The physical nature of the basic concepts of physics

3. Work and Kinetic Energy

Guido F. Nelissen

guido.nelissen@gmail.com

Abstract

The principle of conservation of 'energy' is the ultimate building stone of physics. The problem thereby is (as Richard Feynman has put it in his Lectures on Physics) that we don't have a tight description of what 'energy' is and how and where it is physically present.
In the present textbooks the concept of 'work' is mathematically defined as the product of a force times its displacement, and the concept of the 'kinetic energy' of a moving body is in defined as the product of its mass times half the square of its velocity.
In this paper the author demonstrates that 'work' is the total amount of congruent translational motion that is reversibly transferred between particle systems at different velocity levels (in a given reference frame).
This means that the 'kinetic energy' of a body with a given velocity in a given reference frame is a mathematical expression of its total amount of reversibly transferable congruent translational motion at the given velocity level, with regard to any free body that is at rest in that same reference frame.

1. The indistinct nature of the concept of 'energy'

The principle of conservation of 'energy' is the ultimate building stone of physics. It implies the transformation of different forms of 'energy' into one another while the total amount of 'energy' remains constant.
The problem thereby is that, although the textbooks of physics give exact mathematical definitions for the different forms of 'energy' – kinetic energy, potential energy, gravitational energy, electromagnetic energy, field energy, radiant energy, thermal energy, etc. - one cannot find an accurate definition of the physical nature of 'energy' as such!

In fact, most textbooks simply don't give a definition of 'energy' at all, and in those cases that they do, it is rather woolly. In Wikipedia for instance, 'energy' is defined as "*the ability to do work*". This 'definition' agrees with the definition of Peter Atkins of the University of Oxford, who writes [1] that 'energy' is "*the capacity of a system to do work*". It is however clear that the definition of something as an 'ability', a 'capacity' or a 'potentiality' doesn't exactly tell us what that something is and how it is physically present.

Richard Feynman, in his "Lectures on Physics", blames this vagueness with regard to 'energy' on a fundamental ignorance of its real nature [2]: *"It is important to realize that in physics today, we have no knowledge of what energy is. We do not have a picture that energy comes in little blobs of a definite amount. It is not that way. However, there are formulas for calculating some numerical quantity, and when we add it all together it gives always the same number. It is an abstract thing in that it does not tell us the mechanism or the reasons for the*

various formulas."

In this paper I will reveal the physical nature of kinetic energy and I will give the reasons and the mechanisms for its conservation [i]. To do so we first have to deepen our insight into the laws of motion, making thereby a clear distinction between the real physical data on one hand and the numerical results of their mathematical processing on the other hand.

2. The vagueness of the present definitions of 'work' and 'kinetic energy'

In my paper part 1 "The true nature of linear momentum", I have demonstrated that Newton's laws of motion lead to the concept of 'the quantity of motion', which is nowadays known as the 'linear momentum': $\mathbf{p} = m\mathbf{v}$

Another consequence of Newton's laws of motion is the so-called 'moving energy' [3] of an object with mass 'm' and speed 'v', which is nowadays called the 'kinetic energy' and mathematically defined as: $K = mv^2/2$

The problem is that this definition of 'kinetic energy' is a purely mathematical definition of which the physical meaning is not really clear, as it seems at first sight. In the present textbooks 'kinetic energy' is physically defined as "*something that an object possesses by virtue of having work done on it*" or as "*the capacity of an object to do work by virtue of its speed*". And 'work' is defined as "*something that is done on an object by a force as the object is displaced and which is equal to the change of kinetic energy*".

These definitions of 'work' and 'kinetic energy' are thereby based on a circular reasoning, because it follows automatically from the mathematical equation of kinetic energy, that the work done by a force while displacing a body and the transfer of 'kinetic energy' to that body, are just two different mathematical expressions of one and the same thing.

$$W = \int \mathbf{F}.d\mathbf{s} = \int m.\mathbf{a}.d\mathbf{s} = \int m.d\mathbf{v}.d\mathbf{s}/dt = \int m.\mathbf{v}.d\mathbf{v} = m.v^2/2 - m.v_0^2/2 = K - K_0 = \Delta K$$

But this still doesn't tell us at all what this 'thing' is and how and where it is physically present. This means that we are turning in circles and that we badly need to clarify these fundamental concepts once and for all.

3. The true nature of 'work' and 'kinetic energy'

3.1 The total amount of momentum flow

In section 3 of my paper Part 2: "The true physical nature of force" I have shown that the momentum flow '$\mathbf{Q_p}$' of a body with mass 'm' and velocity 'v' indicates the amount of linear momentum that moves across a given section per unit time:

$$\mathbf{F} = \mathbf{Q_p} = Q_m.\mathbf{v} = (m\mathbf{v})/t = (m\mathbf{v})/(\mathbf{L}/\mathbf{v}) = (m\mathbf{v}/\mathbf{L}).\mathbf{v} = (\mathbf{p}/\mathbf{L}).\mathbf{v}$$

In this expression the linear momentum per unit time, is expressed as the linear momentum per unit length '$\mathbf{p}/\mathbf{L}$' multiplied by the number of unit lengths per unit time '$\mathbf{v}$'.

If the momentum flow '$\mathbf{Q_p}$' of a particle system with mass 'm' and a velocity '$\mathbf{v}$', is equal to the linear momentum that is present in a unit length multiplied by its velocity, then the 'total

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(i) This will be analyzed in my paper on the physical nature of potential energy.

amount of momentum flow' ($Q_{p\,L}$) of the whole particle system must be equal to the linear momentum that is present in its whole length (m**v**) multiplied by its velocity (**v**):

$$\mathbf{F.L} \ = \ \mathbf{Q_p.L} \ = \ p.\mathbf{v} \ = \ m.v^2 \quad \text{(expressed in N.m = J)}$$

This means that 'mv²' is a mathematical expression of the total amount of momentum flow of a particle system with a mass 'm' that is moving with a congruent velocity '**v**'.
This expression corresponds exactly to the specific situation of photons that all move with an invariable speed 'c', in which case the total amount of (kinetic) energy is identical to the total amount of momentum flow, so that we automatically obtain Einstein's mass-energy equation:
$Q_{pL} \ = \ K \ = \ mc^2 \ = \ E$ [ii]

This demonstrates that the energy of a photon must be physically present as congruent kinetic energy, or in other words as congruent motion (see section 6).

3.2 The total amount of reversibly transferrable momentum flow

The way of transferring momentum flow as described in section 3.1, may be okay for photons with an invariable speed and for perfectly elastic billiard balls, but for real macro structures, the accelerations and the consecutive impulsive forces are much too big and will cause a breakdown of those structures.
In section 5.3 "Driving Force (**F$_d$**)" of my paper Part 2 "The physical nature of force", I have demonstrated that in order to accelerate real physical structures, we have to use a steady driving force '**F$_d$**' that consists of a large number of very small successive impulses that produce a steady transfer of increasing momentum flows, so that the velocity increases gradually from '0' to '**v**' and the time interval 't' necessary to transfer the total momentum 'mv' is:

$$t \ = \ \mathbf{L/v_{av}} \ = \ \mathbf{L}/(v/2) \ = \ 2\mathbf{L/v}$$

and the frequency is consequently:

$$f \ = \ N/t \ = \ N\mathbf{v}/2\mathbf{L}$$

so that the driving force is:

$$\mathbf{F_d} \ = \ \Delta\mathbf{Q_p} \ = \ \Delta p_1.\,f \ = \ (m_1.\mathbf{v})(N\mathbf{v}/2\mathbf{L}) \ = \ (Nm_1)v^2/2\mathbf{L} \ = \ m.v^2/2\mathbf{L} \ = \ \mathbf{Q_p}/2$$

This means that we have to maintain a steady transfer of momentum flow '$\Delta\mathbf{Q_M}$' that is equal to half the momentum flow '$\mathbf{Q_p}$' of that body when it steadily proceeds with that same velocity '**v**'. This equation of the driving force can also be written in function of the average speed, as:

$$\mathbf{F_d} \ = \ \Delta\mathbf{Q_p} \ = \ \Delta p_1.\,f \ = \ (m.\Delta\mathbf{v}).(\mathbf{v}/2\mathbf{L}) \ = \ (m.\Delta\mathbf{v/L}).(\mathbf{v}/2) \ = \ (\Delta p/\mathbf{L}).\mathbf{v_{av}}$$

This allows us to reveal the physical nature of the total amount of momentum flow (ΔQ_{pL}), because if the linear momentum that is reversibly transferred by the particles of a unit length multiplied by their average velocity is equal to the transferred momentum flow '$\Delta\mathbf{Q_p}$', then

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(ii) This will be analyzed in my paper on the physical nature of mass.

the linear momentum transferred by the particles in the whole length '**L**' multiplied by their average velocity, is equal to the total amount of momentum flow (ΔQ_{pL}):

$$\Delta Q_{pL} \; = \; \mathbf{F_d}.\mathbf{L} \; = \; \Delta \mathbf{Q_p}.\mathbf{L} \; = \; \Delta \mathbf{p_1}. \textit{v}.\mathbf{L} \; = \; (N m_1.\Delta\mathbf{v}).(\mathbf{v}/2\mathbf{L})\mathbf{L} \; = \; (m.\Delta\mathbf{v}).(\mathbf{v}/2) \; = \; \Delta\mathbf{p}.\mathbf{v_{av}}$$

3.3 The physical nature of work

This demonstrates that 'work', which is classically defined as the force times its displacement ($W = \mathbf{F_d}.\mathbf{L}$), is in fact a mathematical expression of the total amount of momentum flow 'ΔQ_{pL}' that is reversibly transferred by a 'force' particle system with a length '**L**' in its direction of motion:

$$W \; = \; \Delta Q_{pL} \; = \; \mathbf{F_d}.\mathbf{L} \; = \; = \; \Delta\mathbf{p}.\mathbf{v_{av}} \; = \; Q_{pL}/2$$

And which can also be expressed as the total amount of transferred linear momentum ($\Delta \mathbf{p} = m.\Delta\mathbf{v}$) multiplied by the average velocity level ($\mathbf{v_{av}} = \mathbf{v}/2$) at which this linear momentum is transferred, which demonstrates that the total amount of reversibly transferable momentum flow 'ΔQ_{pL}' is equal to half the total amount/capacity of momentum flow of the moving force particle system 'Q_{pL}'.
It is hereby important to stress the fact that in the classic definition of work, the factor '**L**' stands for the displacement of the force (together with the body), whereas in my physical analysis of work, the factor '**L**' stands for the total active length of the force particle system in its direction of motion.

This physical meaning of 'work' as the total amount of reversibly transferred momentum flow, can still be revealed more clearly by analyzing the physical meaning of reversibility with regard to the velocity increase. If we want to increase the velocity of a real physical body in a reversible way, we have to do so with the smallest possible increasing momentum quanta. In mathematics these quanta are the so-called 'infinitesimal' velocity increases 'd**v**', so that:

$$W \; = \; \int \mathbf{F} d\mathbf{L} \; = \; m \int \mathbf{v} d\mathbf{v} \; = \; m\mathbf{v}^2/2$$

When we take this "smallest possible velocity increase" as the natural velocity unit, then we have to fire mass particles with increasing velocity quanta from '1' up to 'v'. It is clear that in this system of extremely small natural velocity quanta, 'v' must represents an incredibly large number.
The total amount of linear momentum (congruent translational motion) that has to be transferred to increase the velocity of the body in a reversible way by means of 'unit' or a 'quantum' velocity increases, from '0' (the initial speed of the body) to 'v' (the final speed of the body), is then equal to the sum of the linear momenta of all the successively fired particles with unit mass and with increasing speeds (Fig. 3.1):
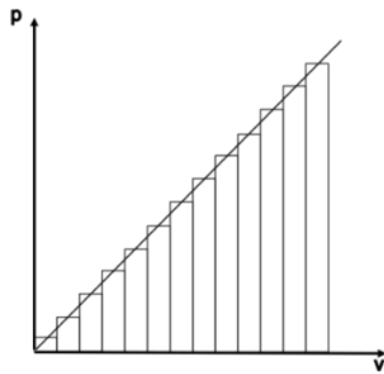
Fig. 3.1

$$\Delta p_{tot} = m(1 + 2 + 3 + ...+ v)$$

$$= m[v(1 + v)/2]$$

which for $v \gg 1$ gives us:

$$\Delta p_{tot} = mv^2/2$$

This physical reasoning demonstrates that the numerical value of the performed work represents in fact the sum of all the linear momenta at all the intermediary velocity levels. This definition of 'work' corresponds to the thermodynamic generation of work by means of an infinite number of reversible Carnot processes, with infinitesimally increasing temperatures, during which the fluid goes through successive equilibrium states that produce the same amount of work as the real cycle [4] (Fig 3.2).
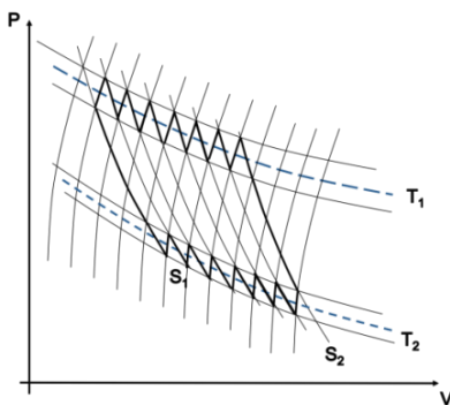

Fig. 3.2

Since I have demonstrated in my paper Part 1 that 'linear momentum' is a mathematical expression of the total amount of congruent translational motion, this means that 'work' is a mathematical expression of the total amount of congruent translational motion that is reversibly transferred between particle systems with at different congruent velocity levels, in the given reference frame.

It follows logically from this conclusion that a 'constant' force on a free body means that we have to transmit a steady stream of impulses with increasingly higher velocities (in the given reference frame).

3.4 The physical nature of 'kinetic energy'

For an initial zero velocity: $\Delta \mathbf{v} = \mathbf{v}$
so that in that case, the 'work' or i.e. the total amount of reversibly transferred congruent/coherent translational motion is equal to:

$$W = \Delta Q_{pL} = \Delta \mathbf{Q_p}.\mathbf{L} = \mathbf{F_d}.\mathbf{L} = (m.v^2/2L).\mathbf{L} = m.v^2/2 = Q_{pL}/2 = \Delta K$$

which is the equation of the 'kinetic energy' of a mass 'm' moving with a speed 'v'.
It follows from this that the work 'W' on a body with a mass 'm' that is propelled from a velocity 'v' to a velocity 'v + Δv' is equal to the difference between of the kinetic energies of this mass at both velocity levels:

$$W = \int m\mathbf{v}d\mathbf{v} = m(\mathbf{v} + \Delta \mathbf{v})^2/2 - mv^2/2 = \Delta K$$
and
$$\Delta K = m(\mathbf{v} + \Delta \mathbf{v})^2/2 – m.v^2/2 = m.\mathbf{v}.\Delta \mathbf{v} + m.\Delta v^2/2 = m.\Delta \mathbf{v}(\mathbf{v} + \Delta \mathbf{v})/2 = m.\Delta \mathbf{v}.\mathbf{v}_{av} = \Delta \mathbf{p}.\mathbf{v}_{av}$$

Which demonstrates that the work done by a driving force on a free body, and the transfer of kinetic energy to that body while exerting this force, are evidently one and the same thing.

This demonstrates that the 'kinetic energy of congruent/bulk motion' of a body with a given velocity is a mathematical expression of its total amount of reversibly transferable congruent translational motion at the given velocity level, with regard to any free body that is initially at rest in the same reference frame.
This definition of the kinetic energy of a moving body is generally considered as a relative datum because it depends on a fortuitously chosen reference frame. But as was already demonstrated in 1669 by Christiaan Huygens [5], it is an absolute, physical datum in regard to any other physical body in the same reference frame is [iii].

3.5 The link between 'force' and 'kinetic energy'

It follows from the former definitions of kinetic energy of bulk motion that 'force', which is classically defined as the transfer of linear momentum per unit time, can also be defined as the transfer of kinetic energy (or i.e. of the total amount of momentum flow) per running meter:

$$\mathbf{F} = \Delta \mathbf{p}/\Delta t = K/\mathbf{L} = Q_{pL}/\mathbf{L} = \mathbf{Q_p}$$

As I have mentioned in my paper part 2 'The true nature of force', this direct link between 'force' and 'energy' by means of the concept of momentum flow was already indirectly demonstrated by Andrea diSessa, who proposed in his paper "Momentum flow as an alternative perspective in elementary mechanics" [6] to use the notion of 'momentum flow' instead of 'force', because momentum flow analysis provides a better appreciation for the distribution mechanism of the 'forces' in static structures involving real bodies. To demonstrate this he worked out a number of examples in which he compared momentum flow with 'force'(while holding an apple in one's hand, in a rope pulled by two elephants, in the

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(iii) The nature of velocity will be analyzed in my paper on the physical nature of Extent, Velocity and Length Contraction.

legs of a racehorse, etc.). But in his last example (E) he comes to a derivation of Bernoulli's law, which as he writes himself, is clearly a question of energy instead of force. This illustrates that the intuitive link between 'force' and 'kinetic energy' that was made by diSessa was an evident link, because from the standpoint of momentum flow, 'force' is nothing else than the transferred (kinetic) energy per running meter.

This similarity between 'force' and 'energy' has however led to a non-proper use of both concepts in the 19$^{\text{th}}$ century. The reason therefore is that originally 'mv²' was referred to as the 'living force' (vis-viva) which was widely used by physiologists to explain the motion of biological bodies at that time [7]. This explains why the first paper that established the conservation of energy: "*On the quantitative and qualitative determination of forces*", that was published in 1841 by Julius Robert von Mayer, used the word 'force' instead of 'energy' [iv]. In that same period, Hermann von Helmholtz (1821 – 1894), also used the word 'force' instead of 'energy' in his paper "On the conservation of force", which shows that the concepts of energy and force were poorly differentiated at that time.

3.6 The difference between 'kinetic energy of bulk motion' and 'linear momentum'

Already in the Newton's time there was a considerable controversy about the relative significance of the 'quantity of motion' (nowadays called the 'linear momentum') and the quantity then called the "living force" that was related to what we now call 'moving' or 'kinetic' energy [8] and the great European physicists, Descartes, Leibnitz and D'Alembert had long discussions whether 'kinetic energy' or 'linear momentum' was the true property that is considered by the conservation laws [9].
My unveiling of the physical nature of 'linear momentum' and 'kinetic energy of bulk motion' allows us to stress the fundamental difference between both physical concepts.

In section 3 of my paper "Part 1: The true nature of linear momentum" I have demonstrated that the conservation of "linear momentum" is a mathematical expression of the physical fact that the particles of a particle system cannot change their 'congruent' translational motion by their own.
This is completely different with the nature of 'kinetic energy' of bulk/congruent motion which is a mathematical expression of the amount of reversibly transferrable translational motion between different velocity levels (in a given reference frame).

To demonstrate this, we first consider the case in which we push a free body with mass 'm' gradually from a zero velocity to a velocity 'Δv'. In that case:

- the increase of the linear momentum is equal to the mass times the velocity increase:
  $\mathbf{p} = m.\Delta\mathbf{v}$
- the total transferred momentum at all the intermediary velocity levels or i.e. the increase of kinetic energy, is equal to the transferred linear momentum times the average velocity at which this momentum is transferred:
  $W = Q_{pL} = \mathbf{p}.\mathbf{v_{av}} = (m.\Delta\mathbf{v}).(\Delta\mathbf{v}/2) = m.\Delta\mathbf{v}^2/2$

(iv) Julius Robert von Mayer (1814 – 1878) was not taken seriously and his achievements were overlooked and credit was later given to James Joule. Mayer tried to commit suicide in 1850, but permanently lamed himself. Later his scientific reputation began to recover and in 1871, he received the Copley Medal of the Royal Society of London for his scientific work.

Then we consider the case in which we push the same body gradually from a velocity 'v' to a velocity '$v + \Delta v$'. In that case:

- the increase of the linear momentum is equal to the mass times the velocity increase:
  $$p = m.\Delta v$$
  Which means that the transfer of linear momentum depends only on the increase of the momentum and is independent of the velocity level at which this increase takes place.

- the increase of kinetic energy is then:
  $$W = Q_{pL} = \Delta p.v_{av} = (m.\Delta v).[v + (v + \Delta v)]/2 = (m.\Delta v).(v + \Delta v/2)$$
  This equation tells us that in order to give a body with a mass 'm' and a velocity 'v' a velocity increase 'Δv', we must add a linear momentum '$m.\Delta v$' at a velocity level '$v + \Delta v/2$', so that:
  $$W = Q_{pL} = \Delta p.v_{av} = m.\Delta v.v + m.\Delta v^2/2$$
  This equation tells us that in order to increase the velocity of a body in a reversible way, we must first make sure that the force particle system finds itself at the same velocity level as the body. So the first term of this equation represents the amount of linear momentum '$m.\Delta v$' at the velocity level 'v' and the second term represents the momentum flow that corresponds to the transfer of the necessary amount of linear momentum '$m.\Delta v$' from velocity level 'v' to level '$v + \Delta v$'.

To illustrate this, we take e.g. the case of an object in space with a mass of '$m = 1000$ kg' of which we want to increase the speed by '$\Delta v = 1$ m/s'.

- When the object has a velocity '$v = 0$ m/s' with regard to the force particle system:
  o The momentum increase is equal to $\Delta p = m.\Delta v = 1000$ Ns
  o The total momentum flow or 'work' is equal to
  $$W = m.\Delta v^2/2 = 500 \text{ Nm}$$

- When the object has a velocity of '$v = 1$ m/s' with regard to the force particle system:
  o The momentum increase is equal to $\Delta p = m.\Delta v = 1000$ Ns
  o The total momentum flow or 'work' is equal to:
  $$W = m.v.\Delta v + m.\Delta v^2/2 = 1000 + 500 = 1500 \text{ Nm}$$

This confirms our conclusion that the kinetic energy of a material object with regard to another material object depends on its relative velocity with respect to that body, which means that the relative velocity between material bodies is a real, physical characteristic.

4. Kinetic energy of internal motion

The advantage of the concept of kinetic energy is that it is a scalar, which means that its use is not restricted to congruent translational motion, but that it can also be used to characterize rotating or vibrating motion.

Translational motion: $K = mv^2/2$

Rotational motion: $v = R\omega$ so that: $K = mR^2\omega^2/2$

Vibrational motion: $x = x_m \cos(\omega t + \theta)$ and consequently $v = -\omega x_m \sin(\omega t + \theta)$

So that: $K = (m\omega^2/2)x_m^2\sin^2(\omega t + \theta)$

In the case of inelastic collisions, the motions of the colliding bodies will be scattered over all possible directions, so that the kinetic energies of the colliding particle systems will be transformed into kinetic energy of isotropic/thermal motion or 'thermal energy' [(v)].

5. The total amount of kinetic energy

In the former section we have seen that kinetic energy is a scalar that characterizes the total amount of reversibly transferable motion between two velocity levels, and that it can also be used to express the total amount of motion of a particle system (its congruent as well as its thermal motion). In that way, the conservation of 'energy' means that the total amount of congruent and internal motion, will be conserved.

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(v) This will be analyzed in my paper on the physical nature of pressure, temperature, thermal energy.