

The physical nature of the basic concepts of physics

2. Force

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Abstract

The concept of the ‘force’ of a particle system, as the vector product of its mass times its acceleration, finds its origin in Newton’s second law of motion and is the fundamental concept of classical physics, since it is the basis of the other fundamental notions such as ‘work’ and ‘energy’. The problem is that this classic concept of a force covers a wide variety of phenomena, which blurs its true nature.

In this paper the author analyses the physical nature of the impulsive force exerted between colliding bodies and of the repetitive impulsive forces, such as the tensile force on a rigid wall and the driving force on a free body. This allows him to define ‘force’ in a general way as the rate at which linear momentum flows, which is a mathematical expression of the transfer rate of congruent translational motion from one body to another.

In the light of the particle nature of matter, this means that the ‘force’ exerted between two particle systems can be expressed as the transfer of congruent translational motion per impulse, times the impulse frequency. This demonstrates that ‘force’ has fundamentally a dynamic character, and that there are no such things as ‘static’ forces.

1. The present concept of ‘force’

For centuries the problem of motion and its causes have been an important subject of Natural Philosophy. In those times, the scientific paradigm was based on Aristotle’s view (384 – 322 BC) that a body was in its natural state when it was at rest and that some ‘action’ was needed to keep a body moving, because otherwise it would naturally come to at rest. This conviction was based on the everyday experience that after a while, all moving objects finally came to a stop. During centuries this daily experience has supported the idea that steady motion needed a permanent ‘action’.

Roughly 2000 years later, on the basis of the extrapolation of the results of experiments with polished blocs and greasy surfaces, Galileo (1564 – 1642) came to exactly the opposite conclusion, namely that in order to change the velocity of a body, an external action is needed, and that no action at all is needed to maintain it!

- This principle of Galileo was adopted by Isaac Newton (1642 – 1727) in his “*Principia Mathematica Philosophiae Naturalis*” (1686), as his ‘first law of motion’, which is also called the ‘law of inertia’ and in which he introduced the concept of ‘force’^[1]: “*Every body persists in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed on it*”.

- In his second law of motion, Newton gives a (mathematical) definition of the concept of ‘force’ as “the rate of change of momentum” of an object:

$$\mathbf{F} = d\mathbf{p}/dt = d(m\mathbf{v})/dt = m.d\mathbf{v}/dt = m\mathbf{a}$$

- And in his third law of motion, Newton tells us that “*the mutual forces of two bodies on each other are always equal in magnitude and opposite in direction*”, or in other words: action equals reaction.

2. The indistinctness of the present concepts of ‘force’

The concept of ‘force’, as it is defined by Newton’s laws of motion, covers a wide variety of different kinds of ‘forces’ such as contact force, impulsive force, tensile force, static force, dynamic force, friction force, drag force, conservative force, non-conservative force, inertial force, centrifugal force, centripetal force, coriolis force, electromagnetic force, weak force, strong force, gravitational force, etc., in a way that makes it difficult to reveal the real physical mechanism behind these different kinds of interactions.

The common property of all these ‘forces’ is that they are based on Newton’s second law of motion, in which force is defined as the time rate of change of motion. This means, however, that according to Newton’s definition, ‘change of motion’ and ‘force’ are used as synonyms: if the motion of a body changes, there is a force on it!

In the centuries after Newton, cases have however been found in which the motion of objects changes without there being any ‘forces’ involved:

- The so-called coriolis ‘force’ is a well-known example of a change of motion without the intervention of any force and it is therefore nowadays called a ‘pseudo force’, in the same way as the centrifugal force and the centripetal force.
- In the case of gravitation also, the motion of gigantic masses undeniably changes, and therefore Newton, in his ‘universal law of gravitation has defined the gravitational ‘force’: $F_g = Gm_1m_2/r^2$. Newton’s gravitational force has allowed us, during centuries, to calculate the motion of falling objects with clockwork precision. Yet, in the early years of the twentieth century, Einstein has demonstrated in his ‘general theory of relativity’, that falling masses are not at all pulled by gravitational ‘forces’, but meander effortlessly toward each other, in a geometric space-time curvature ⁽ⁱ⁾.

In this paper, I will reveal the true nature of the so-called ‘contact forces’ (and the different ways they transfer momentum from one body to another.

3. Force as the transfer of momentum flow

In my paper Part 1 “The true physical nature of linear momentum” I have referred to the study of fluid mechanics in which “the amount that flows across a given section per unit time” is defined as the ‘flow’ (Q).

- I have thereby analyzed the concept of ‘mass flow’ (Q_m) which is a mathematical expression of the amount of ‘mass’ that moves per unit time across a given section.

For a steady stream of particles, such as a fluid or a moving particle cloud with a length ‘ L ’ in its direction of motion and an area ‘ A ’ perpendicular to this length, that consists of ‘ N ’ unit

(i) This will be analyzed in my paper on the physical nature of the gravitation.

particles with unit mass ‘ m_1 ’, that are uniformly spread over its volume ‘ V ’ (= $A \cdot L$), that move in a congruent way with a velocity ‘ v ’, the mass flow is expressed as:

$$Q_m = Nm_1/t = m/t = m(v/L) = \rho Vv/L = \rho Av_c$$

in which ‘ m ’ (= $N \cdot m_1$) is the total mass and ‘ ρ ’ (= Nm_1/V) is the mass density of the fluid.

- Another important application of the ‘flow’ concept, which is currently used in hydraulic engineering, is ‘momentum flow’ (Q_p) which indicates the amount of ‘linear momentum’ that moves per unit time across a section with a given area (A), and is therefore equal to the mass flow (Q_m) times the velocity ‘ v ’:

$$Q_p = Q_m \cdot v = (Nm_1v)/t = (mv)/t = (mv) \cdot (v/L) = mv^2/L = \rho Vv^2/L = \rho Av^2$$

For a steady stream of particles this definition of the ‘momentum flow’ corresponds exactly to Newton’s (mathematical) definition of ‘force’ as “the rate of change of the linear momentum” of an object:

$$F = p/t = m \cdot v/t = (mv) \cdot (v/L) = mv^2/L = \rho Vv^2/L = \rho Av^2 = Q_p$$

This means that a particle (system) with a mass ‘ m ’ moving with velocity ‘ v ’ has a momentum flow:

$$Q_p = mv^2/L = \rho Av^2 = F$$

and that when this particle (system) hits another particle (system) that is at rest in the same reference frame, it will experience a collision, by which its total amount of momentum flow is transferred to that other particle system ⁽ⁱⁱ⁾.

This demonstrates that ‘force’ is not a basic physical phenomenon, but a consequence of conflicting momentum flows.

This demonstration of ‘force’ as ‘momentum flow’ has already been revealed in 1980 by Andrea A. diSessa of the Division for Study and Research in Education of MIT who, mainly for pedagogical reasons, proposed in his paper “*Momentum flow as an alternative perspective in elementary mechanics*” ^[2] to use the notion of ‘momentum flow’ instead of ‘force’, because momentum flow analysis allows a better insight in the intrinsic dynamical nature of ‘force’.

According to diSessa, “*force is simply the flow of the conserved momentum, from one place to the other. Technically speaking, force is the rate with which momentum flows*”.

In his paper he works out a number of examples to demonstrate this, like e.g. “*the case in which I holds a weight in my hand. Newton’s law $F = ma$ tells me that since the acceleration is zero, the force on the weight must be zero and I conclude that my hand is providing an equal and opposite force. In a momentum flow analysis, the apple is pouring momentum through my hand, through my arm and further through my body and through my legs, into the floor*”. In other words, the apple generates a constant downward momentum flow through my hand, and my arm has to generate an opposite upward momentum flow to immobilize it. I will come back to diSessa’s paper in section 6 about the intrinsic dynamic nature of ‘force’.

4. Impulsive force (F_p)

4.1 The average impulsive force

(ii) This will be analyzed in my paper on the physical conservation of kinetic energy in elastic collisions.

The basic way to transfer momentum from one body to another is by means of an elastic collision between both bodies (like in the case of e.g. two colliding billiard balls). In the present textbooks, this problem is solved by means of the definition of ‘impulse’ (\mathbf{J}), which is defined as the change of linear momentum in one stroke ($\Delta\mathbf{p}$):

$$\mathbf{J} = \Delta\mathbf{p} = \int \mathbf{F}_p dt$$

In this classic representation, the average (impulsive) ‘force’ is defined as the change of momentum per unit time ($\mathbf{F}_p = d\mathbf{p}/dt$). In this way, the impulse is defined as the integration of the (impulsive) force over time, which is represented in the Ft-diagram as the area under the curve of force versus time.

One must however realize that it are not the (equal and opposite) impulsive forces that cause the momentum transfer, but that it is in fact the other way around: it is the momentum transfer (together with the elasticity of the colliding bodies) that generates the impulsive forces.

The magnitude of the impulse force ‘ \mathbf{F}_p ’ during the collision, depends indeed on the momentum change ‘ $d\mathbf{p}$ ’ and the time interval ‘ dt ’ during which that momentum change takes place. The problem thereby is that the impulsive force has a very short duration (in the range of milliseconds) and changes in that very short time from zero to a very high peak value, so that it is difficult to measure the exact value of the impulsive force ‘ \mathbf{F}_p ’. In the present textbooks^[3] the impulse is therefore represented as the product of the duration of the stroke ‘ Δt ’ and the average impulsive force ‘ \mathbf{F}_{pav} ’ within that time interval (Fig. 2.1):

$$\mathbf{J} = \Delta\mathbf{p} = \mathbf{F}_{pav}\Delta t$$

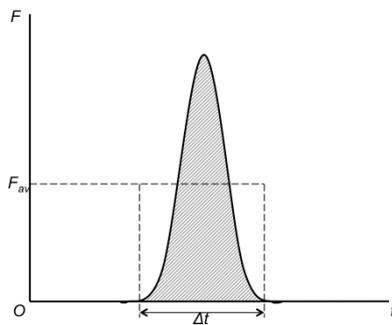


Fig. 2.1

In order to calculate that average impulsive force (\mathbf{F}_{pav}), the present textbooks of physics make an estimation of the time interval ‘ Δt ’ during which the momentum transfer takes place, and calculate on the basis of that estimation the average impulsive force between the colliding bodies:

$$\mathbf{F}_{pav} = \Delta\mathbf{p}/\Delta t.$$

4.2 The calculation of the peak value of the impulsive force

It is clear that this guessing of the peak force is not a very scientific method.

For homogeneous materials, the exact value of the peak force can however be calculated by means of the stress/strain graph of the colliding bodies.

For the working stages of loading of most materials, this stress/strain graph is a straight line, which means that the strain (load per unit area $\sigma = F/A$) is proportional to the stress (stretch

per unit gauge length $\varepsilon = \Delta L/L'$). This linear relationship for most construction materials is generally known as ‘‘Hooke’s law’’.

The slope $\text{tg}\gamma$ of the stress-strain graph (fig. 2.2) is called ‘the modulus of elasticity’ or ‘Young’s modulus’ for the given material and is usually designated by ‘E’ (expressed in N/m^2): $E = \sigma/\varepsilon = (F/A)/(\Delta L/L) = \text{tg}\gamma$

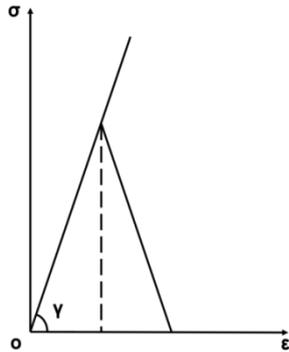


Fig. 2.2

This enables us to calculate the elongation or compression of a material under a given load (e.g. for steel, $E = 200 \cdot 10^9 \text{ N/m}^2$).

These engineering considerations allow us also to calculate the exact value of the peak force that is generated during an elastic collision between e.g. two identical steel blocks, each with a mass ‘m’ of which one has a velocity ‘v’ towards the other, which is at rest in the same reference system. In this case the kinetic energy of the moving block will be gradually used to produce an elastic deformation of the steel blocks along their stress-strain graph.

During an elastic deformation, the force ‘F’ is proportional to the deformation: $F = AE\Delta L/L$ so that:

$$mv^2/2 = (AE/L) \int \Delta L dL = AE\Delta L^2/2L = F\Delta L/2$$

$$\text{or: } mv^2 = F\Delta L$$

From Hooke’s law we know that the material will deform along a straight line with a steep slope ($\text{tg}\gamma = E$), so that:

$$E = \sigma/\varepsilon = FL/A\Delta L$$

$$\text{or: } FL = EA\Delta L$$

We have two equations with two unknowns, F and ΔL , giving:

$$F = v\sqrt{mEA/L} \quad \text{and} \quad \Delta L = v\sqrt{mL/AE}$$

For homogenous blocks with a constant section, the velocity of the deformation will gradually decrease, so that $v_{av} = v/2$ and

$$\Delta t = \Delta L/v_{av} = 2\Delta L/v = 2\sqrt{mL/AE}$$

This example demonstrates that for homogenous materials the peak force and the duration of the impulse during collision are calculable and that even in the case of hard physical impacts, the momentum transfer is not instantaneous, but progresses gradually⁽ⁱⁱⁱ⁾.

5. Repetitive impulsive forces

5.1 Introduction

In the former section I have demonstrated that if we want to increase the speed of a perfectly elastic body, like a billiard ball with mass ‘m’ from ‘0’ to ‘v’, we can do that by means of an appropriate impulse, e.g. by shooting another identical billiard ball to it with the required speed ‘v’. When the second billiard ball (which has a linear momentum with magnitude ‘mv’ and a total amount of momentum flow ‘mv²’) hits the target, it will come to a complete standstill and by doing so it will transfer its total translational motion to the identical target billiard ball, which will now move with the speed ‘v’ (and which consequently has a linear momentum with magnitude ‘mv’ and a total amount of momentum flow ‘mv²’).

This way of transferring momentum flow from one body to another by means of an elastic collision may be okay for billiard balls, but for composite macroscopic structures, the accelerations and the consecutive impulsive forces are much too high and will cause serious damage to those structures.

In the case of composite bodies that are made up from individual elements, the momentum transfer between both colliding particle systems takes place by means of consecutive collisions between their constituent particles. In that way a second time interval comes into play, which is the average time interval between two consecutive collisions (T) between the particles of both particle systems. This can also be expressed by its inverse, as the frequency (f = 1/T) of the successive collisions.

This form of momentum flow is similar to the case in which we fire a steady stream of bullets/particles to a target. When each individual particle has a mass ‘m₁’ and a velocity ‘v’, the frequency ‘f’ with which the particles proceed to the target is equal to the total number of fired particles ‘N’ divided by the total firing time ‘t’. Since the total length ‘L’ of the particle stream is equal to the velocity of the particles times the total firing time, the frequency (f) of the particles proceeding to the target may be expressed as:

$$f = 1/T = N/t = Nv/L$$

In that way, the transfer of momentum per unit time (the momentum flow) ‘Q_p’ is equal to the linear momentum of one particle (p₁) times the frequency (f) with which the particles succeed each other.

$$Q_p = p_1 \cdot f = (m_1 \cdot v) \cdot (Nv/L) = (Nm_1)v^2/L = mv^2/L = mAv^2/V = \rho Av^2 = Q_m \cdot v$$

This means that the impulsive force (F_p) on a body, is equal to the amount of linear momentum (the amount of congruent translational motion) that is transferred per impulse (Δp₁) times the number of impulses per unit time (the frequency) ‘f’:

$$F = \Delta p_1 \cdot f = \Delta Q_p$$

(iii) This will be analyzed in my paper “The physical nature of work, kinetic energy and Planck’s constant”.

This viewpoint demonstrates that ‘force’ in general and even so-called ‘static’ forces and also ‘pressure’, are intrinsically dynamic characteristics because they are the consequences of repetitive collisions of the molecules of a force particle system with the molecules of a body. This is exactly what happens between the moving molecules of a gas and the walls of a pressure tank or a combustion cylinder, or between the jiggling molecules of our hand and the surface of an object when we exert a pressure on it, or when we push it away, etc. (see section 6).

To analyze this further we have to make a distinction between the momentum transfer to a immovable rigid wall (the ‘tensile force’) and the momentum transfer to a body that is free to move (the ‘driving force’).

5.2 Tensile force (F_t)

5.2.1 The case of an immovable elastic wall

This is the case of the transfer of linear momentum (congruent translational motion) by means of repetitive collisions of unit particles with mass ‘ m_1 ’ and velocity ‘ v ’ with an immovable, but perfectly elastic rigid wall.

The classic equation of the final velocity for an elastic collisions gives us in this case:

$$v_{1f} = -v$$

so that the transfer of momentum of the elastic wall to the particle is: $\Delta p_{1p} = -2m_1v$.

This means that the impulse at each collision on an elastic wall is equal to twice the linear momentum (p_1) of a particle that is moving to the wall: $\Delta p_{1w} = 2p_1 = 2m_1v$.

The (tensile) force on that wall, which is per definition the amount of linear momentum (or i.e. of congruent translational motion) transferred per unit time to that wall, is then equal to:

$$F_t = \Delta Q_p = \Delta p_1 \cdot f = (2m_1v)(Nv/L) = 2(m_1N)v^2/L = 2mv^2/L = 2\rho Av^2 = 2Q_p$$

It follows from this that the (tensile) force (F_t) on a perfectly elastic wall is equal to twice the momentum flow (Q_p) to the wall.

It is thereby important to stress the fact that in these equations of the tensile force, ‘ L ’ is the total length of the moving ‘force’ particle system in its direction of motion ($L = vt$).

5.2.2 The case of an immovable inelastic wall

In the specific case of a immovable inelastic wall: $\Delta p_1 = m_1v$

so that in this case the force on the inelastic rigid wall is equal to the momentum flow (Q_p) to that wall: $F_t = \Delta Q_p = \Delta p_1 \cdot f = (m_1v)(Nv/L) = mv^2/L = \rho Av^2 = Q_p$

Which is half of the force on an elastic wall.

5.2.3 A general solution by means fluid mechanics

In fluid mechanics the problem of the force exerted by the steady flow of a fluid on an obstacle (a wall, a deflector, etc.) is generally solved by means of ‘the momentum equation’, which for a steady, uniform flow is written as: $\Sigma F = \rho_2 A_2 v_2 \mathbf{v}_2 - \rho_1 A_1 v_1 \mathbf{v}_1$ (Fig. 2.3)

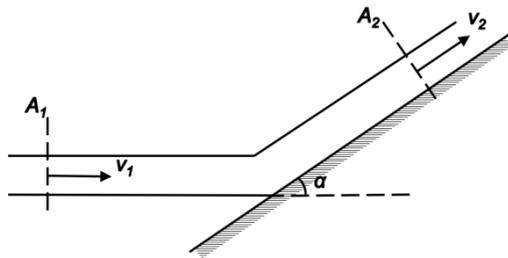


Fig. 2.3

For a stationary obstacle, this leads to a force \mathbf{F} on the wall equal to:

$$\mathbf{F} = \rho A v^2 (1 - \cos \alpha)$$

Here ' α ' is the angle between the direction of the flow (which we take to be horizontal, that is along the x-axis) and the direction of the obstacle.

- For an obstacle that is parallel to the direction of the flow:
 $\alpha = 0^\circ$, so $\cos \alpha = 1$ and $F = 0$
- For an obstacle perpendicular to the flow:
 $\alpha = 90^\circ$, so $\cos \alpha = 0$ and $F = \rho A v^2$
This corresponds to the case of an inelastic wall (section 5.2.2) by which the horizontal component of the flow disappears completely
- For a circular obstacle that deflects the flow over 180 degrees:
 $\alpha = 180^\circ$, so $\cos \alpha = -1$ and $F = 2\rho A v^2$
This corresponds to the case of an elastic wall (section 5.2.1) by which the flow is completely reversed. This knowhow is used in the case of the so-called 'Pelton turbines', which completely reverse the momentum flow and produce in that way a force that is twice that on a flat blade.

5.3 Driving force (\mathbf{F}_d)

In this case the repetitive collisions do not take place against an immovable wall, but against a body that is free to move. In order to increase the motion of such a body, we use forces that consist of a large number (N) of very small successive impulses. This is exactly what is done by heated or pressurized gas in the cylinders of a combustion engine. When a steady, quasi continuous momentum flow from the molecules of a 'force' particle system is transferred in a reversible way to a body with a mass ' m ' that is free to move, that body will experience a steady 'driving force' or 'thrust' (\mathbf{F}_d). This driving force will increase the speed of the body when it acts in its direction of motion, but it can also decrease the speed of the body when it acts against its direction of motion (in which case it is called a 'braking force'), or it can change the direction of motion of the body when it acts perpendicular to its direction of motion (in which case it is called a 'steering force').

In this case the linear momentum ' $m\mathbf{v}$ ' is transferred to a free moving body, while this body covers a distance ' L ' with gradually increasing speed. During this displacement, the velocity is reversibly increasing from '0' to ' \mathbf{v} '. The time interval ' t ' necessary to transfer the total momentum ' $m\cdot\mathbf{v}$ ' is consequently: $t = L/v_{av} = L/(v/2) = 2L/v$

The average frequency with which the consecutive impulses of the particles hit the body is then: $f = N/t = Nv/2L$

This means that the ‘force’, that is the steady transfer of momentum to the body (which is necessarily equal to the increase of the momentum flow of the body while both particle systems are covering the same distance ‘L’) is equal to:

$$F_d = \Delta Q_p = \Delta p_1 \cdot f = (m_1 \cdot v)(Nv/2L) = (Nm_1)v^2/2L = m \cdot v^2/2L = \rho A v^2/2 = Q_p/2$$

This leads to the conclusion that if we want to increase the speed of a mass ‘m’ from zero to ‘v’ over a distance ‘L’, we have to maintain a steady transfer of momentum flow (or a steady transfer rate of congruent translational motion) ‘ ΔQ_p ’ which is equal to only half the momentum flow ‘ Q_p ’ of that body when it steadily proceeds with that velocity ‘v’ (iv):

$$F_d = \Delta Q_p = m \cdot v^2/2L = Q_p/2$$

5.4 Action and reaction

In this section 5, I have demonstrated that the action of a given momentum flow $Q_p = \rho A v^2/2$ on an object, can result in a wide range of transfers of momentum flow to (or of the force on) that object: going from $F = \text{zero}$, over $F = \rho A v^2/2$ and $F = \rho A v^2$, to $F = 2\rho A v^2$, depending on the orientation and the elasticity of that object and on its ability to go with the flow, which means that there is not a connection between the applied momentum flow and the reaction of the object.

Newton’s third law on the other hand, tells us however that the net action on a body is equal to the reaction of that body. Newton’s third law is correct, because it considers the momentum that is effectively transferred to a body, in which case Newton’s third law is in fact an mathematic expression of the conservation of motion.

6. The fundamentally dynamic nature of ‘force’

These finalizations of the concept of ‘force’ clearly underline the fact that Newton’s definition of ‘force’ is not a basic physical phenomenon, but a consequence of conflicting momentum flows when momentum is transferred from one particle system to another.

As I already mentioned in section 3, this causal connection was already developed in 1980 by Andrea A. diSessa of MIT, who proposed in his paper “*Momentum flow as an alternative perspective in elementary mechanics*” to use the notion of ‘momentum flow’ instead of ‘force’, “*because this expresses the fundamentally dynamic character of force as a consequence of repetitive collisions between the molecules of the particle system of the force with the molecules of the particle system of the body*”. As is demonstrated by diSessa, “*this is exactly what happens in a pressure tank between the gas molecules and the walls of the tank or between the jiggling molecules of an object lying on the floor surface. This is as a matter of fact also exactly what happens between the jiggling molecules of our hand and the jiggling molecules of a wall while stretching ourself against that wall, while holding ourself up on a branch of a tree or while holding an apple stationary above the ground*”. The present textbooks tell us that in those cases no ‘work’ has been done, as nothing has been displaced. But we surely get exhausted by doing it!

(iv) This will be analyzed in my paper on the physical nature of work, kinetic energy and Planck’s constant.

I doubt however whether diSessa's plea has had any success, because the role of 'momentum' and consequently of 'momentum flow' is in the present textbooks still being completely minimized and restricted to some peripheral phenomena such as the equations for the final velocities of elastic collisions and the equation for rocket propulsion.

In this paper I have demonstrated that diSessa's representation of 'force' as the transfer of momentum flow doesn't only allow a better comprehension of the observed phenomena, but that it is the exact scientific designation of the dynamic phenomenon that we are used to call 'force'. The bald underestimation of the fundamentally dynamical nature of 'force' as the transmission rate of linear momentum is strikingly demonstrated by the fact that 'momentum' and 'momentum flow' are completely ignored in the present SI-system, so that in those rare occasions that linear momentum is occasionally used, it must be expressed in 'N.s', whereas, 'momentum' (expressed as $M_o = \text{kg.m/s}$) is the true fundamental unit and 'force', which represents the transfer of momentum per unit time, is the derivative unit.

In that way 'force' ought to be expressed as "momentum transfer per unit time" ($M_o/s = N$) which clearly emphasizes that the unity 'N' has a fundamentally dynamic character and that there is no such thing as a 'static' force or 'static' pressure, because both are the consequences of repetitive momentum transfers between the particles of the force object and the particles of the body.

7. Pressure

Pressure, which is generally defined as the force per unit area, is just like 'force' a derivative quantity. In a certain way it is also a purely mathematical quantity, because the chosen 'unit' area (m^2) is after all a fortuitous area and the force on 'a unit' area is therefore of exactly the same nature as the force on any other area 'A', which are however all expressed in 'N'.

This mathematical notion of 'pressure' is therefore mainly an engineering convenience that allows for direct calculation of the force on any given surface in function of the force on a 'unit' area of that surface.

Since a 'unit' area is by definition the same for all kinds of surfaces, this means that for a given material it stands for a same number of molecules, so that the 'pressure' is in that case a comparative indication of the force per molecule (and its mutual bonds) and of the physical stress in the given material.

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