# Fuzzy evidential influence diagram evaluation algorithm

Haoyang Zheng[a], Yong Deng[a,b,c,*], Yong Hu[d]

[a]*School of Computer and Information Science, Southwest University, Chongqing 400715, China*
[b]*Institute of Integrated Automation, School of Electronic and Information Engineering, Xi'an Jiaotong University, Xian, Shaanxi, 710049, China.*
[c]*School of Engineering, Vanderbilt University, TN 37235, USA*
[d]*Big Data Decision Institute, Jinan University, Tianhe, Guangzhou, 510632, China*

**Abstract**

Fuzzy influence diagrams (FIDs) are one of the graphical models that combines the qualitative and quantitative analysis to solve decision-making problems. However, FIDs use an incomprehensive evaluation criteria to score nodes in complex systems, so that many different nodes got the same score, which can not reflect their differences. Based on fuzzy set and Dempster-Shafer (D-S) evidence theory, this paper changes the traditional evaluation system and modifies corresponding algorithm, in order that the influence diagram can more effectively reflect the true situation of the system, and get more practical results. Numerical examples and the real application in supply chain financial system are used to show the efficiency of the proposed influence diagram model.

*Keywords:* Influence diagram, fuzzy set theory, evidence theory, supply chain.

## 1. Introduction

Influence diagram (ID), which is a directed acyclic graph (DAG) composed of nodes and directed arc, is an effective tool to analyze and evaluate risk. The nodes represent the variables in the studied problem while the directed arcs represent the interrelations among variables. Decision-making problems processed by influence diagram can not only

*Corresponding author at: School of Computer and Information Science, Southwest University, Chongqing 400715, China.
*Email address:* ydeng@swu.edu.cn, ydeng@xjtu.edu.cn, prof.deng@hotmail.com (Yong Deng)

be handled by computers, but also it is easily understood by technicians in different fields. Therefore, even if the influence diagram was firstly introduced in 1984 [1], it gradually becomes mature after decades of development: Jensen and Vomlelová [2] expanded the traditional influence diagram into unconstrained influence diagram (UID), which makes the decision-making sequence do not need to be strictly specified. Lauritzen and Nilsson [3] relaxed the non-forgotten restrictions in traditional IDs, and proposed limited memory influence diagram (LMID). Smith *el al.* [4] extended the influence diagram by changing the conditional probability distribution in IDs, so that the IDs can clearly represent the asymmetric decision problem without increasing the time complexity. In addition, some other papers also contribute to the development of IDs [5, 6, 7, 8, 9, 10].

However, there are still many problems in decision-making when using IDs. One is that it is difficult to quantify the dependencies between nodes. Many experts proposed to use non-precision variables to represent the dependencies between nodes [11, 12, 13], and introduced fuzzy set theory into fuzzy influence diagram (FID). However, the FID still shows the shortcoming in one of our example (Section 3.1): all the relations between nodes are evaluated by the same evaluation criteria. But in real life, the relationship among some nodes may be very close, but some may not be close because nodes are always influenced by external environment and sensors used by different nodes are various. Using the same criteria to evaluate node's relationship does not meet the actual situation. Therefore, this paper introduces evidence theory in the description of non-precise variables, so that the relations between nodes can be better expressed in proposed models.

Evidence theory is firstly proposed by Dempster and Shafer [14], which is also known as Dempster-Shafer (D-S) evidence theory. Compare to fuzzy set theory, it can not only handle uncertain information, but also provides Dempster rule to combine uncertain information from different sources. A number of methods based on evidence theory were applied to to many real applications such as decision making under uncertain environment[15, 16, 17, 18], pattern recognition [19, 20, 21], failure analysis [22, 23, 24] and sensor data fusion [25, 26]. It should be pointed out that evidence theory has some open issues, such as conflicting management [27, 28, 29], generating basic probability assignment [30, 31, 32] and dependence evidence combination[33, 34, 35]. The introduction of evidence theory makes up the deficiency of fuzzy influence diagram when describing the state of nodes.

In this paper, a new influence diagram model, which uses fuzzy set theory and evidence theory to process influence diagram, is proposed to evaluate risks in complex system. This paper is organized as follows: in section 2, some preliminaries are briefly introduced, including influence diagram, fuzzy influence diagram and evidence theory. In section 3, the proposed influence diagram is detailed. In section 4, some numerical examples are used to illustrate the efficiency of the proposed method. In section 5, the proposed influence diagram was applied in credit risk assessment of supply chain financial system. The conclusion is given in Section 6 to end the paper.

## 2. Preliminary

### 2.1. Influence diagram

Influence diagrams are one of the graph models for solving complex decision problems based on uncertain information [36]. The influence diagram expresses the dependencies among variables, conditional independence relations and the preference information of the decision maker [1, 37]. Because of the advantages of intuitionistic expression and the ability of filling large amount of information, the influence diagram is widely used in decision analysis and uncertainty reasoning [37, 38, 39]. Influence diagram can be divided into qualitative and quantitative levels.

Qualitative level refers to the graphical structure of influence graph, which can be represented by a directed acyclic graph. The influence diagram contains a total of three types of nodes [40]:

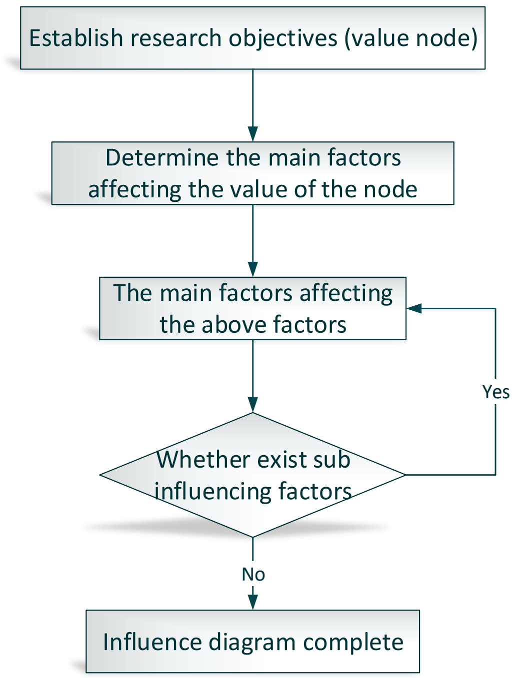



Figure 1: Construction process of influence diagram

Decision nodes describe the actions that a decision system may take. It is generally represented by a rectangle. Chance nodes express uncertainties in the process of decision making, which is represented by a circle. Utility nodes encode the decision makers utility for each state of the nodes parents. It is represented by a diamond.

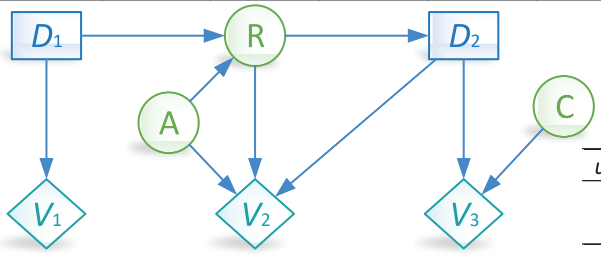
The arc with the arrow is generally connected between nodes. The arrow points to is the child node (successor), while the node connected to the end of the arc is the parent node (predecessor). In the influence diagram, the arcs can be divided into two types: the conditional arc pointed to the natural node (used to represent the probability dependence of the natural node to its predecessor) or the utility node (the functional dependency of the utility node to its predecessor). The other is the information arc pointing to the decision node, which is used to represent the information in the decision node.

At the quantitative level, an influence diagram specifies the list of alternatives for all decision nodes and the state space for natural nodes. Each natural node is attached with

| $P(R\|D_1,A)$ | $(nt,as)$ | $(nt,al)$ | $(nt,am)$ | $(t,as)$ | $(t,al)$ | $(t,am)$ |
|---|---|---|---|---|---|---|
| $b$ | 0 | 0 | 0 | 0 | 0.288 | 0.313 |
| $g$ | 0 | 0 | 0 | 0.182 | 0.565 | 0.437 |
| $e$ | 0 | 0 | 0 | 0.818 | 0.147 | 0.250 |
| $nr$ | 1 | 1 | 1 | 0 | 0 | 0 |

| $A$ | $P(A)$ |
|---|---|
| $as$ | 0.660 |
| $al$ | 0.244 |
| $am$ | 0.096 |

| $D_1$ | $u(D_1)$ |
|---|---|
| $t$ | -\$1B |
| $nt$ | 0 |




| $C$ | $P(C)$ |
|---|---|
| cs | 0.98 |
| cf | 0.02 |

| $u(D_2,C)$ | $cs$ | $cf$ |
|---|---|---|
| $c$ | \$8B | -\$4B |
| $n$ | 0 | 0 |

| $u(R\|D_2,A)$ | $(a,as)$ | $(a,al)$ | $(a,am)$ | $(n,as)$ | $(n,al)$ | $(n,am)$ |
|---|---|---|---|---|---|---|
| $b$ | -M | -M | -M | 0 | 0 | 0 |
| $g$ | \$12B | -\$6B | -\$10B | 0 | 0 | 0 |
| $e$ | \$12B | -\$6B | -\$10B | 0 | 0 | 0 |
| $nr$ | \$12B | -\$6B | -\$10B | 0 | 0 | 0 |

Figure 2: ID representation of reactor problem [36]

a conditional probability table to describe the conditional probability distribution from the state of the node to its predecessor. This conditional probability table can also be considered as the decision maker's decision scheme. Each utility node attaches a utility function, and sets a mapping from the state of the predecessor to a real number, to express the decision maker's preference. Sorting out the above qualitative and quantitative considerations in accordance with the steps in Fig.1, an influence diagram can be constructed.

Fig.2 is a influence diagram of the company's reactor construction revenue [36, 41, 42]. Where $D_1$, $D_2$ are the decision nodes, $A$, $C$ and $R$ are chance nodes, $V_1$, $V_2$ and $V_3$ are utility node. $D_2$ indicates that the company will choose one of three options: "Design advanced reactors ($n$), build tradition reactors ($c$) or not to build ($l$)". If the reactor constructs successfully (no accident), the construction of advanced reactor can get more profits, but advanced reactor will also bring greater risk.

The probability of success ($cs$) is 98%, the probability of failure ($cf$) is 2%. The probability of advanced reactor $A$ to be successful ($as$, 60%), and the probability of occurrence of accident ($al$) is 24.4%, the probability of a major accident ($am$) is 9.6%. If the traditional reactor $C$ is successful, the company can earn \$8B profits. If the reactor fails, the company will lose \$-4B. If the advanced reactor $A$ is successfully operated, the company will earn a profit of \$12B. If the reactor fails, the company will earn \$-6B. If there is a major accident, the company will lose \$10B. Before building the reactor, the company can choose whether or not to test the advanced reactor $D$ (if test $t$, choose not to test $nf$), the test costs \$1B. Test results $R$ can be divided into three cases: Bad ($b$), Good ($g$) and Excellent ($e$). In conditional probability table $P(R|D_1, A)$, $P(R = b|D_1 = t, A = as) = 0$ means when $A = as$, the state of the $R$ can not be $b$. In the utility table $u(R, D_2, A)$, $u(b, a, as) = -M$ (suppose $M$ is a sufficiently large positive number) means it is impossible for the company to build an advanced reactor ($D_2 = a$) when the test result is not good ($R = b$).

Based on this model, the optimal strategy (test advanced reactor) can be calculated by the influence diagram. If the test results are excellent or good, the advanced reactor should be constructed. Otherwise the company should construct the traditional reactor. The expected utility of the optimal strategy is about \$8.13B [36].

### 2.2. *Fuzzy influence diagram*

Although the influence diagram constructed above can deal with the complex decision-making problems, it still remains so many problems, that the constructed model can not make fact-based decision. One of the problems is that, due to the complexity of decision-making environment and different decision-making problems, the constructed influence diagram is difficult to specify precise conditional probability distributions and evaluate utility. However, in quantifying the dependency and utility between nodes, experts found it is more convenient and efficient to rate dependency and utility by imprecise information [12, 43].

According to the different uncertain data obtained by sensors, many papers use different mathematical tools to construct corresponding influence diagrams [11, 13, 44, 45, 46, 47], including the fuzzy influence diagram [48, 49, 50, 51].

The fuzzy influence diagram describes the state, frequency and the nodes relationship by fuzzy theory. Similar to the traditional influence diagram, the fuzzy influence diagram also includes three different types of nodes and two different arcs. But in the quantitative level, fuzzy influence diagram applies the fuzzy theory to reduce the accuracy of data collection requirements. There are three types of uncertain data sets in fuzzy influence diagram: two types of state fuzzy sets and frequency fuzzy sets. Through processing three fuzzy data sets in the use of the evaluation algorithm, the frequency matrix of value node can be obtained.

### *2.2.1. Evaluation algorithm of fuzzy influence diagram*

In this algorithm, the process of risk assessment will not be influenced by decision making, so this influence diagram does not contain decision nodes. For any subsystems in influence diagram, the predecessor can be regarded as a utility node, which is affected by the state frequency of successors and the association with them. The status of the root node is not affected by successors. Therefore, according to whether nodes status are influenced by their successors, nodes can be divided into independent nodes ($X_1$, Fig.3(a)) and dependent nodes ($X_2$, Fig.3(b)).

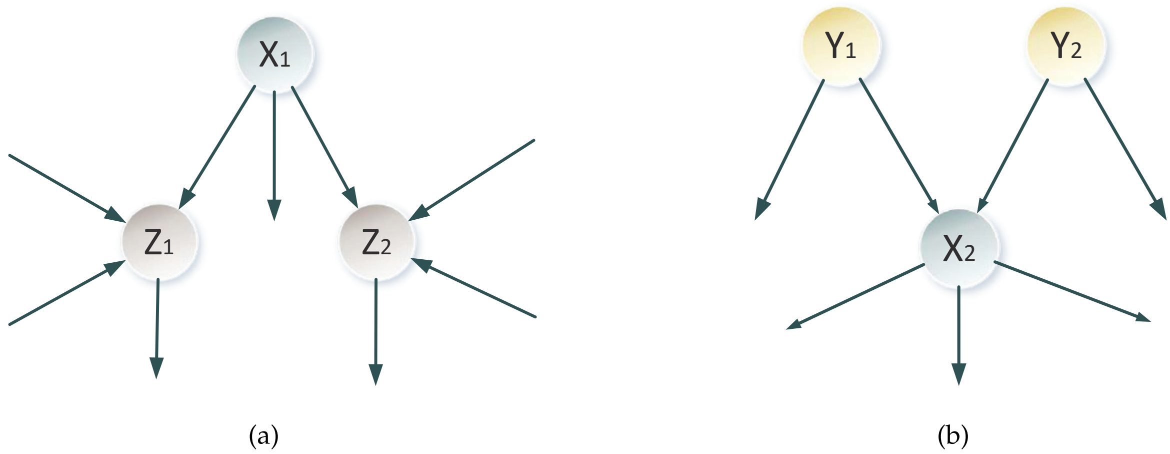



Figure 3: Independent node $X_1$ and dependent node $X_2$

**Definition 1.** The node is an **independent node** if it does not have successors or its state and corresponding frequency is not affected by its successors.

Let $X$ denote the independent node. Suppose the possible state vector of node $X$ is:

$$P_X = \{P_{X_1}, P_{X_2}, ..., P_{X_n}\}^T$$

Where $P_{X_1}, P_{X_1}, ..., P_{X_n}$ construct a fuzzy set to describe the whole states node $X$ may occur. The frequency vector of independent node $X$ is:

$$f_X = \{f_{X_1}, f_{X_2}, ..., f_{X_n}\}^T$$

Where $f_{X_1}, f_{X_2}, ..., f_{X_n}$ are the frequency fuzzy sets corresponding to each possible state in the frequency vector of node $X$. Based on the above data, the frequency matrix of independent nodes $X$ can be calculated:

$$F_X = (P_{X_1} \times f_{X_1}) \bigcup (P_{X_2} \times f_{X_2}) \bigcup ... \bigcup (P_{X_x} \times f_{X_x}) \quad (1)$$

**Definition 2.** The node is a **dependent node** if it is affected by its successors.

In Fig.3(b), suppose $X$ is a node consists of $m$ successors $Y_1$, $Y_2$, ..., $Y_m$, $F_{XP}$ represents the joint frequency matrices of $X$ constructed by all successors:

$$F_{XP} = F_{Y_1} \bigcup F_{Y_2} \bigcup ... \bigcup F_{Y_m} \quad (2)$$

Assuming $R_{XY_1}$ is defined as a fuzzy relation between node $Y_1$ and $X$, which is derived from the following equation:

$$R_{XY_1} = (P_{Y_{11}} \times P_{X_i}) \bigcup (P_{Y_{12}} \times P_{X_i}) \bigcup ... \bigcup (P_{Y_{1n}} \times P_{X_i})$$

Where $P_{Y_{1j}} \in \{P_{Y_{11}}, P_{Y_{12}}, ... , P_{Y_{1n}}\} = P_{Y_1}$, $P_{X_i} \in \{P_{X_1}, P_{X_2}, ..., P_{X_n}\} = P_X$. In general, the fuzzy relation from node $Y_m$ to node $X$ is:

$$R_{XY_m} = (P_{Y_{m1}} \times P_{X_i}) \bigcup (P_{Y_{m2}} \times P_{X_i}) \bigcup ... \bigcup (P_{Y_{mn}} \times P_{X_i}) \quad (3)$$

The union of the fuzzy relations $R_{XY_1}$, ..., $R_{XY_m}$ for node $X$ is

$$R_{XP} = R_{XY_1} \bigcup R_{XY_2} \bigcup ... \bigcup R_{XY_m} \quad (4)$$

By synthesizing the joint frequency matrix $F_{XP}$ and fuzzy relation matrix $R_{XP}$, the frequency matrix of node $X$ can be obtained.

$$F_X = F_{XP} \circ R_{XP} \quad (5)$$

The synthesis of matrix $F_{XP} = (q_{ik})_{m \times l}$ and $R_{XP} = (r_{kj})_{l \times n}$ is derived from the following formula:

$$F_{XP} \circ R_{XP} = (s_{ij})_{m \times n}$$
$$s_{ij} = \vee_{k=1}^{l} (q_{ik} \wedge r_{kj})$$

According to Eq.(5-14), the frequency matrix $F_X$ of the value node $X$ can be obtained. The membership function of each probability value is calculated by Eq.(6), and then the probability function of the random result is completed.

$$\mu_{X_i} = \vee_i (f_{X_{ij}} \times \sum_j f_{X_{ij}}) \quad (6)$$

$$P(X_i) = \frac{\mu_{X_i}}{\sum_i \mu_{X_i}} \quad (7)$$

According to the probability function, the risk can be evaluated by calculating its expectation and variance [48].

### 2.3. D-S Evidence Theory

In the process of using the fuzzy influence diagram, we find the fuzzy influence diagram is still flawed for the information processing. In order to make the result of the influence diagram more realistic, the evidence theory is introduced in the fuzzy influence diagram evaluation algorithm, which makes the experimental results more realistic.

Dempster-Shafer evidence theory, is a mathematical tool to deal with uncertain information. Some basic concepts are introduced below for reader to better understand.

*2.3.1. The frame of discernment*

Let $\Omega$ be the exhaustive set of variables $\theta_i$. If the element in $\Omega$ are mutually exclusive, then set $\Omega$ is called the **frame of discernment** (FD). The frame of discernment is use to describe all circumstances of variable $\theta_i$.

$$\Omega = \{\omega_1, \omega_2, \cdots, \omega_i, \cdots, \omega_N\}$$

Suppose set $\Omega$ contains $N$ elements, thus the number of elements of the power set $2^\Omega$ is $2^N$. Set $2^\Omega$ can be expressed as follow:

$$2^\Omega = \{\emptyset, \{\omega_1\}, \cdots, \{\omega_N\}, \{\omega_1, \omega_2\}, \cdots, \{\omega_1, \omega_2, \cdots, \omega_i\}, \cdots, \Omega\}$$

where $\emptyset$ is an empty set.

*2.3.2. The basic probability assignment*

For any subset $\varepsilon \subseteq 2^\Omega$, assuming it corresponds to a number $m \in [0,1]$, and satisfies:

$$m(\emptyset) = 0 \quad \sum_{\varepsilon \subseteq 2^\Omega} = 1$$

Then set $\varepsilon$ is called **basic probability assignment** (BPA), $m$ is a mass function.

*2.3.3. Belief function and plausibility function*

The **belief function** (*Bel*) of the proposition for $2^\Omega \to [0,1]$ is:

$$Bel(\varepsilon) = \sum_{\delta \subseteq \varepsilon} m(\delta) \qquad \vee\, \varepsilon \subseteq \Omega$$

$Bel(\varepsilon)$ can be regarded as the trust in $\varepsilon$. Besides, belief function also meets the following relationships:

$$Bel(\emptyset) = 0 \qquad Bel(\Omega) = 1$$

The **plausibility function** ($Pl$):$2^{\Omega} \rightarrow [0,1]$ is defined as

$$Pl(\varepsilon) = 1 - Bel(\overline{\varepsilon}) = \sum_{\varepsilon \cap \delta \neq \emptyset} m(\delta) \qquad \vee\, \varepsilon \subseteq \Omega$$

*2.3.4. Dempster rule of combination*

If $m(\varepsilon) > 0, \varepsilon$ is a focal element, and the set of some focal elements is named a **body of evidence** (BOE). Suppose for same evidence $\varepsilon_1$ and $\varepsilon_2$, a different BPA distribution will be derived due to the different sources. To use these evidence effectively, Dempster and Shafer [14] proposed a combination rule to fuse these evidence:

$$m(\varepsilon) = \frac{\sum\limits_{\varepsilon_1 \bigcap \varepsilon_2 = \varepsilon} m_1(\varepsilon_1) m_2(\varepsilon_2)}{1 - K} \tag{8}$$

where $K = \sum\limits_{\varepsilon_1 \bigcap \varepsilon_2 = \emptyset} m_1(\varepsilon_1) m_2(\varepsilon_2)$ is a normalization factor. The combination rule is valid if and only if $m(\emptyset) \neq 1$.

*2.3.5. Yang's evidential network approach*

Based on Dempster rule of combination, some fusion method was prosed to process uncertainty in system. In evidential networks (ENs), Yang *et al.* [52] proposed EN approach to fuse information.

In ENs, all nodes in system can be represented by three states: $\{up\}$ indicates the node is operating normally, $\{down\}$ indicates the node broke down, and $\{up, down\}$ means the node is in an unknown state, that we can not know whether it is working or faulty. Besides, these three states can be expressed by the plausibility function and the belief function of the evidence theory:

$$m_i(up) = Bel_i$$

$$m_i(down) = Pl_i$$

$$m_i(up, down) = Pl_i - Bel_i$$

Where $m_i$ represents the BPA distribution of node $N_i$ in this system. When two nodes ($N_i$ and $N_j$) are connected, there are a total of nine possible connection modes:

$$\begin{array}{ccc} \{up\},\{up\} & \{up\},\{down\} & \{up\},\{up,down\} \\ \{down\},\{up\} & \{down\},\{down\} & \{down\},\{up,down\} \\ \{up,down\},\{up\} & \{up,down\},\{down\} & \{up,down\},\{up,down\} \end{array}$$

Similarly, there are three states for the subsystems, which is constructed by two nodes. The rate of occurrence of these three states can be represented by the plausibility function and the belief function [52]:

$$m_{ij}(system) = \begin{cases} Bel_i \cdot Bel_j & m_{ij} = \{up\} \\ 1 - Pl_i \cdot Pl_j & m_{ij} = \{down\} \\ Pl_i \cdot Pl_j - Bel_i \cdot Bel_j & m_{ij} = \{up, down\} \end{cases} \tag{9}$$

*2.3.6. Belief reliability analysis*

Belief reliability analysis (BRA) methodology [53] is another way to fuse uncertainty. In contrast to the former method, BRA method seeks to eliminate uncertainties in system and represents the results simpler. Through this method, the subsystem $N_{ij}$ can only use {up} and {down} to represent the subsystem state:

$$m_{ij}(system) = \begin{cases} \dfrac{Bel_i \cdot Bel_j}{1-K} & m_{ij} = \{up\} \\ \\ \dfrac{1 - Pl_i \cdot Pl_j}{1-K} & m_{ij} = \{down\} \end{cases} \quad K = Pl_i \cdot Pl_j - Bel_i \cdot Bel_j \tag{10}$$

## 3. The proposed influence diagram model

*3.1. Background of the proposed model*

The fuzzy set theory is widely used in the existing influence diagram to process uncertain information, which reduces the requirement of data precision and enhance the practicability and operability of algorithms. However, due to the strict data classification rules, there still remains limitation to estimate result using fuzzy influence diagram. The following example is a good illustration of the problem.

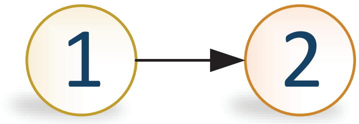



Figure 4: A simple influence diagram

**Example 3.1.** *As is shown in Fig.4, node $N_1$ is independent node and $N_2$ is the predecessor of $N_1$. Fuzzy influence diagram has a mature grading mechanism [48, 54], and the grading rules is shown below:*

Table 1: Frequency fuzzy sets [48]

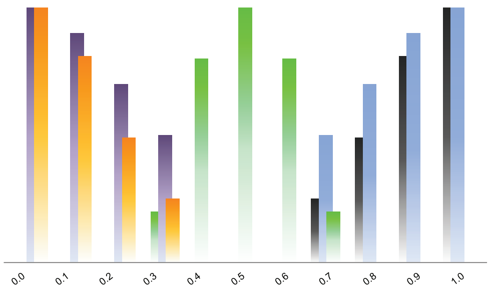
| Fuzzy set | Symbol | Membership degree |
|---|---|---|
| Very high | $VH$ | $\{0.7\|0.25, 0.8\|0.49, 0.9\|0.81, 1.0\|1.0\}$ |
| High | $H$ | $\{0.7\|0.5, 0.8\|0.7, 0.9\|0.9, 1.0\|1.0\}$ |
| Middle | $M$ | $\{0.3\|0.3, 0.4\|0.7, 0.5\|1.0, 0.6\|0.8, 0.7\|0.2\}$ |
| Low | $L$ | $\{0\|1.0, 0.1\|0.9, 0.2\|0.8, 0.3\|0.6\}$ |
| Very low | $VL$ | $\{0\|1.0, 0.1\|0.81, 0.2\|0.64, 0.3\|0.36\}$ |

*The relationship between node 1 and node 2 is represented by Tab.10*

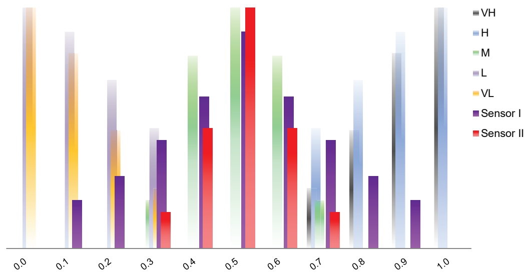
Table 2: Node relationship table

| Node | Name | Node relationship |
|---|---|---|
| 1 | $N_1$ | $G \to 2M$; $M \to 2H$; $B \to 2L$ |

Tab.10 and Tab.3 use three sets of relationships and their corresponding frequency distribution to clearly describe the relationship between $N_1$ and $N_2$. It can be seen from




(a)




(b)

Figure 5: Fuzzy frequency sets of five evaluation criterions and two different sensors

Tab.3 that the traditional fuzzy influence diagram evaluation algorithm divides the frequency into five levels: $VH$, $H$, $M$, $L$ and $VL$. In order to more intuitively express this classification, we plot these five fuzzy frequencies in Fig.5(a). However, in the industrial field, due to the varying surrounding environment of sensor and the difference among sensors themselves, the sensitivity of detector can not be simply classified as the above categories.

**Definition 3.** **Sensitivity** refers to the amount of change of output due to changes in unit input.

The traditional influence diagram dictates that all sensors, which used to collect original data, have the same sensitivity. It is obviously not consistent with the real life. Suppose $N_1$ use two different sensors (Sensor $I$ and Sensor $II$) to detect information, and their frequency distribution is shown in Fig.5(b) (sensor $I$ with purple and sensor $II$ with red). The expectation of the two sensors is the same, but the variance is different. The original frequency distribution obtained by these two sensors can only be represented by fuzzy set $M$, so their frequency distribution of the utility node is the same, in the case of other conditions unchanged. However, according to the proposed model, their utility node frequency distribution should be different (see Section 4.2).

### 3.2. *The proposed fuzzy evidential influence diagram*

In order to make the fuzzy influence diagram more realistic, we introduce the evidence theory to process original data. To get a better fusion result, we modify the frequency fuzzy set in Table 3 as follow:

Table 3: Frequency fuzzy sets [48]

| Fuzzy set | Symbol | Membership degree |
|---|---|---|
| Very high | $VH$ | {0.9\|0.81, 1.0\|1.0} |
| High | $H$ | {0.7\|0.7, 0.8\|0.5, 0.9\|0.2} |
| Middle | $M$ | {0.3\|0.2, 0.4\|0.8, 0.5\|1.0, 0.6\|0.8, 0.7\|0.2} |
| Low | $L$ | {0.1\|0.2, 0.2\|0.5, 0.3\|0.7} |
| Very low | $VL$ | {0\|1.0, 0.1\|0.81} |

Assume the actual fuzzy set frequency vector $M$ of the sensor is (shown in Fig.6 with red):

$\{0|0.48, 0.1|0.5, 0.2|0.55, 0.3|0.67, 0.4|0.8, 0.5|1, 0.6|0.8, 0.7|0.67, 0.8|0.55, 0.9|0.5, 1.0|0.48\}$

By linear fitting, we find the optimal solution of frequency:

$f(VH)$=0.4551; $f(H)$=0.8110; $f(M)$=0.9834; $f(L)$=0.8110; $f(VL)$=0.4551.

Where $f$ means the membership degree of the sensor in the specific frequency fuzzy set. The comparison between the solution with the actual sensor frequency is shown in Fig. The left side of the table is the optimal solution derived from combining five frequency fuzzy sets, and the right column is the ideal sensor actual frequency.

Therefore, we denote the relationship from the state $M$ corresponding to frequency fuzzy set: $M \to \{VH, H, M, L, VL\}$

After obtaining the data, suppose the possible state vector of node $X$ is:

$$P_X = \left\{ \begin{array}{c} \{P_{X_1}^{Pl}, P_{X_2}^{Pl}, ..., P_{X_n}^{Pl}\}^T \\ \{P_{X_1}^{Bel}, P_{X_2}^{Bel}, ..., P_{X_n}^{Bel}\}^T \end{array} \right\}$$

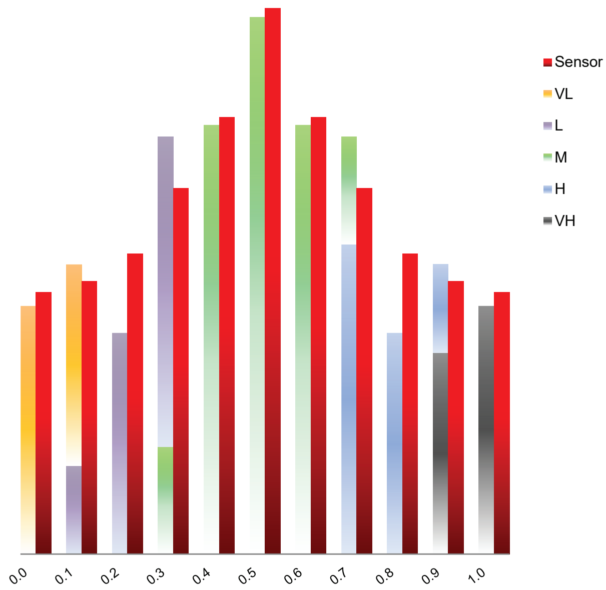



Figure 6: Fusion results comparison

Where $P_{X_n}^{Pl}$ is plausibility value in fuzzy set, $P_{X_n}^{Bel}$ is belief value of $P_{X_n}$. The frequency vector of independent node $X$ is:

$$f_X = \left\{ \begin{array}{c} \{f_{X_1}^{Pl}, f_{X_2}^{Pl}, ..., f_{X_n}^{Pl}\}^T \\ \{f_{X_1}^{Bel}, f_{X_2}^{Bel}, ..., f_{X_n}^{Bel}\}^T \end{array} \right\}$$

The frequency matrix of independent node:

$$F_X = (P_{X_1} \otimes f_{X_1}) \bigcup (P_{X_2} \otimes f_{X_2}) \bigcup ... \bigcup (P_{X_n} \otimes f_{X_n}) \tag{11}$$

The results of the frequency matrix can be obtained by two different fusion methods (described in Section 2.3.5 and 2.3.6):

**Method 1:** The frequency matrix is represented by a plausibility matrix and a belief matrix.

$$P_{X_x} \otimes f_{X_x} = \left\{ \begin{array}{ll} F_X^{Pl} = (\theta_{ij}^{Pl})_{m \times m}, & (\theta_{ij}^{Pl})_{m \times m} = P_{X_n}^{Pl} \times f_{X_n}^{Pl} \\ F_X^{Bel} = (\theta_{ij}^{Bel})_{m \times m}, & (\theta_{ij}^{Bel})_{m \times m} = 1 - P_{X_n}^{Bel} \times f_{X_n}^{Bel} \end{array} \right. \tag{12}$$

Where $n = 1, 2, ..., m$.

**Method 2:** The frequency matrix is represented by a single matrix.

$$P_{X_x} \otimes f_{X_x} = F_X^{Pl} = (\theta_{ij}^{Pl})_{m\times m}, \quad (\theta_{ij}^{Pl})_{m\times m} = \frac{P_{X_n}^{Pl} \times f_{X_n}^{Pl}}{1-(P_{X_n}^{Pl} \times f_{X_n}^{Pl} - P_{X_n}^{Bel} \times f_{X_n}^{Bel})} \tag{13}$$

After obtaining the frequency matrix of the independent nodes, it is brought into the predecessor for subsequent operation.

$$F_{XP} = F_{Y_1} \bigcup F_{Y_2} \bigcup ... \bigcup F_{Y_m} \tag{14}$$

The process of solving the relation matrix is similar to the Eq.(12) and Eq.(13):

$$R_{XY_m} = (P_{Y_{m1}} \otimes P_{X_i}) \bigcup (P_{Y_{m2}} \otimes P_{X_i}) \bigcup ... \bigcup (P_{Y_{mn}} \otimes P_{X_i}) \tag{15}$$

The remaining steps to get frequency matrix of $F_X$ are similar to the traditional FID evaluation algorithm:

$$\begin{aligned} R_{XP} &= R_{XY_1} \cup R_{XY_2} \cup ... \cup R_{XY_m} \\ F_X &= F_{XP} \circ R_{XP} \end{aligned} \tag{16}$$

Besides, the way to get the membership degree of the corresponding frequency $\mu_{X_i}$ is modified:

$$\begin{aligned} \mu_{X_i} &= \sum_j f_{X_{ij}} \times \phi_{X_j} \\ P(X_i) &= \frac{\mu_{X_i}}{\sum_i \mu_{X_i}} \end{aligned} \tag{17}$$

where $\phi_{X_j}$ is the occurrence probability for frequency values $X_1, X_2, ..., X_i, ..., X_n$. For easier to comprehend, the solving process of proposed fuzzy influence diagram is represented by the flow chart (Fig.7).

## 4. Example analysis

### *4.1. Example 1*

The frequency fuzzy set and state fuzzy set are shown in Tab.4-6. According to the above tables, we establish a fuzzy influence diagram (Fig.8) and its corresponding relationship (Tab.7 and Tab.8):

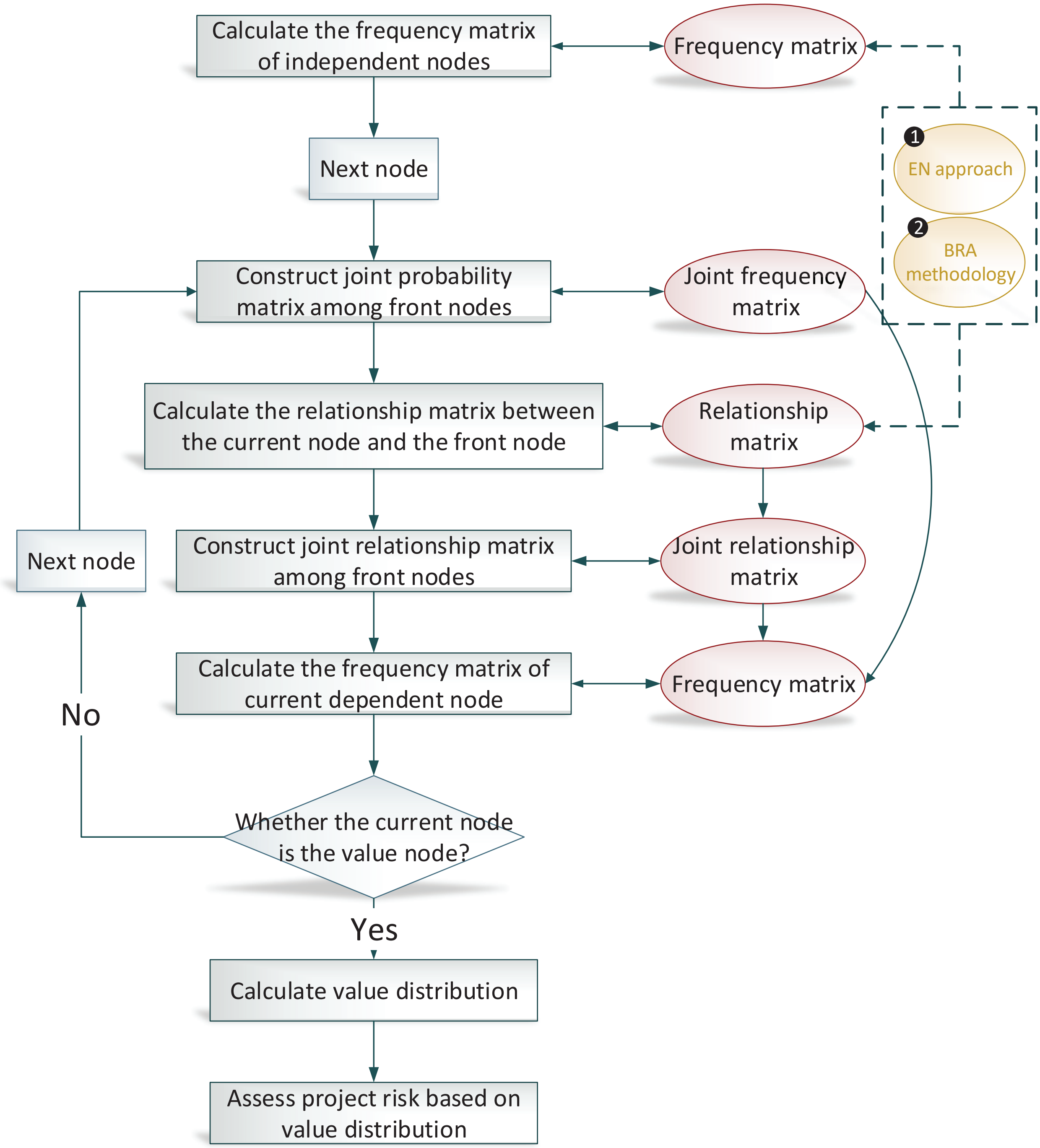



Figure 7: Analysis process of proposed fuzzy influence diagram

Table 4: Frequency fuzzy sets

| Fuzzy set | Symbol | Membership degree |
|---|---|---|
| Very high | $VH$ | {0.7\|0.25, 0.8\|0.49, 0.9\|0.81, 1.0\|1.0} |
| High | $H$ | {0.7\|0.5, 0.8\|0.7, 0.9\|0.9, 1.0\|1.0} |
| Middle | $M$ | {0.3\|0.3, 0.4\|0.7, 0.5\|1.0, 0.6\|0.8, 0.7\|0.2} |
| Low | $L$ | {0\|1.0, 0.1\|0.9, 0.2\|0.8, 0.3\|0.6} |
| Very low | $VL$ | {0\|1.0, 0.1\|0.81, 0.2\|0.64, 0.3\|0.36} |

Table 5: state fuzzy set for independent nodes

| Fuzzy set | Symbol | Membership degree |
|---|---|---|
| Rapid growth | $RG$ | {60%\|0.7, 70%\|0.8, 80%\|1.0, 90%\|0.8} |
| Faster growth | $FG$ | {30%\|0.5, 40%\|0.7, 50%\|1.0, 60%\|0.2} |
| Not much growth | $NG$ | {10%\|0.8, 20%\|1.0, 30%\|0.5, 40%\|0.2} |
| Less growth | $LG$ | {0%\|1.0, 10%\|0.5, 20%\|0.2, 30%\|0.1} |
| Stagnant growth | $SG$ | {0%\|1.0, 10%\|0.2} |

Table 6: State fuzzy set for dependent nodes

| Fuzzy set | Symbol | Membership degree |
|---|---|---|
| Good | $G$ | {Good\|1.0, Middle\|0, Bad\|0} |
| Middle | $M$ | {Good\|0, Middle\|1.0, Bad\|0} |
| Bad | $B$ | {Good\|0, Middle\|0, Bad\|1.0} |

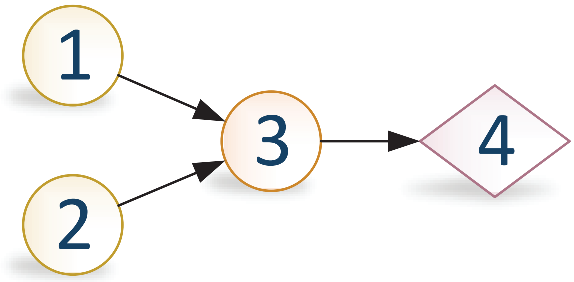



Figure 8: Fuzzy influence diagram

Table 7: Independent node status and frequency evaluation table

| Node | The occurrence frequency of state | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $G$ | $M$ | $B$ | $RG$ | $FG$ | $NG$ | $LG$ | $SG$ |
| 1 | $M$ | $H$ | $L, VL$ | | | | | |
| 2 | | | | $VL$ | $L$ | | $H$ | $VH$ |

Table 8: Node relationship table

| Node | Node relationship |
|---|---|
| 1 | $G \rightarrow 3SG$; $M \rightarrow 3LG$; $B \rightarrow 3FG$ |
| 2 | $RG \rightarrow 3FG$; $FG \rightarrow 3LG$ ; $LG \rightarrow 3SG$ ; $SG \rightarrow 3SG$ |
| 3 | $LG \rightarrow 4LG$; $FG \rightarrow 4RG$; $LG \rightarrow 4FG$; $SG \rightarrow 4SG$ |

This example is solved by Method 2 (BRA methodology). According to Eq.11:

$$F_1 = (M \otimes G) \bigcup (H \otimes M) \bigcup (\{L, VL\} \otimes B)$$
$$F_2 = (VL \otimes RG) \bigcup (L \otimes FG) \bigcup (H \otimes LG) \bigcup (M \otimes SG)$$

$$F_1 = \begin{array}{cc} & \begin{array}{ccc} G & M & B \end{array} \\ \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1.0 \end{array} & \begin{bmatrix} 0 & 0 & 1.00 \\ 0 & 0 & 0.83 \\ 0 & 0 & 0.58 \\ 0.20 & 0 & 0.40 \\ 0.80 & 0 & 0 \\ 1.00 & 0 & 0 \\ 0.80 & 0 & 0 \\ 0.20 & 0.50 & 0 \\ 0 & 0.70 & 0 \\ 0 & 0.90 & 0 \\ 0 & 1.00 & 0 \end{bmatrix} \end{array} \quad F_2 = \begin{array}{cc} & \begin{array}{cccccc} 0\% & 10\% & 20\% & 30\% & 40\% & 50\% \end{array} \\ \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 0 & 0.40 & 0.80 & 1.00 & 0.90 & 1.00 \\ 0 & 0.36 & 0.72 & 0.90 & 0.73 & 0.81 \\ 0 & 0.28 & 0.56 & 0.70 & 0.44 & 0.49 \\ 0.20 & 0.20 & 0.40 & 0.50 & 0.30 & 0.25 \\ 0.80 & 0 & 0 & 0 & 0 & 0 \\ 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0.80 & 0 & 0 & 0 & 0 & 0 \\ 0.50 & 0.40 & 0.20 & 0 & 0 & 0 \\ 0.70 & 0.56 & 0.28 & 0 & 0 & 0 \\ 0.90 & 0.72 & 0.36 & 0 & 0 & 0 \\ 1.00 & 0.80 & 0.40 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The frequency matrix and relationship of node $N_3$ are:

$$\begin{aligned} F_{3P} &= F_1 \bigcup F_2 \\ R_{3P} &= R_{31} \bigcup R_{32} \\ &= (G \otimes SG) \bigcup (M \otimes LG) \bigcup (B \otimes FG) \bigcup (RG \otimes FG) \bigcup (FG \otimes LG) \bigcup (LG \otimes SG) \bigcup (SG \otimes SG) \end{aligned}$$

$$F_{3P} = \begin{array}{c@{}c} & \begin{array}{ccccccccc} \mathrm{G} & \mathrm{M} & \mathrm{B} & 0\% & 10\% & 20\% & 30\% & 40\% & 50\% \end{array} \\ \begin{array}{c} \phantom{0.0} \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1.0 \end{array} & \begin{bmatrix} 0 & 0 & 1.00 & 0.80 & 0.40 & 0.80 & 0.80 & 0.90 & 1.00 \\ 0 & 0 & 0.838 & 0.72 & 0.40 & 0.72 & 0.72 & 0.83 & 0.83 \\ 0 & 0 & 0.58 & 0.56 & 0.40 & 0.56 & 0.58 & 0.58 & 0.58 \\ 0.20 & 0 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 \\ 0.80 & 0 & 0 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0.00 \\ 1.00 & 0 & 0 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0.00 \\ 0.80 & 0 & 0 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0.00 \\ 0.20 & 0.50 & 0 & 0.50 & 0.40 & 0.50 & 0.50 & 0.50 & 0.40 \\ 0 & 0.70 & 0 & 0.70 & 0.40 & 0.70 & 0.70 & 0.60 & 0.40 \\ 0 & 0.90 & 0 & 0.90 & 0.40 & 0.80 & 0.90 & 0.60 & 0.40 \\ 0 & 1 & 0 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0.40 \end{bmatrix} \end{array}$$

$$R_{3P} = \begin{bmatrix} 0 & 0 & 0.10 & 0.20 & 0.30 & 0.40 & 0.50 \\ 0 & 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0.10 & 0.80 & 0.32 & 0.16 & 0 & 0 & 0 \\ 0.20 & 0.80 & 0.64 & 0.32 & 0.20 & 0.12 & 0 \\ 0.30 & 1.00 & 0.80 & 0.48 & 0.60 & 0.36 & 0 \\ 0.40 & 0.60 & 0.48 & 0.72 & 0.90 & 0.54 & 0 \\ 0.50 & 0.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0 \end{bmatrix}$$

Then the frequency matrix of node $N_3$, $N_4$ can be derived from Eq.5:

$$F_3 = F_{3P} \circ R_{3P}$$

$$F_4 = F_{4P} \circ R_{4P}$$

$$F_3 = \begin{bmatrix} & 0 & 10\% & 20\% & 30\% & 40\% & 50\% \\ 0 & 0.80 & 0.64 & 0.80 & 1.00 & 0.60 & 0 \\ 0.1 & 0.72 & 0.64 & 0.80 & 0.83 & 0.60 & 0 \\ 0.2 & 0.56 & 0.56 & 0.58 & 0.58 & 0.58 & 0 \\ 0.3 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 & 0 \\ 0.4 & 0.80 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 1.00 & 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0.80 & 0 & 0 & 0 & 0 & 0 \\ 0.7 & 0.50 & 0.50 & 0.40 & 0.20 & 0.12 & 0 \\ 0.8 & 0.70 & 0.70 & 0.40 & 0.20 & 0.12 & 0 \\ 0.9 & 0.90 & 0.80 & 0.40 & 0.20 & 0.12 & 0 \\ 1 & 1.00 & 0.80 & 0.40 & 0.20 & 0.12 & 0 \end{bmatrix} F_4 = \begin{bmatrix} & 0\% & 10\% & 20\% & 30\% & 40\% & 50\% \\ 0 & 0.80 & 0.40 & 0.80 & 0.80 & 0.90 & 1.00 \\ 0.1 & 0.72 & 0.40 & 0.72 & 0.72 & 0.83 & 0.83 \\ 0.2 & 0.56 & 0.40 & 0.56 & 0.58 & 0.58 & 0.58 \\ 0.3 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 \\ 0.4 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.5 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0 \\ 0.6 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.7 & 0.50 & 0.40 & 0.50 & 0.50 & 0.50 & 0.40 \\ 0.8 & 0.70 & 0.40 & 0.70 & 0.70 & 0.60 & 0.40 \\ 0.9 & 0.90 & 0.40 & 0.80 & 0.90 & 0.60 & 0.40 \\ 1 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0.40 \end{bmatrix}$$

According to Method 1, the frequency matrix $F_4$ can also be represented by the plausibility matrix and belief matrix:

$$F_4^{Pl} = \begin{bmatrix} & 0\% & 10\% & 20\% & 30\% & 40\% & 50\% \\ 0 & 0.80 & 0.40 & 0.80 & 0.80 & 0.90 & 1.00 \\ 0.1 & 0.72 & 0.40 & 0.72 & 0.72 & 0.90 & 0.90 \\ 0.2 & 0.56 & 0.40 & 0.56 & 0.60 & 0.70 & 0.70 \\ 0.3 & 0.40 & 0.40 & 0.40 & 0.50 & 0.50 & 0.50 \\ 0.4 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.5 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0 \\ 0.6 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.7 & 0.50 & 0.40 & 0.50 & 0.50 & 0.50 & 0.40 \\ 0.8 & 0.70 & 0.40 & 0.70 & 0.70 & 0.60 & 0.40 \\ 0.9 & 0.90 & 0.40 & 0.80 & 0.90 & 0.60 & 0.40 \\ 1 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0.40 \end{bmatrix} F_4^{Bel} = \begin{bmatrix} & 0\% & 10\% & 20\% & 30\% & 40\% & 50\% \\ 0 & 0.80 & 0.40 & 0.80 & 0.80 & 0.90 & 1.00 \\ 0.1 & 0.72 & 0.40 & 0.72 & 0.72 & 0.81 & 0.81 \\ 0.2 & 0.56 & 0.40 & 0.56 & 0.56 & 0.56 & 0.49 \\ 0.3 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 & 0.40 \\ 0.4 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.5 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0 \\ 0.6 & 0.80 & 0.40 & 0.80 & 0.80 & 0.60 & 0 \\ 0.7 & 0.50 & 0.40 & 0.50 & 0.50 & 0.50 & 0.32 \\ 0.8 & 0.70 & 0.40 & 0.70 & 0.70 & 0.60 & 0.32 \\ 0.9 & 0.90 & 0.40 & 0.80 & 0.90 & 0.60 & 0.36 \\ 1 & 1.00 & 0.40 & 0.80 & 1.00 & 0.60 & 0.40 \end{bmatrix}$$

Finally, we obtain the utility probability of $N_4$ using Eq.(6), (7). The utility value obtained by the Method 1 should be an interval, while Method 2 gets a value.

| | Method 1 | | Method 2 | |
|---|---|---|---|---|
| | Plausibility | Belief | Probability | Cumulative |
| 0% | 0.2202 | 0.2202 | 0.2202 | 0.2202 |
| 10% | 0.1120 | 0.1120 | 0.1120 | 0.3322 |
| 20% | 0.2003 | 0.2003 | 0.2003 | 0.5325 |
| 30% | 0.2221 | 0.2202 | 0.2202 | 0.7527 |
| 40% | 0.1655 | 0.1621 | 0.1622 | 0.9149 |
| 50% | 0.0886 | 0.0765 | 0.0851 | 1.0000 |

### 4.2. *Example 2*

It is assumed that the sensitivity of sensor used by node $N_1$ decreases due to environmental factors, which changes the state matrix of node $N_1$:

$$\{0.1|0.1, 0.2|0.25, 0.3|0.55, 0.4|0.8, 0.5|1, 0.6|0.8, 0.7|0.55, 0.8|0.25, 0.9|0.1\}$$

In this condition, the traditional fuzzy influence diagram can only express the sensor with a single fuzzy set, but the proposed model can be more accurately described by multiple fuzzy sets (shown in the table below), and the frequency matrices of the utility node obtained by Method 1 and 2 are different (shown in Fig.9):

| Node | The occurrence frequency of state | | |
|---|---|---|---|
| | $G$ | $M$ | $B$ |
| Traditional model | $M$ | $M$ | $M$ |
| Proposed model | $H, M, L$ | $H, M, L$ | $H, M, L$ |

Traditional fuzzy influence diagram

| Frequency\Result | 0% | 10% | 20% | 30% | 40% | 50% |
|---|---|---|---|---|---|---|
| 0 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.40 |
| 0.1 | 0.72 | 0.40 | 0.72 | 0.72 | 0.60 | 0.40 |
| 0.2 | 0.56 | 0.40 | 0.56 | 0.56 | 0.56 | 0.40 |
| 0.3 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| 0.4 | 0.80 | 0.40 | 0.80 | 0.80 | 0.80 | 0.80 |
| 0.5 | 1.00 | 0.40 | 0.80 | 1.00 | 0.90 | 1.00 |
| 0.6 | 0.80 | 0.40 | 0.80 | 0.80 | 0.80 | 0.80 |
| 0.7 | 0.50 | 0.40 | 0.50 | 0.50 | 0.50 | 0.32 |
| 0.8 | 0.70 | 0.40 | 0.70 | 0.70 | 0.60 | 0.32 |
| 0.9 | 0.90 | 0.40 | 0.80 | 0.90 | 0.60 | 0.36 |
| 1.0 | 1.00 | 0.40 | 0.80 | 1.00 | 0.60 | 0.40 |

- [0.8,1.0]
- [0.6,0.8]
- [0.4,0.6]
- [0.2,0.4]
- [0.01,0.2]
- 0

Plausibility frequency matrix

| Frequency\Result | 0% | 10% | 20% | 30% | 40% | 50% |
|---|---|---|---|---|---|---|
| 0 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.40 |
| 0.1 | 0.72 | 0.40 | 0.72 | 0.72 | 0.60 | 0.40 |
| 0.2 | 0.56 | 0.40 | 0.56 | 0.56 | 0.56 | 0.40 |
| 0.3 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| 0.4 | 0.80 | 0.40 | 0.80 | 0.80 | 0.80 | 0.80 |
| 0.5 | 1.00 | 0.40 | 0.80 | 1.00 | 0.90 | 1.00 |
| 0.6 | 0.80 | 0.40 | 0.80 | 0.80 | 0.80 | 0.80 |
| 0.7 | 0.50 | 0.40 | 0.50 | 0.50 | 0.50 | 0.32 |
| 0.8 | 0.70 | 0.40 | 0.70 | 0.70 | 0.60 | 0.32 |
| 0.9 | 0.90 | 0.40 | 0.80 | 0.90 | 0.60 | 0.36 |
| 1.0 | 1.00 | 0.40 | 0.80 | 1.00 | 0.60 | 0.40 |

Belief frequency matrix

| Frequency\Result | 0% | 10% | 20% | 30% | 40% | 50% |
|---|---|---|---|---|---|---|
| 0 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.40 |
| 0.1 | 0.72 | 0.40 | 0.72 | 0.72 | 0.60 | 0.40 |
| 0.2 | 0.56 | 0.40 | 0.56 | 0.56 | 0.56 | 0.40 |
| 0.3 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| 0.4 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.00 |
| 0.5 | 1.00 | 0.40 | 0.80 | 1.00 | 0.60 | 0.00 |
| 0.6 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.00 |
| 0.7 | 0.50 | 0.40 | 0.50 | 0.50 | 0.50 | 0.50 |
| 0.8 | 0.70 | 0.40 | 0.70 | 0.70 | 0.70 | 0.70 |
| 0.9 | 0.90 | 0.40 | 0.80 | 0.90 | 0.90 | 0.90 |
| 1.0 | 1.00 | 0.40 | 0.80 | 1.00 | 0.90 | 1.00 |

Method 2

| Frequency\Result | 0% | 10% | 20% | 30% | 40% | 50% |
|---|---|---|---|---|---|---|
| 0 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.40 |
| 0.1 | 0.72 | 0.40 | 0.72 | 0.72 | 0.60 | 0.40 |
| 0.2 | 0.56 | 0.40 | 0.56 | 0.56 | 0.56 | 0.40 |
| 0.3 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 | 0.40 |
| 0.4 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.44 |
| 0.5 | 1.00 | 0.40 | 0.80 | 1.00 | 0.60 | 0.50 |
| 0.6 | 0.80 | 0.40 | 0.80 | 0.80 | 0.60 | 0.44 |
| 0.7 | 0.50 | 0.40 | 0.50 | 0.50 | 0.50 | 0.32 |
| 0.8 | 0.70 | 0.40 | 0.70 | 0.70 | 0.60 | 0.32 |
| 0.9 | 0.90 | 0.40 | 0.80 | 0.90 | 0.60 | 0.36 |
| 1.0 | 1.00 | 0.40 | 0.80 | 1.00 | | 1.00 |

- [-0.01,-0.2]
- [-0.2,-0.4]
- [-0.4,-0.6]
- [-0.6,-0.8]
- [-0.8,-1.0]

Figure 9: Frequency matrix of the utility node

## 5. Application in credit risk assessment of supply chain financial system

Supply chain finance is the operation process of banks for logistics industry. Surrendering the core enterprises, it controls the capital flow and logistics, translates the risks from an uncontrolled risk of a single enterprise to a manageable risk of the whole supply chain enterprises, to minimize the financial risk.

Supply chain finance is not a business or a product but a financing service to connect suppliers, manufacturers, distributors and end-users. It changes the credit model from the monopoly of banks and other financial institutions to a supply chain around a core enterprise. Through the division of functions and cooperation of related enterprises, it will help to increase the profits of the entire supply chain. As an effective way to solve the financing problem of small and medium enterprises (SMEs), supply chain finance has been paid much attention on banks, SMEs, logistics enterprises, etc.

However, because of the large number of financial factors in the supply chain and the complex operation process, the risk of supply chain finance is often affected by many factors, which is difficult to evaluate risks according to a single business activity. Besides, the risk in supply chain finance is often hidden in the development process of the event, and affected by various uncertain factors (such as external environment and inner factors), and finally lead to credit risk. On the other hand, due to the inherent unpredictability, discontinuity and jumpiness in financial risk evaluation, it is very difficult for the financial industry to assess, manage and control the risks. How to take into account the uncertainties and suddenness of risk accidents and construct a suitable evaluation system for financial risks in supply chain finance has become an urgent problem.

Many scholars study in the supply chain financial risk evaluation. For instance, Lavastre *et al.*[55] studied the data of many companies in France, researched the relevant management staffs of the company, and concluded that risk management is a subject of continuous concern in supply chain management. Randall and Farris [56] examined how firms can use financial management techniques to improve the ability to resist supply chain risk and analyzed how cash flow variables and shared-weighted average cost of capital can reduce the financing costs of supply chain finance. Bernabucci and Robert [57] studied a physical entity in a supply chain and constructed a cash flow risk-related model to prevent cash flow risks. Barsky *et al.* [58, 59] put forward a model for risk analysis based on processing control, which divides business risk into process risk, environmental risk, information technology risk, human resource risk and basic structure risk. Xiong *et al.* [60] used principal component analysis and logistic regression, in order to establish the credit risk evaluation model.

To better evaluate the uncertain and uncontrollable credit risk in supply chain financial, this paper uses the fuzzy set theory, evidence theory, and influence diagram to construct a mathematical model and apply in one of the SMEs supply chain finance project in China. The following section details the process of constructing the model and solve the problem.

### *5.1. Establish supply chain financial credit risk topology*

**Step 1** Determine the value node:

Firstly, judge the credit risk by overall evaluation of the financial supply chain. The increase in the risk return is expressed by a frequency matrix in the value node.

**Step 2** Determine the chance node:

Supply chain finance credit risk mainly arises from five main aspects: the SMEs themselves, the qualification of core enterprise, Supply chain operation status, the state of financing projects and system environment. SMEs and the qualifications of core enterprise are mainly determined by their financial indicators, while the financing projects and the state of financing projects are affected by the system environment [61].

**Step 3** The factors are subdivided into several basic risk factors [62, 63, 64, 65].

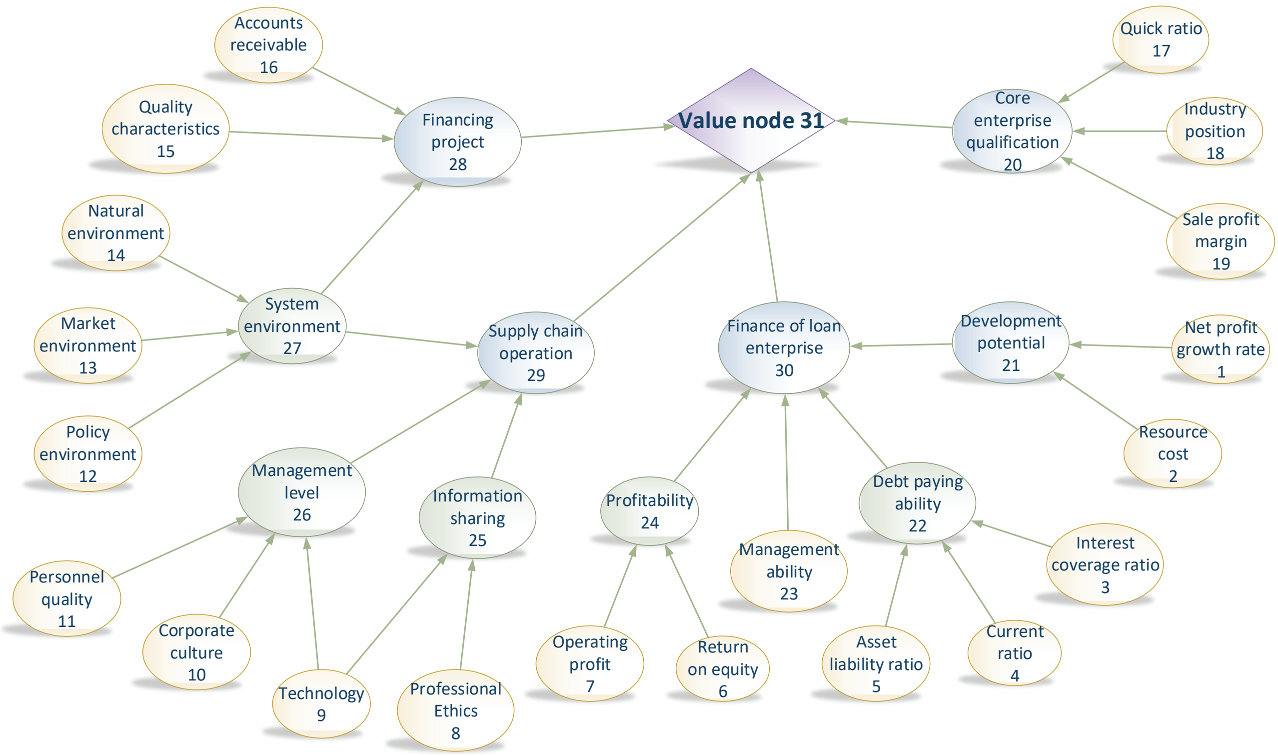



Figure 10: Influence diagram on supply chain financial risks [61]

**Step 4** According to the internal relations of credit risk and the reasoning process above, draw the fuzzy influence diagram of credit risk. The topological structure of fuzzy influence diagram is shown in Fig.10.

### 5.2. *Define fuzzy sets*

The frequency fuzzy set, state fuzzy sets for describing the uncertainty state and independent nodes are shown in Tab.4-6. We summarize the evaluation (Tab.9) and relationship between nodes (Tab.10) by consulting experts, combining existing fuzzy influence diagrams [61, 66, 67, 68, 69] and relevant literature [70, 71, 72].

Table 9: Independent node status and frequency evaluation table

| Node | Name | The occurrence frequency of corresponding state | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $G$ | $M$ | $B$ | $RP$ | $FG$ | $NG$ | $LG$ | $SG$ |
| 1 | Net profit growth rate | | | | $H$ | $M$ | $L$ | | |
| 2 | Resource cost | | | | | | $M$ | | $VH$ |
| 3 | Interest coverage ratio | $H$ | $L$ | $VL$ | | | | | |
| 4 | Current ratio | | | | | $H$ | | $L$ | |
| 5 | Asset liability ratio | | | | | $L$ | | | $H$ |
| 6 | Return on equity | | | | $VH$ | | $M$ | | |
| 7 | Operating profit | | | | $VH$ | | | $L$ | |
| 8 | Professional Ethics | $H$ | | $L$ | | | | | |
| 9 | Technology | | $H$ | $VL$ | | | | | |
| 10 | Corporate culture | $VH$ | | | | | | | |
| 11 | Personnel quality | $H$ | | $L$ | | | | | |
| 12 | Policy environment | $VH$ | | $VL$ | | | | | |
| 13 | Market environment | | $H$ | $VL$ | | | | | |
| 14 | Natural environment | $H$ | $L$ | $VL$ | | | | | |
| 15 | Quality characteristics | $H$ | | $L$ | | | | | |
| 16 | Accounts receivable | | $H$ | $L$ | | | | | |
| 17 | Quick ratio | | | | | $H$ | | | $L$ |
| 18 | Industry position | $H$ | $L$ | | | | | | |
| 19 | Sale profit margin | | | | | $H$ | $M$ | $L$ | |
| 23 | Management ability | $\{H, M\}$ | | $L$ | | | | | |

Table 10: Node relationship table

| Node | Name | Node relationship |
|---|---|---|
| 1 | Net profit growth rate | $RG \to 21FG$; $FG \to 21NG$; $NG \to 21SG$ |
| 2 | Resource cost | $NG \to 21SG$; $SG \to 21FG$ |
| 3 | Interest coverage ratio | $G \to 22FG$; $M \to 22NG$; $B \to 22SG$; |
| 4 | Current ratio | $FG \to 22FG$; $LG \to 22SG$ |
| 5 | Asset liability ratio | $FG \to 22SG$; $SG \to 22NG$ |
| 6 | Return on equity | $RG \to 24RG$; $LG \to 24LG$ |
| 7 | Operating profit | $RG \to 24RG$; $LG \to 24SG$ |
| 8 | Professional Ethics | $G \to 25G$; $B \to 25M$ |
| 9 | Technology | $M \to 25M, 26M$; $B \to 25B, 26B$ |
| 10 | Corporate culture | $G \to 26G$ |
| 11 | Personnel quality | $G \to 26G$; $B \to 26B$ |
| 12 | Policy environment | $G \to 27G$; $B \to 27B$ |
| 13 | Market environment | $M \to 27M$; $B \to 26B$ |
| 14 | Natural environment | $G \to 27G$; $M \to 27G$; $B \to 27M$ |
| 15 | Quality characteristics | $G \to 28G$; $B \to 28B$ |
| 16 | Accounts receivable | $M \to 28M$; $B \to 28B$ |
| 17 | Quick ratio | $FG \to 20\{G, M\}$; $SG \to 20B$ |
| 18 | Industry position | $G \to 20G$; $M \to 20M$ |
| 19 | Sale profit margin | $FG \to 20G$; $NG \to 20G$; $LG \to 20M$ |
| 20 | Core enterprise qualification | $G \to 31\{RG, FG\}$; $M \to 31NG$; $B \to 31SG$ |
| 21 | Development potential | $FG \to 30FG$; $NG \to 30NG$; $SG \to 30SG$ |
| 22 | Debt paying ability | $FG \to 30FG$; $NG \to 30NG$; $SG \to 30SG$ |
| 23 | Management ability | $G \to 30FG$; $B \to 30SG$ |
| 24 | Profitability | $RG \to 30RG$; $FG \to 30NG$; $LG \to 30LG$ |
| 25 | Information sharing | $G \to 29FG$; $M \to 29NG$; $B \to 29SG$ |
| 26 | Management level | $G \to 29FG$; $M \to 29NG$; $B \to 29SG$ |
| 27 | System environment | $G \to 29FG$; $M \to 28NG, 29NG$; $B \to 28NM, 29SG$ |
| 28 | Financing project | $G \to 31FG$; $M \to 31NG$; $B \to 31SG$ |
| 29 | Supply chain operation | $FG \to 31FG$; $NG \to 31LG$; $LG \to 31LG$; $31LG \to 31SG$ |
| 30 | Finance of loan enterprise | $RG \to 31RG$; $FG \to 31FG$; $NG \to 31LG$; $LG \to 31LG$;,$31LG \to 31SG$ |

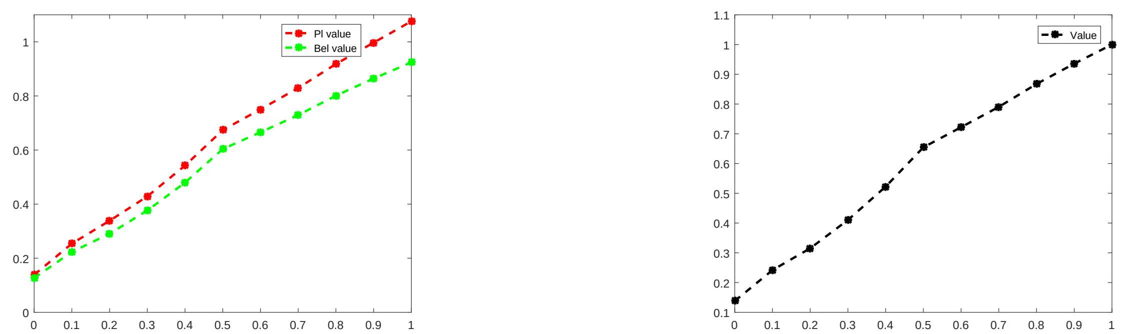
Through Eq.(11)-(17), the probability distribution of value node is derived. Since the calculation process is similar to the example in Section 4.1, the results are given directly below.

### 5.3. *Result evaluation*

From Tab.11 and Fig.11, it can be concluded that the probability of the increment benefit of this company from 0% to 20% is about 34%. The probability of increasing by 40% to 60% is about 32%, by 70% to 100% is about 33%. We find that the probability of the company's profit greatly increase (60%-100%) is less than the probability of low return (0%-40%). Besides, the expected value (result from Method 2):

$$
\begin{aligned}
E(X) &= 0.1386 \times 0\% + 0.1153 \times 10\% + 0.0845 \times 20\% + 0.0900 \times 30\% + 0.1141 \times 40\% \\
&+0.1311 \times 50\% + 0.0767 \times 60\% + 0.0789 \times 70\% + 0.0888 \times 80\% \\
&+0.0456 \times 90\% + 0.0789 \times 100\% \\
&\approx 0.489 < \frac{0\%+10\%+20\%+30\%+40\%+50\%+60\%+70\%+80\%+90\%+100\%}{11} = 0.5
\end{aligned}
$$

so it can be concluded that this company's credit risk is slightly higher than average.




(a) (b)

Figure 11: Result obtained by Method 1 and 2

Table 11: Probability distribution of value node

| | Method 1 | | Method 2 | |
|---|---|---|---|---|
| | Plausibility | Belief | Probability | Cumulation |
| 0% | 0.1386 | 0.1266 | 0.1380 | 0.1380 |
| 10% | 0.1153 | 0.0965 | 0.1038 | 0.2418 |
| 20% | 0.0845 | 0.0674 | 0.0733 | 0.3151 |
| 30% | 0.0900 | 0.0857 | 0.0952 | 0.4103 |
| 40% | 0.1141 | 0.1032 | 0.1121 | 0.5224 |
| 50% | 0.1311 | 0.1242 | 0.1320 | 0.6544 |
| 60% | 0.0767 | 0.0620 | 0.0677 | 0.7221 |
| 70% | 0.0789 | 0.0635 | 0.0683 | 0.7904 |
| 80% | 0.0888 | 0.0717 | 0.0771 | 0.8675 |
| 90% | 0.0789 | 0.0635 | 0.0683 | 0.9358 |
| 100% | 0.0789 | 0.0597 | 0.0642 | 1.0000 |

## 6. Conclusion

In this paper, a new fuzzy evidential influence diagram is proposed to fix the shortcoming in traditional fuzzy influence diagram. Traditional fuzzy influence diagram must use the same evaluation criteria to evaluate nodes, which makes some nodes with the same score but different in their sensitivity can not be distinguished. In Section 4.1, we describe the detailed computational process of the proposed model by an example. Another example in Section 4.2 elaborately explains why the information in traditional fuzzy influence diagram is not accurate, and the correct representation is given by the model we put forward. Besides, we also show the difference between the results of traditional influence diagram and the proposed influence diagram. The real application in credit risk assessment of supply chain financial system is illustrated to show the practicability and efficiency of the proposed influence diagram.

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