

# A Survey of Rational Diophantine Sextuples of Low Height

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A rational Diophantine  $m$ -tuple is a set of  $m$  distinct positive rational numbers such that the product of any two is one less than a rational number squared. A computational search is used to find over 300 examples of rational Diophantine sextuples of low height which are then analysed in terms of algebraic relationships between entries. Three examples of near-septuples are found where a rational Diophantine quintuple can be extended to sextuples in two different ways so that the combination fails to be a rational Diophantine septuple only in one pair.

## Introduction

A rational Diophantine  $m$ -tuple is a set of  $m$  positive rational numbers  $\{a_1, \dots, a_m\}$  such that the product of any two is one less than a rational number squared.

$$a_i a_j + 1 = x_{ij}^2, i \neq j, \quad a_i, x_{ij} \in \mathbb{Q}$$

Diophantus of Alexandria was able to find triples and quadruples of such numbers [1]. Fermat looked at the problem in integers and found solutions for quadruples such as  $\{1,3,8,120\}$  [2]. Euler was then able to extend this to a rational Diophantine quintuple and show that an infinite number of such quintuples exist [3].

$$1, 3, 8, 120, \frac{777480}{8288641}$$

Since 1999 the existence of rational Diophantine sextuples has been known by examples [4.5.6.7] and this year some infinite families of such sextuples have been constructed [8,9,10]. The previously known list of 45 sextuples may have helped in finding these families and it therefore seems worthwhile to extend the search for low height sextuples further (The height of an  $m$ -tuple is defined as the largest numerator or denominator that appears in the  $m$ -tuple when all entries are expressed in lowest form.)

In the case of integer solutions it is known that no Diophantine sextuple exists and there can only a finite number of quintuples of which no examples are known. In contrast very little is known about the limits in the rational case. It is known that any given rational Diophantine quadruple can only be extended to a rational Diophantine quintuple in a finite number of ways [11], but it is not known if a bound on the number of possible ways exists. As a corollary there cannot exist an  $\infty$ -tuple i.e. an infinite sequence of distinct rational numbers such that the product of any two is one less than a square.

In relation to this the present search has turned up two cases of quadruples which can be extended to quintuples in six different ways:

$$\begin{array}{r} 81 \quad 5696 \quad 2875 \quad 4928 \\ \hline 1400' \quad 4725' \quad 168' \quad 3 \end{array}$$

Can be extended to a quintuple using any one of these rationals:

$$\begin{array}{r} 98 \quad 104 \quad 96849 \quad 1549429 \quad 3714303488 \quad 7694337252154322 \\ \hline 27' \quad 525' \quad 350' \quad 1376646' \quad 6103383075' \quad 1857424629984075 \end{array}$$

This second quadruple

$$\begin{array}{r} 152 \quad 2665 \quad 3906 \quad 1224 \\ \hline 357' \quad 2856' \quad 17' \quad 12943 \end{array}$$

Can be extended with any of

$$\begin{array}{r} 1519 \quad 4505 \quad 1959335 \quad 13303605 \quad 73026883629 \quad 515358540182255 \\ \hline 408' \quad 168' \quad 7824984' \quad 1077512' \quad 17054089928' \quad 7116911275416 \end{array}$$

Finally, the Lang conjecture if true would imply that there is an upper bound on the number of ways to extend a quintuple to a sextuple and therefore also that there is an upper bound on the number of elements in a m-tuple [12].

## Search Methodology

The objective is to find and analyse rational Diophantine sextuples of low height. Sextuples of low height are thought to be more likely to have further extensions to rational Diophantine septuples, so if there is a way to find them in large numbers it can be used to search effectively for such septuples.

There are many possible strategies for such a search but they mostly follow the following steps

**Step 1:** Generate large numbers of rational Diophantine triples (or quadruples) of low height. For this survey a brute force search for all examples up to a given height was used to form the main base of triples. It is not hard to generate all triples up to a height of 1000 in this way and a partial search up to a height of 3000 was also used.

**Step 2:** Find ways to extend the triples to as many rational Diophantine quadruples as possible (or quadruples to quintuples.) One way to do this is to resolve the elliptic curve generated by the triple [M,N]. This is the best technique if many extensions are desired but in practice the larger solutions are unlikely to form sextuples except in special cases that are well understood (see below.) It is therefore sufficient to extend using simpler methods. This can be done by a combination of extending triples to regular quadruples[13] and quadruples to regular quintuples [14] by well-known known methods. In addition I give below a parametric solution to extending rational Diophantine pairs to triples that was used extensively in this search.

**Step 3:** Scan all rationals that extend the triple to a quadruple in pairs to see which form quintuples. Checking whether large rational numbers are squares is quite a costly part of the algorithm so it is important to do this only once for each pair and store the results in a Boolean array. This can then be used to quickly check if there are any triples amongst the extension list which would then complete a rational Diophantine sextuple.

Once a sextuple is found it is worthwhile to carry out a longer search up to higher numbers to see if it can be extended further.

The algorithm was implemented in Java using the BigInteger class for the arithmetic on large numbers.

## Extending Rational Pairs to Triples

A critical element of the algorithm is the extension of rational Diophantine pairs to rational Diophantine triples. This can be used to form the base set of triples for extension if desired but it is also effective as a way of extending rational triples to quadruples as an alternative to elliptic curve methods. I.e. to extend a triple take the pair of lowest height and use this method to extend it to triples. These can be checked against the third element of the original triple to see if a rational Diophantine quadruple is formed.

Assume then that that  $\{a, b\}$  is a rational Diophantine pair, i.e.  $ab + 1 = x^2, b > a$

This will be extended to a triple by a rational number  $c$  when

$$ac + 1 = y^2, bc + 1 = z^2$$

This can be solved given  $(y, z)$  such that

$$c = \frac{y^2 - 1}{a} = \frac{z^2 - 1}{b}$$

All solutions can be parameterised by the rational gradient

$$r = \frac{p}{q} = \frac{z - 1}{y - 1}$$

Which leads to a pair of linear equations in  $y$  and  $z$

$$ry - z = r - 1$$

$$by - arz = ar - b$$

Eliminating  $z$

$$y = \frac{2ar - ar^2 - b}{b - ar^2}$$

$$c = \frac{y^2 - 1}{a} = \frac{4(ar - b)r(1 - r)}{(b - ar^2)^2}$$

Or in terms of the integers  $p, q$

$$c = \frac{4(ap - bq)pq(q - p)}{(bq^2 - ap^2)^2}$$

## Search Results

The full table of results consisting of 307 rational Diophantine Sextuples is included in the Annex. The fractions within each sextuple are ordered by weight and the sextuples themselves are ordered by weight.

Each sextuple is followed by symbols indicating algebraic relationships between the fractions. The letters in round brackets, e.g. (a,c,d,f), indicate sub-tuples which are regular pairs, triples, quadruples and quintuples [4]. The six letters represent the fractions as given in the order a,b,c,d,e,f . When the letters appear in square brackets e.g [a,c,d,f] this means that the reciprocals of the numbers satisfy the equation for a regular  $m$ -tuple. Regular sextuples are also shown in square brackets since in this case the inverses always also satisfy the same equation [15]. The possibility that a product of fractions equals one was also tested and was indicated in the one case where it occurred with [abcf = 1].

A special relationship for a triple given in [10] is indicated as <>a b c>>. When this relationship is met the elliptic curve induced by the triple has torsion group  $Z2 \times Z6$  with the result that it can be extended to a rational Diophantine sextuple containing three regular quintuples.

Another relationship equivalent to  $(abcd - 1)^2 = 4(ab + 1)(cd + 1)$  is indicated by <>ab|cd>> [9,10]. When this condition is met the two extensions of the quadruple to a quintuple form a sextuple with two regular quintuples and two regular quadruples.

## Notable Cases

The most notable cases are pairs of sextuples which have a quintuple in common. These form a near-septuple which is deficient only in one pair of numbers whose product is not one less than a square. There are three examples of this in the table from pairs numbered (22,67) (124,125) and (182,183). When combined they form the following near-septuples where only the last two numbers do not produce a square.

243/560 1147/5040 1100/63 7820/567 95/112 38269/6480 196/45  
 (a,b,d,f) (b,c,e,f) (a,b,c,d,g) (a,b,e,g) (a,c,e,g) (b,d,e,g)

7657/420 480/91 441/260 425/1092 191840/273 13/105 43953/39605  
 (a,b,c,e) (a,b,c,f) (b,c,d,f) (a,c,d,e,f) (a,b,c,d,g)

518/45 7344/185 4004/1665 25900/690561 216/185 100/333 166600/37

(a,b,e,f) (a,c,e,f) (a,b,c,g) (a,c,d,e,g) [a,b,c,d,e,f]

There are also pairs of sextuples which share triples. Most cases of this occur when a triple induces an elliptic curve with  $Z2 \times Z6$  torsion and rank greater than zero. Examples in the results table are pairs (172,292) and (293,295). It is known that in this situation there is in fact an infinite family of sextuples containing the same triple [S]. There is one further pair which shares a triple and which is not of this type (153,159).

The search also turned up a notable quintuple which is the first known whose common denominator is a prime number

$$\frac{4}{23}, \frac{374}{23}, \frac{420}{23}, 276, 328440$$

## Classification

The most useful way to classify sextuples is in terms of the pattern of regular quadruples and quintuples as follows (the number of instances is shown)

70 (a,b,c,d,e) (a,b,c,f)  
63 (a,b,c,d,e) (a,b,c,d,f) (a,b,e,f) (c,d,e,f)  
55 (a,b,c,d,e) (a,b,c,d,f)  
50 (a,b,c,d,e) (a,b,c,f) (a,b,d,f)  
18 (a,b,c,d,e) (a,c,d,f) (a,b,e,f)  
11 (a,b,c,d,e)  
10 (a,b,c,d,e) (a,b,c,d,f) (a,b,c,e,f)  
6 (a,b,c,d,e) (a,b,c,f) (a,b,d,f) (a,c,e,f)  
4 (a,b,c,d,e) (a,b,c,d,f) (a,b,e,f)  
4 (a,c,d,e) (a,b,c,f) (b,d,e,f)  
4 (a,b,c,d) (a,b,c,e)  
3 (a,b,c,d)  
2 (a,b,c,d,e) (a,b,c,d) (a,c,d,f) (a,b,e,f)  
1 (a,b,c,d,e) (a,b,c,f) (b,c,d,f) (a,d,e,f)  
1 (a,b,c,d,e) (a,b,c,e) (a,c,d,f) (b,d,e,f)  
1 (a,b,c,d,e) (a,b,c,d,f) (a,b,e,f) (a,c,e,f) (b,d,e,f)  
1 (a,b,c,d,e) (a,b,c,d,f) (a,b,c,e) (a,c,e,f) (b,d,e,f)  
1 (a,b,c,d,e) (a,b,c,d) (a,b,c,f)  
1 (a,b,c,e) (a,b,d,f)  
1 none

While some classes have been analysed and understood there are others with multiple examples that have not yet.

## Acknowledgments

I thank Andrej Dujella for highly useful discussions relating to this work.

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## Annex: Table of Results

|   |   |
|---|---|
| (1) 96/847 287/484 1209/700 129/28 147/25 847/4             | (a,b,c,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b d,f>>         |
| (2) 221/1260 175/324 203/180 81/35 265/28 1120/9            | (a,c,d,e) (a,b,d,e,f) (c,d,e,f)                                 |
| (3) 225/1156 47/60 287/240 225/64 1463/60 512/15            | (b,c,d,e) (b,e,f) (a,b,c,e,f)                                   |
| (4) 39/1760 136/165 275/96 672/55 1010/33 1320              | (a,b,c) (a,c,d,e) (a,b,c,d,f) (a,b,c,e,f) (b,d,e,f) <<a,c b,f>> |
| (5) 32/525 224/867 53/84 273/100 75/7 1953/100              | (a,c,d) (a,b,c,d,e) (a,c,e,f) (b,c,d,e,f)                       |
| (6) 377/1260 119/180 297/140 992/315 175/9 2275/4           | (a,b,d) (a,c,d,e) (a,b,c,d,f) (c,d,e,f)                         |
| (7) 5/36 665/1521 5/4 3213/676 189/4                        | (a,c,d) (a,b,c,e) (a,b,c,d,f) (b,d,e,f) (a,c,d,e,f) <<a,c d,f>> |
| (8) 256/2783 2829/3872 363/184 3627/736 2163/184 989/2      | (a,b,d,e) (a,b,c,d,f) (a,b,c,e,f) (c,d,e,f) <<a,b c,f>>         |
| (9) 56/465 1240/4107 175/93 792/155 3312/155 2635/3         | (a,b,c,e,f) (c,d,e,f)   |
| (10) 925/4116 99/140 25/21 119/60 608/105 693/20            | (a,b,c,e) (b,d,e) (b,c,e,f) (a,c,d,e,f)                         |
| (11) 12/91 429/1372 160/91 1881/364 221/28 4116/13          | (a,b,c,d) (a,d,e) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b e,f>> |
| (12) 171/1372 44/133 480/133 343/76 4257/532 3876/7         | (a,b,c,e) (b,d,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b d,f>> |
| (13) 9/44 91/132 4420/3993 60/11 44/3 1265/12               | (a,b,c,d) (a,b,d,e) (a,b,c,e,f) (a,d,e,f)                       |
| (14) 9/140 220/343 4929/3500 4524/875 35/4 1428/5           | (a,b,c,d) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b e,f>>         |
| (15) 81/1400 104/525 5696/4725 98/27 2875/168 4928/3        | (b,c,d) (a,b,c,d,e) (a,b,d,e,f)                                 |
| (16) 249/2048 3720/6241 715/384 369/128 38/3 920/3          | (a,c,d,e) (a,b,c,e,f) (c,d,e,f) [a,b,c,d,e,f]                   |
| (17) 215/6864 128/429 1235/528 2457/176 781/39 6435/16      | (a,b,c) (a,b,d,e) (a,b,c,e,f) (b,d,e,f)                         |
| (18) 3/80 2220/6889 55/16 28/5 1683/80 1680                 | (a,c,d) (a,c,d,e) (a,b,d,e,f) (c,d,e,f)                         |
| (19) 33/152 7360/5491 4275/2312 1209/152 19/2 1920/19       | (a,b,c,e) (b,c,d,f) (a,d,e,f) [a,b,c,d,e,f]                     |
| (20) 11/152 2139/7448 1368/2401 2958/931 392/19 969/8       | (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c e,f>>         |
| (21) 225/1856 475/1392 512/435 667/240 7749/580 1363/60     | (b,c,d,e) (a,b,c,d,f) (a,b,e,f) (c,e,f) (a,c,d,e,f) <<a,f c,d>> |
| (22) 1147/5040 243/560 95/112 196/45 7820/567 1100/63       | (a,b,c,d) (a,c,d,e) (b,c,d,f) (a,b,d,e,f)                       |
| (23) 6/125 1144/1445 483/160 56/5 375/32 8184/5             | (a,b,c,e,f) (c,d,e,f)   |
| (24) 81/560 196/135 285/112 1100/189 7820/189 8463/80       | (a,b,c,d,e) (b,c,d,f) (a,b,e,f)                                 |
| (25) 559/1980 639/880 1024/495 1071/220 8671/880 3025/36    | (a,c,d) (b,d,e) (a,b,c,d,f) (b,c,e,f) (a,d,e,f)                 |
| (26) 99/532 76/343 2080/1197 1273/252 1372/171 8721/28      | (a,b,c,d) (a,d,e) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b e,f>> |
| (27) 957/7840 3840/5329 426/245 1813/160 1640/49 8745/32    | (a,c,d,e) (a,b,c,d,f) (a,d,e,f)                                 |
| (28) 1860/9317 115/308 28/55 777/220 396/35 825/28          | (a,b,c,d) (b,c,d,e) (b,c,e,f) (a,c,d,e,f)                       |
| (29) 2324/9801 243/484 20/9 5600/1089 4235/324 155/4        | (a,b,d,e) (b,c,d,f) (a,b,c,e,f)                                 |
| (30) 256/2601 2619/9248 395/288 35/8 3179/288 10395/578     | (a,b,c,d) (c,d,e) (a,b,e,f) (a,c,d,e,f)                         |
| (31) 76/1029 5236/11163 420/169 2044/507 370/21 5280/7      | (a,b,c,e,f) (c,d,e,f)   |
| (32) 189/800 8/9 378/289 608/225 325/32 11528/225           | (a,b,d,e) (a,b,c,e,f) (a,d,e,f)                                 |
| (33) 2310/9409 141/88 11704/5547 200/33 341/24 6552/11      | (b,d,e) (a,b,c,d,e) (b,d,e,f)                                   |
| (34) 85/1716 3483/2860 2299/780 7136/2145 12727/540 2080/33 | (a,b,d) (b,c,d,f) (a,b,c,e,f) (a,d,e,f)                         |
| (35) 1224/12943 152/357 2665/2856 1519/408 4505/168 3906/17 | (b,c,d) (a,b,c,d,e) (b,d,e,f)                                   |
| (36) 32/405 205/324 2080/729 14805/3364 153/20 77/5         | (a,b,c,e) (a,b,e,f) (a,c,d,e,f)                                 |
| (37) 3/200 174/169 27/8 136/25 16275/1352 200               | (a,c,d) (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <<a,d c,f>> |
| (38) 77/380 57/55 1632/1045 957/380 893/220 16800/209       | (a,c,e) (a,b,d,e,f) (c,d,e,f)                                   |

|   |   |
|---|---|
| (39) 35/1488 341/768 11439/5776 1280/93 16835/93 9207/16          | (a,b,c,d,e) (a,d,e,f)   |
| (40) 1080/11881 1495/1152 4239/2048 17719/4608 62/9 56            | (b,c,d,f) (a,b,d,e,f)   |
| (41) 32/297 497/3300 583/75 1625/132 18377/1188 9801/100          | (a,b,c,d) (a,d,e) (b,c,d,f) (a,c,e,f) (a,b,d,e,f)                 |
| (42) 15/5488 7843/14000 105/16 528/35 19227/875 1904/5            | (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <<a,d c,f>>           |
| (43) 930/19321 56/93 1131/248 6721/744 840/31 13640/3             | (b,c,d) (a,b,c,d,f) (c,d,e,f)                                     |
| (44) 6292/19881 680/1287 740/143 1026/143 1196/99 16588/9         | (c,d) (b,d,e) (a,b,c,d,e) (c,d,e,f)                               |
| (45) 6496/19965 35/33 215/132 209/60 1953/220 1683/20             | (a,b,c,e) (b,d,e) (a,b,c,d,f) (b,c,e,f)                           |
| (46) 75/656 3567/5120 1155/656 768/205 20503/3280 943/5           | (a,b,c,e) (a,d,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b d,f>>   |
| (47) 17/448 2352/7921 265/448 2145/448 252 23460/7                | (a,b,c,e,f) (c,d,e,f)   |
| (48) 33/140 287/660 1525/924 441/55 3575/84 24288/35              | (b,c,d,e) (a,b,c,d,f) (b,d,e,f)                                   |
| (49) 2376/24367 208/693 391/77 7560/1331 1760/63 5635/99          | (b,c,d,f) (c,e,f) (a,b,c,e,f)                                     |
| (50) 3069/6724 24101/26244 2144/2025 1325/324 3825/529 704/25     | (b,c,d,f) (a,b,c,e,f)   |
| (51) 264/2023 867/2744 83/56 26432/11449 6552/289 1218            | (a,b,c,d,e) (a,b,c,e,f)   |
| (52) 43/324 5760/12769 1147/1296 9243/6724 26752/6561 187/16      | (a,b,c,d,f) (a,c,e,f)   |
| (53) 585/27556 6789/15028 27520/17797 165/52 224/13 741           | (a,b,d,e,f) (a,c,d,e,f)   |
| (54) 549/4900 6272/28227 23375/28812 323/147 741/100 1056/25      | (a,c,d,e) (b,c,d,e,f)   |
| (55) 2232/27455 209/1440 742/855 15200/2601 28971/3040 1312/95    | (a,b,d,e) (a,c,d,f) (b,c,d,e,f)                                   |
| (56) 13/160 30525/30056 165/104 117/40 18837/2890 1280/13         | (a,b,c,e) (a,b,c,d,f) (b,d,e,f) (a,c,d,e,f) <<a,c d,f>>           |
| (57) 57/1540 55/28 224/55 1071/220 5133/385 30855/28              | (a,b,c) (a,b,c,e) (b,d,e) (a,b,c,d,f) (c,d,e,f)                   |
| (58) 4/35 495/1372 18139/31500 26884/7875 140/9 3213/20           | (a,b,c,d) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b e,f>>           |
| (59) 13/55 3267/5780 725/396 2752/495 2387/180 31968/55           | (a,b,c,d,f) (c,d,e,f)   |
| (60) 3692/32805 5832/18605 220/81 970/81 212/5 28152/5            | (a,c,d,e) (a,b,c,d,f) (c,d,e,f)                                   |
| (61) 57/2960 52/185 6205/592 1221/80 33300/841 8580/37            | (a,b,c,d) (b,c,d,f) (a,b,d,e,f)                                   |
| (62) 1525/33396 299/2700 28/69 14725/2484 6804/575 299/12         | (b,c,d,e) (a,b,c,e,f)   |
| (63) 220/3213 98/255 580/357 1836/35 34720/459 34408/105          | (a,b,d,e) (a,b,c,d,f) (b,c,e,f)                                   |
| (64) 17/120 728/2295 122/255 1325/408 5643/680 35224/15           | (a,b,c,d) (a,c,d,e) (a,b,d,e,f)                                   |
| (65) 1147/14196 63/484 1312/2541 483/169 36421/2028 1521/28       | (a,b,c,d) (a,b,c,e,f) (a,d,e,f) (b,c,d,e,f) <<b,c e,f>>           |
| (66) 171/1100 10400/30723 748/1425 37873/11172 1881/196 14700/209 | (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c e,f>>           |
| (67) 1147/5040 243/560 95/112 38269/6480 7820/567 1100/63         | (a,b,d,e) (a,c,d,f)   |
| (68) 693/1520 5525/6384 969/560 39900/10201 988/105 9856/285      | (a,b,c,e) (a,b,c,d,f) (a,b,e,f)                                   |
| (69) 759/2450 64/147 14896/18723 39904/3675 402/25 1625/6         | (b,d,e) (a,b,c,e,f) (a,d,e,f)                                     |
| (70) 232/1875 150/529 209/96 40672/1587 24864/625 3675/32         | (a,b,d,e) (b,c,d,f) (a,c,e,f) (a,b,c,e,f) [a,b,c,d,e,f]           |
| (71) 27720/44521 1035/1408 27863/30720 296/165 14105/4224 286/15  | (b,c,e,f) (a,c,d,e,f)   |
| (72) 15/364 28083/45500 1716/1715 3796/875 273/20 2380/13         | (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c e,f>>           |
| (73) 55/372 8928/11449 799/372 427/93 2880/31 46345/12            | (a,c) (a,c,d) (a,b,c,d,f) (c,d,e,f)                               |
| (74) 899/10080 135/896 171/70 385/72 1135/56 50176/45             | (a,b,c,d) (a,c,d,e) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b e,f>> |
| (75) 25/456 184/285 8866/2565 1007/120 1512/95 51623/1080         | (a,b,c,d) (a,b,d,e) (a,c,e,f) (a,b,d,e,f)                         |
| (76) 2261/37752 29/78 989/1248 52793/24576 819/8 30447/104        | (b,c,e,f) [a,b,c,d,e,f]   |
| (77) 158/13125 133/600 200/189 53053/15000 297/56 56/3            | (a,b,c,d) (a,b,c,e,f) (a,d,e,f) (b,c,d,e,f) <<b,c e,f>>           |
| (78) 851/2520 35775/53816 4536/4805 917/360 1768/315 950/63       | (a,b,c,e) (a,d,e) (b,c,d,f) (a,c,d,e,f)                           |
| (79) 1274/15987 28305/57344 5995/10752 735/512 152/21 288/7       | (b,c,d,e) (a,b,c,e,f) (c,d,e,f)                                   |
| (80) 1695/23276 3/11 143/108 59840/14283 1335/44 29536/297        | (a,b,c,d) (b,c,e,f) (a,d,e,f) [a,b,c,d,e,f]                       |

- (81) 1700/19881 2132/5625 108/25 158/25 572/9 63800/9 \_\_\_\_\_ (c,d) (b,c,d,e) (a,b,c,d,f) (c,d,e,f)
- (82) 37/540 1107/740 4081/2220 800/111 19040/999 63973/60 \_\_\_\_\_ (b,c,d) (a,b,d,e) (a,b,c,d,f) (c,d,e,f)
- (83) 99/2800 10500/5329 52/21 65593/8400 357/16 44772/25 \_\_\_\_\_ (a,c,d,e) (a,b,c,d,f) (c,d,e,f)
- (84) 85/1248 507/544 1071/416 5320/663 3870/221 66304/663 \_\_\_\_\_ (a,b,c,d) (a,b,d,e) (b,c,d,f) (a,b,c,e,f)
- (85) 33/280 77/270 384/385 9275/2376 70499/7560 2625/88 \_\_\_\_\_ (a,b,c,d) (c,d,e) (b,c,e,f) (a,c,d,e,f) [abcf = 1]
- (86) 6525/54872 238/285 209/120 2368/285 2925/152 70889/120 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,d,e,f)
- (87) 627/3584 20480/73101 267/224 1519/384 1105/168 555/14 \_\_\_\_\_ (a,b,c,d,f) (a,d,e,f)
- (88) 17/120 2088/12005 50/147 5499/1960 833/120 73800/361 \_\_\_\_\_ (a,b,c,d) (a,c,d,e) (a,b,d,e,f)
- (89) 184/273 1521/896 38465/19968 760/273 1870/273 82467/1664 \_\_\_\_\_ (a,d,e) (a,b,c,d,e) (b,c,d,f) (a,b,e,f)
- (90) 135/1696 8480/12321 83160/72557 1246/477 928/53 36835/288 \_\_\_\_\_ (b,c,d,e) (a,b,d,e,f)
- (91) 140/1479 85/348 17748/18769 30485/5916 1827/68 85527/116 \_\_\_\_\_ (a,b,c,d,e) (a,b,c,e,f) [a,b,c,e]
- (92) 3335/40344 17856/85805 33/40 45/8 70/3 59059/120 \_\_\_\_\_ (a,b,c,d,e) (c,d,e,f)
- (93) 20075/74892 179/300 86625/27556 1316/75 299/12 836/3 \_\_\_\_\_ (b,d,e)
- (94) 145/572 1173/2860 50820/89557 132/65 364/55 1365/44 \_\_\_\_\_ (a,b,d,e) (a,d,e,f) (b,c,d,e,f)
- (95) 6656/17457 847/624 855/572 377/132 3315/704 91805/1716 \_\_\_\_\_ (a,b,c,d,f) (b,c,e,f)
- (96) 44268/96721 875/972 24893/18900 2324/675 159/28 4884/175 \_\_\_\_\_ (b,c,d,f) (a,b,c,e,f)
- (97) 8/27 19/24 2318/1875 297/200 98371/15000 200/3 \_\_\_\_\_ (a,b,c,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b|d,f>>
- (98) 24/539 220/289 6468/5041 40/11 7332/539 98670/2401 \_\_\_\_\_ (a,b,c,d,f) (a,d,e,f)
- (99) 136/585 1215/1768 4199/3240 3520/1989 98696/6885 910/17 \_\_\_\_\_ (b,c,d,e) (a,b,c,d,f) (a,b,c,e,f) (a,d,e,f) <<a,f|b,c>>
- (100) 7/120 3480/10201 10600/11163 99138/17405 429/40 1064/15 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)
- (101) 3213/99944 1729/1800 4664/2925 56/13 21573/2600 650/9 \_\_\_\_\_ (a,b,c,d,f) (b,c,e,f)
- (102) 6300/100489 876/4805 111/80 231/80 76/5 4515/16 \_\_\_\_\_ (a,b,c,d,e) (c,d,e,f)
- (103) 85/696 58/147 7337/1176 6517/696 360/29 106560/1421 \_\_\_\_\_ (a,b,c,e) (a,d,e) (a,b,c,d,f) (a,c,e,f)
- (104) 177/544 1400/1003 102/59 109032/28577 6840/1003 54315/1888 \_\_\_\_\_ (b,c,e) (a,b,c,d,f) (a,b,e,f)
- (105) 16/275 179/1859 561/400 112875/29744 144/11 825/16 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>
- (106) 154/1593 1480/1593 2679/1888 4960/1593 885/32 114696/1681 \_\_\_\_\_ (a,b,d) (b,c,d,e) (a,b,c,e,f)
- (107) 8845/127008 8/49 13640/3969 1274/81 765/32 72 \_\_\_\_\_ (b,c,d,f) (a,c,e,f) (a,b,d,e,f)
- (108) 1/10 109725/131072 5280/1681 2485/512 31521/2560 288/5 \_\_\_\_\_ (a,b,d,e) (a,b,c,e,f) (a,d,e,f)
- (109) 108/595 820/1071 133280/71289 3618/595 1960/153 9724/35 \_\_\_\_\_ (a,b,c,d,f) (b,d,e,f)
- (110) 6475/133956 187/900 2772/6889 868/225 27/4 100 \_\_\_\_\_ (b,d,e) (a,b,d,e,f) (b,c,d,e,f)
- (111) 43/1700 130272/134657 1743/425 51/4 29667/1700 14875/4 \_\_\_\_\_ (a,b,c,d,f) (c,d,e,f)
- (112) 6580/145119 145/624 1393/624 8944/867 252/13 1287/16 \_\_\_\_\_ (b,c) (b,c,e,f)
- (113) 125/308 3741/7700 343/275 1056/175 148896/8575 759/28 \_\_\_\_\_ (a,b,c,d) (a,b,d,e) (a,c,d,f) (a,b,c,e,f)
- (114) 77/360 755/792 90155/64152 28475/10648 567/110 155584/3645 \_\_\_\_\_ (b,c,e,f) (a,b,d,e,f)
- (115) 357/10400 31122/156025 264/325 1001/800 2125/416 456/13 \_\_\_\_\_ (a,c,d,e) (c,d,e,f) [b,c,d,f]
- (116) 16560/161051 1456/3795 285/506 1472/165 1155/46 8602/15 \_\_\_\_\_ (b,c,d,e) (a,b,c,e,f) (c,d,e,f)
- (117) 7030/79707 2013/1480 164169/67240 2408/555 1295/24 51480/37 \_\_\_\_\_ (a,b,c,d,f) (b,d,e,f)
- (118) 18676/75615 111600/170723 434/435 1180/609 1848/145 4524/35 \_\_\_\_\_ (b,c,d,e) (a,b,c,e,f) (c,d,e,f)
- (119) 536/1827 98/261 143964/170723 40716/35287 1148/261 1276/7 \_\_\_\_\_ (a,b,c,e) (a,b,d,e,f) (b,c,d,e,f)
- (120) 18/91 31465/90168 320/273 403/168 175680/26299 637/24 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <<a,d|c,f>>
- (121) 11/192 35/192 155/27 512/27 1235/48 180873/16 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (c,d,e,f)
- (122) 125/1092 441/1300 2717/525 5408/525 1632/91 187829/1092 \_\_\_\_\_ (a,b,c,d) (a,b,d,e) (a,b,c,d,f) (b,c,e,f)

(123) 165/592 104481/189440 2975/1776 3515/1587 1792/555 1591/240 \_\_\_\_\_ (a,b,c,f) (b,c,d,e,f)  
 (124) 13/105 425/1092 441/260 480/91 7657/420 191840/273 \_\_\_\_\_ (a,b,c,d) (a,c,d,e) (a,b,c,e,f) (c,d,e,f)  
 (125) 425/1092 43953/39605 441/260 480/91 7657/420 191840/273 \_\_\_\_\_ (a,b,c,d,e) (c,d,e,f)  
 (126) 8832/37303 525/968 712/847 2299/1400 197925/42632 504/25 \_\_\_\_\_ (a,b,c,e) (a,b,c,d,f) (a,d,e,f) (b,c,d,e,f) <<b,c|d,f>>  
 (127) 41184/208537 69/217 1023/1372 343/124 6201/868 1271/28 \_\_\_\_\_ (a,b,d,e) (b,d,e,f) (a,c,d,e,f)  
 (128) 23/572 5280/36517 3245/8788 2016/143 211497/11236 975/11 \_\_\_\_\_ (a,b,d,e) (a,c,d,e,f)  
 (129) 3/55 2277/19220 224/165 2223/220 275/12 218816/165 \_\_\_\_\_ (a,c,d,e) (a,b,d,e,f) (c,d,e,f)  
 (130) 70725/220448 741/800 350/243 24648/15625 6944/2187 189/32 \_\_\_\_\_ (b,c,d,e,f)  
 (131) 5600/65559 321/1820 1356/455 1924/105 220864/195 52325/12 \_\_\_\_\_ (a,b,c,d,f) (b,c,e,f)  
 (132) 1216/224287 17472/49729 33/56 2001/56 426/7 1519/8 \_\_\_\_\_ (a,b,c,e,f) (d,e,f) (a,c,d,e,f)  
 (133) 1/100 89001/163216 201/100 225225/40804 425/16 384 \_\_\_\_\_ (a,c) (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
 (134) 34071/229276 2976/13225 219/775 231/124 2375/124 1984/25 \_\_\_\_\_ (a,b,c,e,f) (c,d,e,f)  
 (135) 27/1856 21420/229709 2065/5568 116/3 23693/192 12880/87 \_\_\_\_\_ (a,c,e,f) (a,b,d,e,f)  
 (136) 39/1120 19055/210392 385/104 237069/37570 273/40 3840/91 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
 (137) 65/1764 13/20 1440/637 3724/585 239481/12740 1980/13 \_\_\_\_\_ (a,b,c,d) (a,c,d,e) (a,b,c,d,f) (b,c,e,f) (a,b,d,e,f) <<a,d|b,f>>  
 (138) 164/507 1955/1452 252/121 148044/63001 253700/61347 41745/676 \_\_\_\_\_ (a,b,c,d,f) (b,c,e,f)  
 (139) 147/640 865/3584 190920/255367 488/35 22185/896 1974/5 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (a,d,e,f)  
 (140) 220/13689 805/2916 17649/12500 44/5 801/20 261324/125 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) (c,d,e,f)  
 (141) 704/13005 35/72 13115/20808 2023/1000 267007/104040 54/5 \_\_\_\_\_ (a,b,e) (a,b,c,d,f) (b,c,e,f) (a,d,e,f)  
 (142) 288/715 2522/4125 121220/175071 1260/143 280364/2145 1716/5 \_\_\_\_\_ (a,b,c,d,f) (a,b,e,f)  
 (143) 384/1805 100815/283024 575/784 899/980 339/80 4851/20 \_\_\_\_\_ (a,b,d,e) (c,d,e) (a,c,d,e,f)  
 (144) 280/1581 145080/291737 93/136 170/93 1683/248 26488/1581 \_\_\_\_\_ (a,c,d,e) (a,c,e,f) (b,c,d,e,f)  
 (145) 8463/106480 903/880 108/55 293040/121801 80/11 1265/16 \_\_\_\_\_ (a,b,c,e) (b,c,e,f) (a,c,d,e,f)  
 (146) 609/1840 92/145 660/667 725/368 299061/53360 8832/145 \_\_\_\_\_ (a,b,c,e) (b,d,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b|d,f>>  
 (147) 57232/310249 1161/4802 54/49 76912/21609 245/18 1312/9 \_\_\_\_\_ (b,c,d,e) (a,b,d,e,f)  
 (148) 2760/134689 4836/31205 8/15 314160/54289 306/5 520/3 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)  
 (149) 13731/318500 715/9604 36/65 218436/79625 663/20 980/13 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (a,d,e,f) (b,c,d,e,f) <<b,c|e,f>>  
 (150) 19404/103247 96/253 9204/12397 1748/539 286/23 323748/539 \_\_\_\_\_ (b,c,d,e) (a,b,c,e,f)  
 (151) 78/475 264/247 133/104 832/475 327543/49400 7125/104 \_\_\_\_\_ (a,b,c,e) (c,d,e) (a,b,c,d,f) (b,d,e,f) (a,c,d,e,f) <<a,c|d,f>>  
 (152) 104/1035 40/207 10465/5832 38947/8280 322/5 333963/920 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) [a,b,c,e]  
 (153) 9/140 47/105 347072/176505 608/105 121275/6724 1225/12 \_\_\_\_\_ (a,c,d,e) (a,b,c,d,f) (b,c,e,f) (a,b,d,e,f) <<a,d|b,f>>  
 (154) 5148/17405 28/45 56925/47524 347913/255380 352/45 725/4 \_\_\_\_\_ (a,b,c,e,f)  
 (155) 13728/354571 123/1036 60384/65863 8723/1036 441/37 2479/28 \_\_\_\_\_ (a,b,d,e) (b,d,e,f) (a,c,d,e,f)  
 (156) 35360/390963 5415/19652 1241/1083 301/204 164151/24548 255/4 \_\_\_\_\_ (a,c,d,e) (a,b,d,e,f) (c,d,e,f)  
 (157) 21/352 237/352 398090/236883 280/33 1573/96 4680/11 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,d,e,f)  
 (158) 3/13 41067/128164 805/1404 401632/255879 1105/108 21280/351 \_\_\_\_\_ (a,b,c,d,e) (c,d,e,f)  
 (159) 9/140 98685/149212 143/105 410816/79935 608/105 1225/12 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
 (160) 9/280 22725/298424 286/105 410872/79935 1216/105 1225/24 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
 (161) 65455/411864 103/96 275575/185856 2445/968 213/8 252343/726 \_\_\_\_\_ (b,c,d,e) (b,d,e,f) [a,b,c,d,e,f]  
 (162) 33/340 20445/78608 85/64 417417/98260 512/85 765/16 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
 (163) 53/195 118976/333015 833/780 48375/27508 418817/19500 1287/20 \_\_\_\_\_ (a,b,c,d,e) (a,c,e,f)  
 (164) 6/65 68103/437320 616/65 10920/841 1955/104 975/8 \_\_\_\_\_ (a,b,d,e) (b,c,d,f) (a,c,e,f) [a,b,c,d,e,f]

(165) 1463/9720 575/1848 275275/450456 1118/1155 1512/55 63800/21 (a,b,c,e,f) (a,b,d,e,f)  
(166) 13/99 21105/123596 80960/114921 11/4 513/44 455840/7569 (a,b,c,d,e) (a,c,d,e,f)  
(167) 31/242 1071/3872 999/968 457191/55112 847/72 512/9 (a,b,d,e,f)  
(168) 183/3520 87/220 825/1156 512/55 464448/15895 3025/64 (b,c,d,e) (a,b,c,d,f) (a,c,e,f) (a,b,d,e,f) <<a,f|b,d>>  
(169) 31/858 2992/28431 560/429 471328/48387 702/11 2717/6 (a,b,c,e,f) (a,b,d,e,f)  
(170) 160/861 615/1372 239136/482447 382571/95052 945/164 46189/3444 (a,c,e,f) (b,c,d,e,f)  
(171) 35/1976 13455/43928 28392/6859 545160/71383 817/104 1368/13 (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
(172) 121/1710 62928/561055 315/418 247950/49379 1520/99 41888/855 (a,b,c,d,e) (a,b,c,e,f) (a,c,d,e,f) <<a,c,e>>  
(173) 51/520 104/125 19182/10985 375/104 570059/87880 936/5 (a,b,c,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b|d,f>>  
(174) 31808/583899 55/408 527/216 405/136 2240/459 119/6 (b,c,e) (a,b,c,d,f) (b,c,d,e,f)  
(175) 17/140 153/140 113183/98260 147/85 590400/34391 4800/119 (b,c,d,e) (a,b,c,d,f) (a,c,e,f) (a,b,d,e,f) <<a,f|b,d>>  
(176) 40128/600635 143/840 18200/57963 147/40 200/21 6882/35 (a,b,c,d,e) (a,b,d,e,f) (b,c,d,e,f) <<b,d,e>>  
(177) 6417/614656 495/1472 1421/1104 533/69 23851/768 47104/147 (b,c,d,e) (a,c,d,e,f)  
(178) 205/768 296496/623045 1659/1280 22971/5120 272/15 60 (a,c,d,e) (a,c,e,f) (a,b,c,e,f)  
(179) 1767/22201 32/31 633795/586756 651/484 205/124 3885/124 (a,b,d,e,f) (b,c,d,e,f)  
(180) 5/28 169389/638428 12480/11767 147/64 27195/2116 493/28 (a,b,c,d,f) (a,c,d,e,f)  
(181) 145860/668233 215/388 84/97 99231/40804 1495/388 3492 (b,c,e) (a,b,d,e,f)  
(182) 25900/690561 100/333 216/185 4004/1665 518/45 7344/185 (b,c,d,e) (b,c,e,f) [a,b,c,d,e,f]  
(183) 25900/690561 216/185 4004/1665 518/45 7344/185 166600/37 (a,b,c,d,f) (c,d,e,f)  
(184) 120/539 15990/40931 215028/190969 172/11 699720/40931 484 (a,b,c,d,f) (b,d,e,f)  
(185) 4/185 371/740 375/148 701760/197173 27195/2116 1332/5 (a,b,c) (a,b,c,e,f) (a,c,d,e,f)  
(186) 45375/725788 33117/292820 59/140 1953/605 6560/847 1815/28 (b,c,d,e) (a,b,d,e,f) (c,d,e,f)  
(187) 27962/170625 325/1512 1064/975 91/24 780367/195000 2376/91 (a,b,c,e) (a,b,c,d,f) (a,d,e,f) (b,c,d,e,f) <<b,c|d,f>>  
(188) 77/340 255/448 813135/550256 1536/595 588133/98260 1785/16 (a,b,c,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b|d,f>>  
(189) 115713/878560 175/456 1083/2560 415/114 238165/27744 1536/95 (a,c,d,e) (a,b,c,d,f) (a,b,e,f) (b,c,d,e,f) <<b,f|c,d>>  
(190) 55352/891075 1000/2673 38831/59400 1944/275 781/24 6578/75 (a,b,c,d,e) (b,c,e,f)  
(191) 262405/895272 189/640 685/672 287/120 718487/159870 5120/189 (a,b,c,e) (a,b,c,d,f) (a,d,e,f) (b,c,d,e,f) <<b,c|d,f>>  
(192) 75/544 187/200 269139/228616 768/425 931875/57154 323/8 (b,c,d,e) (a,b,c,d,f) (a,c,e,f) (a,b,d,e,f) <<a,f|b,d>>  
(193) 4/35 36/35 147/80 381225/153328 956403/109520 1300/7 (a,b,d,e) (a,b,c,d,f) (a,b,c,e,f) (c,d,e,f) <<a,b|c,f>>  
(194) 49/300 261/700 301/75 775/84 988000/94269 576/7 (a,b,c,d) (b,c,d,f) (a,b,c,e,f)  
(195) 22/315 92475/298424 351/280 1000384/239805 475/56 3136/45 (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
(196) 17391/779744 50505/143648 1012480/942841 24/7 1615/224 882 (a,c,d,e,f)  
(197) 3/70 134589/1024000 2560/1701 12103/4320 105/8 51987/280 (a,b,c,d,e) (a,c,d,e,f)  
(198) 24/961 437/392 1176/625 1072797/480200 325/8 600066/1225 (a,b,c,d,e) (a,b,c,e,f) (b,d,e,f)  
(199) 53/460 1600/1863 153/115 1128771/557780 9499/1620 124775/2916 (a,b,c,e) (a,b,c,d,f) (b,c,e,f)  
(200) 361/4608 24115/175712 315/152 512/171 1176385/83232 747/38 (b,c,d,e) (a,b,c,d,f) (a,b,e,f) (a,c,d,e,f) <<a,f|c,d>>  
(201) 287/1452 72795/263452 480/343 273/121 1209920/197589 2057/84 (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <<a,d|c,f>>  
(202) 16211/316840 51/640 19/40 375/32 1225371/79210 3072/125 (a,b,d,e) (a,b,c,d,f) (a,c,e,f) (b,c,d,e,f) <<b,d|c,f>>  
(203) 43/400 528/2209 59475/85264 1307523/120409 1488/25 70400 (a,b,c,d,e) (a,b,c,e,f)  
(204) 16352/1339533 266175/784996 1763/1300 800/117 819/25 606803/468 (a,b,c,d,e) (c,d,e,f)  
(205) 336/3025 92713/143143 2057/2800 1379175/327184 48/7 1575/16 (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <<a,c|e,f>>  
(206) 171/1400 119/384 675/224 1470339/443576 1233575/47526 7168/75 (b,c,d,e) (a,b,c,d,f) (a,b,c,e,f) (a,d,e,f) <<a,f|b,c>>

(207) 64/715 465465/1486088 278400/720863 855/286 429/40 417989/40 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) (a,c,d,e,f) <>a,d,e>>  
(208) 5/448 45/28 1526595/817216 682176/89383 931/64 6144/7 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <>a,b|e,f>>  
(209) 57/728 256/2275 1486353/1223768 3003/200 1563975/43706 2275/32 \_\_\_\_\_ (b,c,d,e) (a,b,c,d,f) (a,c,e,f) (a,b,d,e,f) <>a,f|b,d>>  
(210) 154077/1632160 142680/367087 77/160 102/35 1455/224 264040/10201 \_\_\_\_\_ (c,d,e) (a,d,e,f) (b,c,d,e,f) [a,b,c,f]  
(211) 43875/232324 11/45 71200/47961 1633824/623045 551/180 671/20 \_\_\_\_\_ (a,b,c,e,f) (b,c,d,e,f)  
(212) 49/624 12928/6825 468/175 1339/336 265489/15600 1693375/4368 \_\_\_\_\_ (a,c,d,e) (a,b,c,d,f) (b,c,e,f)  
(213) 7480/166341 982100/1700457 56/33 468/77 155232/22801 1870/21 \_\_\_\_\_ (a,b,c,d,f) (a,c,d,e,f)  
(214) 206280/1206143 715/1656 203320/57121 2835/184 322/9 1746955/1656 \_\_\_\_\_ (a,b,c,d,e) (b,d,e,f)  
(215) 184175/1957072 7680/21853 195/64 231/52 91/16 17391/52 \_\_\_\_\_ (a,b,c,e,f) (c,d,e,f)  
(216) 4/105 15351/106580 11925/3388 1971072/357035 1175/84 756/5 \_\_\_\_\_ (a,b,c,e,f)  
(217) 15/56 58/105 7007/3000 2032520/557283 36363/280 32136/35 \_\_\_\_\_ (a,b,c,d,f) (b,c,e,f)  
(218) 15/112 544880/2093809 415/112 444/35 1104/35 4557/80 \_\_\_\_\_ (a,c,d,f) (c,e,f) (a,b,c,e,f)  
(219) 17255/149784 247/408 1122/961 2138455/392088 1653240/106097 504/17 \_\_\_\_\_ (a,c,d,e) (b,c,d,f) (a,b,e,f) [a,b,c,d,e,f]  
(220) 15008/70395 262944/586625 1001/1500 1105/1083 985/156 2180079/260 \_\_\_\_\_ (b,c,d,e) (a,c,d,e,f)  
(221) 1540/11271 26/135 9880/15987 1232/195 2447172/93845 540/13 \_\_\_\_\_ (a,b,c,d,e)  
(222) 2976/40931 8832/24299 2548623/1705636 117/44 73/11 847/4 \_\_\_\_\_ (a,b,d,e,f) (a,c,d,e,f)  
(223) 85/988 117/380 577065/89167 14304/1235 1216/65 2649075/31684 \_\_\_\_\_ (a,b,c,d) (a,b,d,e) (a,b,c,e,f)  
(224) 4255/13328 1443/1360 1280/833 2711415/620944 300/17 14076/245 \_\_\_\_\_ (a,b,c,d,f) (a,b,e,f)  
(225) 315700/2724261 308/705 2088/1645 700/141 846/35 70312/105 \_\_\_\_\_ (b,c,d,e) (a,b,c,e,f) (c,d,e,f)  
(226) 34/105 539/1080 351/280 3131051/1129080 1793600/197589 3200/21 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (a,b,c,e,f) (c,d,e,f) <>a,b|c,f>>  
(227) 404600/3188883 248216/471495 135/136 1346175/506056 1128/85 442/15 \_\_\_\_\_ (a,c,d,e,f)  
(228) 7/1920 283360/3201267 549/640 288/5 28595/384 44950/3 \_\_\_\_\_ (a,c,d,e) (a,b,d,e,f) (c,d,e,f)  
(229) 33/1036 308959/3285748 63/148 405705/155407 3360/37 6845/28 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <>a,c|e,f>>  
(230) 8064/597529 553/1200 819/100 1408/75 16225/48 3337875/16 \_\_\_\_\_ (b,c,d,e) (a,b,c,d,f) (c,d,e,f)  
(231) 80784/3436205 90644/547805 985/576 621/320 100/9 105040/9 \_\_\_\_\_ (a,b,c,d,e) (a,c,d,e,f)  
(232) 216535/3556956 12/65 1637025/2669956 143/60 340/39 2845596/34445 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) (b,c,d,e,f) <>b,d,e>>  
(233) 351/5600 238469/3148704 1925/936 1792/325 3591575/423864 351/14 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <>a,d|c,f>>  
(234) 228655/3609156 33/67 91455/106276 1615/804 384/67 469/12 \_\_\_\_\_ (a,b,d,e) (a,b,c,e,f) (b,d,e,f)  
(235) 335013/3250000 2917200/4053847 2847/2800 413/208 1456/25 53475/91 \_\_\_\_\_ (c,d,e,f) [a,b,c,d,e,f]  
(236) 5577/48020 1175737/4140500 60/169 3058572/1035125 85/4 588/5 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (b,d,e,f) (a,c,d,e,f) <>a,c|e,f>>  
(237) 33/400 1200/2401 833/400 4209897/490000 490633/30625 1200 \_\_\_\_\_ (a,c) (a,b,d,e) (a,b,c,d,f) (a,b,c,e,f) (c,d,e,f) [b,f] <>a,b|c,f>>  
(238) 438219/2286160 375/1088 1536/2125 187/80 4635939/571540 323/20 \_\_\_\_\_ (a,c,d,e) (a,b,c,d,f) (a,b,e,f) (b,c,d,e,f) <>b,f|c,d>>  
(239) 143/720 135/208 4746060/4208893 767/80 13244/585 73600/117 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,d,e,f)  
(240) 84847/4929096 41536/157323 429/472 1064/177 738/59 7493/24 \_\_\_\_\_ (a,c,d,e) (a,b,d,e,f) (c,d,e,f)  
(241) 10773/106580 985/2052 416/285 4969811/1603935 475/12 31465/228 \_\_\_\_\_ (a,b,c,d,e) (a,b,c,e,f)  
(242) 3328/9075 27/50 89/96 898275/468512 5035275/279752 363/8 \_\_\_\_\_ (a,b,c,d,f) (a,b,c,e,f)  
(243) 482560/857157 657/952 5049408/4541161 25905/11662 1085/408 1819/168 \_\_\_\_\_ (a,b,d,f) (b,c,d,e,f)  
(244) 1984/23763 2709473/5234136 78/121 66171/22472 329/24 168 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)  
(245) 703560/5555449 3243/4576 248/143 913/416 7215/352 51510/143 \_\_\_\_\_ (b,c,d,e) (c,d,e,f) [a,b,c,d,e,f]  
(246) 9/784 25/16 176/49 145236/15625 6563649/250000 39984/25 \_\_\_\_\_ (a,b,c) (a,c,d,e) (a,b,c,d,f) (a,b,c,e,f) (b,d,e,f) <>a,c|b,f>>  
(247) 6900/69169 23/100 4463004/6583175 44/23 825/92 341649/2300 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) [a,b,d,e]  
(248) 15500/196677 248/2925 2127944/6734025 12692/2925 378/13 104 \_\_\_\_\_ (b,d,e,f) (a,c,d,e,f)

- (249) 445056/7438235 37125/480508 89/140 2301/1715 343/20 62400/7 \_\_\_\_\_ (a,c,d,e,f) (b,c,d,e,f)  
 (250) 37/1920 13/30 3248105/5575776 7678359/2323240 2475/32 11264/15 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b|e,f>>  
 (251) 364572/4984831 1842800/7932063 540/589 152/93 1364/57 68510/57 \_\_\_\_\_ (a,b,d,e,f)  
 (252) 41/160 1005/1312 9520200/3279721 330/41 3808/205 82041/160 \_\_\_\_\_ (a,b,c,d,e) (b,d,e,f)  
 (253) 3/28 1084243/2957500 189/400 2432/343 9798528/739375 525/16 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,c,e,f) (a,c,d,e,f) <<a,d|c,f>>  
 (254) 22752/175561 5723872/10169721 1888/2209 341/288 917/288 189/2 \_\_\_\_\_ (d,e) (a,c,d,e,f) (b,c,d,e,f)  
 (255) 135/1456 65/336 9780995/1991964 441/52 10192512/1161979 31616/21 \_\_\_\_\_ (a,b,c,e) (a,b,c,d,f) (a,b,d,e,f) (c,d,e,f) <<a,b|d,f>>  
 (256) 24/665 112/95 11451300/5067001 494/35 3600/133 1254396/665 \_\_\_\_\_ (a,b,d,e) (a,b,c,d,f) (b,d,e,f)  
 (257) 2603/8092 61152/83521 11513760/7126567 2037/1156 55/7 1035/28 \_\_\_\_\_ (a,b,d,e) (a,b,c,e,f) (a,d,e,f)  
 (258) 2895200/11916147 362/705 5288976/5217235 1591/1410 400/141 4277/270 \_\_\_\_\_ (a,b,c,d,f) (b,d,e,f)  
 (259) 4884/72361 13856/103933 516/1813 1147/196 1911/148 13765443/13924 \_\_\_\_\_ (a,b,c,d,e) (a,c,d,e,f)  
 (260) 13/600 37/96 2674525/1393944 14697639/2904050 2816/75 12375/8 \_\_\_\_\_ (a,b,c,d) (a,b,c,e,f) (a,b,d,e,f) (c,d,e,f) <<a,b|e,f>>  
 (261) 32/91 60/91 15343900/12215287 1878240/1324801 42/13 12012 \_\_\_\_\_ (a,b,e) (a,b,c,e,f) (a,b,d,e,f)  
 (262) 64/357 91/204 88893/54289 16829280/7859831 7315/204 3485/84 \_\_\_\_\_ (a,b,c,d,f) (a,e,f) (a,b,c,e,f)  
 (263) 5/324 13765633/17634420 334125/217156 928/45 637/5 77737/180 \_\_\_\_\_ (a,b,c,d,f) (a,d,e,f)  
 (264) 53/760 2149056/9667295 6783/4840 1573/760 20203200/1933459 1767/10 \_\_\_\_\_ (a,d) (b,c,d,e) (a,c,d,e,f)  
 (265) 5/93 5800145/21249012 75/124 338272/93615 8151/620 10912/15 \_\_\_\_\_ (a,b,c,d,e) (a,c,d,e,f)  
 (266) 4/153 687420/21439193 5786424/13075193 220/17 170/9 1904/9 \_\_\_\_\_ (a,b,d,e,f) (a,c,d,e,f)  
 (267) 2323776/22770575 125/184 299/392 66/23 248331/9800 159936/575 \_\_\_\_\_ (a,b,c,d,e) (c,d,e,f)  
 (268) 809523/22848400 289/1200 75/68 128/51 15939/1700 819/68 \_\_\_\_\_ (a,b,c,d,f) (b,c,d,e,f)  
 (269) 35/276 559/1380 252/115 24364935/2979076 16454108/1703535 39100/3 \_\_\_\_\_ (a,b,c,d,f) (a,b,c,e,f)  
 (270) 96/1397 10544303/24580596 20677/16764 561/127 1651/132 7754112/15059 \_\_\_\_\_ (a,b,c,d) (a,c,d,e) (a,b,d,e,f)  
 (271) 245/1452 576/845 6592/1815 845/192 6975/676 25073257/162240 \_\_\_\_\_ (a,b,c,e) (a,b,c,d,f) (b,d,e,f)  
 (272) 60/403 273/620 416/465 28225015/27320052 2108/195 25575/52 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)  
 (273) 539/12996 197/252 2528/1575 539/100 29310176/632025 2025/7 \_\_\_\_\_ (b,c,d) (a,b,c,e,f)  
 (274) 1218560/31752611 14391/19712 111/77 34191/19712 2453/448 1155/16 \_\_\_\_\_ (b,d,e) (a,b,c,e,f) (c,d,e,f)  
 (275) 6704800/32536647 495/884 46784/57915 1521/935 13882141/2726460 935/52 \_\_\_\_\_ (a,b,c,d,f) (b,c,d,e,f)  
 (276) 42/65 37290240/33953929 103400/36517 1785/416 793/160 21198177/449440 \_\_\_\_\_ (a,b,c,d,e) (a,c,d,f)  
 (277) 7000/220323 408/875 21307/21000 469/120 1245/56 41724760/171363 \_\_\_\_\_ (b,c,d) (a,b,c,d,e) (a,b,c,e,f)  
 (278) 609/3520 455/2112 11874240/43080851 99/20 320/33 60137/960 \_\_\_\_\_ (a,b,d,e) (a,d,e,f) (b,c,d,e,f)  
 (279) 7/108 4197555/36452668 46038720/41847961 928/189 10605/1444 759/7 \_\_\_\_\_ (a,b,d,e,f) (a,c,d,e,f)  
 (280) 609/4000 1375/1624 4756101/3902240 2304/1015 64096747/6828920 609/10 \_\_\_\_\_ (a,c,d,e) (a,b,c,d,f) (b,c,e,f) (a,b,d,e,f) <<a,d|b,f>>  
 (281) 462/17405 906096/4407865 55/14 1824/385 69745200/5529491 1015/22 \_\_\_\_\_ (a,b,c,d,f) (a,c,d,e,f)  
 (282) 6741/466412 18500148/71081761 781/1292 1820/323 204/19 12255/68 \_\_\_\_\_ (a,c,d,e) (a,b,d,e,f) (c,d,e,f)  
 (283) 7/828 3880332/98203721 1997780/3659103 575/28 756/23 54370505/125316 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) (a,c,d,e,f) <<a,d,e>>  
 (284) 76475/5369496 5075/23064 26/105 109562688/20056715 1664/105 1197/40 \_\_\_\_\_ (a,b,c,e,f) (b,c,d,e,f)  
 (285) 105/3971 39967928/116014441 8/11 830280/151321 336/11 165 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)  
 (286) 39485/427716 49/180 1760/441 1969/180 18981/245 131703165/9604 \_\_\_\_\_ (b,c,d,e) (a,b,c,d,f) (c,d,e,f)  
 (287) 15/112 5525940/10452289 17108/29929 183246060/57377383 1023/112 112 \_\_\_\_\_ (a,b,c,e,f) (a,c,d,e,f)  
 (288) 15838812/186477245 117/980 18275/28812 64/15 196/15 37700/1083 \_\_\_\_\_ (a,b,c,d,e) (b,c,d,e,f)  
 (289) 9/40 55/96 569088/718205 308556175/135261024 175/6 93184/15 \_\_\_\_\_ (a,b,c,d,e)  
 (290) 31/208 384/637 759/637 327544763/73479184 168080976/7687693 105963/784 \_\_\_\_\_ (a,b,c,d,f) (a,b,c,e,f)

(291) 1894464/11411435 85/273 1143/1820 1313/420 330934240/43388427 2695/156 \_\_\_\_\_ (b,c,d) (a,b,c,d,f) (b,c,d,e,f)  
(292) 121/1710 315/418 49732272/49324495 1140800/777689 1520/99 536765922/139445 \_\_\_\_\_ (a,b,c,d,e) (a,b,c,e,f) (a,b,d,e,f) <<a,b,e>>  
(293) 13/140 65/252 656995680/686686819 10778053/2783340 1323/65 259871360/806013 \_\_\_\_\_ (a,b,c,d,e) (a,b,c,e,f) (a,b,d,e,f) <<a,b,e>>  
(294) 55/636 322097589/1001112500 1820/1749 265/132 85932/6625 525548/4125 \_\_\_\_\_ (a,b,c,d,e) (a,c,d,e,f)  
(295) 13/140 65/252 1140270560/660940623 75612609/31233020 1323/65 848946560/160173 \_\_\_\_\_ (a,b,c,d,e) (a,b,c,e,f) (a,b,d,e,f) <<a,b,e>>  
(296) 22059/286720 693/640 925/672 1045/168 1203937280/29334387 1631/30 \_\_\_\_\_ (a,b,c,d) (b,c,d,f) (a,b,c,e,f)  
(297) 12/119 1125176416/1493609907 620/357 39984/18769 1430/357 3060/7 \_\_\_\_\_ (a,c,e) (a,b,d,e,f) (a,c,d,e,f)  
(298) 39387639/1902517372 148/847 1793505/2292196 140/169 105/4 564/7 \_\_\_\_\_ (a,b,d,e,f) (b,c,d,e,f)  
(299) 320/3993 1463/1215 119893085/78873828 2324448448/577605765 2737/660 1485/4 \_\_\_\_\_ (a,b,c,e,f) (a,b,d,e,f)  
(300) 3884608/35127059 795440832/2473350275 1479/2200 341/200 375/88 429/2 \_\_\_\_\_ (a,c,d,e,f) (b,c,d,e,f)  
(301) 5295888/42820019 598107840/2599582453 95/143 968/637 816/143 7605/11 \_\_\_\_\_ (a,c,d,e,f) (b,c,d,e,f)  
(302) 68/345 2087870708/2641419705 891963/1030580 225/92 207900/31487 736/15 \_\_\_\_\_ (a,b,c,d,f) (a,c,d,e,f)  
(303) 117/968 2240/14079 1272705/554216 1675/312 5162119040/109118883 117/2 \_\_\_\_\_ (a,b,c,d,f) (a,b,d,e,f)  
(304) 15/272 8362368/132687361 3858774400/5430569811 87/17 629/48 166445045/1275312 \_\_\_\_\_ (a,b,c,d,e) (a,b,d,e,f) (a,c,d,e,f) <<a,d,e>>  
(305) 569557280/9978995733 2941/35301 80/357 534576/387617 1911/17 2040/7 \_\_\_\_\_ (a,b,c,e,f) (b,c,d,e,f)  
(306) 27/200 136/225 42603976/53421481 61949225800/49314640761 875/72 29982167523/16017800 (a,b,c,d,e) (a,b,c,e,f) (a,b,d,e,f) <<a,b,e>>  
(307) 31/564 67442410112/152855326875 25163424/13140625 475/141 1053/188 22419/100 \_\_\_\_\_ (a,d,e) (a,b,d,e,f) (a,c,d,e,f)