(Precursor for) Quantum Boundary Conditions for Expanding Universe

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Abstract.

Using Hall and Reginatto's condition for a Wheeler De Witt Equation for a Friedman – Walker metric coupled to a (Inflaton) scalar field, we delineate the outer boundary of the value of a scale factor a(t) for quantum effects, in an expanding universe. The inflaton field is from Padamanabhan's reference, " An Invitation to Astrophysics" which yields a nonstandard Potential U(a,phi) which will lead to an algebraic expression for a(t) for the value of the outer boundary of quantum effects in the universe. Afterwards, using the expression a(t)= a(initial) times [t(time)] ^ alpha, with alpha given different values, we give an estimation as to a time, t(time) which is roughly the boundary of the range of quantum effects. How this is unusual? We use the Wheeler De Witt Equation, as a coupling to a given inflaton field, phi, and from there find a different way as to delineate a time regime for the range of quantum effects in an expanding universe.

Key words: Wheeler De Witt equation, inflaton, Friedman-Walker metric, scale factor

I. Introduction

We work with the Wheeler De Witt Equation as given by [1], as part of the work by Hall and Reginatto, in 2016, where an ordering, called p, is used to link a Wheeler De Witt Equation, as given below, to an inflaton, and the Friedman Walker space-time metric, with the inflaton described by [2] and the Friedman Walker metric given in [2,3]

What we are doing is using [1] with its Wheeler De Witt equation is to look at the following

$$\left[\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a} - \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2} - U\left(a,\phi\right)\right]\Psi = 0 \tag{1}$$

The inflaton, ϕ is defined by [2] as given by

$$\begin{aligned}
&\phi = \sqrt{\frac{\alpha}{4\pi G}} \ln\left(\sqrt{\frac{8\pi G V_0}{\alpha\cdot(3\alpha-1)}}t\right) \\
&U\left(a,\phi\right) = V_0 \exp\left(-\sqrt{\frac{16\pi G}{\alpha}}\phi\right) \\
&a = a_{initial} t^{\alpha}
\end{aligned} \tag{2}$$

The wavefunction we use in Eq.(1) we will use the ansatz of

$$\Psi = \Psi_{initial} \exp\left(\beta\cdot a\left(t\right)\phi\left(t\right)\right) \tag{3}$$

‘

These three sets of equations will be referenced, in our article, and will form the template of the subsequent analysis.

II. Looking at how to come up with a polynomial Equation for $a\left(t\right)$

$a\left(t\right)$ as given in Eq.(2) is used to re define the inflaton in Eq.(2) as well as a re definition of the potential U, as in Eq.(2) with the upshot that

$$\begin{aligned}
&\phi(redone) = \sqrt{\frac{\alpha}{4\pi G}} \ln\left(\sqrt{\frac{8\pi G V_0}{\alpha\cdot(3\alpha-1)}}\cdot\left(\frac{a_{initial}}{a}\right)^{1/\alpha}\right) \\
&U\left(a,\phi\right)[redone] = V_0\left(\frac{8\pi G}{\alpha\cdot\left(3\alpha-1\right)}\right)^{-1}\cdot\left(\frac{a_{initial}}{a}\right)^{2/\alpha} \\
&p = -1, \alpha = 1 \\
&\Rightarrow a^2 - a\cdot\left(\frac{\phi\left(t\right)}{\beta\cdot\left(1-\beta^2\right)}\right) - \frac{V_o\cdot a_{initial}}{4\pi G} = 0
\end{aligned} \tag{4}$$

Now what is strange about the bottom quadratic equation for the scale factor, as given in Eq. (4)? We have that, here we are using Eq, (2) in the end to define, here, a ϕ inflaton equation in terms of time, not the scale factor version of it, as given in Eq. (4). If we use this approach, and constrain ourselves to very small time steps, i.e. of the order of Planck scale time (very small) we get then that the range of quantum effects, from an initial $a_{initial}$ to the boundary of quantum gravity effects, is given by, approximately for small $a_{initial}$

$$a(quantum-outer-boundary) \sim \left(\frac{1}{2} \cdot \frac{\beta \cdot \left(1-\beta^2\right) V_0}{\pi G \cdot \left|\phi_{initial}\right|} \right) \cdot a_{initial} \qquad (4)$$

III. Conclusion, we have taken the simplest case, and it could be more complicated.

What we have done is to look at $\alpha = 1$, and also avoided the situation of using the general inflaton

$$\phi(redone) = \sqrt{\frac{\alpha}{4\pi G}} \ln\left(\sqrt{\frac{8\pi G V_0}{\alpha \cdot (3\alpha - 1)}} \cdot \left(\frac{a_{initial}}{a} \right)^{1/\alpha} \right) \qquad (5)$$

in

$$a^2 - a \cdot \left(\frac{\phi(t)}{\beta \cdot \left(1-\beta^2\right)} \right) - \frac{V_o \cdot a_{initial}}{4\pi G} = 0 \qquad (6)$$

Were we to insert Eq. (5) for the inflaton into Eq. (6) we would have a very non linear case, for the scale factor equation. One which could only be deciphered by numerical analysis

If we stick with the above methodology, we still have to consider conditions for which

$$a(quantum-outer-boundary) \sim \left(\frac{1}{2} \cdot \frac{\beta \cdot \left(1-\beta^2\right) V_0}{\pi G \cdot \left|\phi_{initial}\right|} \right) \cdot a_{initial} \qquad (7)$$

Which presumably would be linked to

$$\left(\frac{1}{2} \cdot \frac{\beta \cdot \left(1-\beta^2\right) V_0}{\pi G \cdot \left|\phi_{initial}\right|} \right) > 1 \qquad (8)$$

Indeed, though, if $\alpha \neq 1$ there is no way we could possibly retrieve Eq. (4) above, i.e. we have a numerical problem, one which we will investigate in future papers. In addition, for Eq. (4), Eq.(6) and Eq. (7) we need to remember β comes from Eq. (3) and its value will need to be considered.

What we have though is based upon [1] and the idea of a quantum ensemble and operator-ordering. In order for the readers to get more insights as to the physics inherent in the choice of p, in Eq. (1) the reader is referred to [4,5,6].

Finally, [7,8,9,10,11,12] have issues which need to be reviewed which may in fact, have a ready impact upon Eq. (7), and Eq. (8) above, i.e. [7,8,9] refers to Corda's work with the foundation of gravity, and if or not Gravity is quantum, or purely due to classical General Relativity. In particular the issue of scalar-tensor gravity needs to be investigated, to see if it falsifies Eq. (7) or if it adds new restrictions as to the boundaries.

Note also, that [10] touches upon if or not quantum mechanics is part of a deterministic set up, which would have immediate consequences as to Eq. (3). References [11, 12] as to higher dimensions, should be looked at as far as the fidelity of Eq. (1) to the setup of the universe concluded. i.e. both references postulate higher dimensions. In addition

Ng [13] have it that there would be a wavelength, as part of the derivation of entropy included in the entropy formula of

$$S \sim N(particle-count)\times\left(\ln\left(V/\lambda^3\right)+3/2\right) \tag{9}$$

The answer, as given by Ng, is that if the volume of space, V, is ~ λ^3, and that λ is proportional to the wavelength , then due to the situation of how a massive graviton could at least have accelerated mass values, this will allow for the Ng formula, being changed to

$$S \sim N(particle-count) \tag{10}$$

Does Eq. (7) and Eq. (9) falsify Eq. (9) and Eq. (10) ?

It needs to be answered. And of course all this needs to avoid being in conflict with [14] and the gravity results so derived. Finally, does Eq. (7) and Eq. (8) , not to mention Eq. (4) falsify the conditions given in [15] as to massive gravity ? This question should also be investigated.

After these questions are entertained, and examined, the last supposition, as mentioned should be investigated, i.e. of a different time variable, delineating the amount of time in a quantum regime for the expansion of the universe. IMO, using

$$a = a_{initial} t^{\alpha} \tag{11}$$

And if $\alpha \sim 1$ would lead to a time regime for quantum effects, delineated by

$$t(quantum-outer-boundary) \sim \left(\frac{1}{2}\cdot\frac{\beta\cdot\left(1-\beta^2\right)V_0}{\pi G\cdot\left|\phi_{initial}\right|}\right) \tag{12}$$

Of course, if $\alpha \neq 1$, we would have a different power relationship, very different.

I.e. all these questions need to be investigated in the near future.

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