A New Formalism of Arbitrary Spin Particle Equations

S.R. Shi*

Huiyang Radio and TV station,Daishui,Huiyang,Huizhou,Guangdong,China,516211

(Dated: October 24, 2016)

Abstract

In this paper, a new formalism of arbitrary spin particle equations is constructed. The physical meaning of the new equation is very clear. It's completely expressed by the amounts about spin. It's proved to describe correctly neutrino, photon and electron etc. Then a scalar field is introduced into the new equation. The new equation with the scalar field has an unique characteristic. The scalar field is like a switch. It can control generation and annihilation of particles. This provides a new dynamics mechanism about generation and annihilation of particles. This can also explain why the inflation period universe can be completely described by scalar fields.

PACS numbers: **11.**

Keywords: **Spin,Particle,Equation,Switch,Scalar fields**

1. INTRODUCTION

The various formalisms of arbitrary spin particle equations have been studied by many people in history. Such as Penrose[1–3],Bargmann[4],Wigner[4] etc. In this paper,a new formalism of arbitrary spin particle equations will be proposed. It has many wonderful properties. Now let's start!

2. A NEW FORMALISM OF ARBITRARY SPIN PARTICLE EQUATIONS

Starting from the Lorentz transformation properties of the totally symmetry two-component Weyl[5] spinor tensor $\varphi^{\overbrace{ABCD\cdots}^{2s}}$, the spin matrices $\vec{\sigma}(s)$ of a special representation can be obtained.

$$\vec{\sigma}(s)=(\frac{1}{2}\begin{bmatrix} 0 & 2s & 0 & 0 & 0 \\ 1 & 0 & 2s-1 & 0 & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 2s & 0 \end{bmatrix},\frac{i}{2}\begin{bmatrix} 0 & -2s & 0 & 0 & 0 \\ 1 & 0 & -(2s-1) & 0 & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & -1 \\ 0 & 0 & 0 & 2s & 0 \end{bmatrix},\begin{bmatrix} s & 0 & 0 & 0 & 0 \\ 0 & s-1 & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & -(s-1) & 0 \\ 0 & 0 & 0 & 0 & -s \end{bmatrix}) \quad (1a)$$

$$[\sigma_\alpha(s),\sigma_\beta(s)]=i\varepsilon_{\alpha\beta}{}^{\gamma}\sigma_\gamma(s) \qquad \alpha,\beta,\gamma=x,y,z \quad (1b)$$

$$\vec{\sigma}^2(s)=s(s+1) \qquad s=\frac{1}{2},1,\frac{3}{2},2,\frac{5}{2},\cdots \quad (1c)$$

The above relationships show $\vec{\sigma}(s)$ are really the correct spin matrices. Space-time metric will adopt diag(+,+,+,+) in this paper. A $(2s+1)$-component spinor $\psi(s)$ with Lorentz representation $(s,0)$ or $(0,s)$ can be constructed from spin matrices $\vec{\sigma}(s)$. Its Lorentz transformation $\Lambda[\psi(s)]$ is also obtained.

$$\psi(s)=[\psi^1(s),\psi^2(s),\cdots,\psi^{2s}(s),\psi^{2s+1}(s)]^T \quad (2a)$$

$$\Lambda[\psi(s)]=e^{\frac{1}{2}\varepsilon^{ab}S_{ab}(s)}=e^{(i\vec{\omega}+\varsigma\vec{\epsilon})\cdot\vec{\sigma}(s)},\varsigma=\pm1 \quad (2b)$$

$$\varepsilon^{(ab)}=\begin{bmatrix} 0 & \omega_z & -\omega_y & i\epsilon_x \\ -\omega_z & 0 & \omega_x & i\epsilon_y \\ \omega_y & -\omega_x & 0 & i\epsilon_z \\ -i\epsilon_x & -i\epsilon_y & -i\epsilon_z & 0 \end{bmatrix},S_{(ab)}(s)=i\begin{bmatrix} 0 & \sigma_z(s) & -\sigma_y(s) & -\varsigma\sigma_x(s) \\ -\sigma_z(s) & 0 & \sigma_x(s) & -\varsigma\sigma_y(s) \\ \sigma_y(s) & -\sigma_x(s) & 0 & -\varsigma\sigma_z(s) \\ \varsigma\sigma_x(s) & \varsigma\sigma_y(s) & \varsigma\sigma_z(s) & 0 \end{bmatrix} \quad (2c)$$

$\varsigma=+1$ corresponds to Lorentz representation $(s,0)$ and $\varsigma=-1$ corresponds to Lorentz representation $(0,s)$. $S_{ab}(s)$ is a spin tensor. A new equation with sources can be directly constructed by spin s and spin tensor $S_{ab}(s)$.

$$[s\partial_a+S_{ab}(s)\partial^b]\psi(s)=J_a(s) \qquad J_a(s)=[J_a{}^1(s),J_a{}^2(s),\cdots,J_a{}^{2s+1}(s)]^T \quad (3)$$

The physical meaning of this new equation (3) is very clear. It's completely expressed by the amounts about spin. So it may be called the spin equation. The spin equation (3) can be proved to be completely equivalent to the following Penrose totally symmetric spinor equation[1, 2].

$$(\vec{\sigma}, -i\varsigma)^{a}{}_{A'A}\partial_a\varphi^{\overbrace{ABCD\cdots}^{2s}} = J_{A'}{}^{\overbrace{BCD\cdots}^{2s-1}} \qquad \vec{\sigma} \text{ are Pauli matrices.} \tag{4}$$

proof: Substitute (1a) into (3),get

$$[s\partial_x + i\sigma_z(s)\partial_y - i\sigma_y(s)\partial_z - i\varsigma\sigma_x(s)\partial_\tau]\psi(s) = J_x(s) \tag{5a}$$

$$[s\partial_y - i\sigma_z(s)\partial_x + i\sigma_x(s)\partial_z - i\varsigma\sigma_y(s)\partial_\tau]\psi(s) = J_y(s) \tag{5b}$$

$$[s\partial_z + i\sigma_y(s)\partial_x - i\sigma_x(s)\partial_y - i\varsigma\sigma_z(s)\partial_\tau]\psi(s) = J_z(s) \tag{5c}$$

$$[s\partial_\tau + i\varsigma\sigma_x(s)\partial_x + i\varsigma\sigma_y(s)\partial_y + i\varsigma\sigma_z(s)\partial_z]\psi(s) = J_\tau(s) \tag{5d}$$

$(5a) \pm i(5b)$ and $(5c) \pm i\varsigma(5d)$,get

$$\{[s + \sigma_z(s)](\partial_x + i\partial_y) - [\sigma_x(s) + i\sigma_y(s)](\partial_z + i\varsigma\partial_\tau)\}\psi(s) = J_x(s) + iJ_y(s) \tag{6a}$$

$$\{[s - \sigma_z(s)](\partial_x - i\partial_y) + [\sigma_x(s) - i\sigma_y(s)](\partial_z - i\varsigma\partial_\tau)\}\psi(s) = J_x(s) - iJ_y(s) \tag{6b}$$

$$\{-[\sigma_x(s) - i\sigma_y(s)](\partial_x + i\partial_y) + [s - \sigma_z(s)](\partial_z + i\varsigma\partial_\tau)\}\psi(s) = J_z(s) + i\varsigma J_\tau(s) \tag{6c}$$

$$\{[\sigma_x(s) + i\sigma_y(s)](\partial_x - i\partial_y) + [s + \sigma_z(s)](\partial_z - i\varsigma\partial_\tau)\}\psi(s) = J_z(s) - i\varsigma J_\tau(s) \tag{6d}$$

The following equivalent equation can be obtained from the above equations. At the same time, the relationships of $J_a(s)$ and $\tilde{J}(s)$ are also obtained.

$$(\vec{\sigma} \otimes I_{2s}, -i\varsigma)^a\partial_a\tilde{\psi}(s) = \tilde{J}(s) \qquad \tilde{J}(s) = [\tilde{J}^1(s), \tilde{J}^2(s), \cdots, \tilde{J}^{2s+1}(s)]^T \tag{7a}$$

$$\tilde{\psi}(s) = [\psi^1(s), \psi^2(s), \psi^2(s), \psi^3(s), \psi^3(s), \cdots, \psi^{2s}(s), \psi^{2s}(s), \psi^{2s+1}(s)]^T \tag{7b}$$

$$J_a(s) = N(s)(\vec{\sigma} \otimes I_{2s}, i\varsigma)_a\tilde{J}(s) \quad N(s) = \begin{bmatrix} 2s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2s-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2s-2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2s-1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2s \end{bmatrix} \tag{7c}$$

The inverse inference of the above proof is also correct. So the equation (7) is equivalent to the equation (3). On the other hand, Penrose equation (4) can also equivalently transform to the equation (7). Then the spin equation (3) is equivalent to Penrose equation (4) too. Proof is finished. So the three equations (3),(4) and (7) are all equivalent. The relationships of $\psi(s)$ and $\varphi^{\overbrace{ABCD\cdots}^{2s}}$, $\tilde{J}(s)$ and $J_{A'}{}^{\overbrace{BCD\cdots}^{2s-1}}$ are also obtained.

$$\psi(s) =(\varphi^{\overbrace{11\cdots11}^{2s}}, \varphi^{\overbrace{21\cdots11}^{2s}}, \varphi^{\overbrace{22\cdots11}^{2s}}, \cdots, \varphi^{\overbrace{22\cdots21}^{2s}}, \varphi^{\overbrace{22\cdots22}^{2s}}) \tag{8a}$$

$$\tilde{J}(s) =(J_{1'}{}^{\overbrace{1\cdots11}^{2s-1}}, J_{2'}{}^{\overbrace{1\cdots11}^{2s-1}}, J_{1'}{}^{\overbrace{2\cdots11}^{2s-1}}, J_{2'}{}^{\overbrace{2\cdots11}^{2s-1}}, \cdots, J_{1'}{}^{\overbrace{2\cdots21}^{2s-1}}, J_{2'}{}^{\overbrace{2\cdots21}^{2s-1}}, J_{1'}{}^{\overbrace{2\cdots22}^{2s-1}}, J_{2'}{}^{\overbrace{2\cdots22}^{2s-1}})^T \tag{8b}$$

From the Lorentz transformation properties of $\varphi^{ABCD\cdots}$ and $J_{A'}{}^{BCD\cdots}$, the Lorentz transformations of $\tilde{\psi}(s)$ and $\tilde{J}(s)$ can be obtained.

$$\Lambda[\tilde{\psi}(s)] = e^{(i\vec{\omega}+\varsigma\vec{\epsilon})\cdot\vec{\sigma}(\frac{1}{2})} \otimes e^{(i\vec{\omega}+\varsigma\vec{\epsilon})\cdot\vec{\sigma}(s-\frac{1}{2})} \qquad \Lambda[\tilde{J}(s)] = e^{(i\vec{\omega}-\varsigma\vec{\epsilon})\cdot\vec{\sigma}(\frac{1}{2})} \otimes e^{(i\vec{\omega}+\varsigma\vec{\epsilon})\cdot\vec{\sigma}(s-\frac{1}{2})} \tag{9}$$

Using the relationship ($7c$), rewrite the spin equation (3) to the following formalism.

$$[s\partial_a + S_{ab}(s)\partial^b]\psi(s) = N(s)(\vec{\sigma}\otimes I_{2s}, i\varsigma)_a \tilde{J}(s) \tag{10}$$

Multiply (10) by ∂^a,get

$$\partial_a\partial^a\psi(s) = \frac{1}{s}N(s)(\vec{\sigma}\otimes I_{2s}, i\varsigma)_a \partial^a \tilde{J}(s) \tag{11}$$

So the new equation (3) describes massless particles. Especially, when $s = \frac{1}{2}$, the equation (3) is just the Weyl neutrino equation[5]. When $s = 1$, the equation (3) is just the spinor formalism[1, 2] of Maxwell electromagnetic field equations.

3. THE SPIN EQUATION WITH A SCALAR FIELD

The following new equation (12) is obtained by introducing a scalar field ϕ in the equation (3) . And it will have some new physical meanings.

$$[(s+\phi)\partial_a + S_{ab}(s)\partial^b]\psi(s) = J_a(s) \tag{12}$$

Consider the new equation without sources.

$$[(s+\phi)\partial_a + S_{ab}(s)\partial^b]\psi(s) = 0 \tag{13}$$

Because the equation (13) describes massless particles, we can choose particle's movement direction as z-axis. The plane wave solution of the equation (13) is $\psi(s) = \psi_0(s)e^{\pm(ipz-iEt)}, E = p$. Substitute it and $J_a = 0$ into (3),get

$$[\sigma_x(s) + i\sigma_y(s)](1-\varsigma)p\psi(s) = 0 \qquad [(s+\phi) - \sigma_z(s)](1-\varsigma)p\psi(s) = 0 \tag{14a}$$

$$[\sigma_x(s) - i\sigma_y(s)](1+\varsigma)p\psi(s) = 0 \qquad [(s+\phi) + \sigma_z(s)](1+\varsigma)p\psi(s) = 0 \tag{14b}$$

When $\phi = 0$, the above equations have plane wave solutions. And only the solutions of spin z-component $s_z = -s(\varsigma = 1)$ or $s_z = s(\varsigma = -1)$ exist.

$$\varsigma = 1, \psi(s) = a_p(s) \begin{bmatrix} 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix} e^{(ipz - iEt)} + b_p^+(s) \begin{bmatrix} 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix} e^{-(ipz - iEt)} \tag{15a}$$

$$\varsigma = -1, \psi(s) = a_p(s) \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix} e^{(ipz - iEt)} + b_p^+(s) \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix} e^{-(ipz - iEt)} \tag{15b}$$

$$\varsigma = 1, \sigma_z(s)\psi(s) = -s\psi(s) \qquad \varsigma = -1, \sigma_z(s)\psi(s) = s\psi(s) \tag{15c}$$

When $\phi \neq 0$, the above equations have no plane wave solution. So the scalar ϕ is like a switch. When it's ON($\phi = 0$), particles can exist. When it's OFF($\phi \neq 0$), no particle can exist. So the new equation (12) may be called the switch spin equation. For two degenerate vacuum states, if one is zero then the other must not be zero. The two degenerate vacuum states will have completely different physical meanings. One is correspond to particles state and the other is correspond to no particle state.

Further mathematical analysis can obtain the following results. When $\phi = 0$, the equation (13) is equivalent to the equation (4). When $\phi \neq 0, -(2s+1)$, the equation (13) only has constant solutions. When $\phi = -(2s+1)$, the equation (13) is equivalent to the following equation (16).

$$\partial_a \psi(s) = \frac{1}{2s+1} N(s)(\vec{\sigma} \otimes I_{2s}, i\varsigma)_a \varsigma \tilde{J}(s) \tag{16}$$

Especially when $s = \frac{1}{2}$ and $\tilde{J}(s)$ is a contant, the above equation has a twistor[3] solution.

$$\psi(\frac{1}{2}) = (\vec{\sigma}, i\varsigma)_a x^a \pi \qquad \pi = \frac{1}{2}\varsigma \tilde{J}(\frac{1}{2}) \tag{17}$$

The above solution is just the Penrose twistor[3] projection relation.

4. THE SWITCH SPIN EQUATION IN $N+1$ DIMENSION SPACETIME

Construct the following massive spin equation in $N+1$ dimension spacetime.

$$[(s+\phi)\partial_a + S_{ab}\partial^b]^{\eta}{}_{\kappa}\psi^{\kappa\lambda\mu\cdots} = -\frac{1}{2}m\gamma_a{}^{\eta}{}_{\kappa}\psi^{\kappa\lambda\mu\cdots} \quad s = \frac{1}{2}, S_{ab} = \frac{1}{4}[\gamma_a, \gamma_b] \tag{18}$$

γ_a is a Dirac matrix[6] in $N+1$ dimension spacetime. $\psi^{\kappa\lambda\mu\cdots}$ is a totally symmetry Dirac spinorial tensor[6]. Multiply (18) by γ_a (*not* γ^a), get the following equivalent equation.

$$(\gamma^a\partial_a + m)^\eta{}_\kappa \psi^{\kappa\lambda\mu\cdots} = -2\phi\gamma_1{}^\eta{}_\kappa\partial_{x_1}\psi^{\kappa\lambda\mu\cdots} = -2\phi\gamma_2{}^\eta{}_\kappa\partial_{x_2}\psi^{\kappa\lambda\mu\cdots}$$
$$= \cdots = -2\phi\gamma_N{}^\eta{}_\kappa\partial_{x_N}\psi^{\kappa\lambda\mu\cdots} = -2\phi\gamma_\tau{}^\eta{}_\kappa\partial_\tau\psi^{\kappa\lambda\mu\cdots} \tag{19}$$

When $\phi = 0$, the above equation is the Bargmman-Wigner equation[4] in $N+1$ dimension spacetime. It has plane wave solutions. When $\phi \neq 0$, the above equation has no plane wave solution. In this case, the scalar ϕ is still like a switch. In $N+1$ dimension spacetime there is the following theorem about spin tensor S_{ab}.

$$\begin{aligned}
Theorem: [S_{ab}, S_{cd}] &= [\delta_{cb}S_{ad} - \delta_{da}S_{cb} + \delta_{db}S_{ca} - \delta_{ca}S_{bd}] \\
&\Leftrightarrow S_{ab} = [e^\varepsilon]_a{}^c[e^\varepsilon]_b{}^d e^{\frac{1}{2}\varepsilon^{ij}S_{ij}} S_{cd} e^{-\frac{1}{2}\varepsilon^{ij}S_{ij}}, \forall\varepsilon
\end{aligned} \tag{20}$$

$$\begin{aligned}
proof: [S_{ab}, S_{cd}] &= [\delta_{cb}S_{ad} - \delta_{da}S_{cb} + \delta_{db}S_{ca} - \delta_{ca}S_{bd}] \\
&\Leftrightarrow \frac{1}{2}\varepsilon^{cd}[S_{ab}, S_{cd}] = \frac{1}{2}\varepsilon^{cd}[\delta_{cb}S_{ad} - \delta_{da}S_{cb} + \delta_{db}S_{ca} - \delta_{ca}S_{bd}], \forall\varepsilon\to 0 \\
&\Leftrightarrow 0 = \delta_a{}^c S_{cd} + \delta_b{}^d S_{cd} + \frac{1}{2}\delta_a{}^c\delta_b{}^d\varepsilon^{ij}[S_{ij}, S_{cd}], \forall\varepsilon\to 0 \\
&\Leftrightarrow S_{ab} = (1+\varepsilon)_a{}^c(1+\varepsilon)_b{}^d(1+\frac{1}{2}\varepsilon^{ij}S_{ij})S_{cd}(1-\frac{1}{2}\varepsilon^{ij}S_{ij}), \forall\varepsilon\to 0 \\
&\Leftrightarrow S_{ab} = [e^\varepsilon]_a{}^c[e^\varepsilon]_b{}^d e^{\frac{1}{2}\varepsilon^{ij}S_{ij}} S_{cd} e^{-\frac{1}{2}\varepsilon^{ij}S_{ij}}, \forall\varepsilon\to 0 \\
&\Leftrightarrow S_{ab} = [e^\varepsilon]_a{}^c[e^\varepsilon]_b{}^d e^{\frac{1}{2}\varepsilon^{ij}S_{ij}} S_{cd} e^{-\frac{1}{2}\varepsilon^{ij}S_{ij}}, \forall\varepsilon
\end{aligned}$$

Proof is finished. The above theorem shows that spin tensor S_{ab} is a Lorentz covariant tensor in $N+1$ dimension spacetime. So the equations (3),(12) and (18) are are all Lorentz covariant. A general switch spin equation with general covariant,Lorentz and Yang-Mills[7] covariant can be constructed by spin s and spin tensor S_{ab} in $N+1$ dimension spacetime.

$$[(s+\phi)D_a + S_{ab}D^b]\psi^\sigma = J_a{}^\sigma \tag{21}$$

σ is a Yang-Mills[7] index. D is a covariant derivative about internal and external space. The equations (3),(12) and (18) are its special case. I guesses the scalar ϕ in general case is still like a switch.

5. CONCLUSIONS AND DISCUSSIONS

In this paper, I get a new equivalent formalism of classical particle equations: Spin Equation. This new formalism has a following advantage. If the spin and spin matrix of a particle is known, the equation of the particle can be written immediately. Vice versa, we can immediately know the spin and the Lorentz transformation properties of the particle from the equation. In order to extend this equation, I introduce a scalar into the new equation to get a switch spin equation with new physical meanings. The switch spin equation provides a new concrete dynamics mechanism about generation and annihilation of particles. And it can also explain why the inflation[8] period universe can be completely described by scalar fields. Because in the inflation period the universe fulls scalar fields everywhere, so the other particles with any spin don't exist according to the switch spin equation. In addition, the spin equation (13) includes three different equations in fact. Maybe this provides a new line of thinking to unify five different superstring[9] equations. So this new formalism of particle equations gives us a new way of thinking about physics and the universe.

---

* hytvssr@hotmail.com

[1] Roger Penrose and Wolfgang Rindler. *Spinors and Space-Time, Volume 1: Two-Spinor Calculus and Relativistic Fields.* Cambridge University Press, 1984.

[2] Roger Penrose and Wolfgang Rindler. *Spinors and Space-Time, Volume 2: Spinor and Twistor Methods in Space-Time Geometry.* Cambridge University Press, 1986.

[3] R. Penrose. Twistor algebra. *Journal of Mathematical Physics*, 8(2):345–366, 1967.

[4] V. Bargmann and E. P. Wigner. Group theoretical discussion of relativistic wave equations. *Proceedings of the National Academy of Sciences of the United States of America*, 34(5):211–23, 1948.

[5] H. Weyl. Quantenmechanik und gruppentheorie. *Zeitschrift Fr Physik*, 46(1-2):1–46, 1927.

[6] P.A.M. Dirac. *The Principles of Quantum Mechanics.* Oxford:Clarendon Press, 1930.

[7] C. N. Yang and R. L. Mills. Conservation of isotopic spin and isotopic gauge invariance. *Physical Review*, 96(1):191–195, 1954.

[8] Alan H. Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23(2):347–356, 1981.

[9] Edward Witten. M-theory. *American Mathematical Society Providence Ri*, 1, 1998.