Laws of orbital semimajor axes and periods of exoplanets, and their application

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ABSTRACT

Analyze the data of orbital semimajor axes and periods of planets in extrasolar multi-planets systems, and discover the laws of orbital semimajor axes and periods of exoplanets. Link the laws with the orbital semimajor axis and the period of the pseudo-planet orbiting a star at its surface with Keplerian velocity. Apply the laws to predict the position of unknown exoplanets and to calculate the unknown orbital semimajor axes of known exoplanets, such as those of Kepler-20 and Kepler-102's known planets. Propose that the original reason of the laws is that planets were born from periodic bursts of stars in their youth. The strong stellar wind near the stars pushed them away to their present orbits.

Subject headings: exoplanets, Kepler-20, Kepler-102

1. Introduction

It is well-known that the orbital semimajor axes of planets in our solar system follow the Bode's law: $a_1=0.4, a_n=0.4+0.3\times2^{n-2}(n\geqq2)$. The law works very well for the planets as far as Uranus, but the reason why it works so well is not understood(Weissman 2014). The law can be also shown in a different way:

$$\frac{a_n}{a_{n-1}}=r\,. \qquad (1)$$

This is the law of orbital semimajor axes(Hu 2014). In our solar sysem, there is r=1.73±0.35 and it works very well for the planets as far as Neptune.

Planets in extrasolar multi-planet systems did not follow the Bode's law because they are much closer to their parent star than the planets in our solar system to the Sun, but they usually follow the law of orbital semimajor axes. For example, in HD 10180' system with 6 planets, there is r=2.3606±0.5345.

Because of $a_n^3/P_n^2=(GM_*)/(2\pi)^2$, if planets follow the law of orbital semimajor axes, they follow the law of orbital periods too:

$$\frac{P_n}{P_{n-1}}=\sqrt{r^3}\,. \qquad (2)$$

Are the laws of orbital semimajor axes and periods universal? Are they useful in astronomy? At least, they can guide us to predict the location of unknown planets (or asteroid belts), and to calculate the unknown semimajor axes from the known semimajor axes by using:

$$a_n=a_m\sqrt[3]{(\frac{P_n}{P_m})^2}\,. \qquad (3)$$

This is the formula of semimajor axes. Using it, when the semimajor axis of planet m is known, the unknown semimajor axis of planet n can be calculated. When all semimajor axes are unknown, a pseudo-planet orbiting its parent star at its surface with Keplerian velocity can be assumed with $a_0=R_*$, $v_{*\mathrm{kep}0}=(GM_*/R_*)^{1/2}$, and $P_0=2\pi R_*/v_{*\mathrm{kep}0}$, and semimajor axes of all planets in the system can be calculated using $a_n=a_0(P_n/P_0)^{2/3}$. Let us take a close look.

2. Are the laws of orbital semimajor axes and periods universal?

First, the laws of orbital semimajor axes and periods works well in the Solar System(Spohn et al. 2014), as they are shown in Table 1, with the exception of a_5/a_4=3.41504 and P_5/P_4=6.3069. Using the exception and Bode's law, astronomers found Ceres and other asteroids in the asteroid belt between Mars and Jupiter.

Table 1 Data of the Solar System

Sun's	n=0	n=1, Mercury	n=2, Venus	n=3, Earth	n=4, Mars	n=5, Jupiter	n=6, Saturn	n=7, Uranus	n=8, Neptune
a_n[AU]	0.00464913	0.38710	0.72333	1.00000	1.52366	5.20336	9.53707	19.1913	30.0690
a_n/a_{n-1}		83.263	1.8686	1.3825	1.52366	3.41504	1.83287	2.01228	1.56680
P_n[years]	0.0003179	0.2408	0.6152	1.0000	1.8808	11.862	29.457	84.011	164.79
P_n/P_{n-1}		757.5	2.555	1.625	1.8808	6.3069	2.4833	2.8520	1.9615

Second, the laws of orbital semimajor axes and periods works well in the Kepler-90's (KOI-351's) system(Cabrera et al. 2014), the only known system with 7 planets, as they are shown in Table 2, with the exception of a_3/a_2=3.6 and P_3/P_2=6.851. Using the exception and the laws of orbital semimajor axes and periods, can astronomers find another planet or an asteroid belt between Kepler-90 c and d?

Table 2 Data of the Kepler-90's system with 7 planets

Kepler-90's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f	n=6,g	n=7,h
a_n[AU]	0.0056	0.074	0.089	0.32	0.42	0.48	0.71	1.01
a_n/a_{n-1}		13	1.2	3.6	1.3	1.1	1.5	1.4
P_n[days]	0.14	7.008151	8.719375	59.73667	91.93913	124.9144	210.60697	331.60059
P_n/P_{n-1}		50	1.244	6.851	1.539	1.359	1.686	1.574

Third, the laws of orbital semimajor axes and periods works well in the only known three systems with 6 planets, as they are shown in Table 3(Kane et al. 2014), 4 (Lissauer et al. 2013)and 5(Vogt et al. 2015), with the exception of a_6/a_5=8.287 and P_6/P_5=23.85. Using the exception and the laws of orbital semimajor axes and periods, can astronomers find another planet, or an asteroid belt, or more, between HD 219134 g and h?

Table 3 Data of HD 10180's system with 6 planets

HD 10180's	n=0	n=1,c	n=2,d	n=3,e	n=4,f	n=5,g	n=6,h
a_n[AU]	0.005156	0.06412	0.12859	0.2699	0.4929	1.427	3.381
a_n/a_{n-1}		12.44	2.005	2.099	1.826	2.895	2.369
P_n[days]	0.132	5.75969	16.3570	49.748	122.744	604.67	2205.0
P_n/P_{n-1}		43.6	2.840	3.041	2.467	4.926	3.647

Table 4 Data of Kepler-11's system with 6 planets

Kepler-11's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f	n=6,g
a_n[AU]	0.004951	0.091	0.107	0.155	0.195	0.250	0.466
a_n/a_{n-1}		18.76	1.176	1.449	1.258	1.282	1.864
P_n[days]	0.130	10.3039	13.0241	22.6845	31.9996	46.6888	118.3807
P_n/P_{n-1}		79.3	1.264	1.742	1.411	1.459	2.536

Table 5 Data of HD 219134's system with 6 planets

HD 219134's	n=0	n=1,b	n=2,c	n=3,f	n=4,d	n=5,g	n=6,h
a_n[AU]	0.0036	0.0384740	0.064816	0.14574	0.23508	0.3753	3.11
a_n/a_{n-1}		11	1.685	2.249	1.613	1.596	8.29
P_n[days]	0.0881	3.0931	6.7635	22.805	46.71	94.2	2247
P_n/P_{n-1}		35.1	2.187	3.372	2.048	2.017	23.85

At present, it can be said that at least, the laws of orbital semimajor axes and periods work well in the known four systems with 7 or 6 planets, with only two exceptions. Observational evidences in the future will reveal they are universal or not.

3. Applications of the laws of orbital semimajor axes and periods

Kepler-20' system with 5 planets are listed in exoplanetarchive.ipac.caltech.edu but a_2 and a_4 are not known(Gautier et al. 2011;Fressin et al. 2011). Using (3), the formula of semimajor axes, they can be calucated with known a_1 and P_2/P_1、a_3 and P_4/P_3 respectively, as they are shown in Table 6. The laws of orbital semimajor axes and periods work well in Kepler-20' system, without any exception.

Table 6 Data of Kepler-20's system with 5 planets

Kepler-20's	n=0	n=1,b	n=2,e	n=3,c	n=4,f	n=5,d
a_n[AU]	0.004389	0.04537	*0.06334*	0.0930	*0.1378*	0.3453
a_n/a_{n-1}		10.34	*1.396*	*1.468*	*1.482*	*2.506*
P_n[days]	0.112	3.6961219	6.098493	10.854092	19.57706	77.61184
P_n/P_{n-1}		33.0	1.650	1.780	1.804	3.964

Kepler-102' system with 5 planets are also listed in exoplanetarchive.ipac.caltech.edu but all a_n from n=1 to n=5 are not known(Marcy et al. 2014). Using (3), the formula of semimajor axes, they can be calucated with known a_0 and P_n, as they are shown in Table 7. The laws of orbital semimajor axes and periods work well in Kepler-102' system, without any exception.

Table 7 Data of Kepler-102's system with 5 planets

Kepler-102's	n=0	n=1,b	n=2,c	n=3,d	n=4,e	n=5,f
a_n[AU]	0.0035	*0.055*	*0.067*	*0.086*	*0.12*	*0.16*
a_n/a_{n-1}		*16*	*1.2*	*1.3*	*1.4*	*1.3*
P_n[days]	0.086	5.28696	7.07142	10.3117	16.1457	27.4536
P_n/P_{n-1}		60	1.338	1.458	1.566	1.700

4. Conclusion and discussion

Why all planets in systems with 8, 7, or 6 planets, with only two exceptions, follow the laws of orbital semimajor axes and periods? The original reason is that planets were born from periodic bursts of stars in their youth, as it was suggested at http://vixra.org/abs/1609.0158. The strong stellar wind near the stars pushed them away to their present orbits. The theory of periodic bursts of stars provide a simple self-consistent model to explain the Bode's law and many facts of the solar system and other planetary systems.

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