A Theory of the Gravitational Co-Field and its Application to the Spacecraft Flyby Anomaly

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A co-field to Newton's gravitational field is derived and its properties defined. It is applied to explain "Spacecraft-Earth Flyby Anomalies" discovered during deep space missions launched between 1990 and 2006. The flyby anomaly has been considered a major unresolved problem in astrophysics.

Keywords: Gravitational Co-Field, Spacecraft-Earth Flyby Anomalies, Space Physics, Classical Physics

I. INTRODUCTION

While the general theory of relativity is the accepted theory of gravitation, it is not inconsistent that in the limiting case of low velocities, a co-field to Newton's gravitational equation could be described in a classical form. Such is certainly the case for special relativity and is assumed in this analysis. Historically, there have been numerous efforts to develop a set of gravity related equations analogous to those of Maxwell for electromagnetism. The first of these efforts was by Heaviside in 1893 [1]. A number of more recent papers, categorized as "Gravitoelectromagnetism", extended the analogy to Maxwell's equations to include inputs from general relativity [2]. Basically the quest is for a co-field to Newton's gravitational equation analogous to the magnetic co-field to Coulomb's equation. A difficulty in analysis by analogy is that there has been no recognized body of experimental data to support the results and identify important differences. The effects of the gravitational co-field are either too small to have been observed or have not been recognized. Maxwell and Heaviside interpreted the experimental data involving electricity and magnetism in terms of divergences and curls. A theorem by Helmholtz states that a field can be broken down into an irrotational component and a solenoidal/rotational component. The electromagnetic field exists and the Helmholtz theorem is therefore applicable. It is assumed in this paper that a gravitational, solenoidal co-field also exists. The approach here is to add a purely rotational term to Newton's equation and to apply curls to the resulting equation and its result. This approach is the inverse to that employed by Maxwell. The divergence is non-zero only for the Newtonian term and does not contribute to the co-field development. The resulting analysis is parallel to rather than an analogy to electromagnetism. A similar analysis was done for electromagnetism, where the results are known, to show that the application of the curl and divergence to the modified field equation yields the correct experimental results such as Faraday's induction equation and Ampere's law.

Dimensional analysis or considerations are part of the analysis. For example, the magnetic field which has dimensions M/QT is defined in terms of its dimensional physical quantities multiplied by a constant. The permittivity and permeability in electromagnetism are regarded as constants even though they have physically identifiable dimensions. They are constants only because the electron is the primary interacting object. In gravitation, they are a function of the source mass and its radius.

During the course of the analysis, when the equation for the gravitational co-field was derived, there appeared no criteria to establish its validity until the author discovered the Anderson et al [3] paper on the anomalous velocity changes observed during spacecraft flybys of the earth. It appeared that the proposed theory explained the flyby anomalies. Despite numerous attempts to solve the problem [4], the anomaly had remained the puzzle described by Turyshev and Toth [5].

II. GRAVITATIONAL AND ELECTRICAL FORCE FIELDS

The fundamental basis of this study is that the general total force acting on a moving mass is given by Newton's inverse square force plus a rotational force $m\boldsymbol{a}_\theta = \beta m r\ddot{\theta}\hat{\boldsymbol{\theta}}$. While the term $mr\ddot{\theta}$ is dimensionally correct, a constant multiplier, β, may be required for agreement with experimental results. The proposed total force is thus,

$$\boldsymbol{F}_g = m\mathbf{a}_g = G\frac{mM}{r^2}\hat{\mathbf{r}} + \beta m r\ddot{\theta}\hat{\boldsymbol{\theta}} \tag{1}$$

The generalized field $\boldsymbol{F}/m$ used to develop the field equations of gravitation, with the traditional gravitational constant, G, replaced by $G = 1/(4\pi\varepsilon_{og})$, is

$$\boldsymbol{\Gamma} = \frac{\mathbf{F}_g}{m} = \frac{M}{4\pi\varepsilon_{og} r^2}\hat{\mathbf{r}} + \beta r\ddot{\theta}\hat{\boldsymbol{\theta}} \tag{2}$$

This re-definition of G in equation (2) is done to put the gravitational equation into the same format as for electrical force. The field equation for Electricity and Magnetism, similar to equation (2), is

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\hat{\mathbf{r}} + \beta' \frac{m}{q} r\ddot{\theta}\hat{\boldsymbol{\theta}} \tag{3}$$

The divergence and curl of equation (3) and of its resulting co-field B results in Maxwell's equations. The objective of this paper is to define a co-field to Newton's gravitational equation. In equation (2), the curl of the first term is zero, the second term is non-zero. Conversely, the divergence of the first term is non-zero and the second term is zero. Thus only the curl of equation (2) is applicable to the objective of this paper.

III. PERMITIVITIES AND PERMEABILITIES

In the case of electromagnetism, these quantities are constants, but only because a single primary particle is involved, the electron. For gravitation, one deals with a multiplicity of primary masses and corresponding radii. The objective of this section is to determine the gravitational permeability, μ_{og}. We know that the gravitational permittivity, ε_{og} is a constant which must also be confirmed by the analysis. Dimensional analysis, seldom used in physics but key in fluid mechanics, is used here to define the gravitational permeability and confirm the value of its permittivity. Bolster et al state that "dimensional analysis comes in many forms" [6]. One approach begins by identifying the basic variables in a problem such as force, velocity, density, pressure, and their primary dimensions such as length, mass, time, and charge and converting them to their constituent dimensions of L, M, T, Q. Then from experimental data, and semi-empirical analysis, specific functional values are assigned or fitted to the primary dimensions until agreement with experimental results is achieved.

For example, starting with Coulombs law for the interaction of two electrons and defining the force between then dimensionally

$$F = \frac{q^2}{4\pi\varepsilon_o R^2} = k' M \frac{L}{T^2} \tag{4}$$

where k' is a constant. Let $k'L = r_0$, the Thompson electron radius. Re-arranging and setting $R/T = v = c$ and solving for ε_o

$$\varepsilon_o = \left[\frac{q^2}{4\pi m r_o}\right]\left[\frac{T^2}{R^2}\right] = \frac{1}{\mu_o c^2} \tag{5}$$

$$\mu_o = \left[\frac{4\pi m r_o}{q^2}\right] \tag{6}$$

Inserting the values for the electron in the parenthesis above yields $\mu_o = 4\pi x 10^{-7}$.

Similarly for the gravitational case,

$$F = \frac{mM}{4\pi\varepsilon_{og} R^2} = km\frac{L}{T^2}(\text{dimensionally}) \tag{7}$$

Rearranging and assigning $L = R_o$, where R_o is the effective radius of M, and $R/T = v$

$$\varepsilon_{og} = \left[\frac{M}{k4\pi R_o}\right]\left[\frac{T^2}{R^2}\right] = \frac{1}{\mu_{og} v^2} \tag{8}$$

$$\mu_{og} = \left[\frac{k4\pi R_o}{M}\right] \tag{9}$$

In this formulation μ_{og} is dependent on the major mass in the system and its effective radius. The constant k is retained for subsequent evaluation.

In equation (7), for Newton's equation, $L = R = R_o$, and ε_{og} is proportional to $M\left[\frac{T^2}{{R_o}^3}\right]$, a constant recognized as Kepler's third law. This results in a constant ε_{og} as required.

IV. CALCULATION OF CURL $\mathbf{\Gamma}$

A. Differential Form

The curl of $\mathbf{\Gamma}$ for equation (2) applies only to the second term since the curl of the first term is zero. Because the curl is normal to radial or circular motions, the cylindrical coordinate system is used. $\nabla \times \mathbf{\Gamma}$ in cylindrical coordinates is

$$\nabla \times \mathbf{\Gamma} = \left(\frac{1}{r}\frac{\partial a_z}{\partial \theta} - \frac{\partial a_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial a_r}{\partial z} - \frac{\partial a_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial r a_\theta}{\partial r} - \frac{\partial a_r}{\partial \theta}\right)\hat{\mathbf{z}} \tag{10}$$

If $\nabla \times \mathbf{\Gamma}$ is constrained to be normal to the r-θ plane, then only the z component of equation (10) needs to be considered. In equation (10), assuming a_r is not a function of θ and that $a_\theta = \beta r \ddot{\theta}$,

$$\nabla \times \mathbf{\Gamma} = \left(\frac{1}{r}\frac{\partial r a_\theta}{\partial r}\right)\hat{\mathbf{z}} = \frac{\beta}{r}\frac{\partial \left(r^2 \ddot{\theta}\right)}{\partial r}\hat{\mathbf{z}} \tag{11}$$

Carrying through the differentiation of equation (11) yields:

$$\nabla \times \mathbf{\Gamma} = \frac{\beta}{r}\left[2r\ddot{\theta} + r^2\frac{\partial \ddot{\theta}}{\partial r}\right]\hat{\mathbf{z}} \tag{12}$$

If $\ddot{\theta}$ is not a function of r, the second term of equation (12) vanishes. Then

$$\nabla \times \mathbf{\Gamma} = 2\beta\ddot{\theta}\hat{\mathbf{z}} = \frac{\partial}{\partial t}\left(2\beta\dot{\theta}\right)\hat{\mathbf{z}} = \frac{\partial \mathbf{\Omega}}{\partial t} \tag{13}$$

Thus, $\nabla \times \mathbf{\Gamma}$ produces an induced time dependent angular velocity field $2\beta\dot{\theta}\hat{\mathbf{z}} = 2\beta\omega\hat{\mathbf{z}}$.

$$\Omega = 2\beta\omega \tag{14}$$

The corresponding result for E&M obtained starting with the second term of equation (3) is

$$\nabla \times \mathbf{E} = 2\beta'\frac{m}{q}\ddot{\theta}\hat{\mathbf{z}} = \beta'\frac{\partial}{\partial t}\left(2\frac{m}{q}\dot{\theta}\right)\hat{\mathbf{z}} = -\frac{\partial \mathbf{B}}{\partial t} \tag{15}$$

Experimental data requires that $\beta' = -\frac{1}{2}$.

$$B = \frac{m}{q}\dot{\theta} = \frac{m}{q}\omega \tag{16}$$

B. Integral Form

Given the integral form of the curl of $\mathbf{\Gamma}$: $\oint \mathbf{a}_\theta \cdot d\mathbf{l} = \frac{d}{dt}(\Omega A)$, and incorporating equation (14) yields:

$$\oint \mathbf{a}_\theta \cdot d\mathbf{l} = \frac{d}{dt}\left(2\beta\omega A\right) \tag{17}$$

Assuming an acceleration in a circular orbit, of radius r and area A, normal to the $\mathbf{\Omega}$ field of $2\boldsymbol{\omega}$:

$$a_\theta\left(2\pi r\right) = \frac{d}{dt}\left(2\beta\omega\pi r^2\right) \tag{18}$$

$$a_\theta = \beta\left(2\omega\dot{r} + \dot{\omega}r\right) \tag{19}$$

Since $\dot{\omega}$ and ω are both normal to r and $\dot{r}$, equation (19) stated in vector form, is

$$\mathbf{a}_\theta = \beta \left[2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r}\right] \tag{20}$$

The first term in equation (20) is interpreted as the acceleration of an object moving with velocity $\mathbf{v} = \dot{\mathbf{r}}$ in an angular velocity field $\boldsymbol{\Omega} = 2\beta\boldsymbol{\omega}$. The second term is the acceleration due to a time dependent angular velocity. Both terms in Equation (20) also appear in classical solid mechanics for the acceleration of an object moving in a rotating frame. The first term in the equation is recognized as the Coriolis acceleration. Both terms of equation (20) are designated as "fictitious"accelerations in classical mechanics. They are, in this formulation, terms of mechanical/inertial induction. Since equation (20) yields results found in classical mechanics, but multiplied by a constant β. It is concluded that $\beta = 1$. Thus,

$$\mathbf{a}_\theta = 2\boldsymbol{\omega} \times \dot{\boldsymbol{r}} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} \tag{21}$$

Noting that the first term of equation (21) is the equivalent of the $\mathbf{v} \times \mathbf{B}$ acceleration in Electricity & Magnetism:

$$2\omega \times \dot{\boldsymbol{r}} = \boldsymbol{\Omega} \times \boldsymbol{v} \tag{22}$$

$$\Omega = 2\omega \tag{23}$$

V. CALCULATION OF $\nabla \times \boldsymbol{\Omega}$

In cylindrical coordinates $\boldsymbol{\Omega} = \Omega_r\hat{\mathbf{r}} + \Omega_\theta\hat{\boldsymbol{\theta}} + \Omega_z\hat{\mathbf{z}}$.

$$\nabla \times \boldsymbol{\Omega} = \left(\frac{1}{r}\frac{\partial \Omega_z}{\partial \theta} - \frac{\partial \Omega_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial \Omega_r}{\partial z} - \frac{\partial \Omega_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial r\Omega_\theta}{\partial r} - \frac{\partial \Omega_r}{\partial \theta}\right)\hat{\mathbf{z}} \tag{24}$$

Constraining the curl to the z direction yields:

$$\nabla \times \boldsymbol{\Omega} = \left[\left(\frac{\Omega_\theta}{r}\right) + \left(\frac{\partial \Omega_\theta}{\partial t}\right)\frac{\partial t}{\partial r}\right]\hat{\mathbf{z}} \tag{25}$$

The corresponding equation in E&M for the curl of $\boldsymbol{B}$ is

$$\nabla \times \boldsymbol{B} = \left[\left(\frac{B_\theta}{r}\right) + \left(\frac{\partial B_\theta}{\partial t}\right)\frac{\partial t}{\partial r}\right]\hat{\mathbf{z}} \tag{26}$$

Dimensionally, the terms in parenthesis in equation (26) are those of $\mu_o J$; $\frac{B_\theta}{r}$ is designated as $\mu_o J$.

Starting with $E = \frac{m}{q}r\ddot{\theta}$ and $B = \frac{m}{q}\dot{\theta}$, $\frac{\partial E}{\partial t} = \frac{m}{q}\dot{r}\ddot{\theta} + \frac{m}{q}r\dddot{\theta}$. Neglecting the second term, letting $\dot{r} = c$ and dividing both sides by c^2 yields $\frac{1}{c^2}\frac{\partial E}{\partial t} = \frac{1}{c}\frac{\partial B}{\partial t}$. Thus The second term of equation (26) emerges directly as the displacement current and

$$\nabla \times \mathbf{B} = \left(\mu_o J + \frac{1}{c^2}\frac{\partial E}{\partial t}\right)\hat{\mathbf{z}} \tag{27}$$

The gravitational equivalent is

$$\nabla \times \boldsymbol{\Omega} = \left(\mu_{og} J_m + \frac{1}{v^2}\frac{\partial \Gamma}{\partial t}\right)\hat{\mathbf{z}} \tag{28}$$

VI. INERTIAL DIPOLES

The first term of equation (28) is the basis for an inertial version of the Biot-Savart equation for magnetism :

$$d\Omega = \frac{\mu_{og} i dl \sin\theta}{4\pi R^2} \tag{29}$$

Assuming a mass in a circular orbit, whose current is M/T, and integrating to obtain the value of Ω along the axis yields

$$\Omega = \frac{\mu_{og} i R^2}{2(R^2 + Z^2)^{3/2}} \tag{30}$$

where R is the distance from a point on the circular orbit to a point on the axis and Z is the distance from the center of the orbit to the point on the axis. If $Z = 0$, the value of Ω at the orbit center must be 2ω. Equation (30) with $i = M/T = M\omega/2\pi$ reduces to

$$\Omega = \frac{\mu_{og} M}{4\pi R_o}\omega \tag{31}$$

where R_o is the orbit radius. Since $\mu_{og} = \left[\frac{k 4\pi R_o}{M}\right]$, equation (31) requires that $k = 2$ to obtain $\Omega = 2\omega$ at the the center of the orbit. Thus

$$\mu_{og} = \left[\frac{8\pi R_o}{M}\right] \tag{32}$$

Equation (30), for $Z >> R$ yields the value of Ω along the axis of the dipole created by the mass in a circular orbit

$$\Omega = \frac{\mu_{og}\mu_g}{2\pi Z^3} \tag{33}$$

The radial value of Ω from the axis for the dipole in the equatorial plane is 1/2 the axial value:

$$\Omega = \frac{\mu_{og}\mu_g}{4\pi R^3} \tag{34}$$

The inertial moment μ_g is defined as $\mu_g = iA$, where for a circular orbit, i is the mass current M/T. The period of the orbit is T and A is the area of the orbit. More generally, it can be shown that

$$\mu_g = \frac{I\omega}{2} \tag{35}$$

where I is the moment of inertia about the axis and $\omega = 2\pi/T$ = angular velocity. The moment of inertia is $I = \gamma M {R_{Obs}}^2$, where γ is a constant based on geometry and density distribution. For a uniform sphere it is 2/5, for the earth, the NASA fact sheet lists it as 0.3309 [7]. M is the object mass, R_{0bs} is its observed radius. Setting $\gamma {R_{0bs}}^2 = {R_o}^2$ so that $I = M{R_o}^2$ allows one to treat any rotating object as a point mass in an orbit of effective radius R_o. As an example, consider a rotating sphere and its Ω along its axis from equation (33). For a rotating mass having a value of $\Omega_o = 2\omega$, the Ω field, at a distance Z from the origin along the axis is

$$\Omega = \left[8\pi\frac{R_o}{M}\right]\frac{M{R_o}^2\omega}{4\pi Z^3} = \left[\frac{R_o}{Z}\right]^3 2\omega \tag{36}$$

$$\Omega = \left[\frac{R_o}{Z}\right]^3 \Omega_o \tag{37}$$

Similarly the value of a dipole, Ω radially from the origin in the equatorial plane is

$$\Omega = \frac{1}{2}\left[\frac{R_o}{R}\right]^3 \Omega_o \tag{38}$$

VII. EARTH SPACECRAFT FLYBY ANOMALIES

A small, anomalous change in orbital velocity occurred in a number of spacecraft flybys of the earth. The "flyby"experimental data are the first which provide a direct confirmation of the gravitation co-field. The data from all the flybys were analyzed by Anderson et al [3]. Figure (1) is a schematic of the NEAR flyby which produced the largest measured deflection.

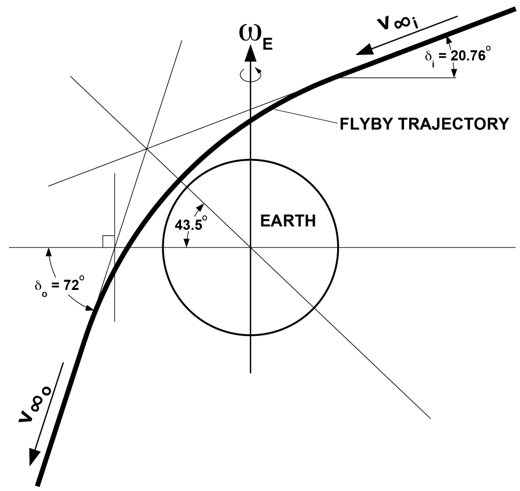

Fig. 1 Schematic of the NEAR spacecraft earth flyby

Anderson, et al. found that the velocity change for the spacecraft flybys of the earth can be described as:

$$\Delta v = 2\omega_E v_\infty \frac{R_E}{c}\left(\cos\delta_i - \cos\delta_o\right) \tag{39}$$

where Δv is the anomalous velocity change, ω_E is the angular velocity of the earth, R_E is the mean radius of the earth and c is the speed of light. The asymptotic velocity vectors and their declinations are designated as V_∞ and δ, respectively. The sub scripts i and o refer to the incoming and outgoing asymptotic velocities and angles. For both the incoming and outgoing asymptotic velocities, $v_\infty \cos\delta$ is normal to the earth's angular velocity axis. Thus equation (39) can be written as

$$\boldsymbol{\Delta v} = (2\boldsymbol{\omega}_E \times \boldsymbol{v}_{\infty i} - 2\boldsymbol{\omega}_E \times \boldsymbol{v}_{\infty o})\frac{R_E}{c} \tag{40}$$

The terms in parenthesis can be interpreted as the difference between the accelerations of the spacecraft between the incoming and outgoing asymptotes. Given that the terms in the parenthesis represent accelerations, the $\frac{R_E}{c}$ term represents an interaction time $\Delta t = 2.124x10^{-2}$ sec. This implies a short effective interaction region to produce the observed velocity change. This change presumably occurs in the vicinity of closest approach. From the Anderson et al description of the data, it is possible to present an interpretation of the data from the perspective of the theory presented in this paper. The starting point is:

$$\boldsymbol{\Delta v} = (\boldsymbol{\Omega} \times \boldsymbol{v}_{\infty i} - \boldsymbol{\Omega} \times \boldsymbol{v}_{\infty o})\frac{R_H\Theta}{V_H} \tag{41}$$

The term $R_H\Theta/V_H$ is the interaction time during which the observed velocity change occurred. R_H is the sum of the earth radius and distance of closest approach, Θ is the subtended arc in radians, and V_H is the velocity at closest approach. Considering the rotating earth as an angular velocity dipole, where along the axis,

$$\Omega = \left[\frac{R_o}{R_H}\right]^3 2\omega_E \tag{42}$$

Incorporating equation (42) into equation (41) yields

$$\Delta \boldsymbol{v} = \boldsymbol{a}_\theta \Delta t = \left[\frac{R_o}{R_H}\right]^3 (2\boldsymbol{\omega_E} \times \boldsymbol{v}_{\infty i} - 2\boldsymbol{\omega_E} \times \boldsymbol{v}_{\infty o}) \left[\frac{R_H \Theta}{V_H}\right] \tag{43}$$

Re-writing

$$\Delta v = 2\omega_E v_\infty (\cos\delta_i - \cos\delta_o) \left\{ \left[\frac{R_o}{R_H}\right]^3 \frac{R_H \Theta}{V_H} \right\} \tag{44}$$

All of the terms of equation (44) are defined except Θ. Parameters for the NEAR flyby include $\Delta v = 1.346x10^{-2} m/s$, $R_H = 6.910x10^6 m$, $V_\infty = 6.851x10^3 m/s$, $V_H = 12.39x10^3 m/s$, $\delta_i = -20.76 deg$, $\delta_o = -71.96 deg$. The angular velocity of the earth, $\omega_E = 7.292x10^{-5}$rad/s, $R_o = 2.108x10^6 m$, as defined by the narrative following equation (35).

The terms in equation (44) are identical to those of equation (39) except for the final parenthesis. Anderson et al found that for all the flybys the approximate effective time of deflection was $R_E/c = 2.12x10^{-2} sec$. The terms in the parenthesis of equation (44) define the R_E/c of equation (39). Thus:

$$\frac{R_E}{c} = \left[\frac{R_o}{R_H}\right]^3 \frac{R_H \Theta}{V_H} = 2.12x10^{-2} sec \tag{45}$$

Equation (45) provides the value Θ of for all the flybys; For the NEAR Flyby, equation (45) yields $\Theta = 7.27x10^{-4}$ radians and $R_H\Theta = 5020$ meters.

The theory developed in this paper thus provides an explanation of the flyby anomaly, which is consistent with the semi empirical equation developed by Anderson et al. Detailed trajectory data including velocity and earth dipole co-field, $\boldsymbol{\Omega}_E$, data along the trajectory are required for an exact calculation of $\Delta\boldsymbol{v}$. The integral $\int \boldsymbol{\Omega}_E \times \boldsymbol{v} dt$ for the interval $v_{\infty i}$ to $v_{\infty o}$ along the trajectory should yield the measured value of $\Delta\boldsymbol{v}$.

VIII. POSSIBLE EXPERIMENTS AND APPLICATIONS OF THE THEORY

The above analysis which explains the flyby anomaly is applicable to any problem in celestial mechanics where an object is moving in the co-field of another. However, the effects are small and in many cases fields involve multiple objects and uncertainties in effective radii and even in the effective angular velocities such as for gaseous rotating objects. One experiment whose data might exhibit effects in the analysis described here is the Gravity Probe B experiment [8]. While this experiment was designed to test several general relativity predictions, the earth orbital environment and satellite velocities involved are all non relativistic.

From the results of the flyby anomaly analysis, it would appear that careful laboratory experiments could additionally confirm the theory. Angular velocities of at least 100,000 rpm are currently state of the art technology. At least two kinds of experiments appear feasible. Both would involve a rotating object such as a sphere or cylinder that would rotate at 100,000 rpm or higher to produce an inertial dipole field. High rotational velocities are required because the $[R_o/R]^3$ attenuation of the Ω field limits the extent of the field and therefore the resulting deflection of objects moving in the dipole field. In the first experiment case, a projectile would be launched across the poles and in the equatorial plane at several distances from the origin. Deflections would be recorded as a function of axial and radial coordinates and projectile velocities. A number of projectile shots would be required for each location and velocity due to random shot dispersion to get an average. In the second type of experiment, the projectile would be replaced by a mono-energetic neutral atom beam. Both sets of experiments would need to be enclosed within high vacuum systems.

IX. SUMMARY AND CONCLUSIONS

The proposed theory defines a classical co-field to Newton's gravitational equation. This is not a contradiction to the general relativity theory of gravity, but should be regarded as a lower limit classical description. It successfully explains the anomalous velocity changes observed in spacecraft flybys of the earth as due to interaction of the spacecraft velocity with the earth's gravitational co-field. It further provides a basis for analyses of astronomical objects moving in gravitational co-fields and for possible laboratory experiments. Given a gravitational field of the form $\boldsymbol{\Gamma} = \dfrac{M}{4\pi\epsilon_{og} r^2}$

$\hat{\mathbf{r}} + r\ddot{\theta}\hat{\boldsymbol{\theta}}$ and applying a curl operator yields a gravitational induction term $d\Omega/dt$ which defines the gravitational co-field $\Omega = 2\omega$. The curl of Ω yields a gravitational form of Amperes Law and associated inertial dipole fields for orbital and for rotating objects. The acceleration of a mass moving with velocity V in an Ω field is given by $\mathbf{a} = \boldsymbol{\Omega}\mathbf{x}\mathbf{V}$. An electromagnetic field analysis, starting with $\mathbf{E} = \dfrac{Q}{4\pi\epsilon_o r^2}\ \hat{\mathbf{r}} + \beta' \frac{m}{q} r\ddot{\theta}\hat{\boldsymbol{\theta}}$ parallel to that done for the gravitational case, successfully reproduced Maxwell's equations. The electrical co-field is $B = \dfrac{m}{q}\ \omega$; the Inertial is $\Omega = 2\omega$. Both are angular velocity fields having a constant multiplier. Thus the relationships involving B and Ω are similar with differences occurring as a result of the different constant multipliers and the fact that μ_{og} is not a constant. It is not a constant because in the gravitational case every problem has different driving masses and radii. In electromagnetism, all interactions are driven by the electron. The fundamental equations governing the gravitational co-field are summarized as:

$$\boldsymbol{\Gamma} = \frac{M}{4\pi\varepsilon_{og} r^2}\hat{\mathbf{r}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} \qquad \text{Gravitational Field equation including Co-Field}$$

$$\nabla \times \boldsymbol{\Gamma} = \frac{\partial \boldsymbol{\Omega}}{\partial t} \qquad \text{Inertial Induction}$$

$$\Omega = 2\omega \qquad \text{Value of Gravitational Co-Field}$$

$$\boldsymbol{a}_\theta = 2\boldsymbol{\omega} \times \boldsymbol{v} + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} \qquad \text{Acceleration of a mass in an } \Omega \text{ field}$$

$$\nabla \times \boldsymbol{\Omega} = \left(\mu_{og} J_m + \frac{1}{v^2}\frac{\partial \boldsymbol{\Gamma}}{\partial t} \right) \hat{\mathbf{z}} \qquad \begin{array}{l}\text{Co-Field interaction with mass currents and fields} \\ \text{(Inertial form of Ampere's law)}\end{array}$$

$$\mu_{og} = \left[\frac{8\pi R_o}{M} \right] \qquad \text{Inertial Permeability}$$

$$\Omega = \frac{\mu_{og}\mu_g}{2\pi Z^3} = \left[\frac{R_o}{Z} \right]^3 \Omega_o \qquad \text{Axial value of dipole along axis from origin}$$

$$\Omega = \frac{\mu_{og}\mu_g}{4\pi R^3} = \frac{1}{2}\left[\frac{R_o}{R} \right]^3 \Omega_o. \qquad \text{Radial value of the dipole in the orbital plane.}$$

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