time interval (Fock, 1964)

"Any wave surface can be considered as formed of points moving along the rays with the speed of light according to (4.3.7).

We thus have the possibility of constructing a wave surface at time t when its form at time t is known.

Let the equation of the wave surface at time t_0 have the form

$$S^0(x_0, y_0, z_0) = 0 \ , \tag{4.3.8}$$

where x_0, y_0, z_0 are coordinates varying over this surface. Knowing the equation of the surface we can calculate the quantities

$$\alpha(x_0, y_0, z_0) = \left(\frac{\left(\frac{\partial S}{\partial x}\right)^0}{\sqrt{\left(\frac{\partial S}{\partial x}\right)^{0\,2} + \left(\frac{\partial S}{\partial y}\right)^{0\,2} + \left(\frac{\partial S}{\partial z}\right)^{0\,2}}} \right)_0, \text{ etc.,} \tag{4.3.9}$$

Here the sign of the right-hand sides is determined by the given direction of wave propagation. The equation of the ray passing through the point (x_0, y_0, z_0) of the initial wave surface is

$$\left.\begin{aligned} x - x_0 &= c\alpha(t - t_0) \\ y - y_0 &= c\beta(t - t_0), \quad (\alpha^2 + \beta^2 + \gamma^2 = 1) \\ z - z_0 &= c\gamma(t - t_0) \end{aligned}\right\}, \tag{4.3.10}$$

The quantities x, y, z give the positions of the point to which the point (x_0, y_0, z_0) moves at time t. Allowing x_0, y_0, z_0 to take on all values which satisfy (4.3.8), we obtain from (4.3.10) all points which at time t lie on the wave surface.

If we solve (4.3.10) for x_0, y_0, z_0 and insert the functions

$$x_0 = x_0(x, y, z, t - t_0), \text{ etc.,} \tag{4.3.11}$$

into the wave surface equation (4.3.8), we get the relation

$$S(x, y, z, t - t_0) = 0 \,, \tag{4.3.12}$$

which is the explicit form of the equation of the wave surface at time t. At $t - t_0$ obviously, $x_0 = x, y_0 = y, z_0 = z$ and equation (4.3.12) reduces to (4.3.8), which is the equation of the initially given wave surface.

From the ray equation (4.3.7) there follows the relation

$$c^2(t-t_0)^2 - \left[(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2\right] = 0, \qquad (4.3.13)$$

which connects the coordinates of the initial and final points on each ray. It is the equation of a sphere centred at the point x_0, y_0, z_0 and of a radius $R = c(t-t_0)$ that increases linearly with time. Just as Hamilton-Jacobi equation, from which we started, this equation expresses the fact that the velocity of light propagation is constant.

For points infinitesimally separated relation (4.3.13) takes on the form

$$c^2 dt^2 - \left(dx^2 + dy^2 + dz^2\right) = 0, \qquad (4.3.13')$$

In this form the equation follows directly from Hamilton's equation (4.3.5)".

A frame for which

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 - \left(\frac{\partial S}{\partial z}\right)^2 = 0, \qquad (4.3.14)$$

is valid may be called inertial in the electromagnetic sense".

3.4. Features of the gravitational field

"The principle of the universal limiting velocity can be made mathematically precise as follows:

For any kind of wave advancing with limiting velocity and capable of transmitting signals the equation offront propagation is the same as the equation for the front of a light wave.

Thus the equation (4.3.14) acquires a general character; it is more general than Maxwell's equations from which we derived it.

The presence of a gravitational field somewhat alters the appearance of the equation of the characteristics from the form (4.3.14), but in this case one and the same equation still governs the propafgation of all kinds of wave fronts travelling with limiting velocity, including electromagnetic and gravitational ones.

Let us considere the expressions

$$(\nabla S)^2 = \sum_{\mu,\nu=0}^{3} g^{\mu\nu} \frac{\partial S}{\partial x_\mu}\frac{\partial S}{\partial x_\nu}, \qquad (4.3.15)$$

$$ds^2 = \sum_{\mu,\nu=0} g^{\mu\nu} \partial x_\mu \partial x_\nu, \qquad (4.3.16)$$

which were obtained from the usual expressions of Relativity Theory by introducing variables x_1, x_2, x_3 and x_0 in place of the space and time coordinates *x, y, z, t.* We established the conditions subject to which the variable x_0 can characterize a sequence of events in time and the variables x_1, x_2, x_3 their location in space.

By itself, the introduction of new variables can naturally not influence the physical consequences of the theory; it is merely a mathematical device.

We shall call equations generally covariant, if they are valid for any arbitrary choice of independent variables.

The most essential characteristic of the gravitational field by which it differs from all other fields known to physics reveals itself in the effect of the field on the motion of a freely moving body or mass point. In a gravitational field all otherwise free bodies move in the same manner, provided the initial conditions of their motion, i.e. their initial positions and velocities, are the same.

According to Newton the gravitational field can be characterized by the gravitational potential $U(x, y, z)$. The gravitational potential produced by an solated spherically symmetric mass M at points exterior to itself is

$$\varphi_g = \frac{\gamma M}{r}, \qquad (4.3.17)$$

where r is the distance from the centre of the mass. The quantity γ is the Newtonian constant of gravitation—in c.g.s. units it has the value

$$\gamma = \frac{1}{15000000}\frac{cm^3}{g \cdot cec^2}, \tag{4.3.18}$$

Thus φ_g has the dimensions of the square of a velocity. We note immediately that in all cases encountered in Nature, even on the surface of the Sun or of super-dense stars, the quantity φ_g is very small compared to the square of the speed of light

$$\varphi_g << c^2 , \tag{4.3.19}$$

In the general case of an arbitrary mass distribution the Newtonian potential U it produces satisfies Poisson's equation

$$\Delta\varphi_g = -4\pi\gamma\rho_m , \tag{4.3.20}$$

where ρ_m is the mass density. The Newtonian potentiall φ_g is fully determined by Poisson's equation together with continuity and boundary conditions which are as follows : the function φ_g and its first derivatives must be finite, singlevalued and continuous throughout space and must tend to zero at infinity.

As a result of the equality of inertial and gravitational mass the equation of motion

$$w = grad\varphi_g , \tag{4.3.21}$$

where w is acceleration, has universal character".

3.5. The space-time interval and the space-time metric

"The phenomenon of universal gravitation forces us to widen the framework of the theory of space and time which was the subject of the Newton theory. The necessity of this widening becomes clear from the following considerations. It follows from the equation of wave front propagation, which can be stated in the form

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left(\frac{\partial S}{\partial x}\right)^2 - \left(\frac{\partial S}{\partial y}\right)^2 - \left(\frac{\partial S}{\partial z}\right)^2 = 0, \tag{4.3.22}$$

that light is propagated in straight lines. But light possesses energy and by the law of proportionality of mass and energy all energy is indissolubly connected with mass. Therefore fight must possess mass. On the other hand, by the law of universal gravitation, any mass located in a gravitational field must experience the action of that field and in general its motion will therefore not be rectilinear, Hence it follows that in a gravitational field the law of wave front propagation must have a form somewhat different from the one given above.

But the equation of wave front propagation is a basic characteristic of the properties of space and time. Hence it follows that the presence of the gravitational field must affect the properties of space and time and their metric is then not a rigid one. This is indeed the conclusion reached in the theory of gravitation which we now begin to construct.

As was shown, the equation of wave front propagation (4.3.22) with some additional assumptions, leads to the following expression for the square of the interval:

$$ds^2 = c^2dt^2 - \left(dx^2 + dy^2 + dz^2\right), \tag{4.3.23}$$

The influence of the gravitational field on the properties of space and time must have the consequence that the coefficients in the equation of wave front propagation and in the expression for the square of the interval will differ from the constant values appearing in (4.3.22) and (4.3.23). We must now find an approximate form for the square of the interval in a gravitational field of Newtonian potential φ_g .

We shall thus now assume that space-time is in the main Euclidean, or rather pseudo-Euclidean, and that any deviation of space-time geometry from Euclidean geometry is a result of the presence of a gravitational field. Whereever there is no gravitational field, geometry must be Euclidean. For an insular distribution of masses the gravitational field must tend to zero at infinity and therefore we have to assume that at points far removed from the masses the geometry of space-time becomes Euclidean.

We shall now try to find a metric such that these equations coincide approximately with the Newtonian equations of motion for a free body in a given gravitational field. If this attempt is successful it will enable us to introduce the hypothesis that in a space-time with given metric a free body (mass point) moves along a geodesic ; in this way the connection between the law of motion and the metric will be established.

As we know, the equation of a geodesic may be derived from the variational principle

$$\delta\int ds = 0\,, \tag{4.3.24}$$

If the squared interval is of the form (4.3.23) we have

$$ds = \sqrt{\left(c^2 - \upsilon^2\right)}dt\,, \tag{4.3.25}$$

or, for small velocities,

$$ds = \left(c - \frac{\upsilon^2}{2c}\right)dt\,, \tag{4.3.26}$$

Inserting (4.3.25) or (4.3.26) into (4.3.24) gives equations that describe motion with constant velocity, which indeed is free motion in the absence of a gravitational field. We can now assume that for small velocities and weak gravitational fields ($U << c^2$) the expression for the interval takes the form

$$ds = \sqrt{\left(c^2 - 2\varphi_g - \upsilon^2\right)}dt\,, \tag{4.3.27}$$

or

$$ds = \left[c - \frac{1}{c}\left(\frac{1}{2}\upsilon^2 + \varphi_g\right)\right]dt\,, \tag{4.3.28}$$

in place of (4.3.25) or (4.3.26). Since neither an additive constant nor a constant multiplier are of any importance in a Lagrangian the variational principle (4.3.24), with ds taken from (4.3.28), gives the same result as the variational principle

$$\delta\int\left(\frac{1}{2}\upsilon^2 + \varphi_g\right)dt = 0\,, \tag{4.3.29}$$

but this did indeed describe free motion of a body in a gravitational field. It is true that just because additive constants and multiplicative factors in a Lagrangian are immaterial equation (4.3.29) could be obtained from (4.3.24) and (4.3.27) with any sufficiently large value of the constant c

These arguments give us good reason to assume that under the conditions

$$\varphi_g << c^2; \quad \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = \upsilon^2 << c^2\,, \tag{4.3.30}$$

the square of the interval differs little from the form

$$ds^2 = \left(c^2 - 2\varphi_g\right)dt^2 - \left(dx^2 + dy^2 + dz^2\right), \tag{4.3.31}$$

The theory of gravitation gives the more exact expression

$$ds^2 = \left(c^2 - 2\varphi_g\right)dt^2 - \left(1 + \frac{2\varphi_g}{c^2}\right)\left(dx^2 + dy^2 + dz^2\right), \tag{4.3.31”}$$

4.0. The description methods of motion of bodies in gravitational field

"We consider (Fock, 1964). a problem of an astronomical type, relating to the motion of celestial bodies in empty space.

Our problem is simplified in the first place by the fact that the metric nowhere deviates greatly from the Euclidean ; the table given below gives an idea of how small the deviation is.

	Sun	Earth	Moon
α	1·48 km	0·443 cm	0·0053 cm
L	696,000 km	6,370 km	1,738 km
$\alpha : L$	2×10^{-6}	7×10^{-10}	3×10^{-11}

where $\alpha = \dfrac{\gamma M}{c^2}$ is the gravitational radius of the mass M . For the Sun, and even more so for the planets, the gravitational radius α is much smaller than the geometric radius L, which may be defined as the radius of a sphere of volume equal to that of the body.

A further simplifying circumstance is that at all significant distances from the bodies, the metric does not depend on the detailed internal structure of the latter, but only on certain overall characteristics. Such characteristics are the total mass of the body, its moments of inertia, the position and velocity of its mass centre and so on. The Newtonian potential of a body depends on these same quantities.

To solve Einstein's equations we shall use a method of approximation. It is based on an expansion of all required functions in inverse powers of the speed of light. An expansion that can formally be so described will, in fact, be an expansion in powers of certain dimensionless quantities, such as φ_g / c^2 and υ^2 / c^2 , where φ_g is the Newtonian potential and υ the square of some velocity, say the velocity of one of the bodies...

If we solve wave equations by introducing corrections for retardation we imply that the dimensions of the system are small compared to the wavelength of the waves emitted, which in this case are gravitational waves."

Let us refer once again to the analysis of the structure of GR, made by M.-A. Tonnela (Tonnelat, 1966):

"All the predictions of general relativity follow from the field equations and the laws of geodesic motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \to g_{\alpha\beta}, \tag{4.4.1}$$

$$\delta \int ds = 0, \tag{4.4.2}$$

The first allow us to define $g_{\mu\nu}$ and put this value in (4.4.2). All the present-day predictions follow from the below mentioned values $g_{\mu\nu}$:

$$ds^2 = \left(1 - \frac{2\gamma\ M}{c^2 r}\right) c^2 dt^2 - \frac{1}{1 - \dfrac{2\gamma\ M}{c^2 r}} dr^2 - r^2 \left(d\vartheta^2 + \sin^2 \vartheta d\varphi^2\right), \qquad (4.4.3)\text{"}$$

From our analysis it follows that the equation (4.4.2) is equivalent to HJE; HJE and the expression for the interval ds are connected between them: according to Fock (see above) from HJE can be obtained ds. But then we must conclude that HJE in some sense is equivalent to HEE (4.4.1). The question arises, if both equivalences can take place, and if so, in what way?

HJE describes the motion of a body in an external field. In a Cartesian coordinate system, it has the form (4.2.5)

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}+\varepsilon_{ex}\right)^2-\left(\frac{\partial S}{\partial x}-p_{x\,ex}\right)^2-\left(\frac{\partial S}{\partial y}-p_{y\,\mathrm{ex}}\right)^2-\left(\frac{\partial S}{\partial z}-p_{z\,\mathrm{ex}}\right)^2=m^2c^2,$$

where the external field is given by functions of energy ε_{ex} and momentum $\vec{p}_{ex}$ (note that, generally, the equation (4.2.5) should be written in covariant form). But the function ε_{ex} and $\vec{p}_{ex}$ must be pre-found from the equation of a field source. Therefore, the equation of the field source in some sense must be equivalent to the HEE. But there are differences here: the HEE gives the metric tensor, while the quantities ε_{ex} and $\vec{p}_{ex}$ are not expressed directly through the metric tensor. To compare with the solutions of general relativity equation, it is necessary to show that this expression is possible, though it, for the solution of HJE this may not be necessary.

4.2. Description of the gravitational field as a perturbation

Let us leave aside the question of the field source equation to the next section, and try to find out if there are in physics, methods of problem solving, which would provide the necessary trajectory, corresponding to (4.4.2) and (4.4.3).

For our search of solution of this problem, the following assumption of M.-A. Tonnela is noteworthy (Tonnela, 1966):

"We assume that the laws of motion can be derived from the geodesic law. The experiment would allow then to recreate with a consistent approximation the structures ($R_{\mu\nu}=0$ in a vacuum)of non-Euclidean space. But we can, on the contrary, suggests that this geodesic law is invalid, or at least, is shown in a simple Euclidean space, which has phenomenological properties, that is, some distortion, or, if desired, a polarization of empty space by means of gravitational field. Then the action of the gravitational field on light will be the result of special interaction, not the propagation of light in the empty space (but a curvilinear one) .

One might think that in this empty Euclidean space - though polarized by matter - there is an "index of emptiness," which justifies the expressions (4.4.1) and (4.4.2), in case of change of the interval $ds'=nds''$".

In other words (see also (Kyriakos, 2012b)), we can consider the gravitational field as a perturbation of the empty (in the absence of fields) physical vacuum, described by Euclidean geometry. In this case, we can use perturbation theory to calculate the motion of bodies in this field.

How does then the statement of the problem look like?

Here we can use the method of Fock (see above), which "is based on an expansion of all required functions in inverse powers of the speed of light. An expansion that can formally be so described will, in fact, be an expansion in powers of certain dimensionless quantities, such as φ_g/c^2 and υ^2/c^2, where φ_g is the Newtonian potential and υ the square of some velocity, say the velocity of one of the bodies".

In the general formulation, this method can be described as follows (Tonnela, 1966):

Assume that the field equations are unknown, i.e. structural conditions are unknown that must be prescribed by a non-Euclidean space. We could search in the reverse order the structure of space, based on experimental results. For this, it would be sufficient to use the power series expansion with the parameter $\varphi_g/c^2=\dfrac{\gamma M}{c^2r}$, characterizing the influence of sources. We obtain:

$$\begin{gathered}ds^2=\left(1+\frac{a_1M}{c^2r}+\frac{a_2M^2}{c^4r^2}+\ldots\right)c^2dt^2-\left(1+\frac{b_1M}{c^2r}+\frac{b_2M^2}{c^4r^2}+\ldots\right)dr^2-\\-\left(1+\frac{c_1M}{c^2r}+\frac{c_2M^2}{c^4r^2}+\ldots\right)r^2d\vartheta^2-\left(1+\frac{d_1M}{c^2r}+\frac{d_2M^2}{c^4r^2}+\ldots\right)r^2\sin^2\vartheta d\varphi^2\end{gathered},\qquad(4.4.4)$$

$$\delta\int ds = 0\,, \tag{4.4.5}$$

In this case the coefficients $a_1, a_2,...; b_1, b_2,...; c_1, c_2,...; d_1, d_2,...,$ should be consistently determined on the basis of perturbation theory: in first approximation - the first coefficient, then the second, and so on. In this case the test can serve the values from (4.4.3) according to general relativity. In particular:

1) Newton's law of gravity and gravitational shift, dictate a_1 (a_1 = - 2);

2) The bending of light rays in a gravitational field, dictates $b_1 = -a_1$ (a_1 = 2, b_1 = 2).

3) the precession of the perihelion of Mercury's, dictates $a_1(b_1 - a_1) + a_2 = 0$ (a_1 = 2, b_1 = 2, a_2 = 0).

All other factors in these problems are equal to zero. It is clear that they are not necessarily equal to zero for other tasks. For example, in the case of the experiments with the gyroscope, the combination ($(2b_1 - a_1)$) arises. Also the new coefficients arise, which are introduced by the form , which does not have spherical symmetry. Such is, for example, the effect of the rotating central body, which introduces the term (*xdy* - *ydx*), the influence of which can be foreseen by theory and measured with experiment.

Our problem can be formulated as follows: how to build HJE, in such a form that on the basis of the perturbation theory, it would be possible to obtain in the interval, the abovementioned terms, which are other than the pseudo-Euclidian interval?

Since the time of Poincaré, the planetary motion in the solar system is considered on the basis of perturbation theory. Recall also that on the basis of perturbation theory many problems of quantum field theory are solved . Is it possible to use this method in this case, selecting as the initial state the one that is given by Newton's theory and adding members, which follow from the relativistic corrections?

To the analysis of this pathway we will devote a separate study. In the next section we will consider the question about the source equation of gravitational field, which can replace HEE equation.

5.0. The equation of gravity of Hilbert-Einstein as a generalization of the wave equation

Thus, we have seen that the motion of bodies in a gravitational field is described by the Hamilton-Jacobi equation if the source is calculated according to the theory of Hilbert-Einstein

Now remember that according to optical-mechanical analogy it is assumed that the HJE associated with wave equation.

On the other hand, in quantum theory the particle-wave duality implies that the Hamilton-Jacobi equation is associated with a wave of an elementary particle, and hence - with the wave equation of particles.

At the same time, we noted (see (Kyriakos, 2012a)), in the theory of Maxwell-Lorentz that the d'Alembert wave equation contains a source of the electromagnetic field and allows to calculate the electromagnetic field of this source. Similarly, the gravity equation of Hilbert-Einstein contains as a source of the gravitational field, the generalization of mass in the form of the energy-momentum tensor, which allows to calculate the corresponding gravitational field.

The question arises**: is it possible to compare in the gravitational theory, the Hamilton-Jacobi equation with a wave equation?**

By analogy with the abovementioned facts we can assume that the Hilbert-Einstein equations is a generalization of the tensor wave equation in covariant record. In other words, we can assume that the covariant HJE and the HEE are two sides of the optical-mechanical analogy (or of dualism wave-particle) in Gravity.

Are there any results to prove this assumption? Yes, indeed, such results exist.

According to (Fock, 1964, p. 194) “the equation:

$$g^{\mu\nu}\frac{\partial S}{\partial x_\mu}\frac{\partial S}{\partial x_\nu}=0, \tag{4.5.1}$$

for the propagation of a gravitational wave-front is the same as the corresponding equation for the front of a light wave in empty space on which the whole theory of space and time, starting from the generally covariant form of Maxwell's equations. Briefly one can say that gravitation is propagated with the speed of light as EM waves”.

Thus (Fock, 1964) “*we see that Einstein's equations are of the type of the wave equation, because their main terms involve the d'Alembert operator*”.

In many textbooks on the theory of gravity is shown that the equation of the HEE in the Newtonian approximation is the inhomogeneous wave equation of D'Alembert:

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}-\frac{\partial^2\varphi_g}{\partial x^2}-\frac{\partial^2\varphi_g}{\partial y^2}-\frac{\partial^2\varphi_g}{\partial z^2}=4\pi\gamma\rho_g, \tag{4.5.2}$$

where φ_g is connected with g_{00} by the relationship

$$g_{00}=1-\frac{2\gamma M}{c_2 r}\equiv 1-\frac{2\varphi_g}{c_2}, \tag{4.5.3}$$

This assumption does not also conflict the alternative gravitation theory of A. Logunov, which gives the same results as the Hilbert-Einstein theory. Indeed, the basic equation of this theory can be represented in the form of a wave equation with a source, like a wave equation of the EM field (see (Logunov, 2002; Kyriakos, 2012a)).

Let us remember also that from the nonlinear theory of elementary particles (NTEP) follows that the sources of the EM field and the gravitational field (electric charge or mass) arise in a nonlinear wave equation of particles. This gives us a reason to look for the equation of gravity as a generalization of the nonlinear wave equations of elementary particles.

Further research on this issue, will be continue in future articles.

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Part 5. Mechanics of electromagnetism. From luminiferous aether to physical vacuum

1.0. Introduction. The Faraday-Maxwell field and the modern idea of field

The present article is basically a summary of all known information about this topic For the comparison of each ideas, we will use quotations from books and articles of the well-known scholars, accompanied by small commentaries. For additional information on the questions examined, the reader can become familiar with books and publications, indicated in the bibliography of this article.

1.1. On the concept of the field and the aether as fundamental field

"In physics (ScienceWeek, 2005) , a field is an entity that acts as intermediary in interactions between particles, and which is distributed over part or all of space, and whose properties are functions of space coordinates, and except for static fields, also functions of time. There is also a quantum-mechanical analog of this entity, in which the function of space and time is replaced by an operator at each point in space-time."

"Isaac Newton (1642-1727) (Wilczek, 1999; Wilczek, 2005; Wilczek, 2008) believed in a continuous medium filling all space, but his equations did not require any such medium, and by the early 19th century the generally accepted ideal for fundamental physical theory was to discover mathematical equations for forces between indestructible atoms moving through empty space.

It was Michael Faraday (1791-1867) who revived the idea that space was filled with a medium having physical effects in itself... To summarize Faraday's results, James Clerk Maxwell (1831-1879) adapted and developed the mathematics used to describe fluids and elastic solids, and Maxwell postulated an elaborate mechanical model of electrical and magnetic fields."

«Maxwell's equations for electricity and magnetism (Wilczek, 2008a) captured a wider range of phenomena… in a precise mathematical world-model. Again, however, the big advance required a readjustment and vast expansion of our perception of reality. Where Newton described the motion of particles influenced by gravity, Maxwell's equations filled space with the play of "fields" or "aethers". According to Maxwell, what our electric and magnetic fields, which exert forces on the matter we observe. Although they begin as mathematical devices, the fields leap out of the equations to take on a life of their own. Changing electric fields produce magnetic fields, and changing magnetic fields produce electric fields. Thus these fields can animate one another in turn, giving birth to self-reproducing disturbances that travel at the speed of light. Ever since Maxwell, we understand that these disturbances are what light is».

That is why this medium is called "luminiferous aether."

"The concept (Wilczek, 2005a) that what we ordinarily perceive as empty space is in fact a complicated medium is a profound and pervasive theme in modern physics. This invisible inescapable medium alters the behavior of the matter that we do see. Just as Earth's gravitational field allows us to select a unique direction as up, and thereby locally reduces the symmetry of the underlying equations of physics, so cosmic fields in "empty" space lower the symmetry of these fundamental equations everywhere. For although this concept of a symmetry-breaking aether has been extremely fruitful (and has been demonstrated indirectly in many ways), the ultimate demonstration of its validity --cleaning out the medium and restoring the pristine symmetry of the equations -- has never been achieved: that is, perhaps, until now."

1.2. Aether and the theory of relativity of Einstein

"In the late 19th century (ScienceWeek, 2005), what we now call "classical" physics incorporated the assumed existence of the "aether", a hypothetical medium believed to be necessary to support the propagation of electromagnetic radiation. The famous Michelson-Morley experiment of 1887 was interpreted as demonstrating the nonexistence of the aether, and this experiment became a significant prelude to the subsequent formulation of Einstein's special theory of relativity. Although it is often stated outside the physics community that the aether concept was abandoned after the Michelson-Morley experiment, this is not quite true, since the classical aether concept has been essentially reformulated into several modern field concepts."

"The achievement of Einstein (1879-1955) (Wilczek, 2005) in his paper on special relativity was to highlight and interpret the hidden symmetry of Maxwell's equations, not to change them. The Faraday-Maxwell concept of electric and magnetic fields, as media or aether filling all space, was retained by Einstein. Later, Einstein was dissatisfied with the particle-field dualism inherent in the early atomic theory, and Einstein sought, without success, a unified field theory in which all fundamental particles would emerge as special solutions to the field equations.

Einstein first purified, and then enthroned, the aether concept. As the 20th century has progressed, its role in fundamental physics has only expanded. At present, renamed and thinly disguised, it dominates the accepted laws of physics."

In modern popular scientific books and textbooks, the widespread view is that aether is a dirty word and a taboo.

STR is very simple. It contains only two postulates: principle of relativity and postulate of constancy of light speed. The content of postulates of STR is in no way connected with presence or absence of aether or electromagnetic field. These postulates lead directly to the Lorentz transformations. This is sufficient to predict all effects, discovered within the framework of electron theory without mention of aether

Some scientists and the majority of popularizers asserted on this base that the electron theory of Lorentz was false, since it relies on aether and on the special assumptions.

The contradictions between the electromagnetic theory of matter of Lorentz and STR of Einstein do not exist. In the theory of Lorentz, the aether is one of the inertial reference systems, which cannot be detected with experiments, i.e., it can not be considered as an absolute frame of reference.

After being introduced to the work of Einstein it is possible to easily ascertain that Einstein himself never rejected aether.. In his articles there are assertions that within the framework of STR, aether is not necessary. There is also an assertion that in nature it is not possible to assign absolute frame of reference. Both assertions are completely correct. But they (for different reasons) do not refute the Lorentz electron theory.

A. Einstein (Einstein, 1920) in his speech "Aether and the theory of relativity" pronounced on May 5, 1920 at the Leyden university, emphasized that the existence of electromagnetic aether does not contradict the special theory of relativity .

The next position which it was possible to take up in face of this state of things appeared to be the following. The aether does not exist at all….

More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny aether. We may assume the existence of an aether; only we must give up ascribing a definite state of motion to it…

The special theory of relativity forbids us to assume the aether to consist of particles observable through time, but the hypothesis of aether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the aether.

The electromagnetic fields appear as ultimate, irreducible realities, and at first it seems superfluous to postulate a homogeneous, isotropic aether-medium, and to envisage electromagnetic fields as states of this medium.

But on the other hand there is a weighty argument to be adduced in favour of the aether hypothesis. To deny the aether is ultimately to assume that empty space has no physical qualities whatever. The fundamental facts of mechanics do not harmonize with this view…

According to our present conceptions the elementary particles of matter are also, in their essence, nothing else than condensations of the electromagnetic field…»

The absence of contradictions between these two theories (the Lorentz electron theory and STR of Einstein) was already undestood by the contemporaries of Einstein.

Several publications of authoritative scholars of this time, such as Kaufman, Laue and Ehrenfest, and later Bohm emphasized that in principle there can not be an experiment that can choose between the approaches of Lorentz and Einstein.

This is quote from article of Ehrenfest - friend of Einstein “To the crisis of hypothesis on light-aether” (*Ehrenfest*, 1913):

“Einstein's theory, denying aether, requires the same as the aether theory of Lorentz. On this base the observer must, according to Einstein's theory, observe on the moving measuring bar, clock et cetera, the same reductions, time difference et cetera, as according to Lorentz's theory. Let us note in this case that such experimentum crucis, which would solve the dispute in favor of one or the other theory, is principally impossible.”

About physical equivalence of Lorentz's theory and the theory of relativity, also wrote Max von Laue (Laue, 1913) - one of the first scientists who supported the theory of Einstein.

“Experimentally, it would be impossible to make a choice between this theory (i.e. the electron theory of Lorentz) and Einstein's theory of relativity. If, however, the Lorentz theory receded into the background - although it still has supporters among physicists - it was done, without a doubt, by reason of philosophical order. Its full agreement with the theory of relativity shows that this will never be possible to locate in a real way, the privileged frame of reference, or, which is the same, we cannot detect any movement of a body relatively to aether.”

1.3. Aether and quantum theory

The following development of physics has shown that the - relativistic in origin - quantum field theory requires also the existence of aether.

“Following Einstein (Wilczek, 2005), Paul Dirac (1902-1984) then showed that photons emerged as a logical consequence of applying the rules of quantum mechanics to Maxwell's electromagnetic aether. This connection was soon generalized so that particles of any sort could be represented as the small-amplitude excitations of quantum fields. Electrons, for example, can be regarded as excitations of an electron field, an aether that pervades all space and time uniformly. Our current and extremely successful theories of the strong, electromagnetic, and weak forces are formulated as relativistic quantum field theories with local interactions.”

Since 50-ies years, Dirac himself spoke many times about this problem

“Physical knowledge (Dirac, 1951) has advanced much since 1905, notably by the arrival of quantum mechanics, and the situation [about the scientific plausibility of aether] has again changed. If one examines the question in the light of present-day knowledge, one finds that the aether is no longer ruled out by relativity, and good reasons can now be advanced for postulating an aether. . . .

We can now see that we may very well have an aether, subject to quantum mechanics and conformable to relativity, provided we are willing to consider a perfect vacuum as an idealized state, not attainable in practice.

From the experimental point of view there does not seem to be any objection to this. We must make some profound alterations to the theoretical idea of the vacuum. . .Thus, with the new theory of electrodynamics we are rather forced to have an aether"

A known contemporary physicist, expert in string theory A.M. Polyakov emphasizes that the ideas of the classical theory of the 19th century did not contradict modern ideas (Polyakov, 1987):

"We have no better way of describing elementary particles than quantum field theory. A quantum field in general is an assembly of an infinite number of interacting harmonic oscillators. Excitations of such oscillators are associated with particles. The special importance of the harmonic oscillator follows from the fact that its excitation spectrum is additive, i.e. if ε_1 and ε_2 are energy levels above the ground state then $\varepsilon_1 + \varepsilon_2$ will be an energy level as well. It is precisely this property that we expect to be true for a system of elementary particles. Therefore we attempt to identify the Hamiltonian of the particles with the Hamiltonian of coupled oscillators (there is a familiar example from solid state physics: the excitations of a crystal lattice can be interpreted as particles – phonons). All this has the flavour of the XIX century, when people tried to construct mechanical models for all phenomena. I see nothing wrong with it because any nontrivial idea is in a certain sense correct. The garbage of the past often becomes the treasure of the present (and vice versa). For this reason we shall boldly investigate all possible analogies together with our main problem…

Elementary particles existing in nature resemble very much excitations of some complicated medium (aether). We do not know the derailed structure of the aether but we have learned a lot about effective Lagrangians for its low energy excitations. It is as if we knew nothing about the molecular structure of some liquid but did know the Navier-Stokes equation and could thus predict many exciting things. Clearly, there are lots of different possibilities at the molecular level"

2.0. The Lorentz microscopic electromagnetic theory, as the electromagnetic theory of matter

The macroscopic theory of electromagnetism (EM) was developed by Maxwell. An attempt to construct a microscopic theory of electromagnetism as a unified theory of matter, has been developed by H. Lorentz

2.1. The age of Maxwell

In his "Treatise on Electricity and Magnetism" (Maxwell, 1873), Maxwell wrote:

«**Stress in dielectrics**. If we now proceed to investigate the mechanical state of the medium on the hypothesis that the mechanical action observed between electrified bodies is exerted through and by means of the medium, as in the familiar instances of the action of one body on another by means of the tension of a rope or the pressure of a rod, we find that the medium must be in a state of mechanical stress.

The nature of this stress is, as Faraday pointed out, a tension along the lines of force combined with an equal pressure in all directions at right angles to these lines. The magnitude of these stresses is proportional to the energy of the electrification.

From the hypothesis that electric action is not a direct action between bodies at a distance, but is exerted by means of the medium between the bodies, we have deduced that this medium must be in a state of stress. We have also ascertained the character of the stress, and compared it with the stresses which may occur in solid bodies.

It must be carefully borne in mind that we have made only one step in the theory of the action of the medium. We have supposed it to be in a state of stress, but we have not in any way accounted for this stress, or explained how it is maintained. This step, however, seems to me to be an important one, as it explains, by the action of the consecutive parts of the medium, phenomena which were formerly supposed to be explicable only by direct action at a distance».

To carry out this step it the work of many great scientists and especially William Thomson (Lord Kelvin), Joseph Larmor, J.J. Thomson and G. Lorentz, as the creators of a classical unified electromagnetic theory of matter, was necessary.

2.2. The age of Lorentz (Rohrlich, 1962)

The mathematical formulation of macroscopic electromagnetic phenomena was beautifully accomplished by J. C. Maxwell about thirty years prior to Lorentz's theory. Lorentz constructed a microscopic theory by using Maxwell's equations and adding to it an expression for the force which a charged particle experiences in the presence of electric and magnetic fields. This microscopic theory is a description of matter in terms of its charged atomic fragments, ions and electrons. The success of this microscopic theory lay in the proof first provided by Lorentz that the macroscopic Maxwell theory can be deduced from this microscopic theory by a suitable averaging process over the motion of the individual ions and electrons. Thus, Lorentz's theory became the primary theory and Maxwell's theory can be reduced to it.

However, Lorentz went beyond this: having successfully described the electromagnetic force acting on a charged particle due to externally present fields, he attempted to describe the structure of an individual electron. His aim was to show that the electron is a completely electromagnetic object. In particular, its mass was to be the mass equivalent of its electromagnetic energy contents; its inertia, i.e., the inertial term in Newton's equations of motion, was to be entirely due to its own electromagnetic field. Accelerating the electron means changing or distorting the field produced by the electron; this requires work. Therefore, the electron exhibits a certain inertia in following the force acting on it.

The starting point of the theory is the Lorentz force and the microscopic equations which, when averaged, produce Maxwell's equations. When the Lorentz force is used to describe the action which the electron's own electromagnetic field exerts on its source, the electron, the following equation of motion is obtained:

$$\frac{4}{3}m\vec{a} - \frac{2}{3}\frac{e^3}{c^3}\dot{\vec{a}} + \text{"structure terms"} = \vec{F}\,, \qquad (5.2.1)$$

Equation (5.2.1) now exhibits three difficulties, associated, respectively, with each of the three terms appearing on the left hand side:

(I)The inertial term differs by a factor 4/3 from Newton's classical "mass times acceleration." This is a kinematic problem which implies that the relationship between momentum and velocity for a particle in Newtonian mechanics differs from that for the completely electromagnetic electron. This would have dire consequences and is an intolerable defect of the theory. Luckily, it can be corrected rather easily. A conscientious merger of this theory with special relativity assures that this factor disappears, since it is incompatible with the relativistic transformation properties. For a finite electron this was first pointed out by Fermi in 1922. It is closely related to the definition of rigidity in special relativity where the difference in simultaneity of relatively moving observers plays an essential role. Unfortunately, Fermi's paper was either never understood or soon forgotten. In any case, the factor 4/3 can still be found in some of today's texts. For point electrons the removal of this factor was later rediscovered several times. Let me summarize then by saying that a relativistic generalization of (5.2.1) will not show this factor; if the non-relativistic theory is derived as the limit of the relativistic one, this factor will disappear from (5.2.1). A study of its origin reveals an unjustified and incorrect assumption about the relationship between the Poynting vector and the momentum of non-radiative electromagnetic fields". (See, for example (Rohrlich, 1997)).

(II) The second difficulty lies in the terms marked "structure terms." These terms depend on the charge distribution and radius of the electron. This dependence is a difficulty here while it is not a difficulty in the first term. In the latter, the mass must be determined experimentally anyway, while here the whole dynamics becomes explicitly dependent on the electron structure. This difficulty can be removed only by eliminating the electron structure altogether. If we assume the particle to be a point particle it will obviously have no structure. Furthermore, if, for this purpose, we let the electron radius r_0 shrink to zero the "structure terms" all vanish. Since no experimental evidence for structure exists (the Coulomb potential is correct down to smallest distances measured) this is a satisfactory procedure indeed. The removal of the structure terms, however, produces a new difficulty, viz., that when the radius shrinks to zero, the electron mass becomes infinitely large. This is the famous problem of the electron self-energy. It exists in the classical theory of the electron as well as in the quantum theory. An at best temporary solution is provided by the renormalization procedure".

(III) There is one more difficulty in the Lorentz theory which is not apparent from the equation of motion. This difficulty relates to the stability of the electron. This problem need not be further discussed since it will be resolved simultaneously with the self-energy problem in any relativistic theory".

As noted Rohrlich, a satisfactory solution to these problem was unknown in the framework of linear theory. As we have shown (see (Kyriakos, 2010, 2011a, 2011b), this problem is solved sequentially in the nonlinear theory of elementary particles (NTEP), and this decision is not contrary to modern notions of quantum field theory."

Let us consider how a mechanical representation of the Faraday-Maxwell theory corresponds to the continuum mechanics.

3.0. Continuum Mechanics as field theory

3.1. General states of continuum mechanics

A fundamental concept of mechanics (Sedov, 1966; 1971) , widely used in the art, is the concept of an elastic medium, which is usually regarded as a solid body. The perfect liquid or gas can also be considered as an elastic body.

The initial model is the model of a solid deformable body, considered as a material continuum. For small particle of continuum the internal energy, free energy, entropy, and other thermodynamic functions can be regarded as a function of strain tensor, temperature and physical constants or variables that characterize the thermal and mechanical properties and the state of matter. The parameters, characterizing the medium, can be tensor quantities.

"Helmholtz (Sommerfeld, 1950) establishes the following fundamental theorem: The most general motion of a sufficiently small element of a deformable (i.e., not rigid) body can be represented as the sum of

1. *translation,*
2. *rotation,*
3. *attention (contraction) in three mutually orthogonal directions.*

The proof is based on the Taylor expansion of the relative displacement of two neighboring points *P* and *O* in terms of their original coordinate differences as the position vector *OP* = *r*.

Let us introduce the symbol *l* for the total change of position of a point of the volume element under consideration *P* . Then we have the sum:

$$\vec{l} = \vec{l}_0 + \vec{l}_1 + \vec{l}_2 , \tag{5.3.1}$$

which indicate that the displacement *l* is compounded of three partial motions:

$\vec{l}_0 = \{\xi_0, \eta_0, \varsigma_0\} = \vec{r}_0$ is *translation*

The displacement l_0, with the components $\xi_0, \eta_0, \varsigma_0$, is the same for all points P of a volume element and is therefore a *translation.*

The central portion of the set (5.3.1), l_1, is a *rotation* (*r* is the position vector):

$\vec{l}_1 = [\vec{\varphi} \times \vec{r}] = \{\xi_1, \eta_1, \varsigma_1\}_0$

This displacement is well known from rigid body kinematics and corresponds to the infinitesimal rotation $\vec{\varphi}$ (the appropriate notation would be $\delta\vec{\varphi}$) whose axis and magnitude arc given by the components $\xi_1, \eta_1, \varsigma_1$. The infinitesimal rotation is not a vector in the proper sense, such as a *polar* vector that characterizes a translatory displacement.

$\vec{l}_2 = \{\xi_2, \eta_2, \varsigma_2\}$ is deformation (strain), which can be represented as: $\vec{l}_2 = \varepsilon_{ik} r_k$, where $i,k = x,y,z$, ε_{ik} is deformation tensor (symmetric). The displacement l_2 is *a linear vector function* of the position vector r . The quantities ε_{ik} are the components of the *strain tensor.* The tensor itself may be symbolised in a similar way as the moment of inertia by the quadratic array. In the present case the tensor is *symmetric*."

4.0. Connection of a continuum theory with the Maxwell-Lorentz theory

Let us show that the Faraday-Maxwell medium can be described just as a mechanical continuum.

4.1. The stress tensor and energy-momentum tensor of mechanics

The relationship between these tensors is the following. The stress tensor is a characteristic of non-relativistic mechanics, while the four-dimensional energy-momentum tensor (which is also known as the 4-momentum tensor) is its relativistic generalization.

4.1.1. *The stress tensor of mechanics* (Feynman et al, 1964a, Ch. 31)

Suppose we have a solid object with various forces on it. We say that there are various "stresses" inside, by which we mean that there are internal forces between neighboring parts of the material. We have talked a little about such stresses in a two-dimensional case when we considered the surface tension in a stretched diaphragm. We will now see that the internal forces in the material of a three-dimensional body can be described in terms of a tensor… Consider a body of some elastic material - say a block of jello

We define the three components of the stress, S_{xy}, S_{yy}, S_{zy} , as the force per unit area in the three directions. Finally, we make an imaginary cut perpendicular to z and define the three components S_{xz}, S_{yz}, S_{zz} . So we have the nine numbers

$$S_{ij} = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}, \qquad (5.4.1)$$

These nine numbers are sufficient to describe completely the internal state of stress, and that S_{ij} is indeed a tensor. Suppose we want to know the force across a surface oriented at some arbitrary angle

The *x*-component S_{xn} of the stress across this plane is equal to ΔF_{xn}, divided by the area, which is $\Delta\sqrt{z\Delta x^2 + \Delta y^2}$, or in general form

$$S_{in} = \sum_j S_{ij} n_j \ , \qquad (5.4.2)$$

where $\vec{n} = n_j$ is the unit vector normal to the face *N.*

The stress tensor - and also its ellipsoid - will, in general, vary from point to point in a block of material; to describe the whole block we need to give the value of each component of S_{ij} as a function of position. So the stress tensor is a *field.* We have had *scalar fields,* like the temperature *T(x, y,* z), which give one number for each point in space, and *vector fields* like $\vec{E}(x,y,z)$*(x, y, z),* which give three numbers for each point. Now we have a *tensor field* which gives nine numbers for each point m space - or really six for the symmetric tensor S_{ij}. A complete description of the internal forces in an arbitrarily distorted solid requires six functions of *x, y,* and *z.*

4.1.2. *The 4-energy-momentum tensor of mechanics* (Feynman et al, 1964a, Ch. 32)

Now, we want to consider tensor in the four dimensions (t, x, y, z) of relativity theory. When we wrote the stress tensor, we defined S_{ij} as a component of a force across a unit area. But a force is equal to the time rate of change of a momentum. Therefore, instead of saying *"* S_{xy} is the *x*-component of the force across a unit area perpendicular to *y*" *we* could equally well say, "S_{xy} is the rate of flow of the x -component of momentum through a unit area perpendicular to *y*." In other words, each term of S_{ij} also represents the flow of the *i*-component of momentum through a unit area perpendicular to the *j*-direction These are pure space components, but they are parts of a "larger" tensor $S_{\mu\nu}$ *in* four dimensions (μ and $\nu = t, x, y, z$) containing additional components like S_{tx}, S_{yt}, S_{tt}, etc. We will now try to find the physical meaning of these extra components.

We know that the space components represent flow of momentum. We can get a clue on how to extend this to the time dimension by studying another kind of "flow" - the flow of electric charge. For the *scalar* quantity, charge, the rate of flow (per unit area perpendicular to the flow) is a space *vector* - the current density vector $\vec{j}$. We have seen that the time component of this flow vector is the density of the stuff that is flowing.

Now by analogy with our statement about the time component of the flow of a scalar quantity, we might expect that with S_{xx}, S_{xy}, S_{xz}, describing the flow of the x-component of momentum, there should be a time component S_{xt} which would be the density of whatever is flowing; that is, S_{xt} should be the density of jo-momentum. So we can extend our tensor horizontally to include a t -component. We have

S_{xt} - density of -momentum,
S_{xx}, S_{xy}, S_{xz} x -flow of x, y, z -momentum components,

Similarly, for the y and z -components of momentum.

In four dimensions there is also a /-component of momentum, which is, we know, energy So the tensor S_{tf} should be extended vertically with S_{tx}, S_{ty}, S_{tz}, where

S_{tx}, S_{ty}, S_{tz} are the x, y, z -flows of energy,

that is, S_{tx} is the flow of energy per unit area and per unit time across a surface perpendicular to the x-axis, and so on.

Finally, to complete our tensor we need S_{tt} which would be the *density* of *energy.*

We have extended our stress tensor S_{ij} of three dimensions to the four-dimensional *stress-energy tensor* $S_{\mu\nu}$.

4.2. The stress tensor and energy-momentum tensor of electromagnetic theory

"This (Sommerfeld, 1964). was Faraday's intimation when he spoke of lines of force as of elastic bands which transmit tension and compression. Maxwell, was also here able to place Faraday's notions into clear mathematical focus. This was the origin 'of Maxwell's stress tensor, which may be expanded relativistically into stress-energy tensor."

4.2.1. *The Maxwell stress tensor*

The stress tensor of the electromagnetic field is called the Maxwell stress tensor.

Stress by definition means the forces acting per unit area in the internal surfaces of a body. The Maxwell stress tensor is the stress tensor of an electromagnetic field. Starting with the Lorentz force law we can derive it using Gauss's law and Faraday's law.

In Gaussian CGS unit, more commonly used in physics textbooks, the Maxwell stress tensor $\sigma = \sigma_{ij}$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

is given by:

$$\sigma_{ij} = \frac{1}{4\pi}\left[E_i E_j + H_i H_j - \frac{1}{2}\left(E^2 + H^2\right)\delta_{ij} \right], \qquad (5.4.4)$$

where E_i is a component of the electric field $\vec{E}$, H_i is a component of the magnetic field $\vec{H}$, and δ_{ij} is the Kronecker delta

In the Maxwell stress tensor, the diagonal elements $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$ represent pressures and the non-diagonal elements $\sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ etc. represent shears. However, the Maxwell tensor has a special property due to the Kronecker delta, δ_{ij}: all the non-diagonal elements vanish. Thus

The Maxwell stress tensor product with a unit vector normal to a surface gives the force per unit area transmitted across the surface by an electromagnetic field.

4.2.2. *The four-tensor of electromagnetic momentum and its mechanical sense*

Since the electromagnetic field also has energy and momentum, it is possible, along with the mechanical tensor of energy-momentum of continuum, $S_{\mu\nu}$, to specify the electromagnetic field tensor of energy-momentum, which we will denote $\tau_{\mu\nu}$, as is customary in our articles on the nonlinear theory of elementary particles (NTEP), publicated in “Prespacetime journal”. Obviously, the physical meaning of the components of the EM tensor will correspond to the physical meaning of the components of the mechanical tensor.

As an example (Feynman et al, 1964a, Ch.32), we will discuss the 4-tensor of energy-momentum not in matter (see above the section 4.1.2), but in a region of free space, which there is an electromagnetic field.

We know that the flow of energy is the Poynting vector $\vec{S} = \frac{c}{4\pi}\vec{E}\times\vec{H}$. So the *x*-, *y*-, and *z*-components of $\vec{S}$ are, from the relativistic point of view, the components S_{tx}, S_{ty}, S_{tz} of our four-dimensional stress-energy tensor. The symmetry of the tensor S_{ij} carries over into the time components as well, so the four-dimensional tensor $\tau_{\mu\nu}$ is symmetric;

$$\tau_{\mu\nu} = \tau_{\nu\mu}, \qquad (5.4.5)$$

The remaining components of the electromagnetic stress tensor $\tau_{\mu\nu}$ can also be expressed m terms of the electric and magnetic fields $\vec{E}$ and $\vec{H}$. That is to say, we must admit stress or, to put less mysteriously, flow of momentum in the electromagnetic field.

The formula for $\tau_{\mu\nu}$ in terms of the fields, recorded through tensor of electromagnetic field, is:

$$\tau_{\mu\nu} = \frac{\varepsilon_0}{2}\left(\sum_{\alpha} F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\sum_{\alpha\beta} F_{\beta\alpha}F_{\beta\alpha} \right), \qquad (5.4.6)$$

Thus, the complete $\tau_{\mu\nu}$ matrix becomes, in abbreviated notation:

$$\tau_{\mu\nu} = \begin{pmatrix} \sigma_{ij} & -\frac{i}{c}\vec{g} \\ -\frac{i}{c}\vec{g} & u \end{pmatrix},$$

где where $\mu,\nu = 1,2,3,4$ and $i,\ j = 1,2,3$, $u = \frac{1}{8\pi}\left(\vec{E}^2 + \vec{H}^2\right)$ is energy density and $\vec{g} = \frac{1}{c^2}\vec{S} = \frac{1}{4\pi c}\left(\vec{E}\times\vec{H}\right)$ momentum density of electromagnetic field.

4.2.3. *The transmission of the force through vacuum*

In four-dimentional form, the Lorentz force is written as follows

$$\left(\vec{f}, f^0\right) = \left\{\rho\left(\vec{E} + \frac{1}{c}\left[\vec{\upsilon}\times\vec{H}\right]\right),\ \frac{\rho}{c}\left(\vec{\upsilon}\cdot\vec{E}\right)\right\}, \tag{5.4.7}$$

Substituting from Maxwell's equations ρ and $\vec{j} = \rho\,\vec{\upsilon}$ in (31.1) we can write this expression in terms of the field strengths only. The same expression can be obtained by using the energy-momentum tensor $\tau_{\mu\nu}$ of the electromagnetic field.

We will show (Sommerfeld, 1964) that the Lorentz force dencity f_μ may be expressed as the four-dimensional divergence of a tensor $\tau_{\mu\nu}$, i.e. that

$$f_\mu = -\frac{1}{4\pi}\frac{\partial\,\tau^\nu{}_\mu}{\partial\,x^\nu} \equiv -\frac{1}{4\pi}\partial_\nu \tau_\mu{}^\nu = \frac{1}{4\pi}\sum_{\nu=1}^{4}\frac{\partial\,\tau_{\mu\nu}}{\partial\,x_\nu}, \tag{5.4.8}$$

where $\tau_{\mu\nu}$ is a four-energy-momentum tensor of Maxwell (superscript denotes the number of the column, the subscript - the number of the line; it is summed by the same indices).

Consider now the time components of Eq. (5.4.8),

$$f_4 = -\frac{i}{c}div_i g - \frac{\partial\,u}{\partial\,x_4}, \tag{5.4.9}$$

Consider now the space components of Eq. (5.4.8), e.g. the first line:

$$\begin{aligned} f_x &= \frac{\partial\,\sigma_{xx}}{\partial\,x} + \frac{\partial\,\sigma_{yx}}{\partial\,y} + \frac{\partial\,\sigma_{zx}}{\partial\,z} - \frac{\partial\,g_x}{\partial\,t} \\ f_y &= \frac{\partial\,\sigma_{xy}}{\partial\,x} + \frac{\partial\,\sigma_{yy}}{\partial\,y} + \frac{\partial\,\sigma_{zy}}{\partial\,z} - \frac{\partial\,g_y}{\partial\,t}, \\ f_z &= \frac{\partial\,\sigma_{xz}}{\partial\,x} + \frac{\partial\,\sigma_{yz}}{\partial\,y} + \frac{\partial\,\sigma_{zz}}{\partial\,z} - \frac{\partial\,g_z}{\partial\,t} \end{aligned} \tag{5.4.10}$$

which, written out in general, becomes

$$f_i = div_i\sigma - i\frac{\partial\,g_i}{\partial\,x_4}, \tag{5.4.11}$$

where $x_4 = ict$, $x_{1,2,3} = x, y, z$. Or conditionally, in vector form the last system equation can be written as

$$\vec{f} = grad\sigma - \frac{\partial\,\vec{g}}{\partial\,t}, \tag{5.4.12}$$

If we omit the last term on the "left, i.e. confine ourselves to a stationary state, we obtain the characteristic equation (5.4.10) of elastic equilibrium in (Sommerfeld, 1950). Just as there the volume force F_x is absorbed and balanced by the stresses σ_{ik}, insofar as they point in the *x*-direction. The Lorentz force density may be completely replaced by these stresses in our case. They are defined throughout the field by the tensor array σ_{ij} (5.4.1), even where, in view of the absence of charge density, the Lorentz force is nonexistent.

We have thus attained the goal set at the beginning of this section, of following up the transmission of the force through vacuum (without the use of a test body).

However, what do we know of the nonstationary state and the term with $\frac{\partial \vec{g}}{\partial t}$ which is then added in (5.4.10)? The answer is given by Eq. 14.1 in (Sommerfeld, 1950), where the corresponding term, there designated by $-\rho \frac{\partial^2 \vec{g}}{\partial t^2}$, represented the inertial resistance of unit volume of the elastic body or, with positive sign, its change in momentum. We learn from this that there exists a momentum per unit volume *g* also in the electromagnetic field, and that it is to be defined, in direction and magnitude, by

$$\vec{g} = \frac{1}{c^2}\vec{S}, \qquad (5.4.13)$$

5.0. Continuum mechanics of the electromagnetic theory of Maxwell

As is known, Maxwell equations can not be written for a medium, in which there is not a parallel translation. Indeed, the first model of a continuous medium, in which the propagation of disturbances is described by Maxwell's equations, was the medium that McCulloch invented. This medium has only rotational elasticity. Later, Kelvin invented quasi-solid model of rotational elastic medium, consisting of vortices, in which the equations of motion coincide with the Maxwell equations. These findings were confirmed by other scientists (Kelly et al.) Modern formalized theory of rotational elastic medium has been developed by Hideo Fukutome.

5.1. The peculiarityof vortex dynamics

A detailed bibliography of vortex dynamics with the comments presented in the book: (Meleshko and Aref, 2007). We will consider only the results that are relevant to our topic.

5.1. 1. *Helmholtz's Vortex Theorems*

In the paper of 1858 (Sommerfeld, 1950) in which Helmholtz gives the kinematic analysis of vortex motion he also completes the dynamic theory of vortices in its essentials. Simplifications in method were found in the following decades, but new results were not discovered.

The main content of Helmholtz's theory are the conservation laws: It *is impossible to produce or* destroy *vortices*, or, expressed in more general terms, *the vortex strength it constant in time.* This theorem is correct under the following conditions: the fluid is inviscid and incompressible; the external forces possess a single-valued potential within the space filled by the fluid. Apart from the conservation of the vortex strength in *time* we see that there is also a *spatial* conservation: the vortex strength is constant *along each vortex line or vortex* tube, which must be either dosed or end at the boundary of the fluid.

5.1.2. *Vortex lines* (Feynman et al, 1964a, Ch. 40)

We have already written down the general equations for the flow of an incompressible fluid when there may be vorticity. They are

$$\text{I. } \vec{\nabla} \cdot \vec{\upsilon} = 0$$

$$\text{II. } \vec{\Omega} = \vec{\nabla} \times \upsilon,$$

$$\text{III. } \frac{\partial \vec{\Omega}}{\partial t} + \vec{\nabla} \times \left(\vec{\Omega} \times \upsilon\right) = 0$$

The physical content of these equations has been described in words by Helmholtz in terms of three theorems. First, imagine that in the fluid we were to draw *vortex lines* rather than streamlines. By vortex lines we mean field lines that nave the direction of $\vec{\Omega}$ and have a density m any region proportional to the magnitude of $\vec{\Omega}$.

Equation I (in the case of an incompressible fluid) expresses the conservation of mass of the liquid.

From II the divergence of $\vec{\Omega}$ is *always* zero - that the divergence of a curl is always zero). So vortex lines are like lines of magnetic field $\vec{B}$ - they never start or stop, and will tend to go in closed loops.

Now Helmholtz described III in words by the following statement: the vortex lines move *with the fluid.* This means that if you were to mark the fluid particles along some vortex lines - by coloring them with ink, for example - then as the fluid moves and carries those particles along, they will always mark the new positions of the vortex lines. In whatever way the atoms of the liquid move, the vortex lines move with them. That is one way to describe the laws.

The law III is really just the law of conservation of angular momentum applied to the fluid.

5.1.3. *General remarks on the dynamics of vortices*

"The dynamics of vortices is indeed a very peculiar one and deviates decisively from the dynamics of mass points (Sommerfeld, 1950, *p. 160).*

To begin with, Newton's first law is altered. The isolated vortex (which is therefore not subjected to "forces") *remains in a state of rest.* A uniform rectilinear motion can only be acquired by association with a second vortex of equal strength but opposite sense of rotation or under the action of a wall at rest.

Thus the relativity principle of classical mechanics according to which the state of rest and of uniform motion arc equivalent is no longer valid. The reason is, of course, that the fluid to which the vortex belongs plays the role of a preferred frame of reference.

The modification of the second law is even more remarkable. The external action originating in a second vortex does not determine the *acceleration* but the ve*locity.* The content of the law of motion of the mass center is shifted accordingly: not the *acceleration,* but the *velocity* of the mass center vanishes. As far as the *law of areas* is concerned the angular momentum of the vortex system is constant as in the ease of a mechanical system in the absence of external forces, but the constant is entirely determined by the vortex strengths, in contradistinction to the mechanics of masses where this constant is a constant of integration that depends on initial conditions which may be chosen freely."

Most effects of vortex dynamics have been confirmed experimentally by the simulation of vortexes by means of linear and circular vortexes in the liquid and gas as well as by gyroscopes (Tait, 1990; Crabtree, 1909).

Let us note two features of the precessional motion of vortexes and gyroscopes. First, the precession does not exhibit an "inertia" (the precession exists as long as the moment of force exists). Second, the rotation axis of precession does not coincide with the moment of force $\vec{M}$ and is perpendicular to it. This means that the force does not do the work.

6.0. Universal elastic-rotational medium

6.1. Elastic-rotational medium of MacCullagh as model of the aether (Sommerfeld, 1950, p. 108)

In 19th century physics , a material carrier was assumed for the optical phenomena, equipped as far us possible with the properties of ordinary elastic bodies. This construction, however, led to difficulties even in the most elementary problem of reflexion and refraction. As early as 1839 MacCullagh tried to drop the connection with the ordinary theory of elasticity with the aim to develop a representation of optics that would be free of the difficulties mentioned. It turned out later that his theory agreed formally with Maxwell's electro-magnetic optics (1864), in particular as far as the optics of transparent bodies is concerned. The following remarks should be considered as an interpretation of MacGullagh's equations.

Let us go back to the beginning of section 3.1. There toe general locomotion of a continuous medium was decomposed into the throe parts of translation, rotation, and deformation. The elastic body responds to a deformation with a stress tensor which is determined by the deformation tensor; it is not sensitive to rotation (and, of course, not to translation). We now try to imagine a "quasi-elastic" body, supposedly insensitive to deformations but *responsive to rotations relative to absolute* space ! Since the rotation has toe character of an antisymmetric tensor we shall assume that the stress acting on the volume element as a result of the rotation w also an antisymmetric tensor, as indicated in the following array:

$$\sigma_{ij} = \begin{pmatrix} 0 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & 0 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & 0 \end{pmatrix}, \quad \sigma_{ij} = -\sigma_{ji},$$

The equation of motion of the quasi-elastic body follows now from (14.1a) (see (Sommerfeld, 1950)).

The system of these equations is of impressive simplicity and symmetry. It is formally identical with, *Maxwell's equations for the empty space!*

What we mean here is not a mechanical *explanation* but, at best, a mechanical *analogy*.

The following historical remark may be of interest. In an extension of MacCullagh's ideas Lord Kelvin, in the eighties (Kelvin, 1910)., developed the concept of the quasi-elastic or, as he sometimes put it, "quasi-rigid" aether. He was not satisfied, only to postulate an aether with reactive responses, but attempted to construct a gyroscopic model that actually would react in the required way. As is well known, a fast spinning top can be so arranged as to acquire directional stability and will then respond to fairly strong moments with small angular changes only. But an aether model, based on gyroscopic effects, becomes desperately complicated. Each volume element has to be equipped with several tops that must be oriented relative to each other in such a way that line desired rotational stiffness is achieved not only for one but for all three axes. A construction as complicated as that would be the only way to realise a "gyrostatic" aether.

6.2. Elastic-rotational medium of Kelvin

The problem of the constitution of the Aether (Lodge, 1909) , and of the way in which portions of it are modified to form the atoms or other constituent units of ordinary matter, has not yet been solved…

Meanwhile there are few physicists who will dissent from Clerk-Maxwel's penultimate sentence in the article "Aether," of which the beginning has already been quoted: "Whatever difficulties we may have in forming a consistent idea of the constitution of the aether, there can be no doubt that the interplanetary and interstellar spaces are not empty, but are occupied by a material substance or body, which is certainly the largest, and probably the most uniform body of which we have any knowledge."…

But now comes the question, how is it possible for matter to be composed of aether? How is it possible for a solid to be made out of fluid? A solid possesses the properties of rigidity, impenetrability, elasticity, and such like; how can these be imitated by a perfect fluid such as the aether must be?

The answer is: they can be imitated by a fluid in motion; a statement which we make with confidence as the result of a great part of Lord Kelvin's work (Kelvin, 1869; 1910).…

A vortex-ring, ejected from an elliptical orifice, oscillates about the stable circular form, as an india-rubber ring would do; thus furnishing a beautiful example of kinetic elasticity, and showing us clearly a fluid displaying some of the properties of a solid…

A still further example is Lord Kelvin's model of a spring balance, made of nothing but rigid bodies in spinning motion. This arrangement utilises the processional movement of balanced gyrostats — concealed in a case and supporting a book — to imitate the behaviour of a spiral spring, if it were used to support the same book…

If the aether can be set spinning, therefore, we may have some hope of making it imitate the properties of matter, or even of constructing matter by its aid.

The estimates of this book, and of Modern Views of Electricity, are that the aether of space is a continuous, incompressible, stationary fundamental substance…

The aether inside matter is just as dense as the aether outside, and no denser. A material unit — say, an electron — is only a peculiarity or singularity of some kind in the aether itself, which is of perfectly uniform density everywhere. What we "sense" as matter is an aggregate or grouping of an enormous number of such units.

The elasticity of the aether,… if this is due to intrinsic turbulence, the speed of the whirling or rotational elasticity must be of the same order as the velocity of light…

The three vectors at right angles to each other, which may be labeled Current, Magnetism and Motion respectively or more generally E, H and $\vec{\upsilon}$, represent the quite fundamental relation between aether and matter, and constitute the link between Electricity, Magnetism and Mechanics. Where any two of these are present, the third is necessary consequence".

Up to this day special attention of scientists, such model of aether attracts, as a "vortex sponge", in the version adopted by Kelvin in 1880, and Fitzgerald in 1885. Under designation "vortex sponge" Kelvin proposed a fine mixture of rotating and non-rotating elements. In 1887 Kelvin proposed analogy between the propagation of light in space and the propagation of disturbances in laminar vortex sponge.

When (Kelly, 1963), vortex tubes follow the fluid, the medium behaves like an elastic solid because of momentum transfer effects arising from the fine-grained vorticity. This character of the medium is altered by displacements which bend the tubes so that they move laterally. The motion of curved tubes relative to the fluid can result in macroscopic vorticity with accompanying rotation of the bulk medium. The mathematical expressions of these effects have the form of Maxwell's curl equations for free space.

6.3.1. ***Kelvin vortex model of the atom*** (Fowler, (2007).

Lord Kelvin proposed that atoms (then considered to be elementary particles) could be described as vortex in aether

The first attempt to construct a physical model of an atom [i.e. elementary particle] was made by William Thomson (later elevated to Lord Kelvin) in 1867. The most striking property of the atom was its *permanence*. It was difficult to imagine any small solid entity that could not be broken, given the right force, temperature or chemical reaction. In contemplating what kinds of physical systems exhibited permanence, Thomson was inspired by a paper Helmholtz had written in 1858 on *vortices*. This work had been translated into English by a Scotsman, Peter Tait (Tait, 1990), who showed Thomson some ingenious experiments with smoke rings to illustrate Helmholtz' ideas. The main point was that in an *ideal* fluid, a vortex line is always composed of the same particles, it remains *unbroken*, so it is ring-like. Vortices can also form interesting combinations-- A good demonstration is provided by creating two vortex rings one right after the other going in the same direction. They can trap each other, each going through the other in succession. This is probably what Tait showed Thomson, and it gave Thomson the idea that atoms might somehow be vortices in the aether.

Of course, in a non ideal fluid like air, the vortices dissipate after a while, so Helholtz' mathematical theorem about their permanence is only approximate. But Thomson was excited because the aether *was* thought an ideal fluid, so vortices in the aether might last forever! This was very aesthetically appealing to everybody- "Kirchhoff, a man of cold temperament, can be roused to enthusiasm when speaking of it.". In fact, the investigations of vortices, trying to match their properties with those of atoms, led to a much better understanding of the hydrodynamics of vortices - the constancy of the circulation around a vortex, for example, is known as Kelvin's law. In 1882 another Thomson, J. J., won a prize for an essay on vortex atoms, and how they might interact chemically. After that, though, interest began to wane - Kelvin himself began to doubt that his model really had much to do with atoms, and when the electron was discovered by J. J. in 1897, and was clearly a component of all atoms, different kinds of non-vortex atomic models evolved.

6.3.2. *Kelvin vortex model of the atom as the primary string theory*

It is fascinating to note (Fowler, (2007). that the most exciting theory of fundamental particles at the present time, *string* theory, has a definite resemblance to Thomson's vortex atoms. One of the basic entities is the closed string, a little loop, which has fields flowing around it reminiscent of the swirl of aethereal fluid in Thomson's atom. And it's a very beautiful theory - Kirchhoff would have been enthusiastic!

For about 20 years (Faddeev and Niemi, 1996) the Kelvin theory was taken seriously, and motivated an extensive study of knots. The results obtained at the time by Tait (Tait, 1990) remain a classic contribution to mathematical knot theory. More recently the idea that elementary particles can be identified with topologically distinct knots has been advanced in particular by Jehle (Jehle, 1972).

Today it is commonly accepted that fundamental interactions are described by stringlike structures (Green et al, 1987), with different elementary particles corresponding to the vibrational excitations of a fundamental string.

We will consider now some mechanical medium that supports Maxwell's equations in terms of the modern theory.

6.3. Elastic-rotational medium of H. Fukutome (Fukutome, 1960)

H. Fucutome (Fukutome, 1960) proposed the, impeccable from a theoretical point of view, unified theory of elementary particles, based upon a renewed idea of “aether”. In the above paper a general classical theory of the relativistic elastic continuum is developed. From the result of the general theory a hypothetical relativistic elastic medium "aether " is introduced. It is an unusual elastic medium which has no mass and no energy in the undeformed state but has mass and energy in the deformed states which are due entirely to the elastic self-interaction of it. It is shown that if the aether is elastically isotropic then it has a conserving quantity called "the Lagrange spin" which has very similar property as that of the iso-spin. It is not conserved if the elastic property of the aether is anisotropic. In this model the elementary particles are viewed as the excited states of the aether and the strong, the electro-magnetic and the weak interactions of them are ascribed to the isotropic, cylindrically symmetric and anisotropic elastic self-interaction of the aether, respectively.

Vortices have been considered in many later attempts to find a connection between mechanics and electromagnetism.

It was shown (see, for example, Kelly, 1963, 1964, 1976) that the vortex sponge to be governed, in restricted cases, by Maxwell's free-space equations. Rotational stability, suggested originally by MacCullagh as a fundamental property of a luminiferous aether, turns out to be a quality of the medium, as do the stresses introduced by Faraday and Maxwell to explain the mechanical actions of electric and magnetic fields. A conventional definition of charge and the laws of Coulomb and Biot complete Maxwell's equations for cases including charges and currents. A model of the magnetic field based on the bulk rotation and the Faraday-Maxwell stresses, combined with the laws of Coulomb and Biot, permits the inference of the Lorentz force. Although numerous gaps occur in the treatment, it seems not unlikely that the vortex sponge has the qualities described by the electromagnetic field equations as well as the mechanical attributes required for a model of these fields.

6.4. Rotaional medium as Yang-Mills field (Ryder, 1985)

Let us examine the rotations of a certain field vector $\vec{F}$ about the 3 axis in the internal symmetry space through an infinitely small angle $\vec{\varphi}$. The meaning of this angle is that $|\vec{\varphi}|$ is the angle of rotation, and $\vec{\varphi}/|\vec{\varphi}|$ is the axis of rotation. Then transition from the initial position of the vector to the final one will be determined by the transformation:

$$\vec{F} \to \vec{F}' = \vec{F} - \vec{\varphi} \times \vec{F}, \tag{5.6.1}$$

We have then a gauge transformation of the first kind, and is, of course, effectively three equations.

$$\delta\vec{F} = \vec{F}' - \vec{F} = -\vec{\varphi} \times \vec{F}, \tag{5.6.2}$$

In contrast to electrodynamics the present case is more complicated, however, and this is directly traceable to the fact that in the present case the rotations form the group SO(3) which is non-Abelian. Its non-Abelian nature is responsible for that fact that $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$: the vector product is not commutative. It will be seen below how this complicates matters. These complications have direct physical consequences.

First note that (3.120) is an instruction to perform a rotation in the internal space of $\vec{F}$ through the same angle $\vec{\varphi}$ at all points in space-time. We modify this to the more reasonable demand that $\vec{\varphi}$ depends on x^{μ} (i.e. that the same relationships are also valid in the four-dimensional space). We then have

$$\vec{\varphi} = \vec{\varphi}(x^{\mu}), \tag{5.6.3}$$

In this case:

$$\partial_{\mu}\vec{F} \to \partial_{\mu}\vec{F}' = \partial_{\mu}\vec{F} - \partial_{\mu}\vec{\varphi} \times \vec{F} - \vec{\varphi} \times \partial_{\mu}\vec{F},$$

or

$$\delta(\partial_{\mu}\vec{F}) = \partial_{\mu}\vec{F}' - \partial_{\mu}\vec{F} = -\partial_{\mu}\vec{\varphi} \times \vec{F} - \vec{\varphi} \times \partial_{\mu}\vec{F}, \tag{5.6.4}$$

Expressed in other words, $\partial_{\mu}\vec{F}$ does not transform covariantly, like $\vec{F}$ does. We must construct a 'covariant derivative'. This will involve introducing a gauge potential $\vec{W}_{\mu}$ analogous to electromagnetic potential A_{μ}.

How should $\vec{W}_{\mu}$ transform? Analogous to the case of A_{μ} we can write

$$\vec{W}_{\mu} \to \vec{W}_{\mu} - \vec{\varphi} \times \vec{W}_{\mu} + \frac{1}{g}\partial_{\mu}\vec{\varphi},$$

or

$$\delta\vec{W}_{\mu} = -\vec{\varphi} \times \vec{W}_{\mu} + \frac{1}{g}\partial_{\mu}\vec{\varphi}, \tag{5.6.5}$$

Note that $\vec{W}_{\mu}$ is a vector in the internal space, whereas A_{μ} only had one component; g is a coupling constant, analogous to electric charge e.

We then write the covariant derivative of the vector $\vec{F}$ as

$$D_\mu \vec{F} = \partial_\mu \vec{F} + g\vec{W}_\mu \times \vec{F}\ , \tag{5.6.6}$$

$\vec{W}_\mu$ is the analogue of A_μ. What is the analogue of the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ Let us call it $\vec{W}_{\mu\nu}$. Unlike $F_{\mu\nu}$, which is a scalar under SO(2), $\vec{W}_{\mu\nu}$ will be a vector under SO(3), and so will transform like $\vec{F}$ itself:

$$\delta\left(\vec{W}_{\mu\nu}\right) = -\vec{\varphi} \times \vec{W}_{\mu\nu},$$

So if we define

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\vec{W}_\mu \times \vec{W}_\nu\ , \tag{5.6.7}$$

then $\vec{W}_{\mu\nu}$ transforms in the required way. The field strength $\vec{W}_\mu$ is a vector, so $\vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu}$ is a scalar and will appear in the Lagrangian, which is, therefore,

$$L = (D_\mu \vec{F}) \cdot (D^\mu \vec{F}) - m^2 \vec{F} \cdot \vec{F} - \frac{1}{4} \vec{W}_{\mu\nu} \cdot W^{\mu\nu}\ , \tag{5.6.8}$$

The equations of motion are obtained by functional variation of this Lagrangian in the usual way from the Euler-Lagrange equation

$$D^\nu \vec{W}_{\mu\nu} = g(D_\mu \vec{F}) \times \vec{F} \equiv g\vec{J}_\mu\ , \tag{5.6.9}$$

This equation is analogous to Maxwell's equation for the 4-current, so that $\vec{W}_{\mu\nu}$ is the 'isospin' gauge field, $\vec{J}_\mu$ is the source or 'matter' term, and instead of ordinary derivatives there are covariant ones. Whereas Maxwell's equations are linear in A_μ, however, this equation is non-linear in $\vec{W}_\mu$.

The non-Abelian generalisation of equations, which are analogous to the homogeneous Maxwell equations:

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0\ , \tag{5.6.10'}$$

is

$$D_\lambda \vec{W}_{\mu\nu} + D_\mu W_{\nu\lambda} + D_\nu W_{\lambda\mu} = 0\ , \tag{5.6.10''}$$

7.0. Unquantized vacuum as a mechanical medium of classical electromagnetic field (Bethe, 1964; Branson, 2012)

7.1. The decomposition of the electromagnetic field on the oscillator as classical quantization of electromagnetic field

The representation of the classical electromagnetic field in the form of the oscillator is not used in the classical electromagnetic theory. It was invented by Dirac to describe radiation of the quanta of the EM wave in form of photons. To do this, Dirac performed the quantization of the obtained classical oscillators and showed that the energy of the electromagnetic field is determined by their number. Changing the number of oscillators indicate their emission or absorption.

Let us describe (very briefly) a process of decomposition of the classical electromagnetic field on the oscillators, as it is usually presented in books on quantum field theory.

7.1.1. *Transverse and Longitudinal Fields*

It is useful to be able to separate the electric $\vec{E}$ and magnetic $\vec{H}$ fields due to fixed charges from the EM radiation from moving charges. This separation is not Lorentz invariant, but it is still.

This field strengths $\vec{E}$ and $\vec{H}$ can simply be written in terms of the 4-vector potential, (which is a Lorentz vector) $A_\mu = \left(\vec{A}, i\varphi\right)$. Gauge symmetry may be used to put a condition on the vector potential $\frac{\partial A_\nu}{\partial x_\nu} = 0$. This is called the Lorentz condition.

As it is known, if the vector of the electromagnetic field to express through the potentials

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\varphi\,,\;\; \vec{H} = \vec{\nabla}\times\vec{A}\,,\;, \tag{5.7.3}$$

then the equations:

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -\frac{4\pi}{c}j\,, \tag{5.7.2a}$$

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = -4\pi\rho\,, \tag{5.7.2b}$$

$$\nabla\vec{A} + \frac{1}{c}\frac{\partial}{\partial t}\varphi = 0\,, \tag{5.7.2c}$$

where ρ is the charge density, and υ is the velocity of charge, are equivalent to Maxwell's equations.

Enrico Fermi showed, in 1930, that A_{II} together with A_0 give rise to Coulomb interactions between particles, whereas $A_\perp$ gives rise to the EM radiation from moving charges.

In this case we must separate the vector potential into the transverse and longitudinal parts, $\vec{A} = A_\perp + A_{II}$ with $\vec{\nabla}\cdot\vec{A}_\perp = 0$ and $\vec{\nabla}\times\vec{A}_{II} = 0$.

In a region in which there are no current source terms, $j_\mu = 0$, we can make a gauge transformation which eliminates A_0. Since the fourth component of A_μ is now eliminated, the Lorentz condition now implies that $\vec{\nabla}\cdot\vec{A} = 0$

Again, making one component of a 4-vector zero is not a Lorentz invariant way of working. We have to redo the gauge transformation if we move to another frame. If $j_\mu \neq 0$, then we cannot eliminate A_0, since (5.7.2).

We will now study the radiation field in a region with no sources so that $\vec{\nabla}\cdot\vec{A} = 0$. We will use the equations

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t},\;\; \vec{H} = \vec{\nabla}\times\vec{A},\;\; \left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{A} = 0\,, \tag{5.7.3}$$

7.1.2. *Fourier Decomposition of Radiation Oscillators*

Our goal is to write the Hamiltonian for the radiation field in terms of a sum of harmonic oscillator Hamiltonians. The first step is to write the radiation field in as simple a way as possible, as a sum of harmonic components. We will work in a cubic volume $V = L^3$ and apply periodic boundary conditions on our electromagnetic waves. We also assume for now that there are no sources inside the region so that we can make a gauge transformation to make $A_0 = 0$ and hence $\vec{\nabla}\cdot\vec{A} = 0$. We decompose the field into its Fourier components at $t = 0$.

Let us expand the potential in the complete system of orthonormal plane waves. For the vector potential, we have

$$\vec{A}(\vec{r},t)=\sum_{k\lambda}{}' \left[q_{k\lambda}(t)\vec{u}_{k\lambda}(\vec{r})+q^{+}_{k\lambda}(t)\vec{u}^{*}_{k\lambda}(\vec{r})\right], \qquad (5.7.4)$$

where functions $\vec{u}_{k\lambda}$ are the plane waves $\vec{u}_{k\lambda} = \frac{\sqrt{4\pi c}}{L^{3/2}}\vec{e}_{k\lambda}e^{i\vec{k}\cdot\vec{r}}$. L^3 is set to normalize the $\vec{u}_{k\lambda}$. $\vec{e}_{k\lambda}$ are real unit vectors $\vec{e}_{k\lambda} = \frac{i\vec{k}}{|\vec{k}|}$, and $q_{k\lambda}(t)$ is the coefficient of the wave with wave vector $\vec{k}$ and polarization index $\lambda = 1,2,3$. Once the wave vector is chosen, the two polarization vectors must be picked so that $\vec{e}_{k1}$, $\vec{e}_{k2}$, and $\vec{k}$ form a right handed orthogonal system. The components of the wave vector must satisfy $k_i = \frac{2\pi n_i}{L}$ due to the periodic boundary conditions.

The prime on the summation sign means that the sum take place over half of all the values $\vec{k}$, so that the function $\vec{u}^{*}_{k\lambda}$ does not duplicate the $\vec{u}_{k\lambda}$. Operators $q^{+}_{k\lambda}$ and $q_{k\lambda}$ are Hermitian associated, so that the operator $\vec{A}(\vec{r},t)$ is real and Hermitian.

Obviously, the following orthogonality relations should be:

$$\int \vec{u}_{k\lambda}\cdot\vec{u}_{k'\lambda'}d\tau = 0, \qquad (5.7.5a)$$

$$\int \vec{u}^{*}_{k\lambda}\cdot\vec{u}^{*}_{k'\lambda'}d\tau = 0, \qquad (5.7.5b)$$

$$\int \vec{u}^{*}_{k\lambda}\cdot\vec{u}_{k'\lambda'}d\tau = 4\pi c^2\delta_{k\lambda}\delta_{k'\lambda'} \qquad (5.7.5c)$$

Similarly we can also expand the scalar potential φ:

$$\varphi(\vec{r},t)=\sum_{k}{}' \left[a_k(t)\, f_k(\vec{r})+a^{+}_k(t)\, f^{*}_k(\vec{r})\right]$$

$$f_k = \frac{\sqrt{4\pi c}}{L^{3/2}}e^{i\vec{k}\cdot\vec{r}}, \qquad (5.7.6)$$

Whence

$$\ddot{a}_k + \omega^2 a_k = \sum_{\mathrm{j}} e_j\, f^{*}_k(\vec{r}_j), \qquad (5.7.7)$$

Lorentz condition (5.7.2b) here takes the form

$$a_k = \omega\, q_{k3}, \qquad (5.7.8).$$

7.2. Energy and the Hamiltonians of the EM field

In this case magnetic field, $\vec{H} = \nabla\times\vec{A}$, is

$$\vec{H}(\vec{\mathrm{r}},\mathrm{t}) = i\sum_{\substack{\vec{\mathrm{k}}\\ \lambda=1,2}}\left[q_{k\lambda}\left(\vec{\mathrm{k}}\times\vec{u}_{k\lambda}\right)-q^{+}_{k\lambda}\left(\vec{\mathrm{k}}\times\vec{u}^{*}_{k\lambda}\right)\right], \qquad (5.7.9)$$

Note that the longitudinally polarized plane waves ($\lambda = 3$) in the expansion of the vector potential $\vec{A}$ do not contribute to the magnetic field. And, vice versa, from (5.7.9) it is clear that the expansion $\vec{H}$ in the plane wave does not contain longitudinally polarized components.

The magnetic force is not a conservative one so we cannot just add a scalar potential.

Let us split $\vec{E}$ in (5.7.1) into the sum of transverse and longitudinal parts. In the free field a longitudinal part of the field is missing

$$\vec{E}_{tr} = -\frac{1}{c}\frac{\partial \vec{A}_{tr}}{\partial t}, \tag{5.7.10a}$$

$$\vec{E}_{\lg} = -\frac{1}{c}\frac{\partial \vec{A}_{\lg}}{\partial t} - \vec{\nabla}\varphi, \tag{5.7.10б}$$

By (5.7.4), we have

$$\vec{E}_{tr}(\mathrm{\vec{r}},\mathrm{t}) = -\frac{1}{c}\sum_{\substack{\mathrm{k}\\ \lambda=1,2}}\left[q_{k\lambda}\vec{u}_{k\lambda} + q^{+}_{k\lambda}\vec{u}^{*}_{k\lambda}\right], \tag{5.7.11).}$$

Thus, the full energy of transverce part of the field (i.e. the free field energy) is:

$$W_{tr} = \frac{1}{8\pi}\int\left(H^2 + E^2_{tr}\right)d\tau, \tag{5.7.12).}$$

To calculate the first term in the integral (5.7.12) we used the equality (5.7.9) and the orthogonality relations (5.7.5). The second term in (5.7.12) is calculated in a similar way with the help of (5.7.11) and (5.7.5).

7.2.1. *The Hamiltonian of the transversal field*

With the above expressions the Hamiltonian of the transverse field can be build

$$\hat{\mathrm{H}}_{tr} = \sum_{\substack{k\\ \lambda=1,2}}{}'\left(p^{+}_{k\lambda}p_{k\lambda} + \omega^2 q^{+}_{k\lambda}q_{k\lambda}\right), \tag{5.7.13}$$

where

$$\dot{q}_{k\lambda} = \frac{\partial \hat{\mathrm{H}}}{\partial p_{k\lambda}} = p^{+}_{k\lambda},\ \dot{q}^{+}_{k\lambda} = \frac{\partial \hat{\mathrm{H}}}{\partial p^{+}_{k\lambda}} = p_{k\lambda}, \tag{5.7.14}$$

Other canonical equation of motion gives

$$\dot{p}^{+}_{k\lambda} = -\frac{\partial \hat{\mathrm{H}}}{\partial q^{+}_{k\lambda}} = -\omega^2 q_{k\lambda},\ \dot{p}_{k\lambda} = -\frac{\partial \hat{\mathrm{H}}}{\partial q_{k\lambda}} = -\omega^2 q^{+}_{k\lambda}, \tag{5.7.15a}$$

Comparing equations (5.7.15a) and (5.7.14), we see that

$$\ddot{q}_{k\lambda} = -\omega^2 q_{k\lambda},\ \ddot{q}^{+}_{k\lambda} = -\omega^2 q^{+}_{k\lambda}, \tag{5.7.15б}$$

Thus, the transverse field is described by an infinite system of oscillators. Equations (5.7.15) are equivalent to classical wave equation, which is derived from Maxwell's equations

7.2.2. Hamiltonian for a charged particles in an electromagnetic field

Let us consider now the relativistic Hamiltonian of system of point particles, each of which has a charge e_j in a given electromagnetic field.

$$\hat{\mathrm{H}}_j = \sqrt{m_j c^2 + \left[c\vec{p}_j - e_j\vec{A}(\vec{r}_j)\right]^2} + e_j\varphi(r_j), \tag{5.7.16}$$

where j labels the particle.

The right side of (5.7.16) must be replaced by potentials due to both external sources and particles themselves.

Using (5.7.4), equation (5.7.2a) can be transformed to

$$\ddot{q}_{k\lambda} + \omega^2 q_{k\lambda} = \frac{1}{c}\sum_{\mathrm{j}} e_j\vec{u}^{*}_{k\lambda}(\vec{r}_j)\cdot\vec{\upsilon}_j, \tag{5.7.17}$$

This is the equation of motion for an oscillator with self frequency ω, and the oscillator is operated by a driving force, caused by charged particles.

Since these fields are no longer free, they will also have longitudinal components. Thus, the equations of motion (5.7.2) are equivalent to equalities (5.7.17), (5.7.6) and (5.7.8).

The Hamiltonian of the complete system is

$$\hat{\mathrm{H}} = \sum_j \hat{\mathrm{H}}_j + \hat{\mathrm{H}}_{\mathrm{lg}} + \hat{\mathrm{H}}_{tr}, \tag{5.7.18}$$

where $\sum_j \hat{\mathrm{H}}_j$ is the Hamiltonian of the particles, (21.14); $\hat{\mathrm{H}}_{tr} + \hat{\mathrm{H}}_{\mathrm{lg}}$ are the Hamiltonians of the field, so that $\hat{\mathrm{H}}_{tr}$ is given by (21.11), and $\hat{\mathrm{H}}_{\mathrm{lg}}$ by

$$\hat{\mathrm{H}}_{\mathrm{lg}} = \sum_k{}' \left(p_{k3}^{+} p_{k3} + \omega^2 q_{k3}^{+} q_{k3}\right) - \sum_k \left(\alpha_k^{+}\alpha_k + \omega^2 a_k^{+} a_k\right), \tag{5.7.19;}$$

where the values α_k are the impulses, canonically conjugated to variables a_k . Canonical equations are fully equivalent to (5.7.17) and (5.7.6). From these can be obtained, for example, the equation (5.7.6).

7.2.3. The Hamiltonian of the Coulomb interaction

Let us consider the part of the Hamiltonian (21.25), which depends on $q_{k3}^{+}, q_{k3}, a_k^{+}, a_k$ (i.e., from longitudinal waves). This dependence is found in the Hamiltonian $\hat{\mathrm{H}}_j$ of j-particles and in $\hat{\mathrm{H}}_{\mathrm{lg}}$. We can write

$$\begin{aligned}\hat{\mathrm{H}}_C &= \hat{\mathrm{H}}_{\mathrm{lg}} + \sum_{\mathrm{j}} e_j f_k^{*}(\vec{r}_j) = \sum_k{}' \left(p_{k3}^{+} p_{k3} + \omega^2 q_{k3}^{+} q_{k3}\right) - \sum_k \left(\alpha_k^{+}\alpha_k + \omega^2 a_k^{+} a_k\right) + \\ &+ \sum_{\mathrm{j}} \sum_k{}' e_j \left[a_k f_k(\vec{r}_j) + a_k f_k^{*}(\vec{r}_j)\right]\end{aligned}, \tag{5.7.20}$$

The summation is easy to perform, and, having made all the necessary operations, we obtain

$$\hat{\mathrm{H}}_C = \frac{1}{2}\sum_{i,j} \frac{e_i e_j}{r_{ij}}, \tag{5.7.21}$$

We see, therefore, that the part of the total Hamiltonian (5.7.18), which depends on the longitudinal component of the field, corresponds to the static Coulomb interaction between the particles and can be expressed in terms of the coordinates of the particles alone.

On this one might argue that we do not fully take into account the dependence of the Hamiltonian (5.7.18) from the longitudinal component, since we did not consider the longitudinal components of the vector potential $\vec{A}(\vec{r}_j)$. But we can show that all the solutions, which satisfy the initial conditions, can be obtained from the new Hamiltonian, which is obtained from (5.7.21) by substituting with $\hat{\mathrm{H}}_C$ an expression (5.7.21) and lowering of longitudinal components $\vec{A}$ in the remaining parts.

Thus, to the values within the total Lagrangian (5.7.18) $\hat{\mathrm{H}} = \sum_j \hat{\mathrm{H}}_j + \hat{\mathrm{H}}_{\mathrm{lg}} + \hat{\mathrm{H}}_{tr}$, taking into account the expressions (5.7.13), (5.7.16), (5.7.19) and (5.7.20) , can be given the following physical interpretation. The first term includes the rest energy and the kinetic energy of the particles. The second term describes the statistical interaction between point particles. The last term is purely the Hamiltonian of the radiation (containing only $A_{\perp}$), and $A_{\perp}$ is the part of the vector potential which satisfies $\vec{\nabla} \cdot \vec{A}_{\perp} = 0$. Note that A_{II} and A_0 appear nowhere in the Hamiltonian. Instead, we have the **Coulomb potential**..

Note that we have above calculated the Hamiltonian density for a classical EM field

All of these formulations, which at a time were a great achievement, suffer from one drawback: the splitting of field on longitudinal and transverse parts is not invariant under Lorentz transformations. Nevertheless, this is the basic approach, which served as starting point for a more elegant method of Schwinger and Feynman.

This is an important calculation because we will use the Hamiltonian formalism to do the quantization of the field.

7.3. Quantization of EM field and physical vacuum hypothesis

The Hamiltonian for the Maxwell field may be used to quantize the field in much the same way that one dimensional wave mechanics was quantized. As we shown the radiation field is the transverse part of the field, $A_{\perp}$, while static charges give rise to A_{II} and A_0. We decomposed also the radiation field into its Fourier components (5.7.4)

Plugging the Fourier decomposition into the formula for the Hamiltonian density and using the transverse nature of the radiation field, we computed the Hamiltonian (5.7.13) . This Hamiltonian will be used to quantize the EM field.

The canonical coordinate $q_{k\lambda}$ and momenta $p_{k\lambda}$ we found for the harmonic oscillator at each frequency. We assume now that a coordinate and its conjugate momentum have the same commutator as in wave mechanics and that coordinates from different oscillators commute:

$$\begin{aligned} &[q_{k\lambda}, p_{k'\lambda'}] = [q^{+}_{k\lambda}, p^{+}_{k'\lambda'}] = ih\delta_{kk'}\delta_{\lambda\lambda'} \\ &[q_{k\lambda}, q_{k'\lambda'}] = [p_{k\lambda}, p_{k'\lambda'}] = 0 \end{aligned}, \tag{5.7.22}$$

As was done for the 1D harmonic oscillator, we write the Hamiltonian in terms of raising and lowering operators that have the same commutation relations as in the 1D harmonic oscillator.

$$\begin{aligned} b_{k\lambda} &= \frac{1}{2}\sqrt{\frac{2\omega}{\hbar}}e^{i\omega t}\left(q_{k\lambda} + \frac{i}{\omega}p^{+}_{k\lambda}\right) \\ b'^{+}_{k\lambda} &= \frac{1}{2}\sqrt{\frac{2\omega}{\hbar}}e^{-i\omega t}\left(q_{k\lambda} - \frac{i}{\omega}p^{+}_{k\lambda}\right) \end{aligned}, \tag{5.7.23}$$

where the operators $b^{+}_{k\lambda}$ and $b_{k\lambda}$ have the same commutators as (5.7.5). Hamiltonian (5.7.13) obtain the following view:

$$\hat{H} = \sum_{\substack{k \\ \lambda=1,2}} \hbar\omega\left(b^{+}_{k\lambda}b_{k\lambda} + \frac{1}{2}\right), \tag{5.7.24}$$

This is just the same as the Hamiltonian that we had for the one dimensional harmonic oscillator. We therefore have the raising and lowering operators, as long as $[b_{k\lambda}, b^{+}_{k\lambda}] = 1$, as we had for the 1D harmonic oscillator.

This means everything we know about the raising and lowering operators applies here. Energies are in steps of $\hbar\omega$ and there must be a ground state. Since (21.55) is the sum of the oscillator Hamiltonians, we can take the operator of particle's number:

$$N_{k\lambda} = b^{+}_{k\lambda}b_{k\lambda}, \tag{5.7.25}$$

7.3.1. *Physical vacuum hypothesis*

When $N_{k\lambda} = 0$, we have energy oscillators $\hat{H} = \frac{1}{2}\hbar\omega$. This is quantum state with the lowest possible energy, which is called the **vacuum state** (or **vacuum**). It contains no real particles.

The theory, developed above, makes it quite easy to describe the radiation of photons. This description is different from the description of the radiation of EM waves in the classical electromagnetic theory only by the postulate of quantization of EM waves.

7.3.2. *The electric dipole radiation approximation*

We can now expand the $e^{-i\vec{k}\cdot\vec{r}} \approx 1 - i\vec{k}\cdot\vec{r} + ...$ term to allow us to compute matrix elements more easily. Since $\vec{k}\cdot\vec{r} \approx \alpha/2$ (where α is fine structure constant) and the matrix element is squared, our expansion will be in powers of α^2 which is a small number. The dominant decays will be those from the zeroth order approximation which is $e^{-i\vec{k}\cdot\vec{r}} \approx 1$ This is called the **electric dipole approximation**.

Beyond the Electric Dipole approximation, the next term in the expansion of $e^{-i\vec{k}\cdot\vec{r}}$ is $i\vec{k}\cdot\vec{r}$. This term gets split according to its rotation and Lorentz transformation properties into the Electric Quadrupole term and the Magnetic Dipole term.

8.0. The theory of aether according to B. Kelvin, J.J. Thomson and H. Lorentz

8.1. Vortex aether, as a set of oscillators

As we have seen, the mathematical basis of representation of the field as a set of oscillators is the Fourier expansion of the electromagnetic field vectors into the sum of exponential functions with imaginary number exponent.

8.1.1. *The description of the harmonic oscillator motion as rotation* (Feynman, 1964b)

Every solution of the differential equation of harmonic oscillator

$$d^2x/dt^2 = -\omega_0 x, \qquad (5.8.1)$$

that exists in the world can be written as $x = A\cos\omega_0 t + B\sin\omega_0 t$.

For the image of coordinates of the point on the plane, we can use a complex number $z = x + iy$. The coordinates of the point are (Fig. 4.1): $x = r\cos\theta,\ y = r\sin\theta$

If the angle θ is set in the function of the parameter (time) t, $\theta = \omega_0 t$, we will obtain a description of the two vibrational motions: $x = r\cos\omega_0 t$ and $y = r\sin\omega_0 t$, on the axis x and the axis y, respectively.

It is easy to see that there is equivalence between simple harmonic motion and uniform circular motion with radius R.

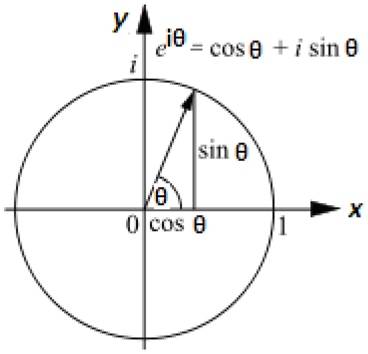

Fig.4.1

The coordinates of the point, moving in a circle at a given point of time are: $x = R\cos\theta,\ y = R\sin\theta$

Here we may note that uniform motion in a circle is closely related mathematically to oscillatory motion. Thus, we can analyze oscillatory motion in a simpler way if we imagine it to be a projection of point moving in a circle. In other words, although the distance y means nothing in the oscillator problem, we may still artificially supplement equation (5.8.1) with another equation using y, and put the two together. If we do this, we will be able to analyze one-dimensional oscillator motion with *circular* motion, which is a lot easier than having to solve a differential equation. We can further simplify the analysis of solution, using complex numbers through *Euler's formula*.

Euler's formula is a mathematical formula in complex analysis that establishes the deep relationship between the trigonometric functions and the complex exponential function:

$$e^{i\theta} = \cos\theta + i\sin\theta, \qquad (5.8.2)$$

In this case, the harmonic motion of a point on two mutually perpendicular coordinates with the amplitude a and angular velocity ω_0 ($\theta = \omega_0 t$) can be written in terms of the exponential function:

$$a\cos\omega_0 t + ia\sin\omega_0 t = ae^{i\omega_0 t}, \qquad (5.8.3)$$

It is easy to see that the mathematical description of the oscillators by means of rotation allows their interpretation as vortices, and vice versa. In this case, the correspondence between the above-described consequences of the expansion of the EM field on the oscillators and the results of the 19th century, are very transparent.

Since mathematics in this case does not bring new results, further we provide a few quotes from the popular works of J.J. Thomson, which introduce us into the circle of these matters. We refer the readers for the mathematical details to the relevant articles of the world-famous scientists: W. Kelvin, J.J. Thomson, H. Lorentz, etc.

(Note that below the terms "vortex", "force tubes", «vortex filaments”, "lines of force" are equivalent).

8.1.2. *Faraday Line of Force Theory* (Thomson, 1909).

Faraday was deeply influenced by the axiom, or if you prefer it, dogma that matter cannot act where it is not. He therefore cast about for some way of picturing to himself the actions in the electric field which would get rid of the idea of action at a distance. He was able to do this by the conception of lines of force.

To Faraday the lines of force were far more than mathematical abstractions — they were physical realities. Faraday materialized the lines of force and endowed them with physical properties so to explain the phenomena of the electric field. Thus he supposed that they were in a state of tension, and that they repelled each other. Instead of an intangible action at a distance between two electrified bodies, Faraday regarded the whole space between the bodies as full of stretched mutually repellent springs. The charges of electricity to which alone an interpretation had been given on the fluid theories of electricity were on this view just the ends of these springs, and an electric charge, instead of being a portion of fluid confined to the electrified body, was an extensive arsenal of springs spreading out in all directions to all parts of the field.

It is easy to develop the method so as to make it metrical. We can do this by introducing the idea of tubes of force. If through the boundary of any small closed curve in the electric field we draw the lines of force, these lines will form a tubular surface… We regard these Faraday tubes as having direction, their direction being the same as that of the electric force,

Maxwell took up the question of the tensions and pressures in the lines of force in the electric field, and carried the problem further than Faraday.

There is a very close connection between the momentum arising from an electrified point and a magnetic system, and the Vector Potential of that system, a quantity which plays a very large part in Maxwell's Theory of Electricity.

8.2. Photon radiation in classical electrodynamics by J.J. Thomson

8.2.1. Effects due to acceleration of the faraday tubes

We have considered the behavior of the lines of force when at rest and when moving uniformly, we shall in this chapter consider the phenomena which result when the state of motion of the lines is changing.

For brevity, we present these results in own words (for more details see (Thomson, 1925; 1930; 1933; 1936) and also in the paper (Kyriakos, 2011b)).

The ends of longitudinal vortices are associated with the charges. These vortices form field of a charged particle and move with it.

The longitudinal vortices retain their own characteristics during the uniform motion of charges. Between two charges, the longitudinal vortices change their shape. This corresponds to the change in mechanical stresses in the space between the charges.

An accelerated movement of longitudinal vortices leads to a new phenomenon. In this case the circular (ring, transverse) vortices are formed. They may be cut off from the charge and exist independently, forming EM waves. This process is called radiation of EM waves.

The theory of photon emission of J.J. Tomsona is supported via following hypothesis of F. Lenard (Lenard, 1921).

8.2.2. The hypothetic model of photons as closed force lines of electric dipole radiation

The solution of Maxwell's equations for the dipole oscillator, as irradiator of EM waves, give a known picture of the force lines of electric and magnetic fields (see Figure 4.2).

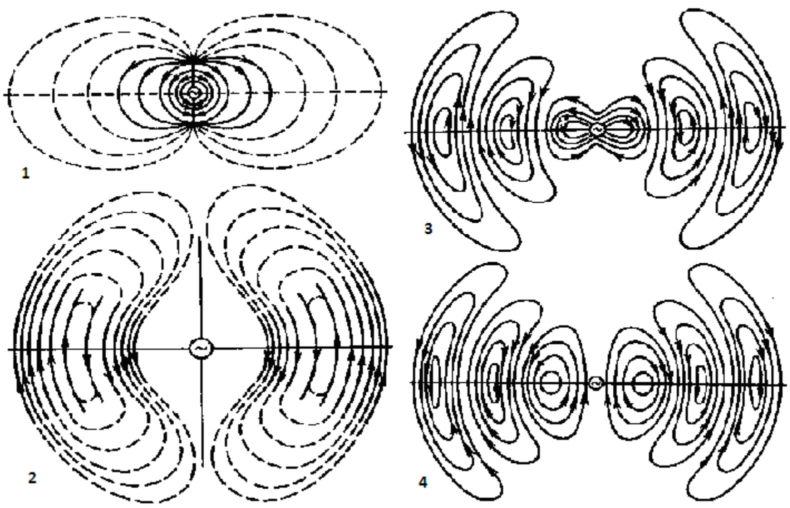

Fig. 4.2

Successive stages of the radiation can be interpreted as stages in the formation and release of ring force lines, which dipole radiates.

As is known, the emission of an electron of a hydrogen atom, if atom regarded as an elementary dipole irradiator, allowed to Lorentz to construct a classical radiation theory of the hydrogen atom. Balmer's experiments confirmed these calculations. Later it was shown that in the first approximation, these results coincide exactly with the quantum-mechanical calculation of emission of photons by hydrogen atom.

Radiation patterns of a electric dipole have the following view (Piestun, 2013, pp. 461-463. Electric dipole antenna) (Fig. 4.3)

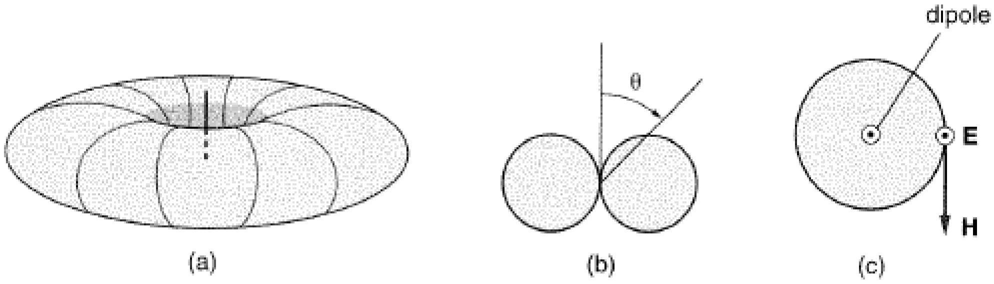

Fig. 4.3. Radiation patterns of a Hertzian dipole verses angle θ, normalized to the maximal radiation. (a) the three-dimensional pattern, (b) the *E*-plane pattern, and the *H*-plane pattern (*E* and *H* are electric and magnetic fields)..

Note that both field components depend on the distance from dipole as $1/r$. No static field has this dependence on r. This type of field is thus different from any electromagnetic field, and is termed the radiation field, or the far field, of the Hertzian dipole. We see here only this field. The field closer to the dipole has other components in addition. All antennas can be considered as large assemblies of elementary electric dipoles.

In 1911 - 1913 Philipp von Lenard had published series of articles, outlining his theory of ring electron and the theory of vortex quantum of light (photon). In the brochure "The principle of relativity, aether, gravity", he wrote, "... that each of the single light wave, emitted by the single oscillating electron, is only one ring of electric force lines, conceivable as a separate aether vortex ring.

8.3. Photon quantization in classical theory by J.J. Thomson

Thomson has calculated the energy of the vortex ring, and then showed that it can be reconciled with the law of Planck.

8.3.1. *Energy of a photon Ring*

If E is the electric polarization in the ring, the energy ε is given by the equation

$$\varepsilon = 2\pi E^2 \cdot (volume\ of\ ring) = 2\pi E^2 \cdot S \cdot 2\pi r^2, \tag{5.8.4}$$

where S is the cross-section of the ring and $2\pi r$ its circumference. As the tube of force came from an electron $ES = pe$, where e is the charge on an electron and p a number not greater than unity; p would be equal to unity if all the lines of force from the electron were done up into one bundle ; it will have a smaller value if there are more bundles than one. Substituting this value for f and writing $S = \pi b^2$, where b is the radius of the cross-section of the ring, we find

$$\varepsilon = 8\pi^2 p^2 e^2 \frac{r^2}{b^2} \frac{1}{2\pi r}, \tag{5.8.5}$$

Thus, if the rings are geometrically similar, their energy will be inversely proportional to their linear dimensions…

8.3.2. *Planck's Law*

There ought on this law to be a very simple relation between the energy of the ring and the frequency of the light of which it is the unit. If E is the energy and ν the frequency, $\varepsilon = h\nu$, where h is Planck's constant $6{,}55 \times 10^{-28.}$

Thus the frequency of the light is directly, and the wavelength inversely, proportional to the energy. This kind of relation might be expected on the view we are discussing, for we have seen that the energy of the ring is equal to (5.8.5).

If the rings are geometrically similar, r/b will be the same for all rings. The frequency of the waves is the same as that of the ring; in geometrically similar rings we should expect the wave-length of the vibration to be proportional to the linear dimension. Hence from (1) we should expect the energy to be inversely proportional to the wave-length; this is Planck’s law. To estimate whether the value of the constant would be anything like Planck's value: let us suppose that the time of vibration of the ring is the time taken by light to travel round the circumference of the core; then $\nu = c/2\pi\, r$, where c is the velocity of light. Hence from (5.8.5)

$$\varepsilon = 8\pi^2 p^3 e^2 \frac{r^2}{b^2 c} \cdot \nu \cong \frac{p^2 r^2}{b} h \cdot \nu , \qquad (5.8.6)$$

So that $pr/b = \pi$, the numerical value of the constant connecting ε and ν would agree well with Planck's value”.

Such a photon has also a spin equal to Planck's constant. In other words the transverse photon of Thomson has the characteristics of a photon of QED: it has an integer spin and helicity equal to 1. The totality of these transverse photons represent transverse electromagnetic waves.

8.4. A comparison with the results of the modern theory and modern terminology

If the longitudinal vortices are compared with longitudinal photons and circular vortices with transverse photons (see the section 7.0), it is easy to identify vortices with virtual photons of the modern theory. At the classical level, to these photons- vortices corresponds a potential - longitudinal and transverse. Now what we have in the framework of QED?

Within mathematics of QED, virtual photon is just one of the spectral lines in the Fourier decomposition of the electromagnetic field. If to sum by all harmonics (i.e., by all possible states of the virtual photon), we get the whole field.

Virtuality of photon means that it was radiated and absorbed so quickly that we can not it observe.

The set of longitudinal virtual photons-vortexes is non other than the electromagnetic field produced by the electrons and through which electrons interact. Interaction lies in that one electron "emits" a virtual photon, and the other "absorbs" its (but this does not mean that the photon leaves one electron and passes into the possession of another. More correctly, but conditionally speaking, photons are not propagate in space; it seems, that they are lengthened). The calculation of this interaction provides the law of Coulomb.

But as the QED shows, the interaction of two real electrons may be accompanied with other processes. For example, the emitted virtual photon can create a virtual electron-positron pair. This couple, exchanging through other virtual photon, is then annihilated, to form a third virtual photon, which absorbed the real second electron. To calculate this process it is needed to draw all possible Feynman diagrams of this kind, make a calculation of the interaction of each one of them, and then sum the results.

It is customary to say that the electron is surrounded by a "coat" of virtual photons. When the electron moves the «coat" shrinks (the higher the speed, the more) in the plane of cross-section of the electron, which is perpendicular to the trajectory (this is shown by J.J. Thomson, based on the idea of the force lines.). At relativistic velocities the field of electron is flat in the transverse direction and can be considered as an electromagnetic wave.

The energy of the “coat” is equal to the energy of electrostatic field. Longitudinal virtual photons have no momentum, but have energy. Virtual photons are transformed into real, if the electron starts to move with acceleration. They say that electron throws off the part of its photon “coat” and emits electromagnetic waves. This radiation takes place due external energy, which is consumed by the acceleration of the electron.

If the electrons, which are surrounded by a “coat”, pass by each other without acceleration, virtual photons collide and generate new particles. In this case the longitudinal virtual photons can turn into a real free photon (or other particles). This event is called the scattering of the electron with the emission/absorption of a photon.

According to Fourier series expansion, the photons have different energies. Moreover, their energy and lifetime correspond to the relationships of uncertainty: $\Delta\varepsilon \cdot \Delta t \geq \frac{1}{2}\hbar$. If the velocity of propagation of virtual photons is the speed of light c, the photon can be spread only on the distance $r = c \cdot \Delta t = \frac{c\hbar}{2\Delta\varepsilon}$. From this follows that the graeter is the "length" of a photon, the lower is the energy of the photon. This means that the two charges interact on long-range by means of low-energy photons.

Let us compare this expression with the expression for the wavelength of photons. Since for the virtual photons are $\varepsilon = \frac{1}{2}\hbar\omega$, we find $r = \frac{c}{\omega}$ or $r = \frac{\lambda}{2\pi} = \lambdabar$. According to this result, a virtual photon is "lengthened" from the electron at a distance no more than one reduced (crossed) wavelength of it. In this case, we must conclude that the photons do not "fly", and at best, open and close like a spring.

Thus, in near layers the virtual photons have high energy, and, thus, a short lifetime and a small length, while in remoted layers they have low energy, and therefore a long lifetime and great length. This corresponds to the Coulomb law: the electric field, near an electron is large, but at a distance it is weak.

It is easy to see that all of this is consistent with the ideas that were developed in the 19th century, if we add here Planck's postulate of quantization of the EM field.

9.0. Experimental results on the detection of vacuum

The first experiments in which these effects were found, were experiments of W. Lamb and R. Rutherford (Lamb and Retherford, 1947), who found, in 1947, the level shift of the atomic electrons (Lamb shift), through which the emission line of the electrons of the atom split. On the basis of quantum mechanics, the calculation of this effect in the same year was made by Hans Bethe (Bethe,1947). In 1948, the calculation of this effect on the basis of the concepts of electrodynamics was given

Another famous - mechanical - manifestation of vacuum fluctuations is the so-called Casimir force, predicted by Hendrik Casimir in 1948. In the experiment, it was measured only in 1997.

9.1. The Welton's calculation of the mean square fluctuation of vacuum in position of a free electron (Welton, 1948).

An intuitive explanation is given for the electromagnetic shift of energy levels by calculating the mean square amplitude of oscillation of an electron coupled to the zero-point fluctuations of the electromagnetic field.

Our starting point is the observation that the quantum-mechanical zero-point variation of the radiation field in empty space gives rise to fluctuating electric and magnetic fields whose mean square values at a point in space are given by the well-known relation

$$\left\langle \mathrm{E}_{0\omega}^2 \right\rangle = \left\langle \mathrm{H}_{0\omega}^2 \right\rangle = \frac{2\hbar c}{\pi}\int_0^\infty k^3 dk\,, \tag{5.9.1}$$

In this equation the variable k refers to the wave number of a quantum, and the contribution to the mean square fluctuation arising from frequencies in the range cdk is therefore explicitly displayed.

Equation (5.9.1) can be simply derived by ascribing to every normal mode of the radiation field an energy which is just the zero-point energy for an oscillator with the frequency of the normal mode. The total energy can be written either as the volume integral of the ordinary electromagnetic energy density or as a sum over normal modes, and Eq, (5.9.1) merely states the equality of these two forms.

It will now be assumed that an otherwise free electron is acted on by these fluctuating fields. The electron will be assumed to move with non-relativistic velocities so that if $\vec{r}$ is its position vector, the equation of motion is

$$m\frac{d^2\vec{r}}{dt^2} = e\vec{E}\,, \tag{5.9.2}$$

The vector $\vec{E}$ is the fluctuating field specified by (5.9.1), Since Eq. (5.9.2) is linear, we can regard it as a classical equation for the quantum-mechanical expectation value of $\vec{r}$. For a given harmonic component of $\vec{E}$ the solution of (5.9.2) is obvious. We perform this integration, find the resulting value of and sum over frequencies using (1). We then obtain a quantity $\left\langle\left(\Delta r^2\right)\right\rangle$, defined as the mean square fluctuation in position of a free electron

$$\left\langle\left(\Delta r^2\right)\right\rangle = \frac{2}{\pi}\frac{e^2}{\hbar c}\left(\frac{\hbar}{mc}\right)^2\int_{k_0}^{\kappa}\frac{dk}{k}\,, \tag{5.9.3}$$

Consider the motion of an electron in a static field of force specified by a potential energy $V(\vec{r})$. The coordinates of the electron consist of two parts: the smooth part $\vec{r}$ and the random part $\Delta\vec{r}$. The instantaneous potential energy is then given by

$$V(r+\Delta r) = \left[1 + \Delta r\cdot\vec{\nabla} + \frac{1}{2}\left(\Delta r\cdot\vec{\nabla}\right)^2 + ...\right]V(r), \tag{5.9.4}$$

The effective potentizl energy in which the particle moves will just be the average of (5) over all values of Δr. Remembering that Δr has an isotropic spatial distribution, we obtain

$$\langle V(r+\Delta r)\rangle = \left[1 + \frac{1}{6}\left\langle(\Delta r)^2\right\rangle\vec{\nabla}^2 + ...\right]V(r), \tag{5.9.5}$$

With the correct selection of the upper and lower limits of integration in (5.9.3) **)** we thus see that the existence of the position fluctuation of the electron will effectively modify the potential in which it moves by the addition of a term proportional to the Laplacian of the potential energy…

The magnitude of this mean square fluctuation in position will be very small for any reasonable kt, but an observable effect will arise when the electron moves in a potential with a large curvature.

For example, in the case of the *Lamb shift* **t**he correction to the energy of a stationary state of the atom with wave function $\psi(\vec{r})$ will be

$$\Delta\varepsilon = \frac{4e}{3}\frac{e^2}{\hbar c}\left(\frac{\hbar}{mc}\right)^2 \ln\frac{mc^2}{\hbar ck_0}|\psi(0)|^2\,, \tag{5.9.5}$$

This expression will be recognized as identical with the expression derived by Bethe for the level shift. The quantity $\hbar ck_0$ should clearly be taken equal to the average excitation energy (17.8Ry) (Ry - Rydbergs) introduced by Bethe, (Bethe, 1947)] in order to obtain approximate agreement with the experimental result of Lamb….

The derivation just given has some attractive features. It gives a convergent result for the physically meaningful part of the reaction of the field on the electron, without the necessity of subtracting two infinite terms."

Also Welton examined with success the several simple processes involving the interaction of electrons with other particles and electron radiation (low energy Compton scattering, the interaction between a spin and a magnetic field).

9.2. Casimir effect

"This startling phenomenon (Lambrecht, 2002; Lambrecht et al, 2006) was first predicted in 1948 by the Dutch theoretical physicist Hendrik Casimir while he was working at Philips Research Laboratories in Eindhoven on – of all things – colloidal solutions (see box on page 30). The phenomenon is now dubbed the Casimir effect, while the force between the mirrors is known as the Casimir force.

All fields – in particular electromagnetic fields – have fluctuations. In other words at any given moment their actual value varies around a constant, mean value. Even a perfect vacuum at absolute zero has fluctuating fields known as "vacuum fluctuations", the mean energy of which corresponds to half the energy of a photon.

They have observable consequences that can be directly visualized in experiments on a microscopic scale. For example, an atom in an excited state will not remain there infinitely long, but will return to its ground state by spontaneously emitting a photon. This phenomenon is a consequence of vacuum fluctuations.

The Casimir force is the most famous mechanical effect of vacuum fluctuations. Consider the gap between two plane mirrors as a cavity (figure 1). All electromagnetic fields have a characteristic "spectrum" containing many different frequencies. In a free vacuum all of the frequencies are of equal importance. But inside a cavity, where the field is reflected back and forth between the mirrors, the situation is different. The field is amplified if integer multiples of half a wavelength can fit exactly inside the cavity. This wavelength corresponds to a "cavity resonance". At other wavelengths, in contrast, the field is suppressed. Vacuum fluctuations are suppressed or enhanced depending on whether their frequency corresponds to a cavity resonance or not.

For two perfect, plane, parallel mirrors the Casimir force is therefore attractive and the mirrors are pulled together (Fig. 4.4). The force, F, is proportional to the cross-sectional area, A, of the mirrors and increases 16-fold every time the distance, d, between the mirrors is halved: $F \sim A/d^4$. Apart from these geometrical quantities the force depends only on fundamental values – Planck's constant and the speed of light.

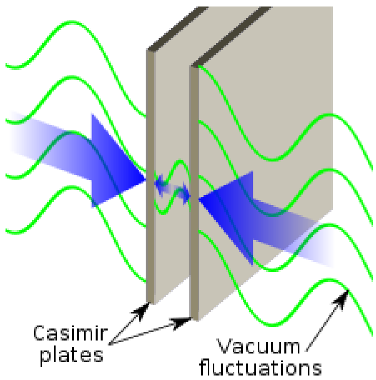

Fig. 4.4

On average the external pressure (great blue arrows) is greater than the internal pressure (little blue arrows). Both mirrors are mutually attracted to each other by what is termed the Casimir force."

Despite the fact that the formula for the Casimir force, does not have the fine structure constant α - the main characteristic of the electromagnetic interaction - this effect, however, has an electromagnetic origin.

9.2.1. *The static Casimir effect*

If there are two fixed plates, two static effects are observed.

1) The Transverse Casimir Force

In the case of smooth plates Casimir force acts perpendicular to the plane.

The transverse Casimir effect was measured more accurately in 1997 by Steve K. Lamoreaux of Los Alamos National Laboratory, and by Umar Mohideen and Anushree Roy of the University of California at Riverside (Lamoreaux, 1997; Mohideen and Roy, 1998)..

2) The Lateral Casimir Force and its demonstration

The lateral Casimir force between a sinusoidally corrugated gold coated plate and large sphere was measured for surface separations between 0.2 μm to 0.3 μm using an atomic force microscope (Chen et al, 2002). The measured force shows the required periodicity corresponding to the corrugations. It also exhibits the necessary inverse fourth power distance dependence. The obtained results are shown to be in good agreement with a complete theory taking into account the imperfectness of the boundary metal. This demonstration opens new opportunities for the use of the Casimir effect for lateral translation in microelectromechanical systems.

9.2.3. *Dynamical Casimir effect (DCE). Simulated emission of photons*

The dynamical Casimir effect is the production of particles and energy from an accelerated boundary, often referred to as a *moving mirror* or *motion-induced radiation* (Fulling and Davies, 1976; Wilson et al, 2011).

"One of the most surprising predictions of modern quantum theory is that the vacuum of space is not empty. In fact, quantum theory predicts that it teems with virtual particles flitting in and out of existence. While initially a curiosity, it was quickly realized that these vacuum fluctuations had measurable consequences, for instance producing the Lamb shift of atomic spectra and modifying the magnetic moment for the electron. This type of renormalization due to vacuum fluctuations is now central to our understanding of nature.

However, these effects provide indirect evidence for the existence of vacuum fluctuations. From early on, it was discussed if it might instead be possible to more directly observe the virtual particles that compose the quantum vacuum. 40 years ago, Moore suggested that a mirror undergoing relativistic motion could convert virtual photons into directly observable real photons. This effect was later named the dynamical Casimir effect (DCE).

Using a superconducting circuit, we have observed the DCE for the first time. The circuit consists of a coplanar transmission line with an electrical length that can be changed at a few percent of the speed of light. The length is changed by modulating the inductance of a superconducting quantum interference device (SQUID) at high frequencies (~11 GHz). In addition to observing the creation of real photons, we observe two-mode squeezing of the emitted radiation, which is a signature of the quantum character of the generation process.

Conclusion

We have shown that: "the garbage of the past often becomes the treasure of the present (and vice versa). For this reason we [can] boldly investigate all possible analogies together with our main problem" (Polyakov, 1987):

1) the theoretical and experimental results of modern classical and quantum field theories can be represented in terms of the mechanics of deformable bodies.

2) the existence of the aether as a continuum with the rotational elasticity, is in accordance with the latest achievements of physics and does not contradict to the quantum field theory.

3) the ideas of the structure of such medium are useful for the development of theory in the modern time.

On the basis of this article, we can formulate the following hypothesis: If we do not take into account the gravitational field, the basis of the existence of all types of matter, their movements and interactions is a single unique fundamental field. For historical reasons, it can be called electromagnetic, because electromagnetic theory was the first and basic field theory.

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