THE GRAVITATIONAL POTENTIAL OF A MULTIDIMENSIONAL SHELL

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Abstract. This paper is an attempt to generalize the well-known expression of the gravitational potential for more than three dimensions. We used the Sneddon-Thornhill approach of the Newton's theorem and then the results are passed through the filter of Poisson's equation. The comparison with other theories implies some restrictions, but the overall results are valid until the experiment will disprove them.

Key words: gravitational potential; extra-dimensions.

1. Introduction

In reference (Sneddon & Thornhill, 1948) the authors were trying to accredit a new demonstration of Newton's theorem and have found some new results concerning the classical gravitational potential. So, the relation:

$$m_{(\alpha)}\Phi_{(r)} + 2\pi\sigma\alpha\gamma_{(\alpha)} = \frac{2\pi\sigma\alpha}{r}\int_{r-\alpha}^{r+\alpha} \beta\Phi_{(\beta)}d\beta \tag{1}$$

has the solutions:

$$\Phi_1 = \frac{A}{r},\ \Phi_2 = \frac{B}{r}e^{-\xi\cdot r},\ \Phi_3 = \frac{C}{r}e^{\xi\cdot r} \tag{2}$$

where Φ are the potentials per unit mass and A=B=C=G, the gravity constant. The reasoning used to create the relation (1) is simple. The gravitational potential created by a point mass, in which all the mass of a spherical material shell is concentrated, is equivalent to the gravitational potential of the spherical shell itself, in a whatever exterior reference point P.

In (Barnes & Keorghi, 1984) one can find another solution for the gravitational potential:

$$\Phi_4 = Dr^2 \tag{3}$$

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result developed in (Răuţ, 2010) for the expansion of the universe case. Finely, in (Răuţ, 2011) these results are generalized for all the measurable cases.

According to (Sneddon & Thornhill, 1948) the well-known Newtonian potential Φ_1 can be obtained from the condition that the solution (2) should be capable to create surfaces of equal potential in spherical shell interior, so that:

$$\gamma_{(\alpha)} r = \int_{\alpha - r}^{\alpha + r} \beta \Phi_{(\beta)} d\beta , \quad \text{for } r<\alpha. \tag{4}$$

In the following we will state that condition (4) is no longer taken into consideration. Since the shell seams to not exist because it has the same potential as the point which is its center, their potentials are equivalent. Thereby the gravitational potential generated by the central point mass it creates equipotent surfaces around its' point exterior whatever the considerate distance is. The interior of the shell is the exterior of the point mass. Thus the gravitational potential somewhere into the shell is the same as the gravitational potential of the point mass correspondent, because they are calculated in the same point, indifferently where it is. In addition, at different scales it is an unforgettable mistake to use Φ_1 , so to presume condition (4) to be not considered seams to be quite reasonable. On the other hand we don't know what the expression of gravitational potential in shell interior is. We can only intuit it. If we imagine that we can minimize the interior of the shell in vicinity of the central point, then solution (2) is valid.

As a consequence, in the following considerations we will show a new approach of this problem. Additionally, we will generalize the solution (2) for more than three dimensions.

2. The N-Dimensional Case

In equation (1) we can neglect the second left term. If the condition (4) is no more valid then the constant γ, corresponding to the additional potential in equation (1), term which can be added to the gravitational potential without the resulted law force to be altered, has now neither signification. In consequence the relation (1) becomes:

$$m_{(\alpha)} \Phi_{(r)} = \frac{2\pi\sigma\alpha}{r} \int_{r - \alpha}^{r + \alpha} \beta \Phi_{(\beta)} d\beta \tag{5}$$

This relation allows the solutions (2) for the gravitational potential per unit mass, as in previous case. The difference now is that with (5) we are not able to generate solutions which to admit other constants than those from (2). In previous case we were forced to make these constants null.

With relation (5) we can now think about the multidimensional case. Suppose:

$$m_{(\alpha)}\Phi_{(r)} = \frac{V_n n\sigma\alpha^{n-2}}{2r}\int_{r-\alpha}^{r+\alpha}\beta\Phi_{(\beta)}d\beta \tag{6}$$

is the relation between the potential of the central point mass and the potential of its corresponding n-dimensional shell. In this relation σ is the 2-dimensional mass density of the n-dimensional shell, thus (6) has no meaning for less than three dimensions. The correspondent equivalent masses $m_{(\alpha)}$ will be defined depending on the gravitational potential. In equation (6) we made the hypothesis that the gravitational potential of the n-dimensional shell is due to one dimension only, defined as a thickness.

A (*n*-1)-dimensional surface, with every dimension defined by the radius *r* is, (Weeks, 1985):

$$S_{n-1} = V_n \frac{n}{r}$$

For *n* even we have:

$$V_n = (1/(n/2)!)\pi^{n/2}r^n$$

and for *n* odd:

$$V_n = (2^n((n-1)/2)!\pi^{(n-1)/2}r^n$$

Equation (6) has the solutions:

$$\Phi_n(r) = \frac{G_n}{r} \tag{7}$$

with the equivalent mass:

$$m_{(\alpha)} = V_n n\sigma\alpha^{n-1}$$

and the corresponding Yukawa-like potentials:

$$\Phi_n^{1Y} = \frac{G_n e^{-\xi r}}{r},\ \Phi_n^{2Y} = \frac{G_n e^{\xi r}}{r} \tag{8}$$

with the equivalent mass:

$$m_{(\alpha)} = V_n n\sigma\alpha^{n-2} \frac{sh(\xi\alpha)}{\xi}$$

The general solution:

$$\Phi = \Phi_n + \Phi_n^{1Y} \tag{9}$$

with the equivalent mass:

$$m\Phi = \sum_i m_i \Phi_i \tag{10}$$

is the same as the one obtained in (Randall & Sundrum, 1999). Nevertheless, (Ehrenfest, 1918) was stated that in a n-dimensional space $\Phi \approx 1/r^{n-2}$. To be in accordance with this statement we must modify the equation (6) as follows:

$$m_{(\alpha)}\Phi_{(r)} = \frac{V_n n\sigma\alpha^{n-2}}{2r^{n-2}} \int_{r-\alpha}^{r+\alpha} \beta^{n-2}\Phi_{(\beta)} d\beta \tag{11}$$

This equation has the solutions:

$$\Phi_n^{'} = \frac{G}{r^{n-2}} \tag{12}$$

and:

$$\Phi_n^{'1Y} = \frac{Ge^{-\xi r}}{r^{n-2}}, \ \Phi_n^{'2Y} = \frac{Ge^{\xi r}}{r^{n-2}} \tag{13}$$

with the same equivalent masses as (7) and (8). The solution (13) is a novel one but in some respect it is equivalent with (8). The universal gravity constant is influenced by scale and this dependence can be expressed by the distance at which the interaction takes place. The lower is the distance the larger is the gravity constant, $G \approx G_n r^{n-3}$. The same comments are valid for (7) and (12). The overall solution in this case must have the equivalent mass (10).

Regarding the solutions (8), they must verify the Poisson's equation:

$$\Delta\Phi = 4\pi G_n(\rho + \rho_{vac})$$

Although they are generated by the matter we must do this compromise to be in agreement with Poisson's equation, as solution (7) is. The solutions (13) must verify the Poisson's equation:

$$\Delta\Phi = 4\pi G(\rho + \rho_{vac})$$

as solution (12) does (Ehrenfest, 1918).

3. Conclusions

In this paper the results given in (Răuţ, 2011) are generalized for the multidimensional case. The results are not found in (El-Nabulsi, 2012) but they are in agreement with (Randall & Sundrum, 1999). Nevertheless, it can occur the situation when (12) and (13) are valid. In any case, the attempt to unify the two solutions can lead to some logical conclusions. On one hand, if the universal gravity constant depends on scale, this dependence can be expressed by the distance at which the interaction takes place. On the other hand, the size of interacting masses can influence the strength of interaction and therefore the physical value of the universal gravity constant. Future experimental results will determine which of (7)-(8) and (12)-(13) solutions are valid.

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