SPECTRA OF NEW JOIN OF TWO GRAPHS

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Abstract. Let G_1 and G_2 be two graph with vertex sets $V(G_1), V(G_2)$ and edge sets $E(G_1), E(G_2)$ respectively. The subdivision graph $S(G)$ of a graph G is the graph obtained by inserting a new vertex into every edges of G. The $SG-vertexjoin$ of G_1 and G_2 is denoted by $G_1 \diamondsuit G_2$ and is the graph obtained from $S(G_1) \cup G_1$ and G_2 by joining every vertex of $V(G_1)$ to every vertex of $V(G_2)$. In this paper we determine the adjacency spectra (respectively Laplacian spectra and signless Laplacian spectra) of $G_1 \diamondsuit G_2$ for a regular graph G_1 and an arbitrary graph G_2

Keywords :Adjacency Spectrum, L - Spectrum, Q -Spectrum, Join of graphs

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1. Introduction

All graphs described in this paper are simple and undirected. Let G $= (V(G), E(G))$ be a graph with vertex set $\{v_1, v_2, \cdots v_n\}$ and edge set $\{e_1, e_2, \cdots e_m\}$. The adjacency matrix of G denoted by $A(G) = (a_{ij})_{n\times n}$ is an $n \times n$ symmetric metrix with

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

Let d_i be the degree of the vertex v_i in G and $D(G) = diag(d_1, d_2, \cdots d_n)$ be the diagonal matrix of G . The Laplacian matrix and signless Laplacian matrix are defined as $L(G) = D(G) - A(G)$ and $Q(G) = D(G) + A(G)$ respectively. The characteristic polynomial of A of G is defined as $f_G(A : x) = det(xI_n - A)$ where I_n is the identity matrix of order n. The roots of the characteristic equation of A are called the *eigenvalues* of G. It is denoted by$\lambda_1(G) \geq \lambda_2(G) \geq \cdots \geq \lambda_n(G)$ and are called $A - Spectrum$ of G. The eigen values of $L(G)$ and $Q(G)$ are denoted by $0 = \mu_1(G) \leq \mu_2(G), \cdots \leq, \mu_n(G)$ and $\nu_1(G) \leq \nu_2(G), \cdots \leq \nu_n(G)$. They are called the $L - Spectrum$ and $Q - Spectrum$ of G. Since $A(G)$,$L(G)$ and $Q(G)$ are real and symmetric, their eigen values are all real numbers. The subdivision graph $S(G)$ [2] of G is the graph obtained by inserting an additional vertex in each edge of G. Equivalently, each edge of G is replaced by a path of length 2.

The incidence matrix of G is the $0 - 1$ matrix R with rows indexed by vertices and column by edges where $R_{ve} = 1$ when the vertex v is an end point of the edge e and 0 otherwise.

In Graph Theory, every graph can be expressed in terms of certain real, symmetric matrices derived from the graph, most notably the adjacency or Laplacian or

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signless Laplace matrices. The idea of spectral graph theory is to exploit the relation between graphs and matrices inorder to study problems with graphs by means of eigen values of some graph matrices. The spectral theory of graphs consists of all the special theories including their interaction. Spectral Graph theory focuses on the set of eigenvalues and eigenvectors, called the spectrum, of these matrices and provides several interesting areas of study.Spectral graph theory is where graph theory and matrix theory meet.

The adjacency spectrum of a graph consist of the eigen value (together with their multiplicities) and the Laplacian (signless Laplacian) spectrum of G consist of the Laplace (signless Laplace) eigen values together with their multiplicities.Two graphs G and H are said to be cospectral if they have the same spectrum.Upto now numerous examples of cospectral but non-isomorphic graphs are reported [3, 9, 10]. But, only few graphs with every special structure have been proved to be determined by their spectrum.A graph is $A-integral$ if the $A-spectrum$ consist only of integers [7, 6, 19] "Which graph are determined by their spectrum?" [11] seems to be a difficult problem in the theory of graph spectrum. The idea of spectral graph theory is to exploit numerous relations between graphs and matrices inorder to study problems with graphs by means of eigen values of some graph matrices. It is a theory in which graphs are studied by means of the eigen values of some matrix(based on adjacency, Laplacian matrices etc).

The characteristic polynomial and spectra of graphs helps to investigate some properties of graphs such as energy [14, 20], number of spanning tree [21, 15], the Kirchoff index [5, 8, 16], Laplace- energy - like invarients [13] and so on.

The energy $E(G)=\sum_{i=1}^{n}|\lambda_i|$, Kirchoff index, $Kf=n\sum_{i=2}^{n}\frac{1}{\mu_i}$ and Laplace-energy - like invarients $LEL=\sum_{i=2}^{n}\sqrt{\mu_i}$

Laplacian matrix has a long history. The first result is by Kirchoff in an 1847 paper concentrated with the electrical network. There exist a vast leterature that studies the Laplacian eigen values and their relationship with various properties of graph [17, 18]. Most of the studies of the Laplacian eigen values has naturally concentrate on external non trivial eigen value. Gutman et al. [20] discovered connection betwen photoelectron spectra of standard hydrocarbones and the Laplacian eigen values of the undelying molecular graphs.

The signless Laplace spectrum performs better in comparison with other commonly used graph matrices. An idea was expressed in [11] among the matrices associated with agraph. The signless Laplacian spectrum seems to be the most convenient for use in studying graph properties [3] .Several paper on the signless Laplacian spectrum are published by D. Cvetkovic. In [7]the spectra of subdivision vertex join and subdivision - edge join are introduced and their $A-Specta$ are computed. In this paper we define $SG-vertex$ join of two graphs and also find the $A-Spectrum$ ($L-spectrum$ and $Q-Spectrum$) of the new join.

2. **Priliminaries**

Lemma 2.1. [4] *Let G be a r-regular graph with an adjacency matrix A and an incidence matrix R. Let $L(G)$ be its line graph. Then $RR^T=A+rI$ and $R^TR=A(L(G))+2I$.*

Proposition 2.2. [4, 1] *Let $M_1.M_2, M_3, M_4$ be respectively $p \times p, p \times q, q \times p, q \times q$ matrix with M_1 and M_4 are invertible then*

$$det \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} = det(M_1)det(M_4 - M_3 M_1^{-1} M_2)$$

$$= det(M_4)det(M_1 - M_2 M_4^{-1} M_3)$$

Definition 2.3. [10] Let G be a graph on n vertices, with the adjacency matrix A. The characteristic matrix $xI - A$ of A has determinant $det(xI - A) = f_G(A : x) \neq 0$,so is invertible. The $A-coronal$, $\Gamma_A(x)$ of G is defined to be the sum of the entries of the matrix $(xI - A)^{-1}$. This can be calculated as

$$\Gamma_A(x) = \mathbf{1}_n^T (xI - A)^{-1} \mathbf{1}_n$$

Lemma 2.4. [10] *Let G be r -regular on n vertices. Then*

$$\Gamma_A(x) = \frac{n}{x - r}$$

Since for any graph G with N vertices, each row sum of the Laplacian matrix $L(G)$ is equal to 0, we have

$$\Gamma_L(x) = \frac{n}{x}$$

Lemma 2.5. [10] *Let G be the bipartite graph K_{pq} where $p + q = n$ then*

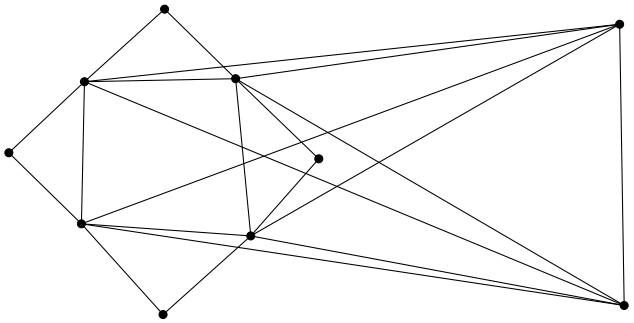
$$\Gamma_A(x) = \frac{nx + 2pq}{x^2 - pq}$$

Proposition 2.6. *Let A be an $n \times n$ real matrix, and $J_{s \times t}$ denote the $s \times t$ matrix with all entries equal to one. Then $det(A + \alpha J_n \times n) = det(A) + \alpha \mathbf{1}_n^T adj(A) \mathbf{1}^n$*
where α is a real number and $adj(A)$ is the adjugate matrix of A.

Corollary 2.7. *Let A be an $n \times n$ real matrix. Then*
$det(xI_n - A - \alpha J_{n \times n}) = (1 - \alpha \Gamma_A(x)) det(xI_n - A)$.

Definition 2.8. Let G_1 be a graph on n_1 vertices and m_1 edges. G_2 be an arbitary graph on n_2 vertices The $SG-vertex$ join of G_1 and G_2 is denoted by $G_1 \diamondsuit G_2$ and is the graph obtained from $S(G_1) \cup G_1$ and G_2 by joining every vertex of $V(G_1)$ to every vertex of $V(G_2)$.Where $S(G_1)$ is the subdivision graph of G_1

Definition 2.9. The $SG - edge$ join of G_1 and G_2 is denoted by $G_1 \triangle G_2$ and is the graph obtained from $S(G_1) \cup G_1$ and G_2 by joining the additional vertices of $S(G_1)$ corresponding to the edges of G_1 with every vertex of $V(G_2)$.



Example $C_4 \diamondsuit K_2$

3. Spectrum of $G_1 \diamondsuit G_2$

Theorem 3.1. *Let G_1 be an r_1 - regular graph on n_1 vertices and m_1 edges. G_2 be an arbitrary graph on n_2 vertices. then,*

$$f_{G_1 \diamondsuit G_2}(A:x) = f_{G_2}(A_2:x)x^{m_1-n_1}(x^2-r_1x-2r_1-n_1x\Gamma_{A_2}(x))\prod_{i=2}^{n_1}(x^2-\lambda_i x-(\lambda_i+r_1))$$

Proof. The adjacency matrix of $G_1 \diamondsuit G_2$ is

$$A = \begin{bmatrix} A_1 & R & J_{n_1\times n_2} \\ R^T & 0_{m_1} & 0_{m_1\times n_2} \\ J_{n_2\times n_1} & 0_{n_2\times m_1} & A_2 \end{bmatrix}$$

where A_1 and A_2 are the adjacency matrix of G_1 and $G_2$and R is the incidence matrix of G_1

The Charactreristic polynomial of $G_1 \diamondsuit G_2 = f_{G_1\diamondsuit G_2}(A:x) =$

$$\begin{vmatrix} xI_{n_1}-A_1 & -R & -J \\ -R^T & xI_{m_1} & 0 \\ -J & 0 & xI_{n_2}-A_2 \end{vmatrix}$$

$$= det\ (xI_{n_2} - A_2)\ det\ S$$

where

$$S = \begin{pmatrix} xI_{n_1}-A_1 & -R \\ -R^T & xI_{m_1} \end{pmatrix} - \left(\begin{pmatrix} -J_{n_1}\times n_2 \\ 0 \end{pmatrix} (xI_{m_2}-A_2)^{-1} \begin{pmatrix} -J_{n_2}\times n_1 & 0 \end{pmatrix} \right)$$

$$= \begin{pmatrix} xI-A_1 & -R \\ -R^T & xI \end{pmatrix} - \begin{pmatrix} \Gamma_{A_2}(x)J_{n_1\times n_1} & 0 \\ 0 & o \end{pmatrix} = \begin{pmatrix} xI-A_1-\Gamma_{A_2}(x)J_{n_1\times n_1} & -R \\ -R^T & xI \end{pmatrix}$$

$$detS = det(xI-A_2)det\left((xI-a_1-\Gamma_{A_2}(x)J-R(xI_{m-1})^{-1}R^T\right)$$

$$= x^{m_1} det\left(xI - A_1 - \Gamma_{A_2}(x)J + \frac{RR^T}{x})\right)$$

$$= x^{m_1} det\left(xI - (A_1 + \frac{RR^T}{x}) - \Gamma_{A_2}(x)J\right)$$

$$= x^{m_1} det\left(xI - (A_1 + \frac{RR^T}{x})\right)\left(1 - \Gamma_{A_2}(x)\Gamma_{A_1+\frac{RR^T}{x}}(x)\right)$$

G_1 is r_1 - regular and row sum of RR^T is $2r_1$
Row sum of $A_1 + \frac{RR^T}{x}$ is $r_1 + \frac{2r_1}{x}$

$$\Gamma_{A_1+\frac{RR^T}{x}} = \frac{n_1}{x - r_1} + \frac{r_1}{x} = \frac{n_1 x}{x^2 - r_1 x - 2r_1}$$

$detS = x^{m_1}\left(1 - \Gamma_{A_2}(x)\frac{n_1 x}{x^2 - r_1 x - 2r_1}\right) det\left(xI - A_1 - \frac{A_1 + r_1 I_{n_1}}{x}\right)$
$= x^{m_1 - n_1}\left(\frac{x^2 - r_1 x - 2r_1 - n_1 x\Gamma_{A_2}}{x^2 - r_1 x - 2r_1}\right) det(x^2 I - A_1 x - A_1 - r_1 I)$
$= x^{m_1 - n_1}\left(\frac{x^2 - r_1 x - 2r_1 - n_1 x\Gamma_{A_2}}{x^2 - r_1 x - 2r_1}\right) \prod_{i=1}^{n_1}\left(x^2 - \lambda_i(G_1)x - \lambda_i(G_1) - r_1\right)$
Here we use the property that $\lambda_1(G_1) = r_1$. Then
$detS = x^{m_1 - n_1}\left(x^2 - r_1 x - 2r_1 - n_1 x\Gamma_{A_2}\right)\prod_{i=2}^{n_1}\left(x^2 - x\lambda_i(G_1) - \lambda_i(G_1) - r_1\right)$
Thus

$$f_{G_1 \diamondsuit G_2}(A : x) = f_{G_2}(A_2 : x)x^{m_1 - n_1}(x^2 - r_1 x - 2r_1 - n_1 x\Gamma_{A_2}(x))\prod_{i=2}^{n_1}(x^2 - \lambda_i x - (\lambda_i + r_1))$$

□

Corollary 3.2. *Let G_1 be an r_1 - regular graph with n_1 vertices and m_1 edges and G_2 be r_2 - regular then the $A - Spectrum$ of $G_1 \diamondsuit G_2$ consists of*

(i) $\lambda_i(G_2), for i = 2, 3, ..., n_2$
(ii) 0 *, repeated* $m_1 - n_1$ *times*
(iii) $\frac{\lambda_i(G_1) \pm \sqrt{(\lambda_i(G_1)+2)^2 + 4(r_1 - 1)}}{2}$ *for* $i = 2, 3..., n_1$
(iv) Three roots of the equation $x^3 - (r_1 + r_2)x^2 + (r_1 r_2 - 2r_1 - n_1 n_2)x + 2r_1 r_2$

Corollary 3.3. *Let G_1 be an r_1 - regular graph with n_1 vertices and m_1 edges and G_2 is $\bar{K_n}$ then the $A - Spectrum$ of $G_1 \diamondsuit G_2$ consists of*

(1) 0 *, repeated* $m_1 - n_1 + p + q - 2$ *times*
(2) $\frac{\lambda_i(G_1) \pm \sqrt{(\lambda_i(G_1)+2)^2 + 4(r_1 - 1)}}{2}$ *for* $i = 2, 3..., n_1$
(3) Four roots of the equation $x^4 - r_1 x^3 - (pq + 2r_1 + n_1(p + q))x^2 + pq(r_1 - 2n_1)x + 2pqr_1$

3.1. **Laplacian Spectrum of $G_1 \diamondsuit G_2$.**

Theorem 3.4. *Let G_1 be an r_1 - regular graph on n_1 vertices and m_1 edges. G_2 be an arbitrary graph on n_2 vertices. then,*

$f_{G_1 \diamondsuit G_2}(L : x) = f_{G_2}(L_2 : x)(\frac{x(x-2)^{m_1}}{x - n_1}(x^2 - (2 + r_1 + n_1 + n_2)x + (2n_1 + 2n_2 + n_1 r_1))$
$\prod_{1=2}^{n_2}(x - n_1 - \mu_i(G_2)\ \prod_{i=2}^{n_1}(x^2 - (2 + \mu_i(G_1 + r_1 + n_2)x + (2n_2 + 3\mu_i(G_1)))$

Proof. The Laplace adjacency matrix of $G_1 \diamondsuit G_2$ is

$$L = \begin{bmatrix} (r_1+n_2)I_{n_1} + L_1 & -R & -J_{n_1 \times n_2} \\ -R^T & 2I_{m_1} & 0_{m_1 \times n_2} \\ -J_{n_2 \times n_1} & 0_{n_2 \times m_1} & n_1 I_{n_2} - L_2 \end{bmatrix}$$

where L_1 and L_2 are the adjacency matrix of G_1 and G_2

The Laplacian Charactreristic polynomial of $G_1 \diamondsuit G_2 = f_{G_1 \diamond G_2}(L:x) =$

$$\begin{vmatrix} (x-r_1-n_2)I_{n_1} - L_1 & R & J \\ R^T & (x-2)I_{m_1} & 0 \\ J & 0 & (x-n_1)I_{n_2} - A_2 \end{vmatrix}$$

$=$

$$det((x-n_1)I_{n_2} - L_2) det S$$

where

$$S = \begin{pmatrix} (x-r_1-n_2)I_{n_1} - L_1 & R \\ R^T & (x-2)I_{m_1} \end{pmatrix} - \left(\begin{pmatrix} J_{n_1 \times n_2} \\ 0 \end{pmatrix} ((x-n_1)I_{m_1} - L_2)^{-1} \begin{pmatrix} J_{n_2 \times n_1} & 0 \end{pmatrix} \right)$$

$$= \begin{pmatrix} (x-r_1-n_2)I - L_1 & R \\ R^T & (x-2)I \end{pmatrix} - \begin{pmatrix} \Gamma_{L_2}(x-n_1)J_{n_1 \times n_1} & 0 \\ 0 & o \end{pmatrix}$$

$$= (x-2)^{m_1} det \left((x-r_1-n_2)I - L_1 - \Gamma_{L_2}(x-n_1)J - \frac{RR^T}{x-2} \right)$$

$$= (x-2)^{m_1} det \left((x-r_1-n_2)I - L_1 - \frac{RR^T}{x-2} \right) (1 - \Gamma_{L_2}(x-n_1)\Gamma_{L_1 + \frac{RR^T}{x-2}}(x-r_1-n_1)$$

$$= (x-2)^{m_1-n_1} \left(1 - \frac{n_2}{x-n_1} \frac{n_1}{x-r_1-n_2} - \frac{2r_1}{x-2} \right) det\left((x-r_1-n_2)(x-2)I - (x-2)L_1 - (A_1 + r_1 I) \right)$$

$$= \frac{x(x-2)^{m_1-n_1}}{x-n_1} \left(x_2 - (n_1+n_2+r_1+2)x + (2n_1+2n_2+n_1 r_1) \right)$$

$$\prod_{i=2}^{n_1} \left(x^2 - (2+\mu_i(G_1)+r_1+n_2)x + (2n_2+3\mu_i(G_1)) \right)$$

Hence

$$f_{G_1 \diamond G_2}(L:x) = f_{G_2}(L_2:x)(x-n_1) \frac{x(x-2)^{m_1-n_1}}{x-n_1} \left(x^2 - (n_1+n_2+r_1+2)x + 2n_1 + 2n_2 + n_1 r_1 \right)$$

$$\prod_{i=2}^{n_1} \left(x^2 - (2+\mu_i(G_1)+r_1+n_2)x + (2n_2+3\mu_i(G_1)) \right)$$

$$f_{G_1 \diamond G_2}(L:x) = x(x-2)^{m_1-n_1} \left(x^2 - (n_1+n_2+r_1+2)x + 2n_1 + 2n_2 + n_1 r_1 \right)$$

$$\prod_{i=2}^{n_2} (x - n_1 - \mu_i(G_2) \prod_{i=2}^{n_1} \left(x^2 - (2+\mu_i(G_1)+r_1+n_2)x + (2n_2+3\mu_i(G_1)) \right)$$

□

3.2. **Signess Laplacian Spectrum of $G_1 \diamondsuit G_2$.**

Theorem 3.5. *Let G_1 be an r_1 - regular graph on n_1 vertices and m_1 edges. G_2 be an arbitrary graph on n_2 vertices. then,*

$$f_{G_1 \diamondsuit G_2}(Q:x) = x(x-2)^{m_1-n_1}\left(x^2-(n_1+n_2+r_1+2)x+2n_1+2n_2+n_1r_1\right)$$

$$\prod_{i=1}^{n_2-1}(x-n_1-\mu_i(G_2))\prod_{i=1}^{n_1-1}\left(x^2-(2+\mu_i(G_1)+r_1+n_2)x+(2n_2+3\mu_i(G_1)\right)$$

Proof. The Signless Laplace adjacency matrix of $G_1 \diamondsuit G_2$ is

$$Q = \begin{bmatrix} (r_1+n_2)I_{n_1}+Q_1 & R & J_{n_1\times n_2} \\ R^T & 2I_{m_1} & 0_{m_1\times n_2} \\ J_{n_2\times n_1} & 0_{n_2\times m_1} & n_1I_{n_2}+Q_2 \end{bmatrix}$$

where Q_1 and Q_2 are the signless Laplace adjacency matrix of G_1 and G_2 respectively.

The proof of the theorem is on similar lines as that of Theorem 3.4

□

Corollary 3.6. *Let G_1 be an r_1 - regular graph on n_1 vertices and m_1 edges. G_2 be an r_2 - regular graph on n_2 vertices then,*

$$f_{G_1 \diamondsuit G_2}(Q:x) = (x-2)^{m_1-n_1}(x^3-ax^2+bx-c)\prod_{i=1}^{n_2-1}(x-n_1-\nu_i(G_2))$$

$$\prod_{i=1}^{n_1-1}(x^2-(2+r_1+n_2+\nu_i(G_1))x+2(r_1+n_2)+\nu_i(G_1))$$

Where

$$a = n_1+n_2+3r_1+2r_2+2$$
$$b = 2n_1+2n_2+4r_1+4r_2+3n_1r_1+2n_2r_2+6r_1r_2$$
$$c = 4(n_1r_1+n_2r_2+2r_1r_2)$$

Acknowledgments.

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